PHYSICAL REVIEW D VOLUME 56, NUMBER 4 15 AUGUST 1997

Cosmic strings in supergravity

J. R. Morris
Physics Department, Indiana University Northwest, 3400 Broadway, Gary, Indiana 46408
(Received 11 April 1997)

It is pointed out that various types of cosmic string solutions that exist in nonsupersymmetric and globally supersymmetric theories, such as D-type gauge strings, F-type global and gauge strings, and superconducting Witten strings, also exist in supergravity models. When the D term and superpotential satisfy some simple conditions allowing the determination of a set of vacuum states with nontrivial topology, the existence of a string embedded within a supersymmetric vacuum with a vanishing cosmological constant can be inferred. Supergravity also admits other string solutions, some of which have no counterparts in globally supersymmetric theories. [S0556-2821(96)06116-4]

PACS number(s): 11.27.+d, 04.65.+e, 98.80.Cq

I. INTRODUCTION

It is now generally recognized that the early universe may have undergone symmetry-breaking phase transitions which spawned the production of topological defects [1,2]. Cosmic strings, in particular, may have played an important role in the subsequent evolution of the universe [1,3]. Furthermore, some realization of supersymmetry, perhaps in the form of supergravity, could present itself as an effective theory for the epochs of cosmic string formation. It is therefore natural to investigate cosmic string solutions in the context of supergravity. Studies of supergravity domain walls [4,5] have indicated that gravitational effects in supergravity can play an important role in the existence and structure of these defects, and we can expect this to be true for cosmic strings, as well.

The bosonic sector of a theory with global supersymmetry (SUSY) can have a more complicated form than that of a nonsupersymmetric theory [6], and this can lead to differences between string solutions in the two different theories. For example, supersymmetric cosmic string models may require the participation of more fields than the nonsupersymmetric counterpart [7], so that the supersymmetry strings have some features that are lacking in the nonsupersymmetric versions. Also, string solutions can emerge from globally supersymmetric theories from a spontaneous symmetry breaking due to either a D term (D-type strings) or an F term (F-type strings) in the scalar potential [8,7]. Since the scalar potential of a supergravity theory can be more complicated and may take quite a different appearance than that of a globally supersymmetric theory with the same form of superpotential, it may not be immediately obvious whether a given superpotential will generate scalar potentials in global supersymmetry and supergravity models that will possess the same types of vacuum manifolds that have the same topology. Therefore, attention is focused here on the scalar potential and vacuum manifold resulting from a given superpotential in supergravity, and comparisons can then be made to the corresponding vacuum manifold arising from the same superpotential in a globally supersymmetric theory containing the same matter chiral superfields. Under certain simple conditions for the D term potential and the superpotential, the same type of string solution will be admitted by both the globally supersymmetric and supergravity theories. However, if these conditions on the superpotential are not met, the two theories can yield quite different solutions. An example can be seen for the case of a constant superpotential, which is dynamically irrelevant in global SUSY, but can lead to spontaneous symmetry breaking in supergravity. Since supergravity can accommodate a negative cosmological constant and global SUSY cannot, there are some string solutions admitted by supergravity that have no counterparts in global SUSY. This result can be viewed as arising from gravitational effects that are included in supergravity, but not in global SUSY.

We concern ourselves mostly with the situation where the cosmological constant Λ vanishes. For the case of a global SUSY theory with a superpotential W, we conclude that a topological string solution is admitted if the vacuum manifold, characterized by the field vacuum configuration $\varphi=\{\varphi^i\}$, possesses a nontrivial topology (such as the topology of S^1) and can be obtained from the conditions (i) $\langle D\rangle=0$ and (ii) $\langle\partial W/\partial\phi^i\rangle=0$. For the case of supergravity, a string solution is admitted if the same kind of vacuum configuration can be obtained from the conditions (i) $\langle D\rangle=0$, (ii) $\langle\partial W/\partial\phi^i\rangle=0$, and (iii) $\langle W\rangle=0$. These conditions are seen to be necessary and sufficient to locate $\Lambda=0$ vacuum states with unbroken supersymmetry. They can be quite useful since the functions D and W are much easier to examine than is the scalar potential.

In the next section we focus upon the bosonic sector and scalar potential of global SUSY theories, and recall some of the various types of string solutions that occur there. Specifically, we present examples of the D-type gauge string, the F-type global or gauge string, and the superconducting Witten string [9], each of which is surrounded by a supersymmetric vacuum with zero cosmological constant $\Lambda=0$. The scalar potential of supergravity is presented in Sec. III, and we show that the same types of $\Lambda=0$ string solutions exist here as well, along with $\Lambda\neq0$ strings. The minimal Kähler potential is extended to a general form in Sec. IV, and a brief summary forms Sec. V.

