Yukawa textures in string unified models with $\mathrm{SU}(4)\otimes\mathrm{O}(4)$ symmetry

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We discuss the origin of Yukawa textures in the string-inspired and string-derived models based on the gauge group $\mathrm{SU}(4)\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$ supplemented by a $\mathrm{U}(1)_X$ gauged family symmetry. The gauge symmetries are broken down to those of the minimal supersymmetric standard model which is the effective theory below 10^{16} GeV. The combination of the $\mathrm{U}(1)_X$ family symmetry and the Pati-Salam gauge group leads to a successful and predictive set of Yukawa textures involving two kinds of texture zeros: *horizontal* and *vertical* texture zeros. We discuss both symmetric and nonsymmetric textures in models of this kind, and in the second case perform a detailed numerical fit to the charged fermion mass and mixing data. Two of the Yukawa textures allow a low energy fit to the data with a total χ^2 of 0.39 and 1.02, respectively, for three degrees of freedom. We also make a first attempt at deriving the nonrenormalizable operators required for the Yukawa textures from string theory. [S0556-2821(97)01617-2]

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I. INTRODUCTION

Over recent years there has been a good deal of activity concerned with understanding the pattern of fermion masses and mixing angles within the framework of supersymmetry and unification (see the next section for a review). The starting point of these analyses is the idea that at high energies the Yukawa matrices exhibit a degree of simplicity, typically involving texture zeros, which can be understood as resulting from some symmetry. The types of symmetry which have been considered include grand unified symmetry to account for the vertical mass splittings within a family and family symmetry to account for the horizontal mass splittings between families. In order to restrict the rather *ad hoc* nature of such models, one may appeal to a rigid theoretical structure such as string theory in terms of which the high energy field theory may be viewed as an effective low energy supergravity model valid just below the string scale. Viewed from this perspective certain classes of unified gauge group and family symmetry appear to be more promising than others, and in addition one may hope to begin to derive the entries of the Yukawa matrices as low energy nonrenormalizable operators which arise from the string theory.

In this paper, guided by the principles outlined in the previous paragraph, we investigate the origin of Yukawa textures in a class of models based on the Pati-Salam $\mathrm{SU}(4)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ symmetry with gauged U(1) family symmetries.

We shall follow both a bottom-up approach, in which the successful textures may be extracted from the known quark and lepton masses and quark mixing angles, *and* a top-down approach in which we shall begin to see how the desired operators may emerge from a particular superstring construction. This model involves both quark-lepton unification, which leads to Clebsch relations to describe the mass relations within a particular family, and a $\mathrm{U}(1)_X$ gauged family symmetry which may account for family hierarchies. Thus we are led to *vertical* and *horizontal* texture zeros which are a feature of this model. In the earlier parts of the paper we shall focus on something we call the string-inspired $\mathrm{SU}(4)\times\mathrm{O}(4)$ $[\sim\mathrm{SU}(4)\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R]$ model which contains many of the features of a realistic string model such as small group representations and a $\mathrm{U}(1)_X$ family symmetry. Within this simplified model we shall relate the high energy textures to the low energy quark and lepton masses and quark mixing angles, and so determine by a bottom-up procedure the operators which are likely to be relevant at high energies. Later on we shall focus on a particular string construction from which we learn how nonrenormalizable operators may be generated from first principles.

The detailed layout of the paper is as follows. In Sec. II we review some ideas concerning Yukawa textures and summarize recent progress in this area. In Sec. III we briefly review the string-inspired $\mathrm{SU}(4)\times\mathrm{O}(4)$ model. In Sec. IV we discuss symmetric textures in the above model. In Sec. V we discuss the nonsymmetric textures. In Sec. VI we perform a full numerical analysis of the nonsymmetric models. In Sec. VII we review the $\mathrm{U}(1)_X$ family symmetry approach to the model and perform an analysis relevant for the full (symmetric and nonsymmetric) model. In the subsequent two sections we present a viable string construction of the model and indicate how the nonrenormalizable operators may arise in the specific string construction. Finally, Sec. X concludes the paper.

II. YUKAWA TEXTURES

The pattern of quark and lepton masses and quark mixing angles has for a long time been a subject of fascination for particle physicists. In terms of the standard model, this pattern arises from 3×3 complex Yukawa matrices (54 real parameters) which result in 9 real eigenvalues plus 4 real mixing parameters (13 real quantities) which can be measured experimentally. In recent years the quark and lepton masses and mixing angles have been measured with increasing precision, and this trend is likely to continue in the future as lattice QCD calculations provide increasingly accurate estimates and B factories come on line. Theoretical progress is less certain, although there has been a steady input of theoretical ideas over the years and in recent times there has been an explosion of activity in the area of supersymmetric unified models. This approach presumes that at very high energies close to the unification scale, the Yukawa matrices exhibit a degree of simplicity, with simple relations at high energy corrected by the effects of the renormalization group (RG) running down to low energy. For example, the classic prediction that the bottom and τ Yukawa couplings are equal at the unification scale can give the correct low energy bottom and τ masses, providing that one assumes the RG equations of the minimal supersymmetric standard model (MSSM) [1].[1] In the context of the MSSM it is even possible that the top, bottom, and τ Yukawa couplings are all approximately equal near the unification scale [3], since, although this results in the top and bottom Yukawa couplings being roughly the same at low energy, one can account for the large top to bottom mass ratio by invoking a large value of $\tan\beta$ defined as the ratio of vacuum expectation values (VEV's) of the two Higgs doublets of the MSSM.

These successes with the third family relations are not immediately generalizable to the lighter families. For the remainder of the Yukawa matrices, additional ideas are required in order to understand the rest of the spectrum. One such idea is that of texture zeros: the idea that the Yukawa matrices at the unification scale are rather sparse, for example, the Fritzsch ansatz [4]. Although the Fritzsch texture does not work for supersymmetric unified models, there are other textures which do, for example, the Georgi-Jarlskog (GJ) texture [5] for down-type quark and lepton matrices:

$$\lambda^E = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & -3\lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & \lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix}. \tag{1}$$

After diagonalization this leads to $\lambda_\tau = \lambda_b$, $\lambda_\mu = 3\lambda_s$, $\lambda_e = \lambda_d/3$ at the scale M_{GUT} which result in (approximately) successful predictions at low energy. Actually the factor of 3 in the 22 element above arises from group theory: It is a Clebsch factor coming from the choice of Higgs fields coupling to this element.

It is observed that if we choose the upper 2×2 block of the GJ texture to be symmetric, $\lambda_{12} = \lambda_{21}$, and if we can disregard contributions from the up-type quark matrix, then we also have the successful mixing angle prediction

$$V_{us} = \sqrt{\lambda_d/\lambda_s}. \tag{2}$$

The data therefore support the idea of symmetric matrices and a texture zero in the 11 position. Motivated by the desire for maximal predictivity, Ramond, Roberts, and Ross (RRR) [6] have made a survey of possible symmetric textures which are both consistent with the data and involve the maximum number of texture zeros. Assuming GJ relations for the leptons, RRR tabulated five possible solutions for up-type and down-type Yukawa matrices. We list them below for completeness.

Solution 1:

$$\lambda^U = \begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}. \tag{3}$$

Solution 2:

$$\lambda^U = \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix}. \tag{4}$$

Solution 3:

$$\lambda^U = \begin{pmatrix} 0 & 0 & \sqrt{2}\lambda^4 \\ 0 & \lambda^4 & 0 \\ \sqrt{2}\lambda^4 & 0 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix}. \tag{5}$$

Solution 4:

$$\lambda^U = \begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \sqrt{3}\lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{6}$$

Solution 5:

$$\lambda^U = \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \sqrt{2}\lambda^4 & \lambda^2/\sqrt{2} \\ \lambda^4 & \lambda^2/\sqrt{2} & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{7}$$

Here $\lambda = 0.22$, and the top and bottom Yukawa couplings have been factored out for simplicity. These textures are valid at the unification scale. All of the solutions involve texture zeros in the 11 entry. Solutions 1, 2, and 4 involve additional texture zeros in the 13=31 positions which are common to both up-type and down-type matrices. Solutions 3 and 5 have no texture zeros which are common to both up-type and down-type matrices, apart from the 11 entry. Thus solutions 1, 2, and 4 involve rather similar up-type and down-type matrices, while solutions 3 and 5 involve very different textures for the two matrices.

[1]The next-to-MSSM (NMSSM) with an additional low energy gauge singlet works just as well [2].

Having identified successful textures,[2] the obvious questions are, what is the origin of the texture zeros and what is the origin of the hierarchies (powers of the expansion parameter λ)? A natural answer to both these questions was provided early on by Froggatt and Nielsen (FN) [8]. The basic idea involves a high energy scale M, a family symmetry group G, and some new heavy matter of mass M which transforms under G. The new heavy matter consists of some Higgs fields which are singlets under the vertical gauge symmetry but nonsinglets under G. These break the symmetry G by developing VEV's V smaller than the high energy scale. There are also some heavy fields which exist in vectorlike representations of the standard gauge group. The vectorlike matter couples to ordinary matter (quarks, leptons, Higgs bosons) via the singlet Higgs boson, leading to "spaghetti-like" tree-level diagrams. Below the scale V the spaghetti diagrams yield effective nonrenormalizable operators which take the form of Yukawa couplings suppressed by powers of $\lambda = V/M$. In this way the hierarchies in the Yukawa matrices may be explained and the texture zeros correspond to high powers of λ.

A specific realization of the FN idea was provided by Ibanez and Ross (IR) [9], based on the MSSM extended by a gauged family $U(1)_X$ symmetry with θ and $\overline{\theta}$ singlet fields with opposite X charges, plus new heavy Higgs fields in vector representations.[3] Anomaly cancellation occurs via a Green-Schwarz-Witten (GSW) mechanism, and the $U(1)_X$ symmetry is broken not far below the string scale [9]. By making certain symmetric charge assignments, IR showed that the RRR texture solution 2 could be approximately reproduced. To be specific, for a certain choice of $U(1)_X$ charge assignments, IR generated Yukawa matrices of the form

$$\lambda^U = \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} \overline{\epsilon}^8 & \overline{\epsilon}^3 & \overline{\epsilon}^4 \\ \overline{\epsilon}^3 & \overline{\epsilon}^2 & \overline{\epsilon} \\ \overline{\epsilon}^4 & \overline{\epsilon} & 1 \end{pmatrix},$$

$$\lambda^E = \begin{pmatrix} \overline{\epsilon}^5 & \overline{\epsilon}^3 & 0 \\ \overline{\epsilon}^3 & \overline{\epsilon} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (8)$$

These are symmetric in the expansion parameters ϵ and $\overline{\epsilon}$, which are regarded as independent parameters. This provides a neat and predictive framework; however, there are some open issues. Although the order of the entries is fixed by the expansion parameters, there are additional parameters of order unity multiplying each entry, making precise predictions difficult. A way to address the problem of the unknown coefficients has been proposed in [11] where it has been shown that the various coefficients may arise as a result of the infrared fixed-point structure of the theory beyond the standard model.

Note that the textures for up-type and down-type matrices are of similar form, although the expansion parameters differ. Also note that there are no true texture zeros in the quark sector, merely high powers of the expansion parameter. Thus this example most closely resembles RRR solution 2 with approximate texture zeros in the 11 and 13=31 positions. However, without the inclusion of coefficients, the identification is not exact. The best fit to RRR solution 2 is obtained for the identification $\epsilon \equiv \lambda^2$, $\overline{\epsilon} \equiv \lambda$ (alternative identifications like $\epsilon \equiv \lambda^2$, $\overline{\epsilon} \equiv 2\lambda^3$ lead to larger deviations). However even this choice does not exactly correspond to RRR solution 2, as can be shown by taking solution 2 and inserting the numerical values of the entries:

$$\lambda^U = \begin{pmatrix} 0 & 1\times 10^{-4} & 0 \\ 1\times 10^{-4} & 0 & 5\times 10^{-2} \\ 0 & 5\times 10^{-2} & 1 \end{pmatrix},$$

$$\lambda^D = \begin{pmatrix} 0 & 5\times 10^{-3} & 0 \\ 5\times 10^{-3} & 2\times 10^{-2} & 2\times 10^{-2} \\ 0 & 2\times 10^{-2} & 1 \end{pmatrix}. \qquad (9)$$

We compare these numbers to the order of magnitudes predicted by the symmetry argument, making the identifications $\epsilon \equiv \lambda^2$, $\overline{\epsilon} \equiv \lambda$:

$$\lambda^U = \begin{pmatrix} 3\times 10^{-11} & 1\times 10^{-4} & 5\times 10^{-6} \\ 1\times 10^{-4} & 2\times 10^{-3} & 5\times 10^{-2} \\ 5\times 10^{-6} & 5\times 10^{-2} & 1 \end{pmatrix},$$

$$\lambda^D = \begin{pmatrix} 5\times 10^{-6} & 1\times 10^{-2} & 2\times 10^{-3} \\ 1\times 10^{-2} & 5\times 10^{-2} & 2\times 10^{-1} \\ 2\times 10^{-3} & 2\times 10^{-1} & 1 \end{pmatrix}. \qquad (10)$$

Comparison of Eqs. (9) and (10) shows that while λ^U is in good agreement, λ^D differs. In Eq. (10), the 23=32 element is an order of magnitude too large. When the unknown couplings and phases are inserted the scheme can be made to work. However, some tuning of the unknown parameters is implicit. This can be avoided by introducing a small parameter δ into all the elements apart from the 33 renormalizable element, so that Eq. (8) gets replaced by[4]

[2]Over the recent years, there has been an extensive study of fermion mass matrices with zero textures [7].

[3]The generalization to include neutrino masses is straightforward [10].

[4]In our scheme we will have a unified Yukawa matrix. This, as we are going to see, will imply a common expansion parameter for the up- and down-type mass matrices and the presence of a factor δ in the up-quark mass matrix as well.

$$\lambda^U=\begin{pmatrix}\epsilon^8 & \epsilon^3 & \epsilon^4\\ \epsilon^3 & \epsilon^2 & \epsilon\\ \epsilon^4 & \epsilon & 1\end{pmatrix},\quad \lambda^D=\begin{pmatrix}\delta\bar{\epsilon}^8 & \delta\bar{\epsilon}^3 & \delta\bar{\epsilon}^4\\ \delta\bar{\epsilon}^3 & \delta\bar{\epsilon}^2 & \delta\bar{\epsilon}\\ \delta\bar{\epsilon}^4 & \delta\bar{\epsilon} & 1\end{pmatrix}. \tag{11}$$

The idea is that the suppression factor δ originates from some flavor-independent physics, while the parameters ϵ and $\bar{\epsilon}$ control the flavor structure of the matrices. For example, suppose we take $\bar{\epsilon}\equiv\lambda$ as in the previous example but scale down the entries by a factor of $\delta=0.2$. Then we would have

$$\lambda^D=\begin{pmatrix}1\times10^{-6} & 2\times10^{-3} & 4\times10^{-4}\\ 2\times10^{-3} & 1\times10^{-2} & 4\times10^{-2}\\ 4\times10^{-4} & 4\times10^{-2} & 1\end{pmatrix}, \tag{12}$$

which provides a better description of the numerical values required by the RRR analysis for solution 2 in Eq. (9), at the expense of introducing the parameter δ. This example indicates that if family symmetries are to give the correct order of magnitude understanding of Yukawa textures without any tuning of parameters, then an extra parameter δ needs to be introduced as above.

Another aspect of the fermion mass spectrum that one would like to understand is that of the mass splitting within a particular family. For example the GJ texture in Eq. (1) provides an understanding of the relationship between the charged lepton and down-type quark Yukawa couplings within a given family, and in the simplest $U(1)_X$ scheme such relations are either absent or accidental, as seen in Eq. (8) where the form of λ^E has been fixed by a parameter choice. Unless such parameters are predicted by the theory, as in the extension of the initial IR scheme that is discussed in [11], the only antidote is extra unification. Then, the leptons share a representation with the quarks, and the magic GJ factors of 3 originate from the fact that the quarks have three colors. For example, the SO(10) model of Anderson *et al.* [12] (with both low energy Higgs doublets unified into a single **10** representation) predicts Yukawa unification for the third family, GJ relations for the charged leptons and down-type masses, and other Clebsch relations involving up-type quarks. As in the IR approach, the approach followed by Anderson *et al.* is based on the FN ideas discussed above. Thus, for example, only the third family is allowed to receive mass from the renormalizable operators in the superpotential. The remaining masses and mixings are generated from a minimal set of just three specially chosen nonrenormalizable operators whose coefficients are suppressed by a set of large scales. The $12=21$ operator of Anderson *et al.* is suppressed by the ratio $(45_1/M)^6$, while the $23=32$ and operators are suppressed by $(45_{B-L}/45_1)^2$ and $(45_{B-L}S/45_1^2)$ where the 45's are heavy Higgs representations. In a complicated multiscale model such as this, the hierarchies between different families are not understood in terms of a family symmetry such as the the $U(1)_X$ of IR. Indeed it is difficult to implement a family symmetry in this particular scheme, as the latest attempts based on global U(2) [13] show. To be embedded into a string model, grand unified theories (GUT's) such as SO(10) require $k>1$ Kac-Moody levels. With these higher Kac-Moody levels, simple orbifold compactifications in which candidate gauge $U(1)_X$ family symmetries are present do not easily emerge. Nevertheless, there has been some progress in this direction and three-family SO(10) and E_6 string-derived models have recently been classified [14]. Here we restrict our discussion to string constructions based on the simpler $k=1$ level of Kac-Moody algebras, which are more ''string friendly.''

