T-duality considerations for hadronic strings

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We show that a simple account of hadronic high energy total cross sections and of the observed masses and flavor-gauge-like couplings of the familiar vector mesons is obtained in a hadronic string model suggested by QCD in the limit of a large number of colors. Our picture involves a Minkowski space which, in addition to the well-established continuous one time and three space dimensions, is endowed with an extra *discrete* space dimension involving a minimal two-point lattice with a spacing of order 10^{-14} cm in one of the two *T*-dual pictures. New mesonic states with characteristic decay modes are expected in such a picture. [S0556-2821(97)02517-4]

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I. INTRODUCTION

With the recent understanding of the role of dualities in string theory [1,2], the hitherto less studied open strings with Chan-Paton (CP) rules [3] have regained interest. These CP rules were originally abstracted from hadron phenomenology, where they form part of the understanding of high energy hadronic total cross sections. In QCD, the theory of strong interactions, hadrons can be pictured as tubes of color flux capped by quarks (or antiquarks) or, in other words, as open strings with quarks at their ends. We wish to explore here whether the recent advances in the theory of open strings may shed some light on the physics of hadrons. We shall see that some interesting new perspectives on some old hadron problems can be gained this way and that an unexpected connection between quark flavors and the structure of Minkowski space-time seems to emerge. In particular, we will find that the validity of hadronic Chan-Paton rules (and of their successful consequences for high energy hadron scattering) and the mass spectrum and flavor-gauge-like coupling pattern of vector mesons are readily explained. We will extend Minkowski space by adding to it a *discrete* spacelike dimension involving a minimal two-point lattice with spacing of order 10^{-14} cm in one of the two *T*-dual pictures. New mesonic states with characteristic decay modes are expected in such a picture.

II. OPEN STRINGS

Let us start from the CP rules, for which there is ample evidence coming primarily from high energy hadron scattering processes at fixed momentum transfer. The Lie group involved in the hadronic CP rules is $U(N)$, N being the number of quark flavors (*not* colors). Consider a process for which, say, both the st and su quark diagrams — i.e., both quark diagrams involving the ''exotic'' s channel — are forbidden because the corresponding CP rule traces vanish [4–7]. Examples of such processes are K^+K^+, K^+p, pp, $\pi^+\pi^+$, ϕp scattering. At the string tree level in all these cases the contribution of the mesonic Regge poles to the scattering amplitude at large s and fixed t (in particular $t=0$) is purely real, so that the characteristic $s^{-1/2}$ contribution to the corresponding total cross section is absent, in agreement with experiments (for an up-to-date analysis of the phenomenology see Ref. [8]). The way this is achieved is by the degeneracy of the odd signature Regge trajectory on which the ρ and ω vector mesons lie with the even signature trajectory on which the tensor mesons f and $a2$ lie. This degeneracy is also experimentally confirmed and there are further instances in which the CP rules are confirmed.

These rules follow from QCD in the limit of a large number of colors in which hadrons appear as strings with quarks at their ends [9,10]. For the purposes of this work we will assume this string picture to hold. Essentially this amounts to assuming that the approach using a large number of colors is already reasonably good for three colors. There is independent evidence [11] in favor of such a string picture. Physically one can think of the strings as tubes of color flux capped by quarks and antiquarks, as was mentioned above. These hadronic strings differ from the fundamental superstrings in that they have massive quarks at their ends [12], in that they are not in a critical dimension, and in that they are not supersymmetric. In spite of all these differences, we will apply T duality to hadronic strings.

