

High energy hadronic total cross sections

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High energy hadronic total cross-section data are found to agree with the predictions of a QCD-string picture. [S0556-2821(97)00617-6]

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I. INTRODUCTION

High energy hadron-hadron scattering amplitudes at fixed momentum transfer have two components: a diffractive component (Pomeron exchange) and a mesonic Regge exchange component. Since the latter decreases with energy, Froisart's unitarity bound becomes a constraint on the diffractive component: at high energies hadronic total cross sections cannot exceed $(\pi/m_\pi^2)\ln^2 s$. Although fits to total cross-section data incorporating $\ln s$ and $\ln^2 s$ terms have been made, it has been noted that the unitarity bound does not rule out a term of the form $X(s/m^2)^\epsilon$ with small but positive ϵ . Indeed, for say $\epsilon=0.08$, $m\approx 1$ GeV, and $X\leq 25$ mb, the unitarity bound violation would not set in [1] until energies of 10^{24} GeV are reached. At such super-Planckian energies the theoretical underpinnings of QCD become meaningless and experiments become impossible. With this in mind, Donachie and Landshoff [2] have successfully fit high energy hadronic total cross sections to expressions of the type

$$\sigma_{AB} = X_{AB} s^\epsilon + Y_{AB} s^{-\eta}, \quad \epsilon = 0.08, \eta = 0.45. \quad (1)$$

For the 10 measured total cross sections $(pp, \bar{p}p, pn, \bar{p}n, \pi^\pm p, K^\pm p, K^\pm n)$ 15 (or 17) parameters (depending on whether or not one requires $X_{np} = X_{pp}$ and $X_{K^+p} = X_{K^+n}$) are needed: five (or seven) X 's and ten Y 's. Taken at face value, these fits and subsequent refinements thereof [3] are at odds with a number of ideas grounded in the quark model and a QCD-string approach to hadron scattering; we have in mind ideas like two-component duality, string tree level exchange degeneracy or Chan-Paton rules, flavor U(3) symmetry, universality of vector-meson couplings. We wish to show that one can successfully implement these ideas as constraints from the beginning and obtain fits different from, but of comparable quality to, those of Refs. [2,3].

II. SOME THEORETICAL IDEAS ON HIGH ENERGY HADRON SCATTERING

We start by explaining how each of the just-mentioned ideas constrain high energy hadronic total cross sections.

A. Universal and flavor U(3) symmetric vector meson coupling pattern

Although in QCD the nine light vector mesons appear as $q\bar{q}$ bound states whose coupling pattern is to be dynamically calculated, it has been known for a long time that the observed pattern closely follows the pattern which would be expected if these mesons were flavor gauge bosons. Specifically this means that (suppressing Lorentz and Dirac indices) in the familiar 3×3 matrix representation, the coupling of these vector mesons V to baryons B is of the form $\text{Tr}(\bar{B}[V, B]) + \text{Tr}(\bar{B}B)\text{Tr}V$ (here the ratio of the two terms' coefficients is determined by requiring the decoupling of the ϕ from the proton). Moreover $g_{\rho^0 \bar{p}p} = \frac{1}{2}g_{\rho^0 \pi^- \pi^+}$, since the third component of the proton's isospin is half that of the positive pion's. Strictly speaking, only at $t = m_\rho^2 \approx m_\omega^2$ does this coupling pattern determine the residue pattern of the odd-signature Regge poles on whose trajectory the vector mesons lie. We will assume that the same pattern is valid also at $t=0$. All this then yields four linear relations between the five odd-charge-conjugation total cross-section combinations. With the notation $\Delta_{AB} = \sigma_{AB} - \sigma_{\bar{A}\bar{B}}$, where σ_{AB} denotes the AB total cross section, these four relations take the familiar form [4,5]

$$\begin{aligned} \Delta_{\bar{p}p} &= 5\Delta_{\pi^- p}, & \Delta_{\bar{p}n} &= 4\Delta_{\pi^- p}, & \Delta_{K^- p} &= 2\Delta_{\pi^- p}, \\ \Delta_{K^- n} &= \Delta_{\pi^- p}. \end{aligned} \quad (2)$$