The $SU(4)\otimes O(4)$ string model can be viewed as the simplest string-friendly unified extension of the standard model which can lead to Clebsch relations of the kind we desire. The Pati-Salam gauge group [15] may be broken without adjoint representations and was considered as a unified string model [16,17] some time ago. This model has recently been the subject of renewed interest from the point of view of fermion masses [18], and an operator analysis has shown that it is possible to obtain desirable features such as Yukawa unification for the third family and GJ-type relations within this simpler model. A particular feature of the published scheme which we would like to emphasize here is the idea of *Clebsch texture zeros* which arise from the group theory of the Pati-Salam gauge group. These Clebsch zeros were used to account for the lightness of the up quark compared to the down quark, for example [18]. However, the operator analysis of [18] did not address the question of the hierarchy between families (no family symmetry was introduced, for example) or the question of the origin of the nonrenormalizable operators. Here we shall introduce a $U(1)_X$ gauge symmetry into the model and combine it with the Clebsch relations previously used, to provide a predictive scheme of fermion masses and mixing angles. We shall also ensure that we obtain the correct order of magnitude for all the entries of the Yukawa matrices from the symmetry-breaking parameter, using structures like that of Eq. (11). In our case the quantity δ will be identified with a bilinear of heavy Higgs fields which are responsible for generating the Clebsch structures, while the parameters such as ϵ will have trivial Clebsch structure (singlets under the vertical gauge group) but will generate family hierarchies from the flavor symmetry. This corresponds to there being two types of heavy Higgs fields: Pati-Salam gauge singlets (corresponding to IR θ and $\bar{\theta}$ fields) which break the $U(1)_X$ family gauge group but leave the Pati-Salam group unbroken, and $H,\bar{H}$ breaking fields whose bilinear forms are $U(1)_X$ singlets but transform nontrivially under the Pati-Salam gauge group, thereby giving interesting Clebsch structures. The nonrenormalizable operators of interest must therefore involve both types of Higgs fields simultaneously. In view of the unusual nature of such operators, we shall provide a string-based discussion of the origin of such operators.

It is worth emphasizing that the main features of the previous analysis [like the assumption of U(1) symmetries, the introduction of singlet fields, etc.] appear naturally in most of the recent string constructions. Therefore, in the final sections of this paper we will try to embed our analysis in the context of realistic string models which are constructed within the free fermionic formulation [19] of the heterotic string. In doing so, we should keep in mind that, in realistic string constructions [19–21] there are usually many constraints and in general the resulting field theory is quite complicated. Moreover (in the language of the fermionic strings [21]), within the same choice of boundary conditions on the string basis vectors of the world-sheet fermions, there are numerous consistent choices of the projection coefficients which result in different Yukawa couplings multiplets and the large number of singlet fields which are usually present. For this reason we shall try to develop a ''string-model''-independent approach and begin by considering a field theory $SU(4)\times O(4)$ model, which possesses the salient features of a realistic string model and at the same time is simpler to work with.

III. STRING-INSPIRED $SU(4)\otimes O(4)$ MODEL

Here we briefly summarize the parts of the model which are relevant for our analysis. For a more complete discussion see [16]. The gauge group is $SU(4)\otimes O(4)$ or, equivalently,

$$SU(4)\otimes SU(2)_L\otimes SU(2)_R. \tag{13}$$

The left-handed quarks and leptons are accommodated in the representations

$$F^{i\alpha a}=(4,2,1)=\begin{pmatrix} u^R & u^B & u^G & \nu \\ d^R & d^B & d^G & e^- \end{pmatrix}^i, \tag{14}$$

$$\overline{F}^i_{x\alpha}=(\overline{4},1,\overline{2})=\begin{pmatrix} \overline{d}^R & \overline{d}^B & \overline{d}^G & e^+ \\ \overline{u}^R & \overline{u}^B & \overline{u}^G & \overline{\nu} \end{pmatrix}^i, \tag{15}$$

where $\alpha=1,\ldots,4$ is an SU(4) index, $a,x=1,2$ are $SU(2)_{L,R}$ indices, and $i=1,2,3$ is a family index. The Higgs fields are contained in the representations

$$h^x_a=(1,\overline{2},2)=\begin{pmatrix} h_2^+ & h_1^0 \\ h_2^0 & h_1^- \end{pmatrix} \tag{16}$$

(where h_1 and h_2 are the low energy Higgs superfields associated with the MSSM). The two heavy Higgs representations are

$$H^{\alpha b}=(4,1,2)=\begin{pmatrix} u_H^R & u_H^B & u_H^G & \nu_H \\ d_H^R & d_H^B & d_H^G & e_H^- \end{pmatrix} \tag{17}$$

and

$$\overline{H}_{\alpha x}=(\overline{4},1,\overline{2})=\begin{pmatrix} \overline{d}_H^R & \overline{d}_H^B & \overline{d}_H^G & e_H^+ \\ \overline{u}_H^R & \overline{u}_H^B & \overline{u}_H^G & \overline{\nu}_H \end{pmatrix}. \tag{18}$$

The Higgs fields are assumed to develop vacuum expectation values (VEV's)

$$\langle H\rangle=\langle\tilde{\nu}_H\rangle\sim M_{\rm GUT},\ \ \langle\overline{H}\rangle=\langle\widetilde{\overline{\nu}}_H\rangle\sim M_{\rm GUT}, \tag{19}$$

leading to the symmetry breaking at $M_{\rm GUT}$:

$$SU(4)\otimes SU(2)_L\otimes SU(2)_R\to SU(3)_C\otimes SU(2)_L\otimes U(1)_Y, \tag{20}$$

in the usual notation. Under the symmetry breaking in Eq. (20), the bidoublet Higgs field h in Eq. (16) splits into two Higgs doublets h_1,h_2 whose neutral components subsequently develop weak scale VEV's,

$$\langle h_1^0\rangle=v_1,\ \ \langle h_2^0\rangle=v_2, \tag{21}$$

with $\tan\beta\equiv v_2/v_1$.

In addition to the Higgs fields in Eqs. (17) and (18) the model also involves an SU(4) sextet field $D=(6,1,1)$ and four singlets ϕ_0 and φ_i, $i=1,2,3$. ϕ_0 is going to acquire an electroweak VEV in order to realize the electroweak Higgs mixing, while φ_i will participate in an extended ''seesaw'' mechanism to obtain light Majorana masses for the left-handed neutrinos. Under the symmetry property $\varphi_{1,2,3}\to(-1)\times\varphi_{1,2,3}$ and $H(\overline{H})\to(-1)\times H(\overline{H})$ the tree-level mass terms of the superpotential of the model read [16]

$$W=\lambda_1^{ij}F_i\overline{F_j}h+\lambda_2HHD+\lambda_3\overline{HHD}+\lambda_4^{ij}H\overline{F}_j\varphi_i+\mu\varphi_i\varphi_j+\mu hh, \tag{22}$$

where $\mu=\langle\phi_0\rangle=O(m_W)$. The last term generates the Higgs mixing between the two SM Higgs doublets in order to prevent the appearance of a massless electroweak axion. Note that we have banned terms which might lead to unacceptably large neutrino-Higgsino mixing [22]. The superpotential, Eq. (22), leads to the neutrino mass matrix [16]

$$\mathcal{M}_{\nu,N^c,\varphi}=\begin{pmatrix} 0 & m_u^{ij} & 0 \\ m_u^{ji} & 0 & M_{\rm GUT} \\ 0 & M_{\rm GUT} & \mu \end{pmatrix} \tag{23}$$

in the basis $(\nu_i,\overline{\nu}_j,\varphi_k)$. Diagonalization of the above gives three light neutrinos with masses of the order $(m_u^{ij})^2/M_{\rm GUT}$ as required and leaves right-handed Majorana masses of the order $M_{\rm GUT}$. Additional terms not included in Eq. (22) may be forbidden by imposing suitable discrete or continuous symmetries, the details of which need not concern us here. The D field carries color and therefore does not develop a VEV but the terms in Eq. (22), HHD and $\overline{HHD}$, combine the color triplet parts of H, $\overline{H}$, and D into acceptable GUT-scale mass terms [16]. When the H fields attain their VEV's at $M_{\rm GUT}\sim10^{16}$ GeV, the superpotential of Eq. (22) reduces to that of the MSSM augmented by neutrino masses. Below $M_{\rm GUT}$ the part of the superpotential involving matter superfields is just

$$W=\lambda_U^{ij}Q_i\overline{U}_jh_2+\lambda_D^{ij}Q_i\overline{D}_jh_1+\lambda_E^{ij}L_i\overline{E}_jh_1+\lambda_N^{ij}L_i\overline{\nu}_jh_2+\cdots. \tag{24}$$

The Yukawa couplings in Eq. (24) satisfy the boundary conditions

$$\lambda_1^{ij}(M_{\rm GUT})\equiv\lambda_U^{ij}(M_{\rm GUT})=\lambda_D^{ij}(M_{\rm GUT})=\lambda_E^{ij}(M_{\rm GUT})=\lambda_N^{ij}(M_{\rm GUT}). \tag{25}$$

Thus, Eq. (25) retains the successful relation $m_\tau=m_b$ at $M_{\rm GUT}$. Moreover, from the relation $\lambda_U^{ij}(M_{\rm GUT})=\lambda_N^{ij}(M_{\rm GUT})$ and the fourth term in Eq. (22), we obtain through the seesaw mechanism the light neutrino masses $=O(m_u^2/M_{\rm GUT})$ which satisfy the experimental limits.

IV. SYMMETRIC TEXTURES

In this section we briefly review the results of the operator analysis of Ref. [18] and then introduce our new approach based on new operators. We discuss the RRR textures as a simple example of the new method.

The boundary conditions listed in Eq. (25) lead to unacceptable mass relations for the light two families. Also, the large family hierarchy in the Yukawa couplings appears to be unnatural since one would naively expect the dimensionless couplings all to be of the same order. This leads us to the conclusion that the λ_1^{ij} in Eq. (22) may not originate from the usual renormalizable tree-level dimensionless coupling. We allow a renormalizable Yukawa coupling in the 33 term only and generate the rest of the effective Yukawa couplings by nonrenormalizable operators that are suppressed by some higher mass scale. This suppression provides an explanation for the observed fermion mass hierarchy.

In Ref. [18] we restricted ourselves to all possible nonrenormalizable operators which can be constructed from different group theoretical contractions of the fields:

$$O_{ij}\sim(F_i\overline{F_j})h\left(\frac{H\overline{H}}{M^2}\right)^n+\text{H.c.},\tag{26}$$

where we have used the fields H and $\overline{H}$ in Eqs. (17) and (18) and M is the large scale $M>M_X$. The idea is that when $H,\overline{H}$ develop their VEV's, such operators will become effective Yukawa couplings of the form $hF\overline{F}$ with a small coefficient of order $M_{\rm GUT}^2/M^2$. We considered up to $n=2$ operators. The motivation for using $n=2$ operators is simply that such higher dimension operators are generally expected to lead to smaller effective couplings more suited to the 12 and 21 Yukawa entries. However, in our field theory approach we shall restrict ourselves to the simple case considering only $n=1$ operators with the required suppression factors originating from a separate flavor sector. We will leave the question of the definite origin of the operators for now. Instead we merely note that one could construct a FN sector to motivate the operators or that one might expect such operators to come directly out of a string theory. In Sec. VII we shall introduce a $\mathrm{U}(1)_X$ family symmetry into the model, which is broken at a scale $M_X>M_{\rm GUT}$ by the VEV's of the Pati-Salam singlet fields θ and $\overline{\theta}$. According to the ideas discussed in Sec. II we shall henceforth consider operators of the form

$$O_{ij}\sim(F_i\overline{F_j})h\left(\frac{H\overline{H}}{M^2}\right)\left(\frac{\theta^n\,\overline{\theta}^m}{M'^{\,n+m}}\right)+\text{H.c.},\tag{27}$$

where M' represents a high scale $M'>M_{\rm GUT}$ which may be identified either with the $\mathrm{U}(1)_X$-breaking scale M_X or with the string scale. We have further assumed the form of the operators in Eq. (26) corresponding to $n=1$ and glued onto these operators arbitrary powers of the singlet fields $\theta,\overline{\theta}$. Note that the single power of $(H\overline{H})$ is present in every entry of the matrix and plays the role of the factor of δ in Eq. (11). However, unlike the previous factor of δ, the factor of $(H\overline{H})$ here carries important group theoretical Clebsch information. In fact Eq. (27) amounts to assuming a sort of *factorization* of the operators with the family hierarchies being completely controlled by the $\theta,\overline{\theta}$ fields as in IR, with m,n being dependent on i,j, and the horizontal splittings being controlled by the Clebsch factors in $(H\overline{H})$. However, this factorization is not complete since we shall assume that the Clebsch factors have a family dependence, i.e., they depend on i,j. We select the Clebsch factor in each entry from phenomenological arguments.

As a first example of our new approach we shall consider the RRR textures discussed in Sec. II. Our first observation is that, restricting ourselves to $n=1$ operators, there are no large Clebsch ratios between the up-type and down-type quarks for any of the operators. This means that it is very difficult to reproduce RRR solutions such as solution 2 where the 12 element of the down-type matrix in Eq. (9), for example, is 50 times larger than its up-type counterpart. Of course this can be achieved by requiring an accurate cancellation between two operators, but such a tuning of coefficients looks ugly and unnatural, and we reject it. On the other hand, the $n=1$ Clebsch coefficients in Table I include examples of *zero Clebsch coefficients*, where the contribution to the up-type matrix, for example, is precisely zero.

TABLE I. When the Higgs fields develop their VEV's at $M_{\rm GUT}$, the $n=1$ operators utilized lead to the effective Yukawa couplings with Clebsch coefficients as shown. We have included the relative normalization for each of the operators. The full set of $n=1$ operators and Clebsch coefficients is given in Appendix A. These $n=1$ operators were used in the lower right-hand block of the Yukawa matrices in the analysis of Ref. [18].