Consider a mesonic string $[ij]$ (baryons can be treated similarly) as an open string with a quark of flavor i at one end and an antiquark of flavor j at the other end, moving in four-dimensional Minkowski space. Let us, for the moment, assume the existence of an ''extended'' Minkowski space, which in addition to its time and three noncompact space dimensions also has a fifth *continuous* space dimension compactified to a circle of radius R. The appearance of such an extra space dimension may be connected with the number of colors going to infinity limit. But a continuous compact fifth dimension implies the existence of a tower of Kaluza-Klein (KK) states and this causes serious problems especially in the closed string glueball sector, which is flooded by KK states at all energies. As we will see in Sec. IV, this problem can be solved by extending Minkowski space by a *discrete* rather than continuous fifth dimension. In this more modest extension the theory remains four dimensional, as expected for a finite number of colors. For now let us temporarily allow the introduction of a compact *continuous* fifth dimension and explore the consequences of this assumption for open strings. This allows us to present in this section the

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open string phenomenology in a familiar setting, which does not get radically changed upon discretizing the fifth dimension. By contrast, in Sec. III we explain the untenable situation which then develops in the closed string glueball sector, and in Sec. IV we deal with this problem by switching to a discrete fifth dimension.

We now proceed to the discussion of open strings in the presence of a fifth dimension in the form of a circle of radius R.

Were we to ignore the quark masses, we could now switch to the T-dual picture in which the radius of the compact dimension's circle is

$$R'=\frac{\alpha'}{R}, \tag{1}$$

where $\alpha'\approx 1\ \mathrm{GeV}^{-2}$ is the hadronic Regge slope. Equivalently, the tension of the hadronic string is $T=1/2\pi\alpha'$. As far as the compact fifth dimension is concerned, the T duals of the Neumann boundary conditions at the end of the original string are then Dirichlet boundary conditions for its dual. This dual string must end [13] on one of a set of N Dirichlet three-branes, N being the number of quark flavors. These N D-branes are N copies of ordinary four-dimensional Minkowski space, located at N given values $\theta_1 R',\theta_2 R',\ldots,\theta_N R'$ of the compact fifth coordinate $\theta R'$. The specific and not necessarily all distinct values of the θ_i reflect the details of the Wilson line included when compactifying the fifth coordinate. If all distinct, they lead to a breaking of the flavor U(N) symmetry to $\mathrm{U}(1)^N$. The ''nondiagonal'' *flavor* gauge bosons present as states of the string thus acquire masses in a pattern characteristic of an adjoint representation Brout-Englert-Higgs (BEH) phenomenon. Specifically the mass m_{ij} of the $[ij]$ vector meson is given by

$$m_{ij}^2=\left(\frac{(\theta_i-\theta_j)R'}{2\pi\alpha'}\right)^2. \tag{2}$$

Thus the K^*, $\rho^\pm$, D^*, D_s^*, B^*, etc., mesons acquire masses, whereas the ρ^0, ω, ϕ, J/ψ, Υ, and toponium remain massless. This is obviously unacceptable and quite reminiscent of the old Sakurai theory of flavor gauging [14]. But in all of this we have ignored the masses of the quarks, acquired through the BEH phenomenon of the standard model. When all the quark masses are set to zero and all the θ_i are equal, then all N^2 vector mesons would be massless gauge bosons. But with *massive constituent* quarks at the ends of the strings there is another contribution to m_{ij}, namely, the sum m_i+m_j of the masses of the constituent quarks of flavor i and j. Then

$$m_{ij}^2=(m_i+m_j)^2+\left(\frac{(\theta_i-\theta_j)R'}{2\pi\alpha'}\right)^2. \tag{3}$$

This is a much more interesting and realistic formula. The first term on the right-hand side gives the familiar equal spacing rule of vector meson masses. The second term now

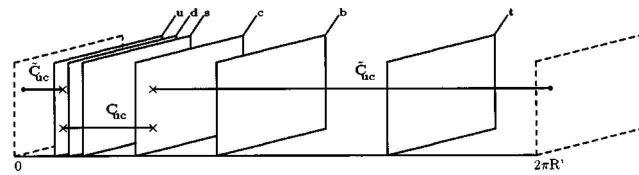

FIG. 1. Configuration of flavor D-branes.