II. GLOBAL SUPERSYMMETRY STRINGS

Let us consider a globally supersymmetric theory with interacting chiral superfields Φ^i and a U(1) vector superfield

A. The matter chiral supermultiplets can be represented as $\Phi^i=(\phi^i,\psi^i,F^i)$ and the vector multiplet by $A=(A^\mu,\lambda_\alpha,\overline{\lambda}_{\dot\alpha},D)$, where F^i and D are the bosonic auxiliary fields, ψ^i is the matter fermion, and λ_α is the photino.[1] For the present we choose the minimal Kähler potential $K=\overline{\phi^i}\phi^i=\Sigma_i\phi^{*i}\phi^i$ which yields the diagonal Kähler metric $K_{i\overline{j}}=\partial^2K/\partial\phi^i\partial\overline{\phi^j}=\delta_{ij}=K^{i\overline{j}}$ and canonical kinetic terms. The bosonic sector of the theory is described by the Lagrangian

$$L_B=K^{i\overline{j}}(D_\mu\phi^i)(D^\mu\phi^j)^*-\tfrac{1}{4}(F_{\mu\nu})^2-V, \tag{1}$$

with a sum over i implied, and where $D_\mu\phi^i=(\partial_\mu+igQ_iA_\mu)\phi^i$ is the gauge covariant derivative of the field ϕ^i with a U(1) charge Q_i, and $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. The scalar potential is

$$V=\sum_i |F^i|^2+\frac{1}{2}D^2=\sum_i \left|\frac{\partial W}{\partial\phi^i}\right|^2+\frac{1}{2}\left[\xi+g\sum_i Q_i\overline{\phi^i}\phi^i\right]^2, \tag{2}$$

where the holomorphic function $W=W(\phi^i)$ is the superpotential and the constant ξ comes from a Fayet-Iliopoulos term [10] that has been included and $\overline{F^i}=-\partial W/\partial\phi^i$, $D=-[\xi+g\Sigma_iQ_i\overline{\phi^i}\phi^i]$. A superpotential of the form $W=W_0+a_i\phi^i+b_{ij}\phi^i\phi^j+c_{ijk}\phi^i\phi^j\phi^k$ allows for renormalizability, and since the constant W_0 is dynamically irrelevant, we can set it equal to zero.

Let us use the derivative notation $X_i=\partial X/\partial\phi^i$, $X_{\overline{j}}=\partial X/\partial\overline{\phi^j}$, $\overline{X}_{\overline{j}}=\partial\overline{X}/\partial\overline{\phi^j}$, $X_{i\overline{j}}=\partial^2X/\partial\phi^i\partial\overline{\phi^j}$, $X_iX_{\overline{i}}=\Sigma_iX_iX_{\overline{i}}$, etc., for some function $X(\phi,\overline{\phi})$, with a sum over repeated indices unless otherwise stated. The vacuum expectation value (VEV) $\langle\phi^i\rangle=\varphi^i$ is located at the minimum of V where

$$V_i=\overline{W}_{\overline{k}}W_{ki}+DD_i=\overline{F^k}F^k_i+DD_i \tag{3}$$

vanishes. We also note that $V\geqslant 0$ so that a negative cosmological constant does not appear. The vacuum state is supersymmetric if $V(\varphi)=0$, but supersymmetry is spontaneously broken by the vacuum if $V(\varphi)>0$. From Eq. (3) it is seen that a nonzero VEV $\varphi^i\neq 0$ can develop from either the F term or the D term in V, resulting in either F-type or D-type strings [8]. Abelian F-type and D-type strings in global SUSY theories are described and discussed in Ref. [8], to which the reader is referred, and a global SUSY model of a local superconducting Witten string, which is an F-type string, is discussed in Ref. [7]. Below we list some of the basic features of these string solutions, and in the next section these types of strings will be reexamined in the context of supergravity.

D-type string. An example of a D-type string is given in [8]. The model contains a primary charged chiral superfield Φ that is involved with spontaneous symmetry breaking, along with other charged chiral superfields (to avoid a gauge anomaly) and possibly neutral chiral superfields. The various chiral superfields interact in such a way that only the one complex scalar field ϕ with U(1) charge Q develops an expectation value, with $W=0$ and $W_i=0$ in the vacuum state. By including a Fayet-Iliopoulos term, setting $\xi=-gQ\eta^2$, and setting to zero the charged scalar fields with vanishing vacuum values, the scalar potential due to the symmetry-breaking field ϕ that arises from the D term is $V_D=\frac{1}{2}g^2Q^2(\overline{\phi}\phi-\eta^2)^2$. This is the type of potential found in the ordinary broken-symmetric Abelian-Higgs model, which admits a Nielsen-Olesen cosmic U(1) gauge string [11]. In the vacuum state, outside the string, $|\phi|=|\varphi|=\eta$ and $D(\varphi)=0$ with $V(\varphi)=0$, which implies that supersymmetry is not broken in the vacuum, and the cosmological constant vanishes, $\Lambda=0$. The structure of the vacuum manifold is determined entirely by the D term in the scalar potential.