Of these five differences, $\Delta_{\pi^- p}$ involves only ρ exchange, whereas the remaining four involve both ρ and ω exchange, predominantly the latter. The derivation of the relations (2) assumes the ρ and ω trajectories' intercepts to be equal: $\alpha_\rho(0) = \alpha_\omega(0) = 1 - \eta$. We first fit, in Fig. 1(a), $\Delta_{\pi^- p}$ to an expression of the form

$$\Delta_{\pi^- p} = \delta_{\pi p} s^{-\eta} \quad (3a)$$

and obtain

$$\eta = 0.54, \quad \delta_{\pi p} = 12.93. \quad (3b)$$

Excellent fits to the remaining four cross-section differences are then obtained by multiplying the function (3) by the integers given in Eqs. (2) (see Fig. 1). We have checked that, not

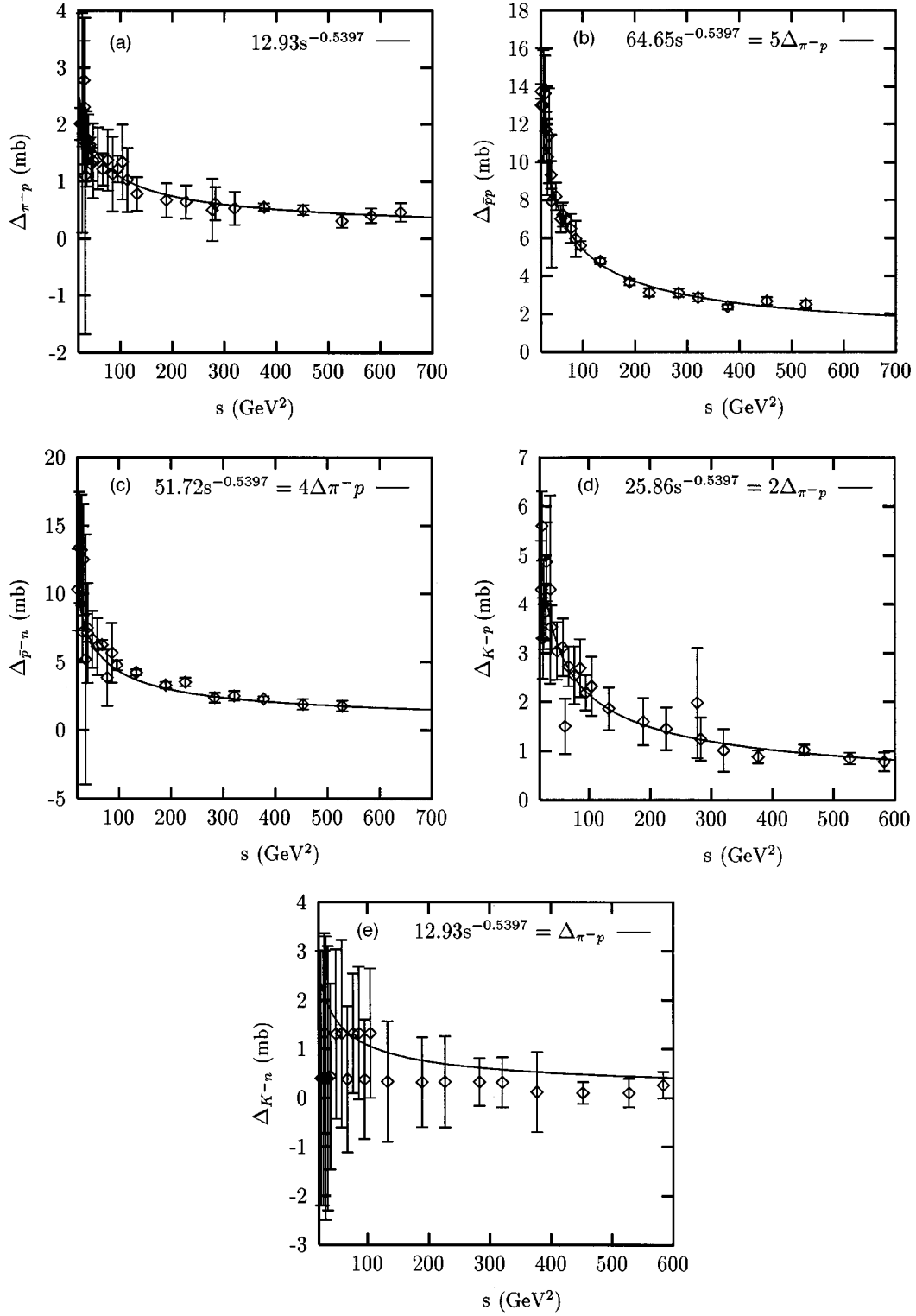


FIG. 1. Single Regge pole fits constrained by Eqs. (2) to the odd signature cross-section differences.