superimposes an adjoint BEH phenomenon over this equal spacing rule. For this mechanism to work, we must have the inequalities

$$\Delta_{ij}:=m_{ij}^2-(m_i+m_j)^2=m_{ij}^2-\frac{(m_{ii}+m_{jj})^2}{4}\geqslant 0,$$

$$i,j=1,\ldots,N. \tag{4}$$

The last expression before the inequality sign determines Δ_{ij} in terms of the experimentally known values of the vector meson masses, so that many of these inequalities can be tested. Let us label the six quark flavors by their usual symbols u,d,s,c,b,t rather than by the numbers $1,2,\ldots,6$. In view of $m_\rho^\pm=m_\rho^0=m_\omega=$ 770–783 MeV, it is not necessary to distinguish the u and d flavors, and so we can use the common notation n for both of these flavors. Using the experimentally known values $m_\rho=0.77$ GeV, $m_\omega=0.782$ GeV, $m_\phi=1.02$ GeV, $m_{J/\psi}=3.097$ GeV, $m_\Upsilon=9.46$ GeV, $m_{K*}=0.892$ GeV, $m_{D*}=2.01$ GeV, $m_{D_s^*}=2.112$ GeV, and $m_{B*}=5.325$ GeV, we then find

$$\Delta_{ns}=-0.005\ \mathrm{GeV}^2,\quad \Delta_{nc}=0.30\ \mathrm{GeV}^2,$$

$$\Delta_{sc}=0.22\ \mathrm{GeV}^2,\quad \Delta_{nb}=2.13\ \mathrm{GeV}^2. \tag{5}$$

Though Δ_{ns} is negative, this is irrelevant given that $|\Delta_{ns}|<(m_\omega)^2-(m_\rho)^2$, which has been neglected as explained above. In other words $\Delta_{ns}=0$ at the level of accuracy intended here. Since

$$\Delta_{ij}=\left(\frac{(\theta_i-\theta_j)R'}{2\pi\alpha'}\right)^2, \tag{6}$$

it then follows that not only do we have $\theta_u=\theta_d$ but even the stronger equalities $\theta_u=\theta_d=\theta_s$. This in turn requires $\Delta_{nc}=\Delta_{sc}$, which holds to within 27% or so. The picture that emerges has three D-branes, the u, d, and s branes essentially coinciding, while the remaining three branes are further away from them. Quantitatively,

$$|\theta_d-\theta_u|R'=|\theta_s-\theta_u|R'=0,$$

$$|\theta_c-\theta_u|R'=3.44\ \mathrm{GeV}^{-1},\quad |\theta_b-\theta_u|R'=9.17\ \mathrm{GeV}^{-1}. \tag{7}$$

At the present stage of top spectroscopy $|\theta_t-\theta_u|R'$ is not determined. The D-brane arrangement corresponding to these θ_i's is presented in Fig. 1. The mass formula (3) accounts well for the observed vector meson masses (there is an experimentally less constrained spin 0 counterpart to all this). It may be worthwhile at this point to take notice of the remarkable changes of Minkowski space under T duality.

Whereas in the ''original'' string picture with compactification radius R there was a unique ambient extended Minkowski space in which all strings — and therefore all string ends — moved, in the T-dual picture with compactification radius R' we find N Dirichlet three-branes on which all strings must end. It is as if Minkowski space in this picture were N-sheeted, the number N of sheets being determined by the number of quark flavors. *A geometrical interpretation of the number of quark flavors thus emerges*. We should stress that this interpretation is tied to the hadronic string considered here and therefore to the large number of quark colors picture which leads to it. It is not to be confused with similar space foliations in a fundamental string theory [13], where the basic scale is dictated by the much smaller Planck length. Multisheeted Minkowski space is also encountered in noncommutative-geometry-based approaches to the standard model [15,16].