	$Q\overline{U}h_2$	$Q\overline{D}h_1$	$L\overline{E}h_1$	$L\overline{N}h_2$
O^A	1	1	1	1
O^B	1	-1	-1	1
O^C	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^D	$\frac{1}{\sqrt{5}}$	$\frac{-1}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^G	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0
O^H	4/5	2/5	4/5	8/5
O^K	8/5	0	0	6/5
O^M	0	$\sqrt{2}$	$\sqrt{2}$	0
O^N	2	0	0	0
O^R	0	$\frac{8}{5}$	$\frac{6}{5}$	0
O^W	0	$\sqrt{\frac{2}{5}}$	$-3\sqrt{\frac{2}{5}}$	0
O^S	$\frac{8}{5\sqrt{5}}$	$\frac{16}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$

Similarly there are *zero Clebsch coefficients* for the down-type quarks (and charged leptons). The existence of such *zero Clebsch coefficients* enables us to reproduce the RRR texture solutions 3 and 5 without fine tuning. Interestingly they are precisely the solutions which are not possible to obtain by the standard IR symmetry approach, which favors solutions 1, 2, and 4 and for which the up-type and down-type structures are similar. Thus our approach is capable of describing the RRR solutions which are complementary to those described by the IR symmetry approach.[5] To take a specific example let us begin by ignoring the flavor-dependent singlet fields, and consider the symmetric $n=1$ operator texture

$$\lambda=\begin{pmatrix} 0 & O^M & O^N \\ O^M & O^W+\mathrm{sd} & O^N \\ O^N & O^N & O_{33} \end{pmatrix}, \qquad (28)$$

where O_{33} is the renormalizable operator and sd stands for a subdominant operator with a suppression factor compared to the other dominant operator in the same entry. Putting in the Clebsch coefficients from Table I we arrive at the component Yukawa matrices, at the GUT scale, of

$$\lambda^U=\begin{pmatrix} 0 & 0 & 2\lambda^U_{13} \\ 0 & \lambda^U_{22} & 2\lambda^U_{23} \\ 2\lambda^U_{13} & 2\lambda^U_{23} & 1 \end{pmatrix}, \qquad (29)$$

$$\lambda^D=\begin{pmatrix} 0 & \sqrt{2}\lambda^D_{12} & 0 \\ \sqrt{2}\lambda^D_{12} & \lambda^D_{22}\sqrt{2}/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (30)$$

$$\lambda^E=\begin{pmatrix} 0 & \sqrt{2}\lambda^D_{12} & 0 \\ \sqrt{2}\lambda^D_{12} & 3\lambda^D_{22}\sqrt{2}/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (31)$$

where λ^D_{22} and λ^E_{22} arise from the dominant O^W_{22} operator and λ^U_{22} comes from a subdominant operator that is relevant because of the texture zero Clebsch in the up sector of O^W_{22}. The zeros in the matrices correspond to those of the RRR solution 5, but of course in our case they arise from the Clebsch zeros rather than from a family symmetry reason. The numerical values corresponding to RRR solution 5 with the correct phenomenology are

$$\lambda^U=\begin{pmatrix} 0 & 0 & 2\times10^{-3} \\ 0 & 3\times10^{-3} & 3\times10^{-2} \\ 2\times10^{-3} & 3\times10^{-2} & 1 \end{pmatrix},$$

$$\lambda^D=\begin{pmatrix} 0 & 5\times10^{-3} & 0 \\ 5\times10^{-3} & 2\times10^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (32)$$

Thus, the hierarchy $\lambda^U_{22}\ll\lambda^D_{22}$ is explained by a Clebsch zero and a suppression factor of the subdominant operator. Using Eq. (32) we can read off the values of the couplings which roughly correspond to a unified matrix of dominant couplings

$$\lambda=\begin{pmatrix} 0 & 3\times10^{-3} & 1\times10^{-3} \\ 3\times10^{-3} & 2\times10^{-2} & 2\times10^{-2} \\ 1\times10^{-3} & 2\times10^{-2} & 1 \end{pmatrix}, \qquad (33)$$

where we have extracted the Clebsch factors. We find it particularly elegant that the whole quark and lepton spectrum is controlled by a unified Yukawa matrix such as in Eq. (33) with all the vertical splittings controlled by Clebsch factors.

At this stage we could introduce a $U(1)_X$ symmetry of the IR kind and the flavor-dependent singlet fields in order to account for the horizontal family hierarchy of couplings in Eq. (33). In the present case we must remember that there is a small quantity δ multiplying every nonrenormalizable entry as in Eq. (11), corresponding to the $n=1$ bilinear $\delta\equiv v\overline{v}/M^2$ which we have required to be present in every nonrenormalizable entry. Thus we can understand Eq. (33) as resulting from a structure such as

$$\lambda=\begin{pmatrix} \delta\epsilon^8 & \delta\epsilon^3 & \delta\epsilon^4 \\ \delta\epsilon^3 & \delta\epsilon^2 & \delta\epsilon \\ \delta\epsilon^4 & \delta\epsilon & 1 \end{pmatrix}, \qquad (34)$$

where we identify $\epsilon\equiv\lambda=0.22$ and set $\delta\approx0.2$ which gives the correct orders of magnitude for the entries, rather similar to the case we discussed in Eq. (12). Here of course the considerations apply to the unified Yukawa matrix, however, not just the down-type quark matrix. The details of the $U(1)_X$ family symmetry analysis are discussed in Sec. VII. Here we simply note that such an analysis can lead to a structure such as the one assumed in Eq. (34).

A similar analysis could equally well be applied to RRR solution 3. In both cases we are led to a pleasing scheme which involves no unnatural tuning of elements and naturally combines the effect of Clebsch coefficients with that of family symmetry suppression, in a simple way. The existence of the Clebsch texture zeros thus permits RRR solutions 3 and 5 which are impossible to obtain otherwise within the general framework presented here.

[5] In [23], two of us used an alternative approach in order to reproduce the structure of solutions 1 and 3 of RRR by the implementation of a symmetry. These solutions were found to lead to the optimal predictions for neutrino masses and mixings. This has been achieved by a proper choice of charges (integer or half-integer) and by imposing residual Z_2 symmetries which forbid different entries in the up- and down-quark mass matrices.

V. NONSYMMETRIC TEXTURES

In this section we update the nonsymmetric textures based on both $n=1$ and $n=2$ operators introduced in Ref. [18] and then extend the new approach introduced in the previous section to the nonsymmetric domain. As in the previous section, we shall begin by ignoring the effect of the singlet fields, which will be discussed in Sec. VII.

As discussed in Appendix B we shall modify the analysis of Ref. [18] to only include the lower 2×2 block *Ansatz*:

$$A_1=\begin{bmatrix} O_{22}^{W}+\mathrm{sd} & 0 \\ O_{32}^{C} & O_{33} \end{bmatrix}. \tag{35}$$

This is then combined with the upper 2×2 blocks considered in Ref. [18]:

$$B_1=\begin{bmatrix} 0 & O^1 \\ O^{Ad} & X \end{bmatrix}, \tag{36}$$

$$B_2=\begin{bmatrix} 0 & O^2 \\ O^{Ad} & X \end{bmatrix}, \tag{37}$$

$$B_3=\begin{bmatrix} 0 & O^3 \\ O^{Ad} & X \end{bmatrix}, \tag{38}$$

$$B_4=\begin{bmatrix} 0 & O^1 \\ O^{Dd} & X \end{bmatrix}, \tag{39}$$

$$B_5=\begin{bmatrix} 0 & O^2 \\ O^{Dd} & X \end{bmatrix}, \tag{40}$$

$$B_6=\begin{bmatrix} 0 & O^3 \\ O^{Dd} & X \end{bmatrix}, \tag{41}$$

$$B_7=\begin{bmatrix} 0 & O^1 \\ O^{Md} & X \end{bmatrix}, \tag{42}$$

$$B_8=\begin{bmatrix} 0 & O^2 \\ O^{Md} & X \end{bmatrix}, \tag{43}$$

where X stands for whatever is left in the 22 position, after the lower 2×2 submatrix has been diagonalized. The Clebsch coefficients of the $n=2$ operators used in Eqs. (36)–(43) are displayed in Table II but we refer the reader to Ref. [18] for the explicit realization of these operators in terms of the component fields for reasons of brevity. The *Ansätze* listed above present problems because of the breakdown of matrix perturbation theory.[6] For purposes of comparison with the new scheme involving only $n=1$ operators, we will recalculate the predictions for each of the models from Ref. [18] numerically in the next section.

[6]When the magnitudes of H_{21}, H_{12}, and H_{22} are calculated they are each of the same order in the down Yukawa matrix, thus violating the hierarchy in Eq. (B1) that was assumed in the calculation of the predictions.

TABLE II. Clebsch coefficients of $n=2$ operators previously utilized.

	$Q\bar{U}h_2$	$Q\bar{D}h_1$	$L\bar{E}h_1$	$L\bar{N}h_2$
O^{Ad}	$\frac{4\sqrt{2}}{25}$	$\frac{12\sqrt{2}}{25}$	$\frac{9\sqrt{2}}{25}$	$\frac{3\sqrt{2}}{25}$
O^{Dd}	$\frac{1}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$
O^{Md}	$\frac{\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$	$\frac{6\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$
O^1	0	$\sqrt{2}$	$\sqrt{2}$	0
O^2	0	$\frac{8}{5}$	$\frac{6}{5}$	0
O^3	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0

We now turn our attention to the new approach introduced in the previous section, based on $n=1$ operators together with singlet fields which for the moment we shall ignore. In this case the 21 operator used in Ref. [18] which gave an up Clebsch coefficient 1/3 times smaller than the down Clebsch coefficient is not available if we only use $n=1$ operators. We must therefore use a combination of two operators in the 21 position that allow the up entry to be a bit smaller than the down entry. We require that the combination provide a Clebsch relation between λ_{21}^{D} and λ_{21}^{E} for predictivity. The two operators cancel slightly in the up sector, but as shown later this cancellation is ~1 and therefore acceptable. The result of this is that the prediction of V_{ub} is lost; however, this prediction was almost excluded by experiment anyway, and a more accurate numerical estimate which does not rely on matrix perturbation theory confirms that V_{ub} in Ref. [18] is too large. So the loss of the V_{ub} prediction is to be welcomed. The Clebsch effect of the 12 operator (with a zero Clebsch effect for the up-type quarks) can easily be reproduced at the $n=1$ level by the operator O^M, for example.

To get some feel for the procedure we will follow, we first discuss a simple example of a nonsymmetric texture, ignoring complex phases for illustrative purposes. Restricting ourselves to $n=1$ operators, we consider the lower block to be A_1 and the upper block to be the modified texture as discussed in the previous paragraph. Thus we have

$$\lambda=\begin{pmatrix} 0 & O^M & 0 \\ O^M+O^A & O^W+\mathrm{sd} & 0 \\ 0 & O^C & O_{33} \end{pmatrix}, \tag{44}$$

where O_{33} is the renormalizable operator. Putting in the Clebsch coefficients from Table IV we arrive at the component Yukawa matrices, at the GUT scale, of

$$\lambda^U = \begin{pmatrix} 0 & 0 & 0 \\ \lambda^U_{21} & \lambda^U_{22} & 0 \\ 0 & \sqrt{2}\lambda^U_{32}/\sqrt{5} & 1 \end{pmatrix}, \tag{45}$$

$$\lambda^D = \begin{pmatrix} 0 & \sqrt{2}\lambda^D_{12} & 0 \\ \sqrt{2}\lambda^D_{21} & \lambda^D_{22}/\sqrt{5} & 0 \\ 0 & -\sqrt{2}\lambda^U_{32}/\sqrt{5} & 1 \end{pmatrix}, \tag{46}$$

$$\lambda^E = \begin{pmatrix} 0 & \sqrt{2}\lambda^D_{12} & 0 \\ \sqrt{2}\lambda^D_{21} & 3\lambda^D_{22}/\sqrt{5} & 0 \\ 0 & -3\sqrt{2}\lambda^U_{32}/\sqrt{5} & 1 \end{pmatrix}, \tag{47}$$

where λ^U_{22} and λ^D_{22} arise from the difference and sum of two operators whose normalization factor of $\sqrt{5}$ has been explicitly inserted, and similarly for λ^U_{21} and λ^D_{21}. To obtain the numerical values of the entries we use some typical GUT-scale values of Yukawa couplings and Cabibbo-Kobayashi-Maskawa (CKM) elements (see Ref. [18]) as follows:

$$\lambda_{33}=1,\ \lambda_c=0.002,\ \lambda_s=0.013,\ \lambda_\mu=0.04,$$

$$\lambda_u=10^{-6},\ \ \lambda_d=0.0006,\ \lambda_e=0.0002, \tag{48}$$

$$V_{cb}=0.05,\ V_{us}=0.22,\ V_{ub}=0.004, \tag{49}$$

where we have assumed

$$\alpha_s=0.115,\ m_b=4.25,\ \tan\beta=55,\ m_t=180\ \text{GeV}. \tag{50}$$

The textures in Eqs. (45), (46), and (47) imply that the 22 eigenvalues are just equal to the 22 elements (assuming matrix perturbation theory is valid—see later) and $\lambda^U_{32}=V_{cb}/2=0.025$. Thus we have $\lambda^U_{22}=0.004$, $\lambda^D_{22}=0.03$. The remaining parameters are determined from the relations

$$\lambda_u=0,\ \lambda_d=3\lambda_e=\lambda^D_{21}\sqrt{2}\lambda^D_{12}/\lambda_s,$$

$$V_{ub}=\lambda^U_{21}V_{cb}/\lambda_c. \tag{51}$$

Note that the up quark mass looks like it is zero, but in practice we would expect some higher dimension operator to be present which will give it a small nonzero value. We thus have three equations and three unknowns, and solving we find $\lambda^U_{21}=2\times10^{-4}$, $\lambda^D_{21}=2\times10^{-3}$, $\lambda^D_{12}=3\times10^{-3}$. The difference between λ^U_{21} and λ^D_{21} requires suppression of O^A caused by the Clebsch zero in the dominant operator O^M. Thus the unified Yukawa matrix involves operators with the approximate numerical coefficients

$$\lambda = \begin{pmatrix} 0 & 3\times10^{-3} & 0 \\ 3\times10^{-3} & 1.5\times10^{-2} & 0 \\ 0 & 2.5\times10^{-2} & 1 \end{pmatrix}, \tag{52}$$

where we have extracted the Clebsch factors, and the 22 and 21 values in Eq. (52) refer to each of the two operators in this position separately. The numerical values in Eq. (52) are not dissimilar from those in Eq. (33); in particular, the upper 2×2 block is symmetrical with the same values as before. In this case the lower 2×2 block has a texture zero in the 23 position, as well as the 31 and 13 positions, but otherwise the numerical values are very similar to those previously obtained in Eq. (33). Thus this particular nonsymmetric texture can be described by a structure of the kind

$$\lambda = \begin{pmatrix} \delta\epsilon^{big} & \delta\epsilon^3 & \delta\epsilon^{big} \\ \delta\epsilon^3 & \delta\epsilon^{1\text{ or }2} & \delta\epsilon^{big} \\ \delta\epsilon^{big} & \delta\epsilon & 1 \end{pmatrix}, \tag{53}$$

where we identify $\epsilon\equiv\lambda=0.22$ and set $\delta\approx0.1$ as before. Can such a structure for the ϵ's be obtained from the $U(1)_X$ symmetry? This will be discussed in Sec. VII.

There is no reason to restrict ourselves to nonsymmetric textures with a zero in the 13 and 31 positions, as assumed in Ref. [18]. For example, the following texture is also viable, amounting to a hybrid of the symmetric case considered in Eq. (28) and the nonsymmetric lower block just considered:

$$\lambda = \begin{pmatrix} 0 & O^M & O^N \\ O^M & O^W & 0 \\ O^N & O^C & O_{33} \end{pmatrix}. \tag{54}$$

Here, O_{33} is the renormalizable operator. We now perform a general operator analysis of the nonsymmetric case, assuming $n=1$ operators for all nonzero entries (apart from the 33 renormalizable entry). In this general analysis there are two classes of texture: those with universal texture zeros in the 13 and 31 positions (essentially $n=1$ versions of the textures considered in Ref. [18]) and new textures with nonzero entries in the 13 and/or 31 position. For now we will not consider the cases with operators in the 13 or 31 positions for reasons of brevity. In the general analysis we repeat the above procedure, being careful about phases, and obtain some numerical estimates of the magnitude of each entry which will be explained in terms of the $U(1)_X$ family symmetry as discussed in the next section.