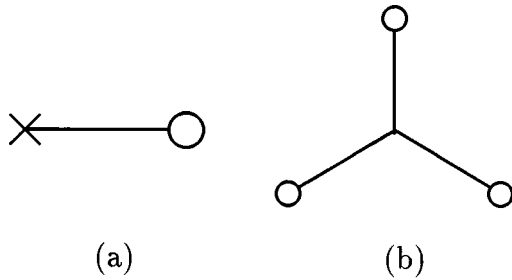


FIG. 2. Mesonic and baryonic strings ($\times = \bar{q}$, $\circ = q$).

surprisingly, the parameters obtained in the fits of Refs. [2, 3] also obey the constraints imposed upon them by Eqs. (2).

We should point out that here and throughout this paper we normalize coefficients so that s is measured in GeV^2 and we use the data of Ref. [3]. All the above fits, as well as all those to follow in Sec. III, were made using data for $s \geq 20 \text{ GeV}^2$. Points at lower energies are also displayed in some of the figures 1, 3, 5.

B. Exchange degeneracy/Chan-Paton rules

In the limit of large number of colors, QCD reduces to a string theory in which mesons are open strings with a quark at one end and an antiquark at the other [Fig. 2(a)]. The strings themselves are tubes of color-electric flux. Baryons are also viewed as systems of three strings with a quark at each of the three open ends, the three other ends meeting at a node [Fig. 2(b)]. When this string picture applies, hopefully for three colors already, then hadronic amplitudes obey duality (we use this word in the sense that the sum of the resonance contributions in the s channel gives rise to the imaginary part of the Regge contribution in the crossed t channel). This, in turn, requires the degeneracy of the odd and even t -channel mesonic Regge pole trajectories and the equality of their residues, for otherwise the amplitude for, say, pp scattering would have a Regge pole contribution with nonvanishing imaginary part even though there are no

pp resonances. The near degeneracy of the observed ω and ρ meson masses on the one hand, and of the f and a_2 meson masses on the other, is as required by exchange degeneracy. The equality of the isospin $I=1$ residues yields further constraints, to wit

$$\sigma_{pn} - \sigma_{pp} = 0, \quad \sigma_{K^+n} - \sigma_{K^+p} = 0. \quad (4)$$

As can be seen from Fig. 3, the kaon difference is compatible with zero, but the $pn - pp$ difference, though very small, appears not to strictly vanish. It can be fit to a combination of a Pomeron-Regge cut and of a small Regge pole term, though the individual contributions of these terms are hard to determine from the data. Even a pure cut gives a good fit. We can therefore safely assume that at string tree level both equations (4) are obeyed. By combining with the just-discussed $I=1$ exchange degeneracy relations their isospin $I=0$ counterparts, one imposes the full Chan-Paton rules and this then requires the absence of a mesonic Regge exchange contribution in pp , pn , K^+p , and K^+n total cross sections. This requirement is strongly violated in the fits of Refs. [2,3]. The reason for this is simple to understand. Before its ultimate rise at very high energies, any of these cross sections, σ_{pp} in particular, decreases with energy. In a fit of type (1) this is only possible if a Regge term is present and the particular Regge term needed to fit the decrease in the low energy σ_{pp} turns out to be very large: $\approx 7\Delta_{\pi-p}$. The fits of type (1) make two simplifying, but otherwise arbitrary assumptions, one concerning the nature of the Pomeron as a unique "effective" Regge pole, and the other concerning the absence of Regge-Regge cuts. We shall see below that by relaxing these assumptions, fits of comparable quality which do not violate this $I=0$ exchange degeneracy requirement are readily obtained.