Though the mass formula (3) is quite realistic, our main interest in this picture comes from the fact that it accounts naturally for the universal coupling of vector mesons, usually referred to as vector meson dominance. It has been known for a very long time indeed [14] that the vector mesons considered here couple as if they were flavor U(N) gauge bosons. For instance, the ρ^0 couples twice as strongly to π^+ mesons as to protons, as expected for a gauge boson of isospin, the third component of the π^+ meson's isospin being double that of the proton. This coupling pattern has not been derived from QCD so far. The *true* gauge bosons of QCD are of course the gluons, and in QCD the vector mesons are composite states whose coupling pattern and mass spectrum are to be worked out numerically, say, by lattice QCD calculations. The string picture considered here has the advantage of automatically yielding the right coupling pattern for the vector mesons, which appear here as massive gauge bosons [in the limit of vanishing quark masses and coincident D-branes for all flavors, they become massless and U (N) gauge invariance is manifest]. The string picture also corrects the unacceptable mass spectrum produced by a pure adjoint BEH phenomenon for these vector mesons. The only other work which deals with both these vector meson problems is that based on hidden local symmetries of a nonlinear σ model [17]. It is not clear whether or how this work relates to the string approach taken here.

III. THE MAGNITUDE OF THE COMPACTIFICATION RADIUS AND THE GLUEBALL PROBLEM

We have now come to the point where we have to confront the question whether the fifth dimension introduced in these arguments is at all acceptable. To meaningfully discuss this problem we first have to estimate the compactification radius R or its dual R'. This can be done in the following manner. In addition to the string stretched between the D-branes i and j, which contributes the Δ_{ij} term to the square of the vector meson mass, there is another ''complementary'' string which goes in the opposite direction along the θ circle; e.g., the complementary string of the string marked C_{uc} in Fig. 1 is the string $\widetilde{C}_{uc}$. For this string not to correspond to a lighter state, we need $2\pi - |\theta_i - \theta_j| \geqslant |\theta_i - \theta_j|$ or $|\theta_i - \theta_j| \leqslant \pi$ for all i and j. Therefore we must have

$$R' \geqslant \frac{1}{\pi}\max(|\theta_i - \theta_j| R'). \tag{8}$$

From Eq. (5), using Eqs. (3) and (4), it can be seen that $|\theta_b - \theta_n| R' / |\theta_c - \theta_n| R' \approx 2.7$, which is very close to $m_b/m_c = m_\Upsilon / m_{J/\psi} = 3.06$, so that it appears that $|\theta_i - \theta_j| R'$ increases more or less linearly with the (absolute value of the) mass difference of the flavor i and flavor j quarks, at least up to the b quark. We do not know how this extends to the top quark and will therefore consider two cases: (i) $|\theta_t - \theta_n| R' / |\theta_b - \theta_n| R' \approx m_t / m_b$ and (ii) $|\theta_t - \theta_n| R' / |\theta_b - \theta_n| R' \approx 1$.

In case (i) the maximum in Eq. (8) sets in for $i = t$ and $j = u$, and, using $m_t = 175$ GeV, this maximum can be estimated at some 108 GeV^{-1}; so $R' \geqslant 21$ F. If the spacing of the levels of the mesonic KK tower were determined by R'^{-1}, this would certainly be the end of the story. *For open strings* the spacing of the levels of the mesonic KK tower is dictated [13] by R^{-1}, with R given by Eq. (1), i.e., $R \leqslant 1/108$ GeV$^{-1} \approx 0.002$ F. In other words, on the ''R side'' we are dealing with with a compactification scale comparable to the weak scale *or smaller*. On the ''R' side,'' on the other hand, the large scale of 20 F appears and one would expect this scale, or the equivalent $1/R' = 10$ MeV mass scale, also to manifest itself in the theory. It does indeed and in a disastrous way at the level of closed strings, i.e., of glueballs. Such closed string glueballs are an inescapable consequence of unitarity (which forbids a pure open string theory). Unlike the open strings, these closed string glueballs can wind around the small radius R circle and give winding modes. If the lightest glueball has mass m_G (typically around 2 GeV), then these R-winding or, equivalently, R'-KK modes involve the masses $\sim m_G (1 + n^2/R'^2 m_G^2)^{1/2}$, $n = 1,2,3,\ldots$. These modes are very closely spaced, with the resulting glueball sector hardly distinguishable from a continuum. Though the couplings of these glueball states to ordinary $q\bar{q}$ mesons are Okubo-Zweig-Iizuka (OZI) suppressed, by their sheer number these glueball states would dominate production processes to an unacceptable degree. In case (ii) the details change somewhat: 10 MeV becomes ~ 1 GeV, but this glueball disaster persists. It is in the glueball sector that the theory betrays its five-dimensional nature. Though at the level of open strings everything worked out for the best at least at experimentally available energies, at the glueball level the continuous fifth dimension cannot be tolerated. This glueball flood is due primarily to the fact that a continuous fifth dimension produces infinite KK towers, which cannot be cut off. To cut off the KK towers, we next explore the possibility that the fifth dimension introduced in this stringy hadron phenomenology is really *discrete*, somewhat along the lines of noncommutative geometry [15,16].