With the above discussion in mind, we consider the new scheme in which the dominant operators in the Yukawa matrix are O_{33}, O^C_{32}, O^W_{22}, O_{21}, $\widetilde{O}_{21}$, and O_{12}, where the last three operators are left general and will be specified later. We are aware from the analysis in Ref. [18] that O_{12} must have a zero Clebsch coefficient in the up sector. A combination of two operators must then provide a nonzero O_{21} entry to provide a large enough V_{ub}, and an additional, much more suppressed operator elsewhere in the Yukawa matrix gives the up quark a small mass. At $M_{\rm GUT}$ therefore, the Yukawa matrices are of the form

$$\lambda^I = \begin{bmatrix} 0 & H_{12}e^{i\phi_{12}}x^I_{12} & 0 \\ H_{21}x^I_{21}e^{i\phi_{21}}+\widetilde{H}_{21}\widetilde{x}^I_{21}e^{i\widetilde{\phi}_{21}} & H_{22}x^I_{22}e^{i\phi_{22}} & 0 \\ 0 & H_{32}x^I_{32}e^{i\phi_{32}} & H_{33}e^{i\phi_{33}} \end{bmatrix}, \tag{55}$$

where only the dominant operators are listed. The I superscript labels the charge sector and x_{ij}^{I} refers to the Clebsch coefficient relevant to the charge sector I in the ijth position. ϕ_{ij} are unknown phases and H_{ij} is the magnitude of the effective dimensionless Yukawa coupling in the ijth position. Any subdominant operators that we introduce will be denoted below by a prime and it should be borne in mind that these will only affect the up matrix. So far, the known Clebsch coefficients are

$$\begin{gathered} x_{12}^{U}=0, \\ x_{22}^{U}=0,\quad x_{22}^{D}=1,\quad x_{22}^{E}=-3, \\ x_{32}^{U}=1,\quad x_{32}^{D}=-1,\quad x_{32}^{E}=-3. \end{gathered} \tag{56}$$

We have just enough freedom in rotating the phases of $F_{1,2,3}$ and $\overline{F}_{1,2,3}$ to get rid of all but one of the phases in Eq. (55). When the subdominant operator is added, the Yukawa matrices are

$$\lambda^{U}=\begin{bmatrix} 0 & 0 & 0 \\ H_{21}^{U}e^{i\phi_{21}^{U}} & H_{22}'e^{i\phi_{22}'} & 0 \\ 0 & H_{32}x_{32}^{U} & H_{33} \end{bmatrix},$$

$$\lambda^{D}=\begin{bmatrix} 0 & H_{12}x_{12}^{D} & 0 \\ H_{21}^{D} & H_{22}x_{22}^{D} & 0 \\ 0 & H_{32}x_{32}^{D} & H_{33} \end{bmatrix},$$

$$\lambda^{E}=\begin{bmatrix} 0 & H_{12}x_{12}^{E} & 0 \\ H_{21}^{E} & H_{22}x_{22}^{E} & 0 \\ 0 & H_{32}x_{32}^{E} & H_{33} \end{bmatrix}, \tag{57}$$

where we have defined

$$\begin{aligned} H_{21}^{U}e^{i\phi_{21}^{U}} &\equiv H_{21}x_{21}^{U}e^{i\phi_{21}}+\tilde{H}_{21}\tilde{x}_{21}^{U}e^{i\tilde{\phi}_{21}}, \\ H_{21}^{D,E} &\equiv H_{21}x_{21}^{D,E}e^{i\phi_{21}}+\tilde{H}_{21}\tilde{x}_{21}^{D,E}e^{i\tilde{\phi}_{21}}. \end{aligned} \tag{58}$$

We may now remove ϕ_{22}' by phase transformations upon $\overline{F}_{1,2,3}$ but ϕ_{21}^{U} may only be removed by a phase redefinition of $F_{1,2,3}$, which would alter the prediction of the CKM matrix $V_{\rm CKM}$. Thus, ϕ_{21}^{U} is a physical phase; that is, it cannot be completely removed by phase rotations upon the fields. Once the operators $O_{21}, \tilde{O}_{21}, O_{12}$ have been chosen, the Yukawa matrices at $M_{\rm GUT}$ including the phase in the CKM matrix are therefore identified with $H_{ij}, H_{22}', \phi_{21}^{U}$.

VI. NUMERICAL ANALYSIS OF MASSES AND MIXING ANGLES FROM NONSYMMETRIC TEXTURES

In this section we discuss the numerical procedure used to analyze the nonsymmetric cases introduced in the previous section. We shall perform an analysis on the new approach based on $n=1$ operators only, and also reanalyze and update the original scheme of Ref. [18] for comparison.

The basic idea is to do a global fit of each considered *Ansatz* to m_e, m_μ, m_u, m_c, m_t, m_d, m_s, m_b, $\alpha_S(M_Z)$, $|V_{ub}|$, $|V_{cb}|$, and $|V_{us}|$ using m_τ as a constraint. We use the approximation that the whole supersymmetry (SUSY) spectrum of the MSSM lies at $M_{\rm SUSY}=m_t$ and that the MSSM remains a valid effective theory until the scale $M_{\rm GUT}=10^{16}$ GeV. Not wishing to include neutrino masses in this analysis, we simply set the right-handed Majorana neutrino mass of each family to be 10^{16} GeV so that the neutrinos are approximately massless and hence their masses do not affect the renormalization group equations (RGE's) below $M_{\rm GUT}$. Recall the parameters introduced in Eq. (57): $\phi_{21}^{U}\equiv\phi$, $H_{21}^{U}\equiv H_{21}'$, $H_{21}^{D}\equiv H_{21}$, $H_{22}', H_{22}, H_{12}, H_{32}, H_{33}$. The values of these eight parameters plus α_S at the GUT scale are determined by the fit.

The matrices λ^{I} are diagonalized numerically and $|V_{ub}(M_{\rm GUT})|, |V_{us}(M_{\rm GUT})|$ are determined by

$$V_{\rm CKM}=V_{U_L}V_{D_L}^{\dagger}, \tag{59}$$

where V_{U_L} and V_{D_L} are the matrices that act upon the $(u,c,t)_L$ and $(d,s,b)_L$ column vectors, respectively, to transform from the weak eigenstates to the mass eigenstates of the quarks. We use the boundary conditions $\alpha_1(M_{\rm GUT})=\alpha_2(M_{\rm GUT})=0.708$, motivated by previous analyses based on gauge unification in SUSY GUT models [24]. $\lambda_{u,c,t,d,s,b,e,\mu,\tau}$, $|V_{us}|$, and $|V_{ub}|$ are then run[7] from $M_{\rm GUT}$ to 170 GeV$\approx m_t$ using the RGE's for the MSSM. Below $M_{\rm GUT}$ the effective field theory of the standard model allows the couplings in the different charge sectors to split and run differently. The λ_i are then evolved to their empirically derived running masses using three-loop QCD $\otimes$one-loop QED [18]. m_τ^e and $\lambda_\tau^p(m_\tau)$ then[8] fix $\tan\beta$ through the relation [12]

$$\cos\beta=\frac{\sqrt{2}m_\tau^e(m_\tau)}{v\lambda_\tau^p(\lambda_\tau)}, \tag{60}$$

where $v=246.22$ GeV is the VEV of the standard model Higgs field. Predictions of the other fermion masses then come from

$$m_{c,t}^{p}\approx\lambda_{c,t}^{p}(m_{c,t})\frac{v\sin\beta}{\sqrt{2}},$$

$$m_{d,s,b}^{p}\approx\lambda_{d,s,b}(m_{1,1,b}^{e})\frac{v\cos\beta}{\sqrt{2}},$$

[7] All renormalization running in this paper is one loop and in the modified minimal subtraction ($\overline{\rm MS}$) scheme. The relevant renormalization group equations are listed in Ref. [18].

[8] The superscript e upon masses, mixing angles, or diagonal Yukawa couplings denotes an empirically derived value, whereas the superscript p denotes the prediction of the model for the particular fit parameters being tested.

TABLE III. Results of best-fit analysis on models with $n=1$ operators only. Note that the input parameters $H_{ij}, H_{ij}', \cos\phi$ shown are evaluated at the scale M_{GUT}. All of the mass predictions shown are running masses, apart from the pole mass of the top quark, $m_t^{\mathrm{phys}} \equiv m_t[1+4\alpha_S(m_t)/3\pi]$. The CKM matrix element predictions are at M_Z.

Model	1	2	3	4	5
O_{12}	O^M	O^W	O^R	O^R	O^R
$O_{21}+\widetilde{O}_{21}$	O^M+O^A	O^G+O^H	O^M+O^A	O^G+O^H	O^R+O^S
$H_{22}/10^{-2}$	2.88	2.64	2.69	2.67	6.15
$H_{12}/10^{-3}$	2.81	4.41	2.13	0.70	1.21
$H_{21}/10^{-3}$	1.30	5.97	1.76	4.33	1.91
$\cos\phi$	0.87	1.00	0.20	1.00	0.61
H_{33}	1.18	1.05	1.05	1.07	4.6
$H_{22}'/10^{-3}$	1.91	1.87	1.87	1.87	2.87
$H_{21}'/10^{-3}$	1.94	1.62	1.63	1.66	0.76
$\alpha_S(M_Z)$	0.119	0.118	0.118	0.118	0.126
m_d/MeV	6.25	1.03	8.07	4.14	11.9
m_s/MeV	158	150	154	152	228
m_c/GeV	1.30	1.30	1.30	1.30	1.30
m_b/GeV	4.24	4.25	4.25	4.25	4.13
m_t^{phys}/GeV	182	180	180	180	192
$\lvert V_{us}\rvert$	0.2211	0.2215	0.2215	0.2215	0.2215
$\lvert V_{ub}\rvert/10^{-3}$	3.71	3.51	3.50	3.52	3.50
$\tan\beta$	59.5	58.3	58.3	58.5	65.7
χ^2/N_{DF}	0.34	1.16	0.13	0.55	1.84

$$m_{e,\mu}^p \approx \lambda_{e,\mu}(m_{1,\mu}^e)\frac{v\cos\beta}{\sqrt{2}}, \tag{61}$$

where $m_1 \equiv 1$ GeV. There are 12 data points and 9 parameters, and so we have three degrees of freedom (DF). The parameters are all varied until the global χ^2/N_{DF} is minimized. The data used (with 1σ errors quoted) are [25]

$$\begin{aligned}
m_e &= 0.510\,999 \text{ MeV},\\
m_\mu &= 105.658 \text{ MeV},\\
m_\tau &= 1.7771 \text{ GeV},\\
m_c &= 1.3 \pm 0.3 \text{ GeV},\\
m_t^{\mathrm{phys}} &= 180 \pm 12 \text{ GeV},\\
m_d &= 10 \pm 5 \text{ MeV},\\
m_s &= 200 \pm 100 \text{ MeV},\\
m_b &= 4.25 \pm 0.1 \text{ GeV},\\
|V_{ub}| &= (3.50 \pm 0.91)\times 10^{-3},\\
|V_{us}| &= 0.2215 \pm 0.0030,\\
\alpha_S(M_Z) &= 0.117 \pm 0.005.
\end{aligned} \tag{62}$$

$|V_{cb}|$ is fixed by H_{32} which does not influence the other predictions to a good approximation and so $|V_{cb}|$ and H_{32} effectively decouple from the fit. We merely note that in all cases, to predict the measured value of $|V_{cb}|$, $H_{32}\sim 0.03$. Note that no errors are quoted upon the lepton masses because m_τ is used as a constraint on the data and because m_e, m_μ were required to be satisfied to 0.1% by the fit. In this way we merely use the lepton masses as three constraints, using up three DF. We did not perform the fit with smaller empirical errors on the lepton masses because of the numerical roundoff and minimization errors associated with high χ^2 values generated by them. Also, 0.1% is a possible estimate of higher loop radiative corrections involved in the predictions. Note that no other theoretical errors were taken into account in the fit. The largest ones may occur in derivations of m_b due to the large λ_b coupling [26] and the nonperturbative effects of QCD near 1 GeV. It is not clear how to estimate these errors since the error on m_b depends upon soft parameters which depend on the SUSY-breaking mechanism in a very model-dependent way and nonperturbative QCD is an unsolved problem. The correlations between the empirical estimations of the current quark masses are also not included. A potentially large error could occur if the *Ansätze* considered are not exact but are subject to corrections by higher dimension operators. We discuss this point further in Sec. VII.

The results obtained from this analysis are given in Table III. Out of 16 possible models that fit the texture required by Eqs. (56) and (55), 11 models fit the data with $\chi^2/N_{\mathrm{DF}}<3$. Out of these 11 models, 5 fit the data with $\chi^2/N_{\mathrm{DF}}<2$ and these are displayed in Table III. The operators listed as $O_{12}, O_{21}, \widetilde{O}_{21}$ describe the structure of the models and the entries $H_{22}, H_{12}, H_{21}, \cos\phi, H_{33}, H_{22}', H_{21}'$ are the GUT-scale input parameters of the best-fit values of the model. The estimated 1σ deviation in $\alpha_S(M_Z)$ from the fits is ± 0.003

TABLE IV. Predictions of best-fit analysis on models from Ref. [18] with $n=2$ operators included. All of the mass predictions shown are running masses, apart from the pole mass of the top quark. The CKM matrix element predictions are at M_Z.

Model	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8
$\alpha_S(M_Z)$	0.123	0.123	0.123	0.124	0.123	0.124	0.125	0.124
m_d/MeV	7.58	9.12	4.64	6.18	7.49	3.63	3.53	4.53
m_s/MeV	215	240	179	210	217	179	200	187
m_c/GeV	1.29	1.38	1.35	1.16	1.29	1.32	0.86	1.31
m_b/GeV	4.19	4.17	4.19	4.19	4.19	4.18	4.20	4.19
$m_t^{\rm phys}$/GeV	188	189	189	189	188	189	190	189
$\lvert V_{us}\rvert$	0.2212	0.2213	0.2214	0.2212	0.2212	0.2215	0.2212	0.2214
$\lvert V_{ub}\rvert/10^{-3}$	4.52	4.37	4.05	4.22	4.56	3.74	3.85	3.98
$\tan\beta$	63.2	63.6	63.4	63.7	63.2	63.8	64.3	63.6
$\chi^2/N_{\rm DF}$	0.95	0.96	1.00	1.05	0.97	1.16	1.87	1.04

and the other parameters are constrained to better than 1% apart from $\cos\phi$, whose 1σ fit errors often cover the whole possible range. Out of the predictions shown in Table III, m_d discriminates between the models the widest. $\alpha_S(M_Z)$ takes roughly central values, apart from model 5 for which the best fit is outside the 1σ errors quoted in Eq. (62) on $\alpha_S(M_Z)$. $m_s,|V_{ub}|$ are within 1σ of the data point and $m_c,|V_{us}|$ are approximately on the central value for all 5 models. Models 3, 1, and 4 are very satisfactory fits to the data with $\chi^2/N_{\rm DF}<1$. We conclude that the χ^2 test has some discriminatory power in this case since, if all of the models were equally good, we would statistically expect to have 11 models with $\chi^2/N_{\rm DF}<1$, 3 models with $\chi^2/N_{\rm DF}=1$–2, and 2 models with $\chi^2=2$–3 out of the 16 tested.

We now briefly return to the original models with upper blocks given by B_{1-8} in Eqs. (36)–(43) [18]. After again isolating the only physical phase to λ_{21}^U, a numerical fit analogous to the above was performed using the same data in Eq. (62). The main difference in the fit with these models is that there are now four degrees of freedom in the fit (since there is one less parameter). All eight models in question fit the data with $\chi^2<2$ and these are displayed in Table IV. We do not display the best-fit input parameters because they are largely irrelevant for the discussion here. 1σ fit deviations of $\alpha_S(M_Z)$ are again 0.003 for B_{1-8}. Note that whereas these models are able to fit $|V_{us}|,m_s,m_d,m_b,m_c$ fairly well, their predictions of $\alpha_S(M_Z)$ are high and outside the 1σ empirical error bounds. $|V_{ub}|$ is naturally high in these models (as found in Ref. [18]) and this forces $\alpha_S(M_Z)$ to be large, where $|V_{ub}|$ may decrease somewhat. To fit m_b with a high $\alpha_S(M_Z)$ requires a large H_{33} element and this is roughly speaking why $m_t^{\rm phys}$ is predicted to be quite high. In each model the high value of $\alpha_S(M_Z)$ required is the dominant source of χ^2 apart from B_7, where m_c is low.

In comparison to the new scheme with $n=1$ operators only, the old scheme with $n=2$ operators fits the data pretty well, although not quite as well as models 1,3,4. The old scheme also has one more prediction than the new one. However, the preferred models are the ones incorporating the $U(1)_X$ symmetry since they go deeper into the reasons for the zeros and hierarchies in the Yukawa matrices.

VII. $\mathbf{U(1)_X}$ FAMILY SYMMETRY IN THE $\mathbf{SU(4)\times O(4)}$ MODEL

In our discussion of the symmetric textures, we assumed that we could obtain the same structure as IR. Of course, as we have already mentioned, the case we are examining is different in two aspects: (a) The fermion mass matrices of the different charge sectors have the same origin, and thus the same expansion parameter, and (b) all differences between these sectors arise from Clebsch factors. As a starting point, we will therefore briefly repeat the IR analysis for symmetric mass matrices in our framework; we then go on to consider the nonsymmetric case, with the goal of being able to reproduce the numerical values (at least to an order of magnitude) of the successful *Ansätze* given in the previous section.

The structure of the mass matrices is determined by a family symmetry $U(1)_X$, with the charge assignment of the various states given in Table V. The need to preserve $SU(2)_L$ invariance requires left-handed up and down quarks (leptons) to have the same charge. This, plus the additional requirement of symmetric matrices, indicates that all quarks (leptons) of the same ith generation transform with the same charge α_i. Finally, lepton-quark unification under $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ indicates that quarks and leptons of the same family have the same charge (this is a different feature as compared to IR, where quarks and leptons of the two lower generations have different charges under the flavor symmetry). The full anomaly-free Abelian group involves an additional family-independent component $U(1)_{\rm FI}$, and with this freedom $U(1)_X$ is made traceless without any loss of generality.[9] Thus we set $\alpha_1=-(\alpha_2+\alpha_3)$. Here we consider the simplest case where the combination $H\bar{H}$ is taken to have zero charge. This is consistent with our requirement that it play no role in the mass hierarchies, other than leading to a common factor δ for all nonrenormalizable entries.