F-type string. An F-type string model can be built from one neutral superfield $\mathcal{Z}=(Z,\psi_Z,F_Z)$ and two charged chiral superfields $\Phi_\pm=(\phi_\pm,\psi_\pm,F_\pm)$ with $Q_++Q_-=0=Q_Z$. A Fayet-Iliopoulos term is not included and the superpotential is taken to be $W=\lambda\mathcal{Z}(\Phi_+\Phi_--\eta^2)$. The lowest energy state of the theory that spontaneously breaks the U(1) symmetry is a supersymmetric vacuum state with $V=0$, determined by the conditions $F^i=0$, $D=0$, and is characterized by $|\langle\phi_\pm\rangle|=|\varphi_\pm|=\eta$, $\langle Z\rangle=0$. Using the simplifying ansatz $\phi_+=\phi$, $\phi_-=\overline{\phi}$, the D term vanishes and scalar potential reduces to

$$V=\lambda^2(\overline{\phi}\phi-\eta^2)^2+2\lambda^2\overline{Z}Z(\overline{\phi}\phi). \tag{4}$$

For the case in which Z is set equal to its vacuum value of zero, the model reduces to a broken-symmetric Abelian-Higgs model, admitting a U(1) gauge string associated with the field ϕ. Since $\phi=\phi_+=\overline{\phi_-}$, we see that the two fields $\phi_\pm$ conspire to form this F-type string. In this case the vacuum manifold for ϕ and Z is determined entirely by the F terms, where $F_+=\lambda Z\phi_-\to\lambda Z\overline{\phi}$, $F_-=\lambda Z\phi_+\to\lambda Z\phi$, $F_Z=\lambda(\phi_+\phi_--\eta^2)\to\lambda(\overline{\phi}\phi-\eta^2)$. Again, the vacuum is supersymmetric and $\Lambda=0$. [Note that we could replace the U(1) gauge symmetry in this model with a global U(1) symmetry, remove the vector supermultiplet, and take $g\to 0$ to get a model of a global string.]

Superconducting Witten string. We can construct a model of a $\mathrm{U}(1)\times\mathrm{U}(1)'$ superconducting global string, with long range gauge fields outside of the string, from two complex scalar fields $\sigma_\pm$ transforming nontrivially under a global group $\mathrm{U}(1)'$, two complex scalars $\phi_\pm$ transforming nontrivially under a local group U(1), and a neutral scalar Z. The fields $\sigma_\pm$ have $\mathrm{U}(1)'$ global charges $Q_\pm$ with $Q_++Q_-=0$, and the fields $\phi_\pm$ have U(1) local charges $q_\pm$ with $q_++q_-=0$. A Fayet-Iliopoulos term is not included and the superpotential, in terms of the scalar fields, is taken to be

$$W=\lambda Z(\sigma_+\sigma_--\eta^2)+(cZ+m)\phi_+\phi_-. \tag{5}$$

[1] Aside from a spacetime metric with signature given by $(+,-,-,-)$, we use, for the most part, the notation and conventions of Wess and Bagger [6]. Units are chosen where $M=M_P/\sqrt{8\pi}=1$, where $M_P=G^{-1/2}$ is the Planck mass. Factors of M can be reintroduced by dimensional considerations.

The scalar potential is $V=\Sigma_k|W_k|^2+\frac{1}{2}D^2$, where $D=gq_+(\overline{\phi_+}\phi_+-\overline{\phi_-}\phi_-)$, and the individual W_k terms are given by

$$\frac{\partial W}{\partial\sigma_\pm}=\lambda Z\sigma_\mp\,,$$

$$\frac{\partial W}{\partial\phi_\pm}=(cZ+m)\phi_\mp\,, \tag{6}$$

$$\frac{\partial W}{\partial Z}=\lambda(\sigma_+\sigma_--\eta^2)+c\phi_+\phi_-\,.$$

The supersymmetric vacuum state with $V=0$ that spontaneously breaks the $\mathrm{U}(1)'$ symmetry but respects the $\mathrm{U}(1)$ symmetry is located where $W_k=0$ and $D=0$, i.e., where the fields take values $\sigma_+\sigma_-=\eta^2$, $\phi_+=\phi_-=0$, and $Z=0$. One can adopt the ansatz $\sigma_+=\sigma$, $\sigma_-=\bar{\sigma}$, $\phi_+=\phi$, $\phi_-=\bar{\phi}$, so that the scalar potential can be written in terms of the fields σ, ϕ, and Z, and a superconducting string solution can be inferred. Here, again, $\Lambda=0$, since the vacuum is globally supersymmetric.