IV. DISCRETE FIFTH DIMENSION

As suggested at the end of the previous section, we now proceed to explore the consequences of a discrete fifth dimension. As we saw, the string theory has two T-dual pictures, involving circles of radii R and R'. Here we replace these two circles by two periodic lattices: one with N points, period $2\pi R$, and spacing a; the other with N' points, period

$2\pi R'$, and spacing a'. This way, $Na=2\pi R$ and $N'a'=2\pi R'$. Because of the periodicity of the lattice, on the ''unprimed'' side the fifth component of the momentum carried by closed strings is quantized in units of $1/R$. Thus the KK modes carry a fifth component of the momentum $n/R=2\pi n/Na$, $n\in\mathbb{Z}$, whereas the winding modes carry a fifth component of the momentum $wR/\alpha'=w/R'$, $w\in\mathbb{Z}$. Because we are dealing with a lattice, the possible values of the fifth component of the momentum are in the interval $[-\pi/a,\pi/a]$. For the extremal values in this interval to be compatible with the KK quantization of the momentum, we must choose N a positive even integer. The KK modes then range over the integer multiples of $1/R$ in the interval $[-N/2R,N/2R]$ and we also have $|wR/\alpha'|\leqslant N/R$. Similar statements, with primed and unprimed quantities interchanged, hold on the primed side as well. The KK (winding) modes on the unprimed side are to be identified with the winding (KK) modes on the primed side with $w\leftrightarrow n'$ and $w'\leftrightarrow n$. Again, for the extremal modes, this requires

$$\frac{N}{R}=\frac{N'}{R'} \tag{9a}$$

or, equivalently,

$$a=a'. \tag{9b}$$

The spacings of the primed and unprimed lattices must be the same.

The T-duality condition on the compactification radii, Eq. (1), now becomes

$$\frac{4\pi^2\alpha'}{a^2}=NN', \tag{10}$$

so that the square of the lattice spacing is rationally related to 2π times the string tension. When the fifth dimension is discrete, Eq. (6) becomes

$$|\nu_i'-\nu_j'|a=2\pi\alpha'\sqrt{|\Delta_{ij}|}, \tag{11}$$

where ν_i' is the position of the flavor i membrane on the primed lattice. Equations (10) and (11) yield

$$|\nu_i'-\nu_j'|=\sqrt{NN'\alpha'|\Delta_{ij}|}. \tag{12}$$

Equations (10)–(12) will be very useful towards obtaining the lattice spacing and the sizes of the two lattices. First of all we get an idea of possible values of N'. Since all $|\nu_i'-\nu_j'|\leqslant N'/2$, Eqs. (10) and (12) require