If the light Higgs h_2,h_1, responsible for the up and down

[9]Since we assume that the 33 operator is renormalizable, the relaxation of the tracelessness condition does not change the charge matrix since any additional FI charges can always be absorbed into the Higgs h_i charges.

TABLE V. $U(1)_X$ charges assuming symmetric textures.

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	h_1	h_2	H	$\overline{H}$
$U(1)_X$	α_i	α_i	α_i	α_i	α_i	α_i	$-2\alpha_3$	$-2\alpha_3$	x	$-x$

TABLE VI. $U(1)_X$ charges for nonsymmetric textures.

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	h_1	h_2	H	$\overline{H}$
$U(1)_X$	β_i	α_i	α_i	β_i	α_i	α_i	$-\beta_3-\alpha_3$	$-\beta_3-\alpha_3$	x	$-x$

quark masses, respectively, arise from the same bidoublet $h=(1,2,2)$, then they have the same $U(1)_X$ charge so that only the 33 renormalizable Yukawa coupling to h_2,h_1 is allowed, and only the 33 element of the associated mass matrix will be nonzero. The remaining entries are generated when the $U(1)_X$ symmetry is broken. This breaking is taken to be spontaneous, via standard model singlet fields, which can be either *chiral* or *vector* ones; in the latter case, which is the one studied in IR, two fields θ and $\overline{\theta}$, with $U(1)_X$ charge -1 and $+1$, respectively, and equal VEV's are introduced. When these fields get a VEV, the mass matrix acquires its structure. For example, the 32 entry in the up quark mass matrix appears at $O(\epsilon)$ because U(1) charge conservation only allows the term $c^c t h_2(\theta/M_2)^{\alpha_2-\alpha_3}$ for $\alpha_2>\alpha_3$ or $c^c t h_2(\overline{\theta}/M_2)^{\alpha_3-\alpha_2}$ for $\alpha_3>\alpha_2$. Here $\epsilon=(\langle\theta\rangle/M_2)^{|\alpha_2-\alpha_3|}$ where M_2 is the unification mass scale which governs the higher dimension operators. In IR, a different scale, M_1, is expected for the down quark and lepton mass matrices.

In our case, however, all charge and mass matrices have the same structure under the $U(1)_X$ symmetry, since all known fermions are accommodated in the same multiplets of the gauge group. The charge matrix is of the form

$$\begin{pmatrix} -2\alpha_2-4\alpha_3 & -3\alpha_3 & -\alpha_2-2\alpha_3 \\ -3\alpha_3 & 2(\alpha_2-\alpha_3) & \alpha_2-\alpha_3 \\ -\alpha_2-2\alpha_3 & \alpha_2-\alpha_3 & 0 \end{pmatrix}. \tag{63}$$

Then, including the common factor δ which multiplies all nonrenormalizable entries, the following pattern of masses is obtained (for vectorlike singlets):

$$\lambda^{u,d,\ell}\approx\begin{pmatrix} \delta\epsilon^{|2+6a|} & \delta\epsilon^{|3a|} & \delta\epsilon^{|1+3a|} \\ \delta\epsilon^{|3a|} & \delta\epsilon^{2} & \delta\epsilon \\ \delta\epsilon^{|1+3a|} & \delta\epsilon & 1 \end{pmatrix}, \tag{64}$$

where[10] $a=\alpha_3/(\alpha_2-\alpha_3)$. We emphasize that the entries in Eq. (64) describe the magnitudes of the dominant operators, and do not take the Clebsch zeros of the different charge sectors into account. Note the existence of a single expansion parameter, for all three matrices. Another interesting point is that a unique charge combination a appears in the exponents of all matrices, as a result of quark-lepton unification. Actually, unlike what appears here, in most schemes the lepton mass matrix is described in the generic case by two parameters. For $a=1$, one generates the structure in Eq. (34) for the unified fermion mass matrices.

Before passing to the nonsymmetric case, let us make a few comments on the possibility of having chiral or vector singlets, as well as on the charge of the Higgs fields. Suppose first that θ is a chiral field. From the form of the charge matrix, we observe that if the 22 and 23 entries have a positive charge, α_3 is negative (for all these entries to be nonvanishing at the same time). Moreover, the hierarchy 1:3 between the 23 and 12 elements indicates that α_2 would have to be zero in the chiral case, and thus the 13 element would tend to be larger than desired. We can say therefore that the symmetric case with vector fields generates the mass hierarchies in a more natural way.

Concerning the h_1,h_2 Higgs fields, there are two kinds originating from free fermionic string models: those coming from Neveu-Schwarz sector, which in general have integer (including zero) $U(1)_X$ charges, and those arising from twisted sectors, which usually carry fractional $U(1)_X$ charges. Which of these cases acquire VEV's is decided from the phenomenological analysis. For example, to obtain the structure of Eq. (34) we see that the charges of $h_{1,2}$ may not be zero, since in such a case the 12 element which is proportional to the Higgs charge would be unacceptably large. For the nonsymmetric case of course this feature does not necessarily hold. Finally, the $H,\overline{H}$ fields [the SU(4) Higgs fields] tend to be nonsinglets under extra $U(1)_X$ symmetries. We now proceed to discuss the *nonsymmetric case*, which in the framework of $U(1)_X$ symmetries has been extensively studied in [27]. Here, we will examine what constraints one may put on the various possibilities for nonsymmetric textures, in the model under study.

The charge assignment for this case appears in Table VI. Fields that belong to the same representation of $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ are taken to have the same charge. Again, it is clear that all fermion mass matrices will have the same structure. With this charge assignment we may proceed as in the symmetric case, and calculate the possible mass matrices that may arise. The charge matrix is now

$$\begin{pmatrix} -\alpha_2-2\alpha_3-\beta_2-2\beta_3 & \alpha_2-\alpha_3-\beta_2-2\beta_3 & -\beta_2-2\beta_3 \\ -\alpha_2-2\alpha_3+\beta_2-\beta_3 & \alpha_2-\alpha_3+\beta_2-\beta_3 & \beta_2-\beta_3 \\ -\alpha_2-2\alpha_3 & \alpha_2-\alpha_3 & 0 \end{pmatrix}. \tag{65}$$

[10] In this simplest (and more predictive) realization, $h_b\approx h_t$ and therefore we are in the large $\tan\beta$ regime of the parameter space of the MSSM.

We now want to find which charge assignments may generate a mass matrix as close as possible to the form in Eq. (53), keeping in mind that there is no reason to restrict ourselves to nonsymmetric textures with a zero in the 13 and 31 positions.

In what follows, we will check whether it is possible to generate the hierarchies in the effective low energy Yukawa couplings required by our *Ansätze* and the data. The required couplings are detailed in Table III. Initially, we determine if we can obtain the correct structure by chiral singlet fields. We assume for a starting point that for the 32 entry we have $\alpha_2-\alpha_3>0$ (without a loss of generality since we can always choose the sign of one entry in the charge matrix). The 23 entry has to be small [it is assumed to be zero in the *Ansätze* in Eq. (55)], indicating that (a) either $\beta_2-\beta_3<0$ or (b) $\beta_2-\beta_3$ is positive and large ($\geqslant 2$). Case (b) is excluded, since it would indicate that the 22 charge, which is always the sum of the 23 and 32 charges, would be unacceptably large as well (which implies that $H_22<H_32$, in contradiction to the fits in Table III). What about case (a)? A negative number must not dominate the 22 entry in the chiral case, and thus $|\beta_2-\beta_3|$ would have to be smaller than $|\alpha_2-\alpha_3|$. This clearly contradicts the required hierarchy between the 22 and 32 elements and so the required couplings can not be naturally described by a model with only a chiral $\mathrm{U}(1)_X$ Higgs θ.

For this reason we are going to look for solutions in the case of vector singlets, where it is the absolute value of the charges that matters. Here, the important difference from the previous case is that a solution with a small and positive $\alpha_2-\alpha_3$ and a large negative $\beta_2-\beta_3$ is allowed. The 23 and 32 elements have the correct hierarchy, while the 22 element can also be sufficiently small, as a result of a cancellation between terms of opposite sign, with the negative contribution being dominant. What can we say about the rest of the structure and how restrictive should we be when looking for solutions? We could allow for a small asymmetry between the 12 and 21 entries. Actually, λ_{12}^D can be slightly larger than λ_{21}^D. This, combined with the fact that there are unknown coefficients of order unity, indicates that we can have an asymmetry of order ϵ between the 12 and 21 entries. We will discuss solutions with such an asymmetry, even in the case that $\lambda_{12}^D<\lambda_{21}^D$, due to this coefficient ambiguity as well as the ambiguity in the experimental value of the up and down quarks. We also need not drop solutions with a large 13 or 31 entry, if they are compatible with the numerics.

On this basis, we have looked for solutions in the following way: For the charges of the elements 12-21-22-32 we made all possible charge assignments (such that lead to a maximum fourth power in terms of the expansion parameter for the resulting mass matrices, for the 12 and 21 entries). This fixes all charges $\alpha_2,\alpha_3,\beta_2,\beta_3$ each time. We then looked at what the charges of the other entries are and whether the generated hierarchies are consistent with the phenomenology.

The restrictions we require in order to identify a viable solution are (in addition to of course that the only renormalizable term is in the 33 position)

$$|\mathrm{charge}(11)|>|\mathrm{charge}(12)|,$$

$$|\mathrm{charge}(11)|>|\mathrm{charge}(21)|,$$

$$|\mathrm{charge}(21)|>|\mathrm{charge}(22)|,$$

$$|\mathrm{charge}(12)|>|\mathrm{charge}(22)|,$$

$$|\mathrm{charge}(13)|>|\mathrm{charge}(22)|,$$

$$|\mathrm{charge}(31)|>|\mathrm{charge}(22)|,$$

$$|\mathrm{charge}(32)|\leqslant|\mathrm{charge}(22)|O(\epsilon),$$

$$|\mathrm{charge}(12)|\approx|\mathrm{charge}(21)|O(\epsilon),$$

$$|\mathrm{charge}(23)|>|\mathrm{charge}(22)|. \tag{66}$$

Then, we end up with the following possibilities:

Case 1:

$$\alpha_2=-2/3,\quad \alpha_3=-5/3,\quad \beta_2=-2,\quad \beta_3=0,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^6 & \delta\epsilon^3 & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^4 & \delta\epsilon & 1 \end{pmatrix}. \tag{67}$$

Case 2:

$$\alpha_2=-1,\quad \alpha_3=-2,\quad \beta_2=-2,\quad \beta_3=0,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^7 & \delta\epsilon^3 & \delta\epsilon^2 \\ \delta\epsilon^3 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^5 & \delta\epsilon & 1 \end{pmatrix}. \tag{68}$$

Case 3:

$$\alpha_2=-4/3,\quad \alpha_3=-7/3,\quad \beta_2=-2,\quad \beta_3=0,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^8 & \delta\epsilon^3 & \delta\epsilon^2 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^6 & \delta\epsilon & 1 \end{pmatrix}. \tag{69}$$

Case 4:

$$\alpha_2=-4/3,\quad \alpha_3=-1/3,\quad \beta_2=0,\quad \beta_3=-2,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^6 & \delta\epsilon^3 & \delta\epsilon^4 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & 1 \end{pmatrix}. \tag{70}$$

Case 5:

$$\alpha_2=-4/3,\quad \alpha_3=-7/3,\quad \beta_2=-3,\quad \beta_3=0,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^9 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^3 & \delta\epsilon^2 & \delta\epsilon^3 \\ \delta\epsilon^6 & \delta\epsilon & 1 \end{pmatrix}. \tag{71}$$

Case 6:

$$\alpha_2=-1,\ \alpha_3=-2,\ \beta_2=-7/3,\ \beta_3=-1/3,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^8 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^3 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^5 & \delta\epsilon & 1 \end{pmatrix}. \tag{72}$$

Case 7:

$$\alpha_2=-5/3,\ \alpha_3=-8/3,\ \beta_2=-3,\ \beta_3=0,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^{10} & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^4 & \delta\epsilon^2 & \delta\epsilon^3 \\ \delta\epsilon^7 & \delta\epsilon & 1 \end{pmatrix}. \tag{73}$$

Case 8:

$$\alpha_2=-4/3,\ \alpha_3=-7/3,\ \beta_2=-7/3,\ \beta_3=-1/3,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^9 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^6 & \delta\epsilon & 1 \end{pmatrix}. \tag{74}$$

Case 9:

$$\alpha_2=-4/3,\ \alpha_3=-1/3,\ \beta_2=-1/3,\ \beta_3=-7/3,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^7 & \delta\epsilon^4 & \delta\epsilon^5 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & 1 \end{pmatrix}. \tag{75}$$

Let us also list for completeness a few cases with a larger splitting between the 21 and 12 entries [up to $O(\epsilon^2)$]:

Case 10:

$$\alpha_2=-4/3,\ \alpha_3=-1/3,\ \beta_2=1/3,\ \beta_3=-5/3,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^5 & \delta\epsilon^2 & \delta\epsilon^3 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & 1 \end{pmatrix}. \tag{76}$$

Case 11:

$$\alpha_2=-2/3,\ \alpha_3=-5/3,\ \beta_2=-7/3,\ \beta_3=-1/3,$$

$$Y_{u,d,\ell}=\begin{pmatrix} \delta\epsilon^7 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^2 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^4 & \delta\epsilon & 1 \end{pmatrix}. \tag{77}$$

Of course, here we also have the cases with the opposite charge assignment.[11] Among the various choices, we see that the charge of the Higgs fields $h_{1,2}$ is always different from zero and there are cases where the 13 and 31 elements are large. We may now examine the results of Table III in the context of the $\mathrm{U}(1)_X$ symmetry discussion above. We take all models that fit the data with $\chi^2/N_{\mathrm{DF}}<1$, i.e., models 1, 3, and 4. We define in each of these models H_{ij}^{emp} as being the dimensionless and dominant effective coupling constants in the $\mathrm{SU}(4)\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R$ unified Yukawa matrix for the best fit parameters.

Then, model 1 has

$$H_{ij}^{\mathrm{emp}}\sim\begin{bmatrix} 0 & 0.003 & 0 \\ 0.001 & 0.03 & 0 \\ 0 & 0.03 & 1 \end{bmatrix}. \tag{78}$$

We see that case 1 above does not fit this pattern very well if all dimensionless couplings are ~ 1 because, in case 1, H_{21} is suppressed in comparison to H_{12}. Cases 4 and 9 do not possess approximate texture zeros in the 31 position and this would affect $|V_{ub}|$ strongly. Similar objections can be raised about other cases, except for cases 2, 7, and 8. Case 2 with $\epsilon=0.21,\delta=0.14$ yields

$$\begin{bmatrix} 2\times10^{-6} & 0.001 & 6\times10^{-3} \\ 0.001 & 0.03 & 6\times10^{-3} \\ 6\times10^{-5} & 0.03 & 1 \end{bmatrix}, \tag{79}$$

which fits Eq. (78) well apart from a factor of ~ 3 in the 12 position. The next subdominant operator in the 22 position needs to be 2×10^{-3} according to Table III. The values of ϵ and δ used in Eq. (79) give the subdominant operator in the 22 position to be $\sim 6\times10^{-3}$. This is acceptable, but a closer match occurs for the next higher dimension operator, which has magnitude $\sim 10^{-3}$. An ambiguity occurs in that we have not set the normalization of the subdominant operator due to its numerous possibilities and so the original discrepancy factor of ~ 3 could easily be explained. Below, we do not consider the numerical size of the subdominant operator because it is clear that some operator can be chosen that will fit the required number well. If the charge assignments under the $\mathrm{U}(1)_X$ symmetry were the same as in this case, we would have succeeded in explaining why the assumption of texture zeros was valid. For example, the 13 element in Eq. (79) being 6×10^{-3} instead of zero only affects mixing angle and mass predictions by a small amount. We have also explained the hierarchies between the elements

[11]The presence of fractional charges implies the existence of residual discrete symmetries after the breaking of the Abelian symmetry.

in terms of the different mass scales involved in the nonrenormalizable operators by not having to choose dimensionless parameters of less than 1/3 (or greater than 3). Case 7 with $\epsilon=0.36,\delta=0.08$ gives

$$\begin{bmatrix} 2\times10^{-6} & 0.001 & 4\times10^{-3} \\ 0.001 & 0.01 & 4\times10^{-3} \\ 6\times10^{-5} & 0.03 & 1 \end{bmatrix}. \tag{80}$$

We should note that at this level we may naively expect 8% corrections to the constraint in Eq. (80) through the next order of δ operators in each element. We could have attempted to include these possible errors in the numerical fits but we did not due to the fact that they are very model dependent. Deeper model building in terms of constructing the nonrenormalizable operators out of extra fields or examining underlying string models would be required to explain why this should not be the case. It should also be borne in mind that explanations for exact texture zeros can be made in this context by setting fractional $U(1)_X$ charges on the heavy fields in the operators or by leaving certain heavy fields out of the FN model. Case 8 with $\epsilon=0.36,\delta=0.08$ gives the same results as in Eq. (80), except with the 22 element as 0.03.