A point to be made here, which will be useful in the next section, is that for each of the examples above for $\Lambda=0$ strings, we have not only $W_k(\varphi)=0$, but also $W(\varphi)=0$.

III. SUPERGRAVITY STRINGS

Now consider a supergravity theory accommodating matter chiral superfields $\Phi^i=(\phi^i,\psi^i,F^i)$ and the vector multiplet $A=(A^\mu,\lambda_\alpha,\overline{\lambda}_{\dot\alpha},D)$. We again choose the minimal Kähler potential $K=\overline{\phi^i}\phi^i$ so that $K_i=\overline{\phi^i}$, $K_{\bar{i}}=\phi^i$, and $K_{i\bar{j}}=K^{i\bar{j}}=\delta_{ij}$. The bosonic sector of the theory is described by

$$e^{-1}\mathcal{L}_B=-\tfrac{1}{2}R+K_{i\bar{j}}(D_\mu\phi^i)(D^\mu\phi^j)^*-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}-\mathcal{V}, \tag{7}$$

where $e=(-\det g_{\mu\nu})^{1/2}$. Upon defining $D_iW=W_i+K_iW$, the scalar potential can be written as

$$\mathcal{V}=e^K\mathcal{U}+\tfrac{1}{2}D^2, \tag{8}$$

where

$$\mathcal{U}=K^{i\bar{j}}(D_iW)(D_jW)^*-3W^*W=|D_iW|^2-3|W|^2, \tag{9}$$

with an implied sum over the repeated indices i and j, and $D=g\Sigma_iQ_i\overline{\phi^i}\phi^i$, as before. For the Kähler potential $K=\overline{\phi^i}\phi^i$ we have the operator $D_i=\partial/\partial\phi^i+\overline{\phi^i}$, so that we can also write $\mathcal{U}$ in terms of W and W_i as

$$\mathcal{U}=|W_i|^2+(|\phi^i|^2-3)|W|^2+(\phi^iW_i\overline{W}+\overline{\phi^iW_i}W). \tag{10}$$

We notice that the supergravity scalar potential, having gravitational contributions, is more complicated than the scalar potential for global SUSY, and that $\mathcal{V}$ can be positive, negative, or zero. Whereas the signal of spontaneous supersymmetry breaking is given by $W_i\neq0$ or $D\neq0$ in global SUSY, in supergravity the signal is given by $D_iW\neq0$ or $D\neq0$.

If we reinsert factors of $M=M_P/\sqrt{8\pi}$ by introducing the constant $\kappa=M^{-1}$ with $W\to\kappa^3W$, $D_iW\to\kappa^2W_i+\kappa^4\overline{\phi^i}W$, and $\mathcal{V}\to\kappa^{-4}\mathcal{V}$, we can write the scalar potential as

$$\mathcal{V}=\exp(\kappa^2\overline{\phi^i}\phi^i)\{|W_i+\kappa^2\overline{\phi^i}W|^2-3\kappa^2|W|^2\}+\tfrac{1}{2}D^2. \tag{11}$$

In the small κ (large M) limit we could expand $\mathcal{V}$ as $\mathcal{V}\approx|W_i|^2+\frac{1}{2}D^2$ with $O(\kappa^2)$ corrections; i.e., the supergravity scalar potential should just be the global SUSY potential with $O(\kappa^2)$ corrections, which would presumably be small. This would lead us to believe that, for low symmetry-breaking energy scales, any superpotential that gives rise to string solutions in global SUSY will give rise to the same string solutions in supergravity, and that if no strings are predicted by global SUSY, then none will be predicted by supergravity either. This seems reasonable if all field vacuum values are small compared to M, but this need not be the case, in general. Also, a more general Kähler potential could be used, to which global SUSY and supergravity can have different sensitivities. This situation is examined later, where the minimal Kähler potential is replaced by a general Kähler potential. However, we can use the *exact* form of $\mathcal{V}$ to see that under certain simple conditions involving the D term potential and the superpotential, a $\Lambda=0$ global SUSY string with unbroken supersymmetry in the vacuum will also show up in supergravity as a $\Lambda=0$ supergravity string with unbroken supersymmetry in the vacuum, *regardless* of the scale of the symmetry breaking. This is not guaranteed to be the case, however, if these conditions are not met.