$$N'\geqslant 4N\alpha'\max|\Delta_{ij}|, \tag{13}$$

the discrete equivalent of Eq. (8). Using the values of α' given above and of the largest $|\Delta_{ij}|$ known so far, namely, $|\Delta_{bn}|$ [see Eq. (5)], we find $N'\geqslant 8N$. We are thus naturally led to the ''minimal'' case $N=2$ and to values of N' in excess of 16. As a first example, set $N=2, N'=50$ (both even as was explained above). Using $\alpha'\approx 1\ \mathrm{GeV}^{-2}$ we find $a=1.26\times 10^{-14}$ cm and with $|\nu_b'-\nu_n'|=15$ we obtain the value $|\Delta_{bn}|=2.25\ \mathrm{GeV}^2$, which closely reproduces the values in Eq. (5). We will find other interesting lattice configurations in the next section. Before we discuss these further choices of the lattice parameters, let us now see how this discrete fifth dimension solves the glueball problem of the continuous case. In the continuous case the KK towers were infinite and given their small spacing the production processes were swamped with glueballs. In the lattice case, by contrast, the KK tower consists of N' states, i.e., of a *finite* number of states. If the lowest glueball state's mass is $m_G\sim 2$ GeV, then its KK tower will consist of N' states of masses $m_n=\sqrt{m_G^2+(2\pi n/N'a)^2}$ where the integer n runs from $-N'/2$ to $+N'/2$. For the example just discussed this means 50 states in the mass interval 2 – 5.4 GeV. The production of each of these states is OZI suppressed by, say, a factor 0.1. We would therefore expect the production of a continuumlike spectrum, but not with the immense cross section obtained when the fifth dimension was a continuous circle.

We should mention that both for open and closed strings, besides the KK towers, one expects the characteristic exponentially growing stringlike spectrum. A stringlike exponentially exploding hadron spectrum is indicated by experiment, as was already known to Hagedorn [18]. The absence of tachyons places severe constraints on the Fermi-Bose imbalance of this spectrum [19], which again seem to be obeyed [11,20] in the region in which hadron spectroscopy is reliable.

V. TESTS OF THE HADRONIC STRING MODEL WITH A DISCRETE FIFTH DIMENSION

We saw in Sec. III the constraints which follow in the continuous case from the requirement that the mesons correspond to the lighter of a ''complementary'' pair (e.g., strings C_{uc} and $\tilde{C}_{uc}$ in Fig. 1). These complementary pairs exist in the discrete case as well. For the $[ij]$ meson which extends over $|\nu_i'-\nu_j'|$ primed lattice sites, its complementary string extends over $N'-|\nu_i'-\nu_j'|$ sites. Though heavier, this complementary $\tilde{C}_{ij}$ string should nevertheless exist. Its mass is given by

$$\tilde{m}_{ij}^2=(m_i+m_j)^2+\left(\frac{(N'-|\nu_i'-\nu_j'|)a}{2\pi\alpha'}\right)^2. \tag{14}$$

We also record here the discrete analogue of Eq. (3) for the mass of the ''original'' (shorter) meson states:

$$m_{ij}^2=(m_i+m_j)^2+\left(\frac{|\nu_i'-\nu_j'|a}{2\pi\alpha'}\right)^2. \tag{15}$$

One can see from the last two equations that, as in the continuous case, both the ''original'' and the ''complementary'' strings carry fractional amounts of the unprimed KK quantum of the fifth component of the momentum. Such a fractional charge is possible in the familiar fashion [13] because of the U(N) and of the D-branes it entails. Note that for ''diagonal'' $[ii]$ type mesons, like the ρ, Υ, etc., Eq. (14) yields their first (and for $N=2$ their last) *unprimed* KK mode. We now have to see how the existence of these complementary strings is reflected in meson spectroscopy. We must compare their predicted masses with meson spectroscopic data. We proceed in four steps: (a) We set $N=2$

TABLE I. Lattice parameters.

N'	a (F)	$\lvert\nu'_c-\nu'_n\rvert$	$\lvert\nu'_b-\nu'_n\rvert$
20	0.199	3	9
30	0.162	4	11
40	0.140	5	13
50	0.126	5	15
60	0.115	6	16
70	0.106	6	17

and choose for N' an even integer $\geqslant 16$; we also impose $N' \leqslant 70$ so as not to obtain an overly crowded glueball spectrum, in line with our discussion in Sec. IV above; (b) from Eq. (10) we determine the lattice spacing a; (c) from Eqs. (15) and (4) we determine the best integers $|\nu'_i-\nu'_j| \leqslant N'/2$ which fit the observed Δ_{ij} of Eq. (5); (d) We insert the results of the previous three steps into Eq. (14) and obtain the complementary meson masses.

The results of the first three steps are displayed in Table I and the complementary meson masses obtained in step (d) are in Table II.