From Table III we see that model 3 (the model that fits the data the best) has

$$H_{ij}^{\text{emp}}\sim\begin{bmatrix} 0 & 0.002 & 0 \\ 0.002 & 0.03 & 0 \\ 0 & 0.03 & 1 \end{bmatrix}. \tag{81}$$

Choosing $\epsilon=0.26,\delta=0.12$ in case 2 gives a good match to Eq. (81):

$$\begin{bmatrix} 9\times10^{-8} & 0.002 & 8\times10^{-3} \\ 0.002 & 0.03 & 8\times10^{-3} \\ 6\times10^{-5} & 0.03 & 1 \end{bmatrix}. \tag{82}$$

Case 7 with $\epsilon=0.40,\delta=0.07$ or case 8 with the same ϵ and δ both give a fairly good match as well.

Model 4 is different in the sense that it possesses a hierarchy between the 12 and 21 entries of the effective Yukawa couplings:

$$H_{ij}^{\text{emp}}\sim\begin{bmatrix} 0 & 0.0007 & 0 \\ 0.004 & 0.03 & 0 \\ 0 & 0.03 & 1 \end{bmatrix}. \tag{83}$$

Here, case 1 with $\delta=0.2,\epsilon=0.15$ predicts

$$\begin{bmatrix} 3\times10^{-7} & 0.0007 & 4\times10^{-3} \\ 0.004 & 0.03 & 4\times10^{-3} \\ 10^{-5} & 0.03 & 1 \end{bmatrix}, \tag{84}$$

an extremely good match to Eq. (83). Case 6 with $\epsilon=0.28,\delta=0.11$ provides a good match also.

Thus we see that we can explain the hierarchies and texture zero structures of the models that fit the data best. In general, it seems likely that we have enough freedom in setting charges to attain the required hierarchies for the Yukawa matrices.

VIII. STRING MODEL

In the following, we will present a semirealistic string model which provides an existence proof of how previously described nonrenormalizable operators may be generated from first principles using string theory. Before this, let us briefly comment on how the basic features of the $U(1)_X$ symmetries that we have discussed arise in string constructions.

In realistic free fermionic string models [19,17] there are some general features: At a scale $M_{\text{string}}\sim5g_{\text{string}}\times10^{17}$ GeV, one obtains an effective $N=1$ supergravity model with a gauge symmetry structure which is usually a product of non-Abelian groups times several U(1) factors. The non-Abelian symmetry contains an observable and a hidden sector. The massless superfields accommodating the Higgs and known chiral fields transform nontrivially under the observable part and usually carry nonzero charges under the surplus U(1) factors. The latter act as family symmetries in the way described above. Some of them are anomalous, but it turns out that one can usually define new linear U(1) combinations where all but one are anomaly free. The anomalous U(1) is broken by the Dine-Seiberg-Witten mechanism [28], in which a potentially large Fayet-Iliopoulos D term is generated by the VEV of the dilaton field. A D term, however, breaks supersymmetry and destabilizes the string vacuum, unless there is a direction in the scalar potential which is D flat and F flat with respect to the nonanomalous gauge symmetries. If such a direction exists, some of the singlet fields will acquire a VEV, canceling the anomalous D term, so that supersymmetry is restored. Since the fields corresponding to such a flat direction typically also carry charges for the nonanomalous D terms, they break all U(1) symmetries spontaneously. For the string model in Ref. [17], the expected order of magnitude for the VEV of the singlet fields is $\langle\Phi_i\rangle\sim(0.1\text{–}0.3)\times M_{\text{string}}$. Thus, their magnitude is of the right order to produce the required mass entries in the mass matrices via nonrenormalizable operators.

As an application of the above procedure, we will make a first attempt to derive the relevant operators for the mass matrices of the model based on the work in Ref. [17]. The string model is defined in terms of nine basis vectors $\{S,b_1,b_2,b_3,b_4,b_5,b_6,\alpha,\zeta\}$ and a suitable choice of the Gliozz-Scherk-Olive (GSO) projection coefficient matrix. The resulting gauge group has a Pati-Salam $[SU(4)\times SU(2)_L\times SU(2)_R]$ non-Abelian observable part, accompanied by four U(1) Abelian factors and a hidden $SU(8)\times U(1)$ symmetry.

In the following, for convenience, we denote a set of complex right fermions with the letters $\{\overline{\Psi}^{1\cdots5},\overline{\varphi}^{1\cdots6},\overline{\eta}^{123},\overline{z}^{12}\}$ and real right fermions with $\{\overline{y}^{1\cdots6},\overline{\omega}^{1\cdots6}\}$. Now, a specific model is defined in terms of a set of boundary conditions on the phases picked up by the fermions when parallel transported around noncontractible loops. The model is derived from the basis [17]

$$S=\{\psi^{\mu},\chi^{12\cdots 6};0\cdots 0\},$$

$$b_1=\{\psi^{\mu},\chi^{12},y^{3456}\overline{y}^{3456};\overline{\Psi}^{1\cdots 5}\,\overline{\eta}^{1}\},$$

$$b_2=\{\psi^{\mu},\chi^{34},y^{12}\overline{y}^{12}\omega^{56}\overline{\omega}^{56};\overline{\Psi}^{1\cdots 5}\,\overline{\eta}^{2}\},$$

$$b_3=\{\psi^{\mu},\chi^{56},\omega^{1234}\overline{\omega}^{1234};\overline{\Psi}^{1\cdots 5}\,\overline{\eta}^{3}\},$$

$$b_4=\{\psi^{\mu},\chi^{12},y^{36}\overline{y}^{36},\omega^{45}\overline{\omega}^{45};\overline{\Psi}^{1\cdots 5}\,\overline{\eta}^{1}\},$$

$$b_5=\{\psi^{\mu},\chi^{34},y^{26}\overline{y}^{26},\omega^{15}\overline{\omega}^{15};\overline{\Psi}^{1\cdots 5}\,\overline{\eta}^{2}\},$$

$$b_6=\{0,0,y^{6}\overline{y}^{6},\omega^{15}\overline{\omega}^{15};\overline{\Psi}^{1\cdots 5}\,\overline{\eta}^{123}\overline{\varphi}^{123}\overline{z}^{1}\},$$

$$\zeta=\{0,\cdots 0;\overline{z}^{12}\overline{\varphi}^{1\cdots 6}\},$$

$$\alpha=\{y^{46}\overline{y}^{46},\omega^{46}\overline{\omega}^{46};\overline{\Psi}^{123}\overline{\eta}^{12}\overline{z}^{12}\}.$$

All world-sheet fermions appearing in the basis are assumed to have periodic boundary conditions, while those not appearing are antiperiodic. An immediate consequence of using only *periodic* and *antiperiodic* boundary conditions is that the resulting gauge symmetry is in general a product of SO(n) groups. Thus, in the above basis, for example, the complex world sheet fermions $\overline{\Psi}^{1\cdots 5}$ define an SO(10) symmetry which is broken by the last vector α into SO(6)$\otimes$O(4). Now, bearing in mind that this part will be interpreted as the observable gauge symmetry, we observe the isomorphies SO(6)$\sim$SU(4), O(4)$\sim$SU(2)$\otimes$SU(2). The two SU(2)'s are going to accommodate the left and right components of the matter fields. Thus, the resulting gauge symmetry is isomorphic to the Pati-Salam (PS) gauge group. Thus the complete symmetry of the model under the above choice is

$$[\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)^3]_o\times[\mathrm{SU}(8)\times\mathrm{U}(1)]_h, \tag{85}$$

where the subscripts (o,h) denote the observable and hidden parts, respectively. With the specific choice of the projection coefficient matrix in [17], one obtains three chiral families in the $(4,2,1)+(\overline{4},1,2)$ representations of the PS symmetry and two Higgs pairs transforming as $(4,1,2)+(\overline{4},1,2)$, all arising from the sectors $b_{1,2,3}$ and b_4,b_5.

In particular, the massless spectrum contains three $(F_{1,3,4})_L=(4,2,1)$ representations obtained from the sectors $b_{1,3,4}$, which accommodate the left-handed fermion fields. There are five $(\overline{4},1,2)$ representations $(\overline{F}_{1,4,5},\overline{F}_2,\overline{F}_2')_R$ named after the corresponding sectors and two $H_{4,5}=(4,1,2)$ arising from the sectors $b_{4,5}$. Thus, two linear combinations of the $\overline{F}_i$ will play the role of the GUT Higgs $\overline{H}$, while the remaining three $\overline{F}$'s accommodate the right-handed fermions. The spectrum includes also bidoublets $h_i=(1,2,2)_i$, sextets $D_i=(6,1,1)_i$, and a sufficient number of singlet fields Φ_{ij},ξ_j,ζ_k. A certain number of singlets should develop VEV's in order to satisfy the flatness conditions and give masses to unwanted color triplets and exotic states.

In addition, one obtains fractionally charged states which arise in nonstandard representations of the Pati-Salam (PS) symmetry, namely, (1,1,2) and (1,2,1) and one pair $(4,1,1)+(\overline{4},1,1)$. Finally, under the hidden gauge group, one obtains ten irreducible representations $Z_i,\overline{Z_i}$ sitting in the **8** of SU(8) while carrying quantum numbers under all five U(1) symmetries of the model. All states are divided to those arising from the Neveu-Schwarz (NS) and Ramond (R) sectors. In particular the NS sector gives the graviton multiplet as well as the singlet fields Φ_i,Φ_{ij}, sextets and the bidoublets $h_3,\overline{h}_3$.

IX. CALCULATION OF TREE-LEVEL AND NONRENORMALIZABLE OPERATORS IN THE STRING MODEL

To calculate the superpotential of the model, one needs to obtain vertex operators for all physical states of the theory. To construct vertex operators for the states of a given model, every world-sheet fermion has to be represented by a conformal field. In the case that a representation of the model can be fully factorized in a left- and a right-moving piece, one can pair them up to bosonized fields. Now, according to the definition of the supersymmetry generator S in the above basis of our model, one can conclude that the left-moving fields χ^i can be bosonized $(\chi^1\pm\imath\chi^2)/\sqrt{2}=\exp\{\pm\imath S_{12}\}$ and similarly for the $\chi^{3,4}$ and $\chi^{5,6}$ pairs. $N=1$ supersymmetry implies the existence of an extra current, which is expressed in terms of S_{ij} as [37]

$$J(q)=\imath\partial_q(S_{12}+S_{34}+S_{56}) \tag{86}$$

and which is extended to three U(1)'s generated by S_{12},S_{34},S_{56}.

The Yukawa couplings in four-dimensional superstring models correspond to expectation values of the form

$$\left\langle\int d^2q_1\int d^2q_2\int d^2q_3 V_1^F(q_1)V_2^F(q_2)V_3^B(q_3)\right\rangle, \tag{87}$$

where the $V_i^{F,B}$ are the vertex operators for the fermionic (F) and the bosonic (B) states, while $q_{1,2,3}$ are the two-dimensional coordinates. Thus, a vertex operator for any physical state is a collection of conformal fields that represent the quantum numbers of the state under all symmetries of the model. The piece of the vertex operators involving the bosonized left-moving fields χ^i is given for the bosons by $V_{-1}^B\sim\exp\{\alpha S_{12}\}\exp\{\beta S_{34}\}\exp\{\gamma S_{56}\}$. Similarly, for the fermions, $V_{-1/2}^F\sim\exp\{(\alpha-1/2)S_{12}\}\exp\{(\beta-1/2)S_{34}\}\exp\{(\gamma-1/2)S_{56}\}$. The subscripts $-1,-1/2$ refer to the corresponding ghost numbers. The total ghost number should add up to -2, and thus in trilinear terms the nonvanishing couplings are proportional to the correlator $\langle V^FV^FV^B\rangle$. In nonrenormalizable contributions, the remaining vertex operators $V_4^B\cdots V_n^B$ have to be ''picture changed'' in the zero picture [37]. In general, a particular correlator is nonvanishing, only if it is invariant under the three U(1)'s. In addition it has to respect the usual (right-moving) gauge invariance and other global symmetries. For example, pure NS couplings are possible only at the tree level. The same is true for higher order couplings involving only Ramond fields, and so on. A complete list of rules is found in [37].

If we imply the well-defined set of rules to calculate the Yukawa interactions in the present string model, we obtain the following tree-level terms that are relevant to our discussion:

$$\mathcal{W}\to F_{4L}\overline{F}_{5R}h_{12}+\frac{1}{\sqrt{2}}F_{4R}\overline{F}_{5R}\overline{\zeta}_2+\overline{F}_{3R}F_{3L}h_3+\overline{\xi}_1h_3h_{12}$$
$$+\xi_4h_3\overline{h}_{12}+\overline{\Phi}_{12}h_{12}h_{12}+\Phi_3\overline{h}_{12}h_{12}+\xi_1\overline{h}_3\overline{h}_{12}$$
$$+\overline{\xi}_4\overline{h}_3h_{12}+\Phi_{12}\overline{h}_{12}\overline{h}_{12}+\overline{\Phi}_3h_{12}\overline{h}_{12}+\cdots, \tag{88}$$

where the ellipsis stands for terms involving exotic and hidden fields and other couplings irrelevant for our purpose. The F-flatness conditions are derived for the complete tree-level superpotential, which is given in [17] and involves in total 18 singlet fields. Five of these fields, namely, $\Phi_{1,\ldots,5}$, have zero quantum numbers under the U(1) groups, while the rest of the fields (denoted by $\xi_{1,2,3,4}$, $\overline{\xi}_{1,2,3,4}$, $\zeta_{1,2}$, $\overline{\zeta}_{1,2}$, Φ_{12}, $\overline{\Phi}_{12}$, $\Phi_{\overline{12}}$, $\overline{\Phi}_{\overline{12}}$) have nontrivial quantum numbers.

From the above, it is clear that only a few Yukawa couplings are available for fermion mass generation at the tree level. The missing terms are expected to be obtained from nonrenormalizable (NR) terms. In the case of the PS symmetry we expect NR terms of the form

$$\overline{F}Fh\frac{\overline{H}H\Phi_i\Phi_j}{M_{\text{string}}^4},\ \text{etc.,} \tag{89}$$

which act as effective mass operators once the fields H, $\overline{H}$, and $\Phi_{i,j}$ get VEV's. The scale where the Higgs fields $H,\overline{H}$ obtain their VEV's is determined from phenomenological requirements and renormalization group analysis [38] of the particular model. Moreover, the singlet VEV's are not completely arbitrary since they should satisfy the D- and F-flatness conditions. In general, the D-flatness conditions read

$$\sum_i Q_X^i|\langle\Phi_i\rangle|^2+\frac{g^2}{192\pi^2}\text{Tr}\,\{Q_{\text{U}(1)_X}\}M_{\text{Pl}}^2=0, \tag{90}$$

$$\sum_i Q_n^i|\langle\Phi_i\rangle|^2=0, \tag{91}$$

where $\langle\Phi_i\rangle$ are the singlet VEV's and g stands for the unified gauge coupling at M_{string}. $\text{U}(1)_X$ in Eq. (90) is the anomalous U(1) combination and Q_X^i the corresponding $\text{U}(1)_X$ charge of the singlet Φ_i. Equation (91) holds for all the nonanomalous U(1) symmetries of the particular model. From relations (90) and (91), it is clear that the order of magnitude of the VEV's of the singlet fields is determined by the Tr term. Thus, we expect that

$$\langle\Phi\rangle^2=O\left(\frac{g^2\,\text{Tr}(Q_X)}{192\pi^2}\right)M_{\text{Pl}}^2. \tag{92}$$

In particular, for the string model in Ref. [17], $\text{Tr}[Q_X]=72$, and therefore the order of magnitude for the singlet fields is $\langle\Phi_i\rangle\sim(0.1\text{–}0.3)\times M_{\text{string}}$. (See also Appendix C for the details.) This indicates that the singlet VEV's have the correct magnitude, in order to produce the required mass entries in the mass matrices via the nonrenormalizable operators of Eq. (89). We also note here that the spontaneous breaking of the anomalous U(1) symmetry introduces one more mass scale M_X in the theory, which is characterized by the magnitude of the related singlet VEV's. Thus, one naturally expects the hierarchy $M_{\text{string}}\geqslant M_X\geqslant M_{\text{GUT}}$.