A. Vacuum states

The vacuum states, labeled by $\langle\phi^i\rangle=\varphi^i$, are located where $\mathcal{V}_i=\partial\mathcal{V}/\partial\phi^i$ vanishes. From Eq. (8),

$$\mathcal{V}_i=e^K(\mathcal{U}_i+\overline{\phi^i}\mathcal{U})+D\partial D/\partial\phi^i. \tag{12}$$

From Eq. (10) we can display $\mathcal{U}_i$ as

$$\mathcal{U}_i=(\overline{W_k}+\phi^k\overline{W})W_{ki}+[(|\phi^k|^2-2)\overline{W}+\overline{\phi^k}\overline{W_k}]W_i+\overline{\phi^i}|W|^2. \tag{13}$$

Solutions to $\mathcal{V}_i=0$ are given by

$$\mathcal{U}_i+\overline{\phi^i}\mathcal{U}=0,\quad D\frac{\partial D}{\partial\phi^i}=(gQ_i\overline{\phi^i})D=0. \tag{14}$$

We note that the field ϕ^i develops a nonzero VEV if the curvature of $\mathcal{V}$ in the ϕ^i direction (i.e., the mass term for ϕ^i) is negative at the origin, e.g., $\mathcal{V}^{(0)}_{i\bar{i}}=(\partial^2\mathcal{V}/\partial\phi^i\partial\overline{\phi^i})|_{\phi=0}<0$, where ϕ collectively represents the set $\{\phi^k\}$. Assuming a superpotential of the form

$$W=W_0+a_i\phi^i+b_{ij}\phi^i\phi^j+c_{ijk}\phi^i\phi^j\phi^k+\cdots, \tag{15}$$

we can calculate

$$\mathcal{V}^{(0)}_{i\bar{j}} = \delta_{ij}\mathcal{U}^{(0)} + \mathcal{U}^{(0)}_{i\bar{j}} + \tfrac{1}{2}[\partial^2(D^2)/\partial\phi^i\partial\overline{\phi^j}]|_0$$
$$= \delta_{ij}(\overline{a}_k a_k - 2|W_0|^2) - a_i\overline{a}_j + 4b_{ki}\overline{b}_{kj} + gQ_i\delta_{ij}\xi, \quad (16)$$

where ξ comes from a Fayet-Iliopoulos term, giving

$$\mathcal{V}^{(0)}_{i\bar{i}} = -2|W_0|^2 - |a_i|^2 + \sum_k (|a_k|^2 + 4|b_{ki}|^2) + gQ_i\xi. \quad (17)$$

The corresponding curvature for the case of global SUSY is

$$V^{(0)}_{i\bar{i}} = 4\sum_k |b_{ki}|^2 + gQ_i\xi. \quad (18)$$

We can therefore see that whereas the constant term W_0 is innocuous in global SUSY, in a supergravity model it contributes toward a destabilization of the normal vacuum state $\langle\phi^i\rangle = 0$, so that for a large enough value of $|W_0|$, spontaneous symmetry breaking is induced. Therefore, although a particular superpotential may not lead to a symmetry breaking in global SUSY, due to an invisibility of W_0, the same superpotential can lead to a symmetry breaking in supergravity. However, we will see that when the cosmological constant vanishes and the D term and superpotential satisfy certain simple conditions, the vacuum states in global SUSY will coincide with those in supergravity.

B. $\Lambda = 0$ strings

From Eq. (8), we can note that for a spontaneously broken symmetry the cosmological constant will vanish, i.e., $\langle\mathcal{V}\rangle = \Lambda = 0$, provided that $\langle\mathcal{U}\rangle = \langle D\rangle = 0$. In this case, by Eq. (14), the condition $\mathcal{V}_i = 0$ is satisfied when the conditions $\mathcal{U}_i = 0$ and $(gQ_i\overline{\phi^i})D = 0$ are simultaneously satisfied. For $\langle\phi^i\rangle \neq 0$, $\langle D\rangle$ can vanish either because of symmetry or because of a symmetry-breaking Fayet-Iliopoulos term. Assuming this to be the case, we are left with the condition $\mathcal{U}_i = 0$. This condition will be met automatically if the field expectation values $\langle\phi^i\rangle = \varphi^i$ can be obtained from the conditions $W = 0$ and $W_i = 0$ when $\{\phi^k\} = \{\varphi^k\}$. Therefore, if a globally supersymmetric theory possesses a nontrivial vacuum configuration $\varphi = \{\varphi^i\}$ that can be extracted from the conditions

$$\text{(i)}\ \langle D\rangle = 0, \quad \text{(ii)}\ \langle W_i\rangle = W_i(\varphi) = 0,$$
$$\text{(iii)}\ \langle W\rangle = W(\varphi) = 0, \quad (19)$$

and as a result admits a string solution, then the same vacuum configuration φ appears in the supergravity theory and hence a string solution appears there, although, due to differences in the forms of the scalar potentials and fermionic couplings, the two string solutions will in general be quantitatively different. We can briefly look at the D-type string, the F-type string, and the superconducting Witten string of the previous section for specific examples.