III. EXPERIMENTAL TEST OF PRINCIPLES A AND B

Before we get to these new fits, we must first analyze in some detail the full implications of the assumptions A and B above (Secs. II A and II B). There are 10 measured total

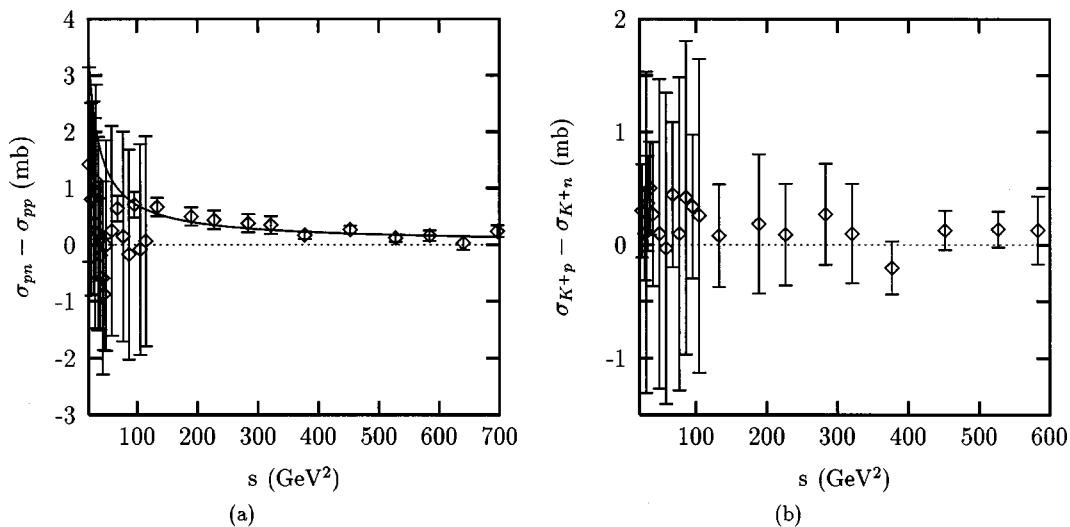
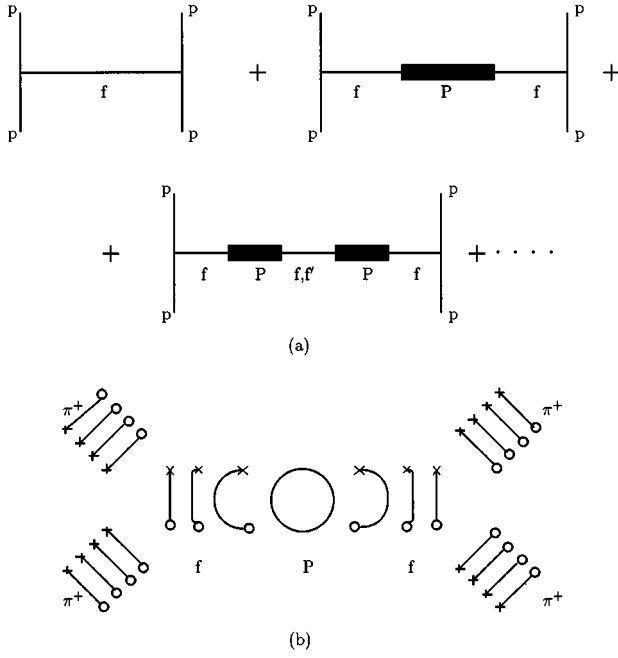


FIG. 3. Test of the $I=1$ exchange degeneracy relations (4). The solid curve in (a) represents a pure Pomeron-Regge cut fit.