One possible choice of nonzero VEV's is

$$\langle\overline{\Phi}_{\overline{12}}\rangle,\langle\Phi_{12}\rangle,\langle\xi_1\rangle,\langle\overline{\xi}_2\rangle, \tag{93}$$

and $\langle Z_5\rangle,\langle Z_8'\rangle\neq 0$ of the hidden fields. Solving the flatness conditions (Appendix C), one finds that the order of magnitude of the singlet VEV's is $\sqrt{a_u/\pi}$ in Planck units. It is easy to see that the choice (93) satisfies trivially the F-flatness conditions. We should point out, however, that this choice is not unique. There are other cases which also satisfy conditions (90) and (91), and hopefully a solution which meets the phenomenological requirements does exist. The nonzero VEV's in Eq. (93) provide all dangerous color triplets with masses from tree-level superpotential terms. Here we would like to investigate if they are also capable of producing the relevant operators for the fermion masses. This computation will prove to be a rather hard task mainly due to the rapidly increasing number of NR operators as the calculation proceeds to higher orders. We will see, however, that the pattern of the fermion mass matrices described in the previous sections is basically obtained.

We will first start the examination of the tree-level superpotential. Because of the string symmetries and the U(1) charges of the superfields, as can be seen from Eq. (88) only three terms relevant to the fermion masses exist at the three level:

$$\mathcal{W}\to F_{4L}\overline{F}_{5R}h_{12}+\frac{1}{\sqrt{2}}F_{4R}\overline{F}_{5R}\overline{\zeta}_2+\overline{F}_{3R}F_{3L}h_3. \tag{94}$$

Here, h_{12},h_3 are bidoublets and $\overline{\zeta}_2$ is a singlet, while the $F_{L,R}$ chiral fields have been presented previously. We may give a nonzero VEV to one of the two bidoublet Higgs fields (or to a linear combination $\cos\theta h_{12}+\sin\theta h_3$) and support one generation with masses at the tree level. Since there are more than one doublets in the spectrum, first, we should determine the massless state along the chosen flat direction. At the tree level, the bidoublet Higgs mass matrix obtained from the relevant terms is

$$(h_3,h_{12},\overline{h}_3,\overline{h}_{12})\begin{pmatrix}0&\overline{\xi}_1&0&\xi_4\\ \overline{\xi}_1&\overline{\Phi}_{12}&\overline{\xi}_4&\varphi_3\\ 0&\overline{\xi}_4&0&\xi_1\\ \xi_4&\varphi_3&\xi_1&\Phi_{12}\end{pmatrix}\begin{pmatrix}h_3\\ h_{12}\\ \overline{h}_3\\ \overline{h}_{12}\end{pmatrix}, \tag{95}$$

with $\varphi=\Phi_3/2$. In order to have at least one nonzero eigenvalue we impose $\det[m_h]\equiv(\xi_1\overline{\xi}_4-\overline{\xi}_1\xi_4)^2=0$, which is satisfied for any value of the $\Phi_3, \Phi_{12}, \overline{\Phi}_{12}$ VEV's, provided $\xi_1 \overline{\xi}_4 = \overline{\xi}_1 \xi_4$. The choice (93) is consistent with these requirements. Moreover, it leaves h_3, h_{12} massless at three level. We then let h_{12} develop a VEV and give masses to the top, bottom, and τ particles living in the $F_{4L}, \overline{F}_{5R}$ representations. The h_3 bidoublet is expected to receive a mass from a NR term. Thus, to proceed further, we need the contributions of the nonrenormalizable terms. As in the tree-level case, a nonvanishing NR term of the superpotential must obey all the string selection rules [37] and be invariant under all the gauge and global symmetries. Since here we discuss the fermion masses, we are primarily interested in those operators contributing to the corresponding matrices. At fourth order, we find no relevant terms. At fifth order, there are several operators which in principle could contribute to the fermion mass matrices. We list them here:

$$\overline{F}_{3R} F_{3L} h_3 \xi_1 \overline{\xi}_2, \quad \overline{F}_{3R} F_{3L} h_3 \Phi^2_{3,4,5}, \tag{96}$$

$$\overline{F}_{1R} F_{1L} \overline{h}_{12} \overline{\zeta}_2 \Phi_2, \quad \overline{F}_{5R} F_{4L} h_{12} \Phi^2_{1,2}, \tag{97}$$

$$F_{5R} F_{4L} \overline{F}_{2R} \overline{F}'_{2R} h_{12} \tag{98}$$

(scaled with the proper powers of M_{string}). Let us analyze the above contributions in terms of the particular flat direction chosen here. It is clear that, irrespective of the choice of the singlet VEV's, the terms (96) do not add a new contribution since they constitute small corrections to the already existing tree-level term $\overline{F}_3 F_3 h_3$. Moreover, within the given choice of our flat direction, $\langle \Phi_i \rangle = \langle \overline{\zeta}_2 \rangle = 0$, the terms (97) do not also generate any new fermion mass term. Thus, there is only one term which contributes to the fermion mass matrices, namely, the operator of Eq. (98). This is a $n=1$ operator according to our classification in the previous sections. We have already interpreted $\overline{F}_{5R}$ and F_{4L} as the left and right components of the third fermion generation. Up to now we have not determined which of $\overline{F}_{2R}$, $\overline{F}'_{2R}$ is going to play the role of the second family. The fifth order operator still leaves this undetermined since both fields enter in the operator in a symmetric way. Thus there are two options. Either we set $\langle \overline{F}_{2R} \rangle = 0$, or we have to rename the fields so that $F_{3L}, \overline{F}_{3R}$ are the third generation fermions and h_3 the massless Higgs boson. In the first case we retain the fifth order contribution to the mass matrix, while in the second we have a unique choice for the second family and the Higgs boson; i.e., $\overline{H} = \overline{F}'_{2R}$ and $\overline{F}_{2R}$ accommodates the right-handed fields of the second generation.

In order to calculate the contribution of the operator in Eq. (98) to the mass matrices, we should properly contract the various fields involved in the NR term. In principle, the numerical coefficient in front of the desired operator is a linear combination of the Clebsch-Gordan coefficients presented in Table I, each of them multiplied by a different phase factor. Our ignorance about the numerical coefficients of the mass matrix entries has been minimized in the unknown phase factors. However, we can make a natural assumption that the largest contributions come from contractions occurring first for the fields belonging to the same sector. Recalling now that $\overline{F}_{2R}$ and $\overline{H} \equiv \overline{F}'_{2R}$ originate from the same sector b_2, while F_{4L}, $H \equiv F_{5R}$ and h_{12} are obtained from $b_{4,5}$. We find that

$$(F_{4L} H)(\overline{F}_{2R} \overline{H}) h_{12} \rightarrow O^G \rightarrow \left\{ \frac{2}{\sqrt{5}} Q d^c h_d, \frac{4}{\sqrt{5}} \ell e^c h_d \right\}, \tag{99}$$

i.e., this operator contributes to down quark and charged lepton mass matrices. Since this contribution is the second largest after the tree-level term $F_{4L} \overline{F}_{5R} h_{12}$, we identify Eq. (99) with the 23 entry of the corresponding mass matrices. It is clear, therefore, that in this picture $F_{5R}, \overline{F}_{2R}$ accommodate the right components of the third and second generations, respectively, whereas F_{4L} contains the left fermions of the heavy generation. In order to obtain nonzero s-quark and μ masses, we need to fill in the 32 entry of the down and charged lepton mass matrices with a higher order operator, so that the 2×2 lower block of the corresponding mass matrices exhibits a structure of the asymmetric type considered in the previous section,

$$\mathcal{F}_{\mathcal{R}} \mathcal{M}_{d,e} \mathcal{F}_{\mathcal{L}} = (\overline{F}_{2R}, \overline{F}_{5R}) \begin{pmatrix} \eta_{22} & \langle H \overline{H} \rangle \\ \eta_{32} & 1 \end{pmatrix} h_{12} \begin{pmatrix} F_{iL} \\ F_{4L} \end{pmatrix}, \tag{100}$$

where η_{ij} stand for higher order NR contributions, while F_{iL} represents in general one of the two remaining left-handed fourplets $F_{(1,3)L}$. To determine which of the latter will accommodate the second generation and calculate η's, one has to proceed above the fifth order and find the relevant nonvanishing correlators. For example, choosing a new flat direction in which $\overline{\zeta}_2, \Phi_2$ singlets develop nonzero VEV's while interpreting $F_{3L}, \overline{F}_{3R}$ as the third generation, η_{22} may arise from the fifth order NR operator $F_{1L} \overline{F}_{1R} \overline{h}_{12} \overline{\zeta}_2 \Phi_2$ in Eq. (96). Interestingly, this is an $n=0$ operator according to our classification; however, it is suppressed compared to a tree-level term due to the presence of the ''effective'' flavor factor $\delta^2 = \langle \overline{\zeta}_2 \Phi_2 \rangle / M^2_{\text{string}}$. Furthermore, higher NR terms will certainly involve $n=2$, 3, etc., operators. Thus, it is clear that the above procedure will require $n \geqslant 2$ operators and the analysis will be more involved than the field theory model described in the our earlier sections.

As a matter of fact, a detailed analysis requires also the examination of all possible flat directions as well as calculation of the nonrenormalizable contributions to even higher orders, since one has to ensure that the necessary Weinberg-Salam doublets living in some combination of our h_3, h_{12} bidoublets remain massless up to this order. For the moment, in our first approach to this model, we have been able to show that the rather complicated string construction stays in close analogy with the field theory approach presented in the early sections.

X. CONCLUSIONS

We have examined Yukawa textures within a string-inspired $SU(4) \times O(4)$ model extended by a gauged $U(1)_X$ family symmetry and nonrenormalizable operators above the unification scale of the form in Eq. (27). These operators factorize into a factor $(H\overline{H})$ and a factor involving the singlet fields $\theta, \overline{\theta}$. The singlet fields $\theta, \overline{\theta}$ break the $U(1)_X$ symmetry and provide the horizontal family hierarchies while the $H, \overline{H}$ fields break the $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ symmetry and give the vertical splittings arising from group-theoretic Clebsch relations between different charge sectors. The factor $(H\overline{H})$ also provides an additional flavor-independent suppression factor δ which helps the fit. The quark and lepton masses and quark mixing angles are thus described at high energies by a single unified Yukawa matrix whose flavor structure is controlled by a broken $U(1)_X$ family symmetry, and all vertical splittings controlled Clebsch factors. An important feature of the scheme is the existence of Clebsch zeros which allow an entirely new class of textures to be obtained. For example, the RRR solutions 3 and 5 may be reproduced by this scheme which are complementary to the RRR solution 2 favored by the IR approach.

In addition to the symmetric textures we have also performed a completely new analysis of the nonsymmetric textures which are motivated by the string construction. A global fit to the fermion mass spectrum with three DF is described, in which three models in Table III are singled out with $\chi^2/N_{\rm DF}<1$.[12] At this level of difference of χ^2 between models, the χ^2 test is subject to large statistical fluctuations. Therefore, we do not statistically distinguish between the fits in Tables III and IV since both contain good fits to the data with $\chi^2/N_{\rm DF}<1$. However, we have a theoretical preference for the models in Table III since these models result from the operators in Eq. (27) where the family hierarchies are accounted for by the $U(1)_X$ symmetry, as explained in Sec. VII. By contrast, the models in Table IV result from the operators in Eq. (26) and are essentially an updated version of those previously considered in Ref. [18].

The string analysis performed in the later sections of the paper lends some support to the approach followed in this model. In the string model, the U(1) family symmetries are a consequence of the string construction, but there are four of them, with one being anomalous. There are several singlets (charged under the family group) to take the role of the θ fields and the $n=1$ operators involving a factor of $H\overline{H}$ are clearly expected in the effective theory below the string scale. We have shown that operators such as O^G which were simply pulled out of thin air in the earlier parts of the paper may in fact originate from string theory. As an example we constructed explicitly the lower 2×2 block in Eq. (100) which has the characteristic asymmetric structure of the Yukawa textures considered earlier. It will be noted, however, that the lower 2×2 block in Eq. (100) does not correspond precisely to the *Ansatz* in Eq. (35). Within the given string construction, such an *Ansatz* does not appear to be possible. The reason is the extra U(1) symmetries and the other discretelike symmetries (selection rules) left over in the low energy model. A new string construction with a new boundary condition on the string basis is required in order to make contact with the phenomenologically preferred *Ansätze*. This will be the subject of future work.

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APPENDIX A: $n=1$ OPERATORS

The $n=1$ operators are by definition all of those operators which can be constructed from the five fields $F\overline{F}hH\overline{H}$ by contracting the group indices in all possible ways, as discussed in this appendix. After the Higgs fields H and $\overline{H}$ develop VEV's at $M_{\rm GUT}$ of the form $\langle H^{\alpha b}\rangle=\langle H^{41}\rangle=v_H$, $\langle\overline{H}_{\alpha x}\rangle=\langle\overline{H}_{41}\rangle=\overline{v}_H$, the operators listed in this appendix yield effective low energy Yukawa couplings with small coefficients of order $M^2_{\rm GUT}/M^2$. However, as in the simple example discussed previously, there will be precise Clebsch relations between the coefficients of the various quark and lepton component fields. These Clebsch relations are summarized in Table VII, where relative normalization factor has been applied to each. The table identifies which SU(4) and SU(2) structures have been used to construct each individual operator by reference to Eqs. (A3) and (A4).

The $n=1$ operators are formed from different group-theoretical contractions of the indices in

$$O^{\alpha\rho y w}_{\beta\gamma x z}\equiv F^{\alpha a}\overline{F}_{\beta x}h^y_a\overline{H}_{\gamma z}H^{\rho w}. \tag{A1}$$

It is useful to define some SU(4)-invariant tensors C and $SU(2)_R$-invariant tensors R as

$$(C_1)^\alpha_\beta=\delta^\alpha_\beta,$$

$$(C_{15})^{\alpha\rho}_{\beta\gamma}=\delta^\gamma_\beta\delta^\rho_\alpha-\tfrac{1}{4}\delta^\alpha_\beta\delta^\rho_\gamma,$$

$$(C_6)^{\rho\gamma}_{\alpha\beta}=\epsilon_{\alpha\beta\omega\chi}\epsilon^{\rho\gamma\omega\chi},$$

$$(C_{10})^{\alpha\beta}_{\rho\gamma}=\delta^\alpha_\rho\delta^\beta_\gamma+\delta^\alpha_\gamma\delta^\beta_\rho,$$

$$(R_1)^x_y=\delta^x_y,$$

$$(R_3)^{wx}_{yz}=\delta^x_y\delta^w_z-\tfrac{1}{2}\delta^x_z\delta^w_y, \tag{A2}$$

where δ^α_β, $\epsilon_{\alpha\beta\omega\chi}$, δ^x_y, ϵ_{wz} are the usual invariant tensors of SU(4), $SU(2)_R$. The SU(4) indices on $C_{1,6,10,15}$ are contracted with the SU(4) indices on two fields to combine them

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[12] By comparison a recent paper [29] performed a global χ^2 analysis for some SO(10) models, including the mass and mixing data. With three DF, they obtain a $\chi^2/N_{\rm DF}\sim 1/3$ for the best model. While our fit to model 3 in Table III, for example, has a smaller $\chi^2/N_{\rm DF}$ than this, it is difficult to make a comparison as in Ref. [29] quark mass correlations from the data, as well as the effect of large $\tan\beta$ on m_b, have been included. Also note that these involve the soft terms, and thus a larger number of parameters are involved in the fit.

TABLE VII. When the Higgs fields develop their VEV's, the $n=1$ operators lead to the effective Yukawa couplings with Clebsch coefficients as shown.