As far as the mesons complementary to the light mesons are concerned, these are all predicted to have masses between 3 and 6 GeV, where no meaningful spectroscopic data are available. Therefore in Table II we show only the heavy mesons. For these we obtain masses which are not much above the heaviest observed mesons and for the $b\bar{b}$ case are actually in the accessible range. The typical decay modes of these complementary mesons is to the ''original mesons'' (whose complementaries they are) and a real or virtual glueball which then decays to pions. These complementary mesons are not to be confused with the radial excitations of the ''originals.'' As can be seen from Table II, the most promising sectors where complementary strings might be found are those of the B, B_s, and $b\bar{b}$ mesons.

We again stress that the glueball proliferation is solved in all the cases of Tables I and II in the same way as outlined in the last section.

By inspecting Tables I and II we can observe some important features. First of all, the ratio $(|\nu'_b-\nu'_n|)/(|\nu'_c-\nu'_n|)$ is close to 3 for all choices of N', as was already noted in the continuous case in Sec. III. Next we notice that the values obtained for $\tilde{m}_\Upsilon$ tend to favor the larger values of N', those above 40. This raises a number of questions. First of all, if the distance between the t and uds D-branes were much larger than that between the b and uds D-branes, then much larger N' values would come into play ($N' \geqslant 300$). With such large N' the glueball proliferation problem would resurface. This is best seen if we recall from Sec. IV that the length of the mass interval covered by the N' resonances in the KK tower increases, for large N', as $\sqrt{N'}$. As explained in Sec. IV, 50 closely spaced, almost continuumlike, OZI-suppressed KK resonances represent the cross section equivalent of, say five ''normal'' resonances over a 3.4 GeV mass interval. By contrast, $N'=300$ OZI-suppressed resonances would then correspond to 30 ''normal'' resonances over a 10.4 GeV mass interval and the contribution to the cross section would indeed be too large. With the values of N' in the tables we are thus constrained to the alternative (ii) of Sec. III.

TABLE II. Complementary meson masses.

N'	$\tilde{m}_{J/\Psi}$ (GeV)	$\tilde{m}_\Upsilon$ (GeV)	$\tilde{m}_D$ (GeV)	$\tilde{m}_B$ (GeV)
20	3.62	9.97	3.31	5.40
30	4.25	10.22	3.87	5.67
40	4.81	10.46	4.36	5.94
50	5.30	10.70	4.90	6.20
60	5.75	10.93	5.30	6.50
70	6.17	11.16	5.74	6.80

All this raises the much deeper question as to how the values of N' and of the ν'_i, the locations of the D-branes on the primed lattice, are determined. We have no insight at present into this important question. The comparable question for the unprimed lattice has been disposed of by making the ''minimal'' choice $N=2$. N' between 20 and 70 is then somewhat removed from the self-dual point. For fundamental strings this would be hard to understand, but then we are *not* dealing with fundamental strings here.

VI. CONCLUSIONS

In this paper we proposed a string picture of hadrons, as suggested by the large number of colors limit of QCD. This has automatically brought with it the Chan-Paton rules which are so useful in understanding the phenomenology of high energy hadron scattering at fixed momentum transfer and therefore of hadronic total cross sections. Expanding Minkowski space by an additional discrete space dimension, we were led to a simple picture of vector meson masses and couplings. This picture led us to the prediction of ''complementary'' mesonic strings which would be heavier than the usual mesons and would decay into these ordinary mesons and glueballs. In the Υ, B, and B_s sectors such complementary mesons lie sufficiently close to the experimentally accessible region as to make a search for such states possible.

To get agreement with the meson spectrum, we were led to a set of values for the parameters which describe the discrete fifth dimension: a lattice spacing of $\sim 10^{-14}$ cm and periodic lattices with 2 points in one picture and 20–70 points in the picture T dual to it. It would be very interesting to understand the theoretical reasons for the appearance of these particular lattices.

We should stress that on account of the discrete fifth dimension, we dealt throughout with a four-dimensional situation with an enriched spectrum. There are *cutoff* KK towers but all that does not produce the kind of proliferation encountered in the presence of a continuous fifth dimension where the KK towers would become infinite.

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