FIG. 4. The f -dominated Pomeron.

cross sections and Eqs. (2) and (4) provide six linear relations among them, thus leaving four independent combinations, which we choose as Δ_{π^-p} , σ_{pp} , σ_{π^+p} , and σ_{K^+p} . Of these, the odd charge-conjugation combination Δ_{π^-p} is dominated, as was already mentioned, by the exchange of the ρ Regge pole, and this is well borne out by the data, as was known for decades. So we really have to fit only the remaining three cross sections. To do so, let us consider each of them separately. Let us start with σ_{pp} . We write the generic formula

$$\sigma_{pp} = P_p(s) + Y_p s^{-\eta} + Z_p s^{-\lambda}, \quad (5)$$

where $P_p(s)$ is the Pomeron contribution, $Y_p s^{-\eta}$ is the f - ω - a_2 - ρ Regge contribution, and $Z_p s^{-\lambda}$ is a contribution due to Regge-Regge cuts and to the f' Regge pole. Now let us consider each of these terms. First of all, the Regge contribution $Y_p s^{-\eta}$ would be absent at the string tree level. At this level the other two terms would be absent as well. Indeed in a string approach, the Pomeron is “ f -dominated” [6,7] at both ends. In other words, one of the protons emits an open f string, which closes up into a Pomeron and then reopens into another f string which gets absorbed by the other proton (see Fig. 4). This process gets iterated and at the next and later steps involves the f' as well, as shown in Fig. 4. The consecutive steps are suppressed by OZI rule breaking so that the ensuing breaking of exchange degeneracy is small. There is strong evidence in favor of this “ f -dominated” Pomeron in the photoproduction of the ρ , ω , ϕ , J/ψ vector mesons [7]. For us the important point is that *exchange degeneracy is exact only at the string tree level*. Its small breaking is caused primarily by Pomeron- f - f' mixing. As such, the first two terms in Eq. (5) are expected to be there, with the understanding that the coefficient of the Regge term is small when compared to that in the fit to

Δ_{π^-p} . The last term in Eq. (5) represents the contribution of Regge-Regge cuts and of the f' Regge pole term induced by the string loop effect of Pomeron- f - f' mixing (see Fig. 4). Both the Regge-Regge cuts and the f' pole have an intercept ~ 0 , so $\lambda \approx 1$ in Eq. (5).

We now turn to the Pomeron term $P_p(s)$. This term is a stand-in for the Pomeron Regge pole and for the multi-Pomeron cuts, as was already pointed out in Ref. [2]. There all this complexity was lumped into a unique power law s^ϵ , for reasons of simplicity, rather than on the basis of any theoretical considerations. Here we will relax this “simplicity” constraint and set

$$P_p(s) = X_p s^\epsilon + C_p s^\mu, \quad 0 \leq \mu < \epsilon. \quad (6)$$

The best fits we obtain for $\mu=0$, so that the last term in Eq. (6) will be a constant.

We will thus simultaneously fit σ_{pp} , σ_{π^+p} , and σ_{K^+p} to the forms

$$\sigma_{pp} = X_p s^\epsilon + C_p + Y_p s^{-\eta} + Z_p s^{-\lambda},$$

$$\sigma_{\pi^+p} = X_\pi s^\epsilon + C_\pi + (Y_\pi + \delta_{\pi p}) s^{-\eta} + Z_\pi s^{-\lambda},$$

$$\sigma_{K^+p} = X_K s^\epsilon + C_K + Y_K s^{-\eta} + Z_K s^{-\lambda}. \quad (7)$$

The two old parameters η and $\delta_{\pi p}$, which appear here, have been determined above from fitting the odd charge conjugation combinations of total cross sections: $\eta=0.54$, $\delta_{\pi p}=12.93$. That it is precisely $\delta_{\pi p}$ which appears in Eq. (7) is a straightforward consequence of assumptions *A* and *B* above. The 14 new parameters which appear in Eq. (7) are fortunately not all independent or unconstrained. First, all the Y 's originate in exchange degeneracy breaking and must therefore be small compared to $\delta_{\pi p}$. The value of λ must, as we saw, be near 1. The quark model determines the ratios

$$\frac{3X_\pi}{2X_p} \approx \frac{3C_\pi}{2C_p} \approx 1 \quad (8a)$$

and the “ f -dominated Pomeron” requires

$$\frac{X_K}{X_\pi} \approx \frac{C_K}{C_\pi} \approx \frac{1}{2} \left(1 + \frac{m_\rho^2}{m_\phi^2} \right) = 0.886. \quad (8b)$$