	SU(2)	SU(4)	$Q\overline{U}h_2$	$Q\overline{D}h_1$	$L\overline{E}h_1$	$L\overline{N}h_2$
O^A	I	I	1	1	1	1
O^B	II	I	1	-1	-1	1
O^C	I	II	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^D	II	II	$\frac{1}{\sqrt{5}}$	$\frac{-1}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^E	III	III	0	2	0	0
O^F	II	III	$\sqrt{2}$	$-\sqrt{2}$	0	0
O^G	III	IV	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0
O^H	IV	IV	4/5	2/5	4/5	8/5
O^I	V	V	0	0	0	2
O^J	VI	V	0	0	$\frac{4}{\sqrt{5}}$	$\frac{2}{\sqrt{5}}$
O^K	V	VI	8/5	0	0	6/5
O^L	IV	VI	$\frac{16}{5\sqrt{5}}$	$\frac{8}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$
O^M	III	I	0	$\sqrt{2}$	$\sqrt{2}$	0
O^N	V	III	2	0	0	0
O^O	V	IV	$\frac{2}{\sqrt{5}}$	0	0	$\frac{4}{\sqrt{5}}$
O^P	I	VI	$\frac{4\sqrt{2}}{5}$	$\frac{4\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$
O^Q	II	VI	$\frac{4\sqrt{2}}{5}$	$-\frac{4\sqrt{2}}{5}$	$-\frac{3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$
O^R	III	VI	0	$\frac{8}{5}$	$\frac{6}{5}$	0

	SU(2)	SU(4)	$Q\overline{U}h_2$	$Q\overline{D}h_1$	$L\overline{E}h_1$	$L\overline{N}h_2$
O^S	VI	VI	$\frac{8}{5\sqrt{5}}$	$\frac{16}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$
O^T	IV	I	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$
O^U	VI	I	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$
O^V	V	I	$\sqrt{2}$	0	0	$\sqrt{2}$
O^W	III	II	0	$\sqrt{\frac{2}{5}}$	$\frac{-3}{\sqrt{\frac{2}{5}}}$	0
O^X	IV	II	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$	$\frac{-6\sqrt{2}}{5}$
O^Y	VI	II	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{-6\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$
O^Z	V	II	$\sqrt{\frac{2}{5}}$	0	0	$-3\sqrt{\frac{2}{5}}$
O^a	I	III	$\sqrt{2}$	$\sqrt{2}$	0	0
O^b	IV	III	$\frac{4}{\sqrt{5}}$	$\frac{2}{\sqrt{5}}$	0	0
O^c	VI	III	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0	0
O^d	I	IV	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$
O^e	II	IV	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{5}}$	$-2\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$
O^f	VI	IV	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{8}{5}$	$\frac{4}{5}$
O^g	I	V	0	0	$\sqrt{2}$	$\sqrt{2}$
O^h	II	V	0	0	$-\sqrt{2}$	$\sqrt{2}$
O^i	III	V	0	0	2	0
O^j	IV	V	0	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$

into **1**, **6**, **10**, **15** representations of SU(4), respectively. Similarly, the $SU(2)_R$ indices on $R_{1,3}$ are contracted with $SU(2)_R$ indices on two of the fields to combine them into **1**, **3** representations of $SU(2)_R$.

The SU(4) structures in Table VII are

$$\begin{aligned}
&(\mathrm{I}) \quad (C_1)^{\beta}_{\alpha}(C_1)^{\gamma}_{\rho},\\
&(\mathrm{II}) \quad (C_{15})^{\beta\chi}_{\alpha\sigma}(C_{15})^{\gamma\sigma}_{\rho\chi},\\
&(\mathrm{III}) \quad (C_6)^{\omega\chi}_{\alpha\rho}(C_6)^{\beta\gamma}_{\omega\chi},\\
&(\mathrm{IV}) \quad (C_{10})^{\omega\chi}_{\alpha\rho}(C_{10})^{\beta\gamma}_{\omega\chi},\\
&(\mathrm{V}) \quad (C_1)^{\beta}_{\rho}(C_1)^{\gamma}_{\alpha},\\
&(\mathrm{VI}) \quad (C_{15})^{\gamma\chi}_{\alpha\sigma}(C_{15})^{\beta\sigma}_{\rho\chi},
\end{aligned} \tag{A3}$$

and the SU(2) structures are

$$\begin{aligned}
&(\mathrm{I}) \quad (R_1)^{z}_{w}(R_1)^{x}_{y},\\
&(\mathrm{II}) \quad (R_3)^{zq}_{wr}(R_3)^{xr}_{yq},\\
&(\mathrm{III}) \quad \epsilon^{xz}\epsilon_{yw},\\
&(\mathrm{IV}) \quad \epsilon_{ws}\epsilon^{xt}(R_3)^{sq}_{yr}(R_3)^{zr}_{tq},\\
&(\mathrm{V}) \quad (R_1)^{z}_{y}(R_1)^{x}_{w},\\
&(\mathrm{VI}) \quad (R_3)^{zq}_{yr}(R_3)^{xr}_{wq}.
\end{aligned} \tag{A4}$$

The operators are then given explicitly by contracting Eq. (A1) with the invariant tensors of Eq. (A2) given by Table VII and Eqs. (A3) and (A4).

APPENDIX B: REVIEW OF ANALYSIS OF REF. [18]

In Ref. [18] we assumed that the Yukawa matrices at M_X are all of the form

$$\lambda^{U,D,E,N}=\begin{pmatrix} O(\epsilon^2) & O(\epsilon^2) & 0 \\ O(\epsilon^2) & O(\epsilon) & O(\epsilon) \\ 0 & O(\epsilon) & O(1) \end{pmatrix}, \tag{B1}$$

where $\epsilon \ll 1$ and some of the elements may have approximate or exact texture zeros in them. First, we examine closer the assumption that the operator in the 33 position of the Yukawa matrices is the renormalizable one. It has been suggested in the past that the large value of $\tan\beta$ required by the constraint

$$\lambda_t(M_{\rm GUT})=\lambda_b(M_{\rm GUT})=\lambda_\tau(M_{\rm GUT}), \tag{B2}$$

such as is predicted by the renormalizable operator, leads to some phenomenological problems. One such problem is that a moderate fine-tuning mechanism is required to radiatively break the electroweak symmetry in order to produce the necessary hierarchy of Higgs VEV's $v_1/v_2 \approx m_t/m_b$ [30,31]. One could set about trying to extend the present model in a manner that would lead to an arbitrary choice of $\tan\beta$, for example, by introducing extra Higgs bidoublets. This route has its disadvantages in that a low value of $\tan\beta$ has been shown [32] in most schemes to be inconsistent with $\lambda_b(M_{\rm GUT})=\lambda_\tau(M_{\rm GUT})$ unification if the τ neutrino mass constitutes the hot dark matter requiring the Majorana mass of the right-handed τ neutrino to be $M_R^{\nu_\tau}\sim 10^{12}$ GeV. To a very good approximation, the largest diagonalized Yukawa coupling in λ^I is equal to its 33 entry λ^I_{33}. (One may obtain small $\tan\beta$ solutions consistent with m_b-m_τ unification and an intermediate neutrino scale, in specific models: Either large mixing in the μ-τ charged leptonic sector has to occur [33] or the Dirac-type Yukawa coupling of the neutrino has to be very suppressed [34].)

To force things to work in a generic scheme, one solution could be to use a nonrenormalizable operator in the 33 position which has some Clebsch factor $x>1$ such that

$$\lambda_t(M_{\rm GUT})=x\lambda_b(M_{\rm GUT})=x\lambda_\tau(M_{\rm GUT}). \tag{B3}$$

Equation (B3) would preserve the bottom-τ Yukawa unification, but lower the prediction of $\tan\beta$ due to the larger contribution to the top Yukawa coupling. It may only be reasonable to examine $n=1$ operators in this context since we know that the third family [18] Yukawa coupling is ~ 1 and higher dimension operators could be expected to provide a large suppression factor. Systematically examining the $n=1$ operators we find that only the operator O^U_{33}, which leads to the prediction

$$\lambda_t(M_{\rm GUT})=2\lambda_b(M_{\rm GUT})=2\lambda_\tau(M_{\rm GUT}), \tag{B4}$$

can decrease $\tan\beta$. The change is minimal, from 56.35 to 55.19 for $\alpha_S(M_Z)=0.117$ and $M_R^{\nu_\tau}\sim 10^{12}$ GeV. The reason that the change is minimal is due to the fact that the Yukawa couplings are approximately at their quasifixed points [35] and so even a large change to $\lambda_{t,b,\tau}(M_X)$ produces only a small change in $\lambda_{t,b,\tau}(m_t)$, which are the quantities that require a high $\tan\beta$ through the relations in Eq. (61). Another possibility would be to include O^M_{33}, O^V_{33} which would allow arbitrary $\tan\beta$ (in particular intermediate $\tan\beta \sim 10$–20). However, this would reduce the predictivity of the scheme as $\tan\beta$ would become an input. One might also be skeptical about whether a parameter ~ 1 could be generated by a nonrenormalizable operator in a perturbative scheme. It would certainly require the heavy mass scales M to be very close to the VEV's $H, \bar{H}, \theta, \bar{\theta}$ and we might therefore naively expect large corrections to any calculation based on this model. We thus abandon these ideas and continue with the usual renormalizable operator in the 33 position of the Yukawa matrices that leads to Eq. (B2). We note in any case that a recent analysis [36] explains that in gauge-mediated supersymmetry-breaking models, the radiative mechanism of electroweak symmetry breaking can be such that no fine-tuning occurs for large $\tan\beta$. In these models high $\tan\beta$ admits solutions of the hot dark matter problem in which the Yukawa couplings unify [32].

The hierarchy assumed in Eq. (B1) allows us to consider the lower 2×2 block of the Yukawa matrices first. In diagonalizing the lower 2×2 block separately, we introduce corrections of order ϵ^2 and so the procedure is consistent to first order in ϵ. We found several maximally predictive *Ansätze* that were constructed out of the operators whose Clebsch coefficients are listed in Table IV for the $n=1$ operators. The explicit $n=1$ operators in component form are listed in Appendix A. We label the successful lower 2×2 *Ansätze* A_i:

$$A_1=\begin{bmatrix} O^D_{22}-O^C_{22} & 0 \\ O^C_{32} & O_{33} \end{bmatrix}, \tag{B5}$$

$$A_2=\begin{bmatrix} 0 & O^A_{23}-O^B_{23} \\ O^D_{32} & O_{33} \end{bmatrix}, \tag{B6}$$

$$A_3=\begin{bmatrix} 0 & O^C_{23}-O^D_{23} \\ O^B_{32} & O_{33} \end{bmatrix}, \tag{B7}$$

$$A_4=\begin{bmatrix} 0 & O^C_{23} \\ O^A_{32}-O^B_{32} & O_{33} \end{bmatrix}, \tag{B8}$$

$$A_5=\begin{bmatrix} 0 & O^A_{23} \\ O^C_{32}-O^D_{32} & O_{33} \end{bmatrix}, \tag{B9}$$

$$A_6=\begin{bmatrix} O^K_{22} & O^C_{23} \\ O^M_{32} & O_{33} \end{bmatrix}, \tag{B10}$$

$$A_7=\begin{bmatrix} O^K_{22} & O^G_{23} \\ O^G_{32} & O_{33} \end{bmatrix}, \tag{B11}$$

$$A_8=\begin{bmatrix} 0 & O^H_{23} \\ O^G_{32}-O^K_{32} & O_{33} \end{bmatrix}. \tag{B12}$$

We now note that solutions A_{2-8} require a parameter $H_{23}\sim O(1)$ to attain the correct λ_μ and V_{cb}. Any calculation based on the hierarchy assumed in Eq. (B1) is therefore inconsistent and so we discard these solutions. We also note that O_{32} only has the effect of fixing V_{cb} to a good approximation and so can consist of any operator in Table VII that has a different Clebsch coefficient for up quark and down quark Yukawa couplings. The precise operator responsible for V_{cb} has no bearing on the rest of the calculation and we therefore just make an arbitrary choice of O_{32}^C for the rest of this paper. We also note that for the phenomenologically desirable and predictive relation

$$\frac{\lambda_{22}^D(M_{\mathrm{GUT}})}{\lambda_{22}^E(M_{\mathrm{GUT}})}=3 \tag{B13}$$

to hold, we may replace $O_{22}^D - O_{22}^C$ in A_1 with $O_{22}^W + O_{22}^C$, $O_{22}^X + O_{22}^D$ or any other combination of two operators which preserves Eq. (B13) and allows λ_{22}^U to be smaller and independent of $\lambda_{22}^{D,E}$. In fact, the preferred solution is that the dominant operator in that position be O_{22}^W which does not give a contribution to the up quark mass. Then, a subdominant operator would be responsible for the entry λ_{22}^U and would therefore be suppressed naturally by one or more powers of ϵ.

APPENDIX C: FLATNESS CONDITIONS IN THE STRING MODEL

We give here the constraints on the various singlet VEV's obtained from the F- and D-flatness conditions in the string spectrum of the model in Sec. VIII. From the F flatness of the superpotential one derives 18 conditions, which are

$$\begin{gathered}
\overline{\xi_1}\,\overline{\xi_4}=0,\\
\xi_1\xi_4=0,\\
\xi_2\overline{\xi_3}+\zeta_1^2+\zeta_2^2=0,\\
\overline{\xi_2}\xi_3+\overline{\zeta_1^2}+\overline{\zeta_2^2}=0,\\
\xi_i\overline{\xi_i}+\zeta_1\overline{\zeta_1}+\zeta_2\overline{\zeta_2}=0,\\
\zeta_1\overline{\zeta_2}+\overline{\zeta_1}\zeta_2=0,\\
2\overline{\Phi}_{\overline{12}}\zeta_1+\tfrac{1}{2}\Phi_3\overline{\zeta_1}+\Phi_4\overline{\zeta_2}=0,\\
2\Phi_{\overline{12}}\overline{\zeta_1}+\tfrac{1}{2}\Phi_3\zeta_1+\Phi_4\zeta_2=0,\\
2\overline{\Phi}_{\overline{12}}\zeta_2+\tfrac{1}{2}\Phi_3\overline{\zeta_2}+\Phi_4\overline{\zeta_1}=0,\\
2\Phi_{\overline{12}}\overline{\zeta_2}+\tfrac{1}{2}\Phi_3\zeta_2+\Phi_4\zeta_1=0,\\
\overline{\Phi}_{12}\xi_4+\tfrac{1}{2}\Phi_3\overline{\xi_1}=0,\\
\Phi_{12}\overline{\xi_4}+\tfrac{1}{2}\Phi_3\xi_1=0,\\
\overline{\Phi}_{\overline{12}}\overline{\xi_3}+\tfrac{1}{2}\Phi_3\overline{\xi_2}=0,\\
\Phi_{\overline{12}}\xi_3+\tfrac{1}{2}\Phi_3\xi_2=0,\\
\Phi_{\overline{12}}\overline{\xi_2}+\tfrac{1}{2}\Phi_3\overline{\xi_3}=0,\\
\overline{\Phi}_{\overline{12}}\xi_2+\tfrac{1}{2}\Phi_3\xi_3=0,\\
\overline{\Phi}_{12}\xi_1+\tfrac{1}{2}\Phi_3\overline{\xi_4}=0,\\
\Phi_{12}\overline{\xi_1}+\tfrac{1}{2}\Phi_3\xi_4=0.
\end{gathered} \tag{C1, C2}$$

Now, a possible choice of nonzero singlet VEV's which satisfy the system (C2) is

$$\langle\Phi_{12}\rangle,\langle\overline{\Phi}_{\overline{12}}\rangle,\langle\xi_1\rangle,\langle\overline{\xi_2}\rangle\neq 0, \tag{C3}$$

accompanied by nonzero VEV's of the two hidden [octets under $SU(8)_h$] fields

$$\langle Z_5\rangle,\langle\overline{Z_3'}\rangle\neq 0. \tag{C4}$$

Taking all other singlet and hidden field VEV's equal to zero, the D-flatness conditions read [39]

$$|Z_5|^2-2|\overline{\Phi}_{\overline{12}}|^2-|\xi_1|^2-|\overline{\xi_2}|^2+\frac{3\alpha_u}{2\pi}=0, \tag{C5}$$

$$\tfrac{1}{2}|Z_5|^2-|\overline{\xi_2}|^2=0, \tag{C6}$$

$$2|\xi_1|^2-|\overline{\xi_2}|^2-2|\overline{\Phi}_{\overline{12}}|^2+\tfrac{3}{2}|\overline{Z_3'}|^2+|Z_5|^2=0, \tag{C7}$$

$$2|\Phi_{12}|^2+|\xi_1|^2-\tfrac{1}{2}|\overline{Z_3'}|^2=0. \tag{C8}$$

The scale of the nonzero singlet VEV's is determined by the above conditions. There are five equations to determine four parameters, and thus one has the freedom to fix one of the nonzero VEV's in Eqs. (C3) and (C4) from phenomenological requirements. In any case, from the above equations it turns out that the natural scale of the nonzero VEV's are of the order $(\alpha_u/\pi)M_{\mathrm{Pl}}$. For $\alpha_u\sim 10^{-1}$ one can see that their magnitude is of the required order to contribute in the mass operators.

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