1. D-type string

As in the global SUSY case, the superpotential for this model satisfies $\langle W\rangle = 0$, $\langle W_i\rangle = 0$. A Fayet-Iliopoulos term is included, and the scalar field ϕ that is nonvanishing in the vacuum gives a nontrivial contribution to the D term, so that we can write, as in the case of global SUSY (where we again set to zero the scalar fields with vanishing vacuum values), $D = gQ\overline{\phi}\phi + \xi = gQ(\overline{\phi}\phi - \eta^2)$. For the vacuum state where $|\langle\phi\rangle| = |\varphi| = \eta$, we have, as before, $\langle D\rangle = 0$, $\langle W\rangle = 0$, $\langle W_i\rangle = 0$, indicating the existence of a D-type string in the supergravity theory. Also, since $D_iW = 0$ in the vacuum state, supersymmetry is respected in the vacuum. Furthermore, from Eq. (17), we see that $\mathcal{V}^{(0)}_{\phi\bar{\phi}} = -(gQ\eta)^2$.

2. F-type string

For this model we have $D = gQ_+(\overline{\phi_+}\phi_+ - \overline{\phi_-}\phi_-)$ and the superpotential is $W = \lambda Z(\phi_+\phi_- - \eta^2)$. The W_i are given by $W_Z = \lambda(\phi_+\phi_- - \eta^2)$, $W_\pm = \lambda Z\phi_\mp$, so that $W_i = 0$ is satisfied by $Z = 0$, $\phi_+\phi_- = \eta^2$, and consequently $W = 0$. The condition $\langle D\rangle = 0$ implies that $|\langle\phi_+\rangle| = |\langle\phi_-\rangle| \equiv |\langle\phi\rangle|$, in accordance with the ansatz implemented previously. Since $D_iW|_{\phi=\varphi} = W_i(\varphi) + \varphi^i W(\varphi) = 0$, we have an F-type string surrounded by supersymmetric vacuum.

3. Superconducting Witten string

The superpotential is given by Eq. (5), and by Eq. (6) we have $\sigma_+\sigma_- = \eta^2$, $\phi_\pm = 0$, $Z = 0$ as solutions of $W_i = 0$, and consequently $W(\varphi) = 0$. Requiring $\langle D\rangle = 0$ gives $|\langle\phi_+\rangle| = |\langle\phi_-\rangle|$ as in our ansatz. $D_iW = 0$ in the vacuum, so that the supergravity theory admits a solution describing a superconducting string embedded in supersymmetric vacuum.

C. Anti–de Sitter $(\Lambda < 0)$ strings

Global SUSY can not accommodate a negative cosmological constant (anti–de Sitter spacetime), since $V = |W_k|^2 + \frac{1}{2}D^2 \geqslant 0$, but supergravity can, and consequently supergravity can admit $\Lambda < 0$ string solutions describing strings embedded in anti–de Sitter spacetime that have no counterparts in global SUSY. As a simple example we take a model with a single chiral superfield Φ invariant under a global U(1) symmetry, so that $D = 0$. We choose a real constant superpotential $W = W_0$. By Eqs. (8) and (10) the scalar potential takes the form

$$\mathcal{V} = W_0^2 e^K(|\phi|^2 - 3). \quad (20)$$

The vacuum state is located by $|\phi| = \sqrt{2}$. The global U(1) symmetry is broken, and a global string solution [12] is admitted. Since $D_\phi W = \overline{\phi}W_0$ does not vanish in the vacuum, supersymmetry is also broken. However, in the core of the string where $\phi \to 0$, supersymmetry is apparently restored. This seems to be the opposite of the case with the $\Lambda = 0$ strings above, where the vacuum is supersymmetric and supersymmetry is broken in the string core. For this supermassive anti–de Sitter string, $|\langle\phi\rangle| \sim M$, and gravity is expected to play an important role.