Equation (7) then really introduces only seven parameters and the small departures from unity of the other just-mentioned combinations of parameters. With all this in mind we now present our fits of type Eq. (7) in Fig. 5. The corresponding values of the parameters are

$$\epsilon = 0.135, \quad \eta = 0.54, \quad \lambda = 1.01,$$

$$X_p = 6.26, \quad C_p = 24.4, \quad Y_p = 0.88, \quad Z_p = 196,$$

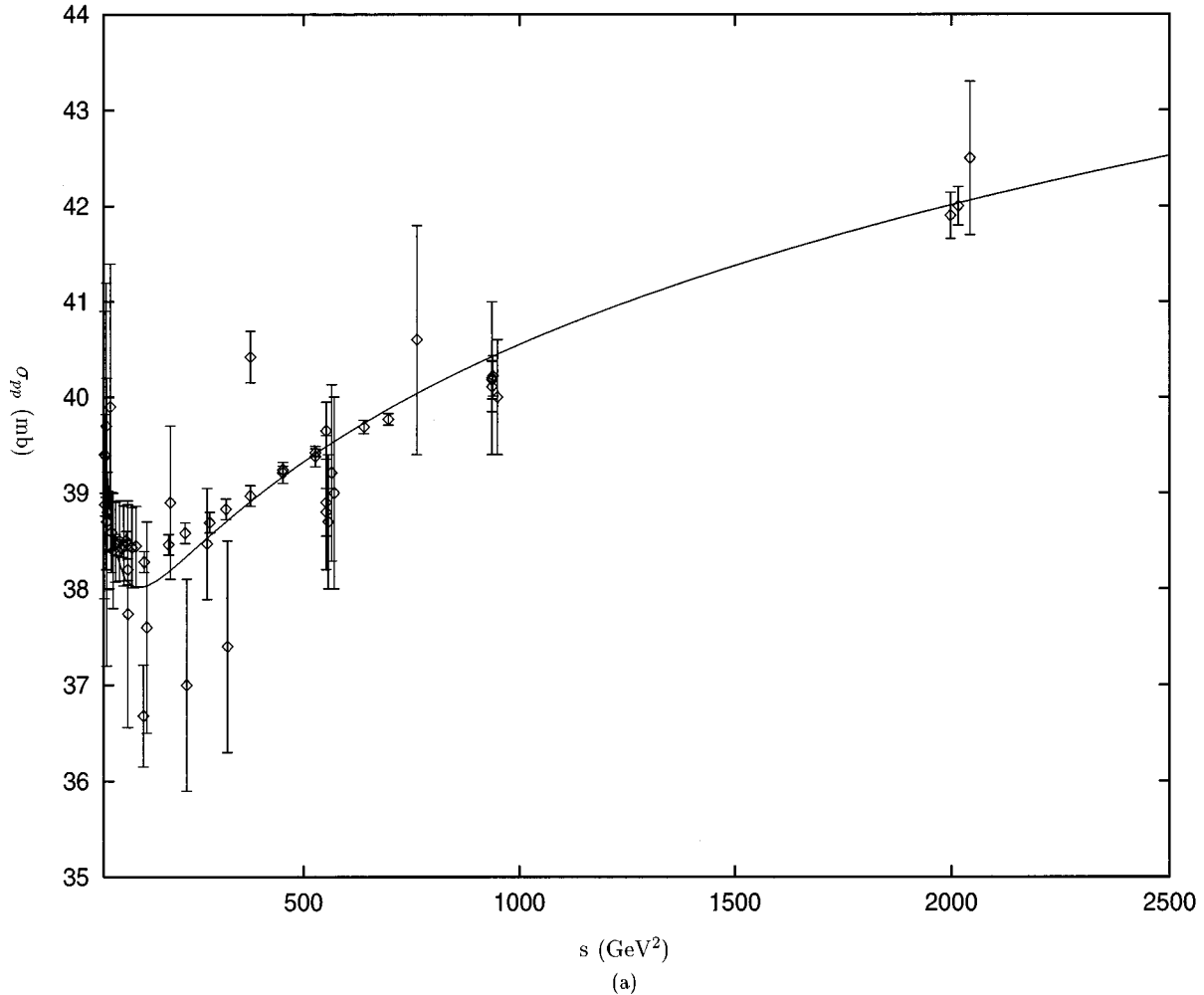


FIG. 5. Fit of the form Eqs. (7) and (8) to the three independent cross sections σ_{pp} , σ_{π^+p} , and σ_{K^+p} . The parameters of this fit are given in Eq. (9).

$$\frac{3X_\pi}{2X_p} = 1.04, \quad \frac{3C_\pi}{2C_p} = 0.84, \quad Y_\pi = -1.9, \quad Z_\pi = 51,$$

$$\frac{X_K}{X_\pi} = 0.9, \quad \frac{C_K}{C_\pi} = 0.83, \quad Y_K = -0.88, \quad Z_K = 0.12. \quad (9)$$

This fit is of comparable quality to the fits of Refs. [2,3]. It has the following characteristic features.