IV. GENERAL KÄHLER POTENTIAL AND $\Lambda=0$ STRINGS

For a general Kähler potential $K(\phi,\overline{\phi})$ the scalar potential for global SUSY is

$$V=K^{i\overline{j}}W_i\overline{W_{\overline{j}}}+\tfrac{1}{2}D^2, \tag{21}$$

where $K^{i\overline{j}}=(K^{-1})_{i\overline{j}}$ and $D=(g\Sigma_i Q_i K_i\phi^i+\xi)$. This potential is minimized by a set of field configurations $\varphi=\{\varphi^i\}$ when

$$V_k=(K^{i\overline{j}})_k W_i\overline{W_{\overline{j}}}+K^{i\overline{j}}W_{ik}\overline{W_{\overline{j}}}+D\frac{\partial D}{\partial\phi^k} \tag{22}$$

vanishes. The scalar potential for supergravity is

$$\mathcal{V}=e^K\mathcal{U}+\tfrac{1}{2}D^2=e^K[K^{i\overline{j}}(D_iW)(D_jW)^*-3|W|^2]+\tfrac{1}{2}D^2, \tag{23}$$

where $D_iW=W_i+K_iW$. This scalar potential is minimized when

$$\mathcal{V}_k=e^K(\mathcal{U}_k+K_k\mathcal{U})+D\frac{\partial D}{\partial\phi^k} \tag{24}$$

vanishes, with

$$\mathcal{U}_k=(\partial_k K^{i\overline{j}})(D_iW)(D_jW)^*+K^{i\overline{j}}[(\partial_k D_iW)(D_jW)^* +(D_iW)\partial_k(D_jW)^*]-3\overline{W}W_k, \tag{25}$$

where $\partial_k=\partial/\partial\phi^k$.

Now let us assume that D vanishes in the vacuum, $\langle D\rangle=0$, and that the cosmological constant vanishes, $\Lambda=0$. For global SUSY, $\Lambda=0$ requires that W_i vanish in the vacuum; i.e., the vacuum values $\langle\phi^i\rangle=\varphi^i$ can be determined from the condition $W_i(\varphi)=0$. If we furthermore require that $W(\varphi)=0$, then we see that $(D_iW)|_\varphi=0$, and therefore $\mathcal{U}=0$ and $\mathcal{U}_k=0$, so that $\mathcal{V}$ is minimized by the vacuum field configuration φ. We therefore conclude that if the superpotential W possesses a global or local U(1) symmetry which is spontaneously broken by the vacuum, and (i) $\langle D\rangle=0$ and (ii) $W_i(\varphi)=0$, then a $\Lambda=0$ string solution exists in the global SUSY theory. If, in addition, we have that (iii) $W(\varphi)=0$, then a $\Lambda=0$ string solution exists in the supergravity theory. (However, it does not necessarily follow that a $\Lambda=0$ supergravity string solution does *not* exist if these conditions are *not* satisfied.) We note that although these conditions are stated for a general Kähler potential, the condition $\langle D\rangle=0$ may constrain the form of K when a local U(1) symmetry is present.

Actually, the conditions (i)–(iii) above are *necessary* and *sufficient* conditions to locate the $\Lambda=0$ vacuum states of the theory with *unbroken supersymmetry*. This can be easily seen by noticing that unbroken supersymmetry requires that $\langle D\rangle=0$ and $\langle D_iW\rangle=0$. Then, from Eq. (9), it follows that $\langle\mathcal{U}\rangle=-3\langle|W|^2\rangle$, and therefore $\Lambda=0$ implies that $\langle W\rangle=W(\varphi)=0$. Since $D_iW=W_i+K_iW=0$ in the vacuum, it follows that $\langle W_i\rangle=0$. If conditions (i)–(iii) are simultaneously satisfied, then they necessarily locate the $\Lambda=0$ supersymmetric vacuum states of a theory. If this set of vacuum states has a vacuum manifold characterized by a topology where the first homotopy group is nontrivial, then a topological string solution must exist. However, if the conditions (ii) and (iii) do not hold simultaneously, then the vacuum states of the theory do not simultaneously have unbroken supersymmetry and $\Lambda=0$. String solutions may exist, but they will not be $\Lambda=0$ strings embedded within supersymmetric vacuum.