The exchange degeneracy breaking parameters Y_p , Y_π , and Y_K are indeed very small compared to $\Delta_{\pi p}$.

The Pomeron exponent ϵ is larger than in most previous fits, but such a larger value was already contemplated in Ref. [8] in the context of very high energies. In any case, even with this larger exponent the Froissart bound is comfortably obeyed even beyond the Planck energy.

The constraints (8) are well obeyed by the leading Pomeron terms (X coefficients) and obeyed at the 15% level for the subdominant terms (C coefficients).

At first sight the f' -Regge-Regge-cut parameter Z_p appears large. Z_p plays for our fit a role similar to that of the large exchange degeneracy breaking Y_p parameter in Refs. [2,3]. This f' -Regge-Regge-cut term falls much faster with energy than an ordinary Regge term, so it makes sense to compare its low energy contribution in the pp amplitude, $\mathcal{CUT} \sim 196s^{-\lambda+1} \approx 196$, to the *nonvanishing real part* of the pp Regge term which is $\text{Re}(R) \sim 5\Delta_{\pi p}s^{-\eta+1} \approx 65s^{0.46}$. Even at the low value $s = 40 \text{ GeV}^2$, we find $\mathcal{CUT}/\text{Re}(R) \approx 0.55$ and this ratio decreases as $s^{-0.46}$. The Regge cut and f' contributions are thus consistently smaller than the Regge pole contributions. Similar arguments can be made for the πp and Kp amplitudes as well. The important new feature here is that this f' -Regge-Regge-cut term represents a *theoretically expected* exchange degeneracy and duality violation.

IV. CONCLUSIONS

From all this we conclude that all hadronic total cross-section data are compatible with the stringy principles A and B above. The reason for the apparent discrepancy between

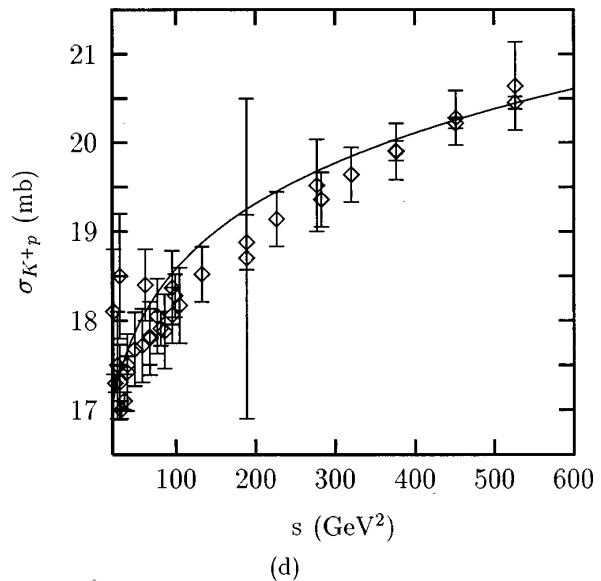
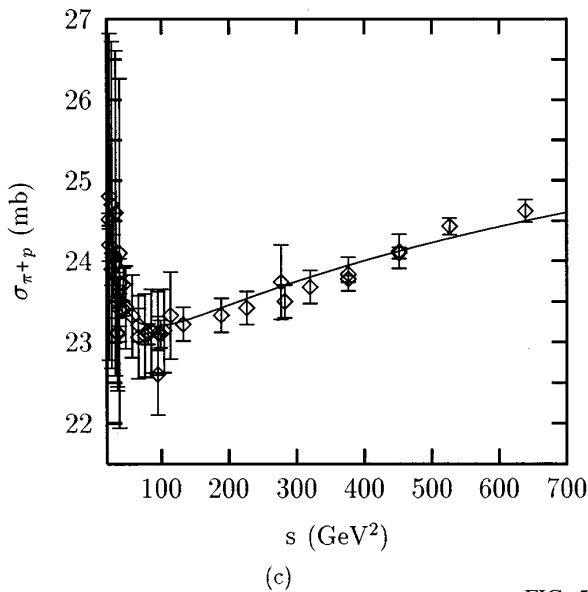
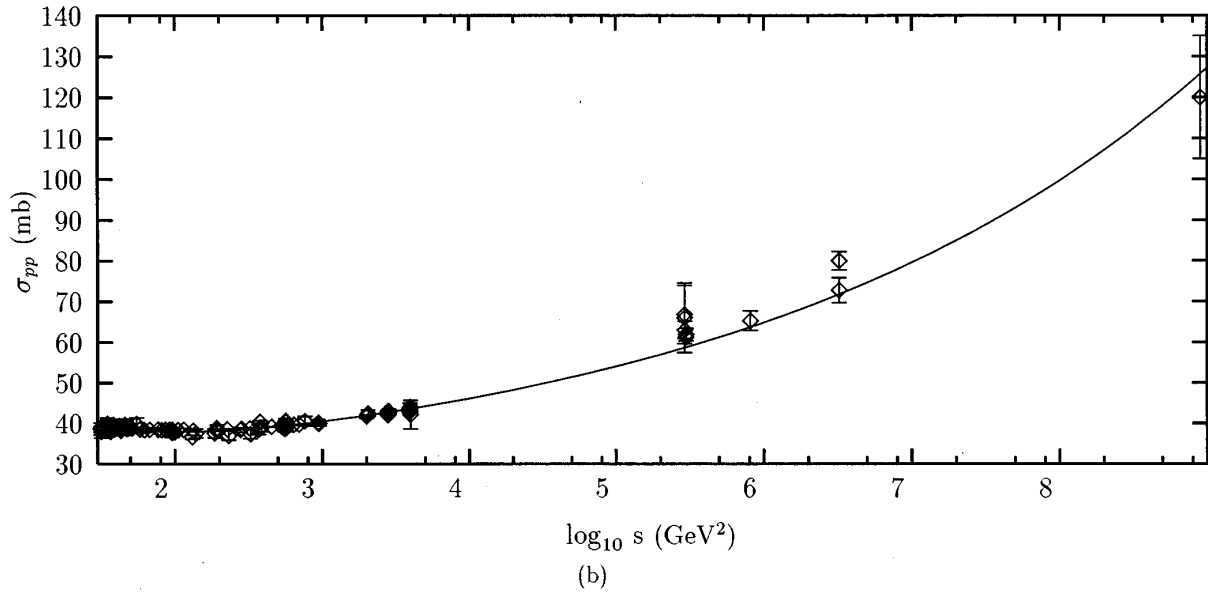


FIG. 5 (Continued).

the fits of Refs. [2,3] and these principles is that for simplicity, they (i) ignored Regge-Regge cuts and (ii) fit the Pomeron to a single power law. The remarkable power of the principles A and B is that, even after abandoning these simplifications, the necessary number of parameters did not increase. Actually, after all constraints were met, we found this number to have *decreased*.

In a forthcoming paper we shall further explore these

principles in the light of recent developments in open string theory.

ACKNOWLEDGMENT

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- [1] P. D. B. Collins and F. D. Gault, Phys. Lett. **73B**, 330 (1978).
 [2] A. Donnachie and P. V. Landshoff, Phys. Lett. B **296**, 227 (1992).
 [3] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996); <http://pdg.lbl.gov/>.
 [4] K. Johnson and S. B. Treiman, Phys. Rev. Lett. **14**, 189 (1965).

- [5] P. G. O. Freund, Phys. Rev. Lett. **15**, 929 (1965).
 [6] R. Carlitz, M. B. Green, and A. Zee, Phys. Rev. Lett. **26**, 1515 (1971).
 [7] C. E. Carlson and P. G. O. Freund, Phys. Rev. D **11**, 2453 (1975).
 [8] CDF Collaboration, F. Abe *et al.*, Phys. Rev. D **50**, 5550 (1994).