Of course, we expect that if the global SUSY or supergravity theory possesses a $\Lambda=0$ string solution, then the Kähler-transformed theory will also have a string solution, since the bosonic Lagrangians are invariant under Kähler transformations. I.e., for global SUSY L_B and V are invariant under the Kähler transformation[2] $K(\phi,\overline{\phi})\rightarrow G(\phi,\overline{\phi})=K+F(\phi)+\overline{F(\phi)}$ and $W\rightarrow W$, where $F(\phi)$ is an analytic function of the ϕ^i. For supergravity, however, $\mathcal{L}_B$ and $\mathcal{V}$ are invariant under the transformation $K\rightarrow G$ provided that $W\rightarrow\widetilde{W}=e^{-F}W$. If $\langle D\rangle$ remains zero under a Kähler transformation, and if $W(\varphi)=0$, $W_i(\varphi)=0$, then $\widetilde{W}_i(\varphi)=0$ and $\widetilde{W}(\varphi)=0$, provided that $F_i(\varphi)$ is finite, so that the conditions for W are also satisfied for $\widetilde{W}$. However, if we consider the Kähler *inequivalent* theories generated by the replacements $K\rightarrow G$, $W\rightarrow W$, or $K\rightarrow K$, $W\rightarrow\widetilde{W}$, then we see that the conditions on the superpotential can still be maintained, since $\widetilde{W}(\varphi)=e^{-F}W(\varphi)=0$ and $\widetilde{W}_i(\varphi)=e^{-F}[W_i(\varphi)-F_iW(\varphi)]=0$. In other words, *different* theories that have the *same* superpotential, but *different* Kähler potentials, or *different* theories that have the *same* Kähler potential but *different* superpotentials W and $\widetilde{W}$ related by $\widetilde{W}=e^{-F}W$ can yield the same type of string solution, provided that the conditions (i)–(iii) above hold. For example, for an F-type global string model with a superpotential $W=\lambda Z(\phi_+\phi_--\eta^2)$, with $|\langle\phi_\pm\rangle|=\eta$, $\langle Z\rangle=0$, conditions (i)–(iii) are satisfied, with $D=0$, so that any other theory generated by a superpotential $\widetilde{W}=\exp[-F(Z,\phi_+,\phi_-)]W$ should also yield a global string solution, assuming that $\langle F_i\rangle$ is finite. Since $F(\phi)$ is arbitrary, we see that there are an infinite number of superpotentials that can generate the same type of string solution. (Renormalizability considerations in global SUSY, however, can reduce the spectrum of acceptable superpotentials.)

V. SUMMARY

Cosmic strings may have been produced during symmetry-breaking phase transitions in the early universe. It is also possible that supersymmetry, perhaps in the form of supergravity, was effectively realized at early epochs. It is then natural to examine cosmic strings in supersymmetric theories. To determine whether a field theory will yield a topological string solution, it is necessary to have information about the topology of the vacuum manifold. This information is embedded within the scalar potential, and so it is sufficient to focus attention upon this function. The scalar

[2]We avoid a Kähler gauge where $G(\phi,\overline{\phi})=K(\overline{\phi}\phi)+\ln W+\ln\overline{W}$, since G becomes undefined when $W=0$.

potential in a supersymmetric theory is constructed from terms that depend upon the functions D, W, and K, where W is the superpotential and K is the Kähler potential. The functions D and W can yield useful information about the vacuum states of the theory if these functions satisfy certain simple conditions. This can be advantageous, since the functions D and W are generally much simpler in structure than is the scalar potential. The existence of cosmic string solutions in a supersymmetric theory can be inferred when a topologically nontrivial vacuum manifold can be extracted directly from these functions.

The scalar potentials of global SUSY and supergravity have different forms for a given superpotential, with the supergravity version being generally more complicated than the global SUSY one. Therefore, it is not immediately obvious whether the existence of a string solution in a global SUSY theory implies the existence of a string solution in the supergravity theory or vice versa. In fact, because supergravity can support a negative cosmological constant and global SUSY cannot, there can be string solutions in supergravity that have no counterparts in global SUSY. Also, even in the case of $\Lambda=0$, if the energy scale of the symmetry breaking is large (say, $\varphi \sim M$), then the gravitational effects inherent in supergravity may be important.

From an examination of the structure of scalar potentials the following conclusions have been drawn for the situation where the cosmological constant Λ vanishes. If a vacuum manifold with nontrivial topology (nontrivial first homotopy group) can be determined from the conditions (i) $\langle D\rangle=0$ and (ii) $\langle \partial W/\partial\phi^i\rangle=0$, then a topological string solution is admitted in a globally supersymmetric theory. This will be a $\Lambda=0$ string embedded within supersymmetric vacuum. If, in addition, the condition (iii) $\langle W\rangle=0$ is satisfied, then the existence of a topological string solution in a supergravity theory can be inferred. Therefore, when conditions (i)–(iii) are satisfied, a $\Lambda=0$ string solution (which asymptotically relaxes into a vacuum with unbroken supersymmetry) that emerges from the global SUSY theory is expected to be accommodated by a string solution of the same type in the corresponding supergravity theory, and vice versa, although the two solutions are expected to be quantitatively different, due to the different structures of the scalar potentials. Using these conditions, it has been shown that several types of string solutions, such as D-type strings, F-type strings, and superconducting Witten strings, exist in gauge-invariant supergravity models.

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