Osp(1|2) Chern-Simons gauge theory as 2D $N=1$ induced supergravity

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We demonstrate the close relationship between Chern-Simons gauge theory with the gauge group Osp(1|2) and $N=1$ induced supergravity in two dimensions. More precisely, the inner product of the physical states in the former yields the partition function of the latter evaluated in the Wess-Zumino supergauge. It is also shown that the moduli space of flat Osp(1|2) connections naturally includes a super Teichmüller space of super Riemann surfaces. [S0556-2821(97)04516-5]

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It is important to quantize two-dimensional (2D) induced (super)gravity because it describes the dynamics of a string world sheet induced by the motion of (super)strings in the background without critical dimensions [1]. A number of interesting phenomena such as the fractal structure have been clarified by vigorous works made in the light-cone gauge [2,3] and in the conformal gauge [4,5]. A local expression of the generally covariant action for 2D induced bosonic gravity was obtained by Verlinde [6] in terms of the Beltrami coefficients. More precisely, he showed that the partition function of the induced 2D gravity is obtained from the inner product of physical states in SL(2,**R**) Chern-Simons gauge theory. He also showed explicitly how we can extract the Teichmüller parameters of Riemann surfaces from the holonomy of flat SL(2,**R**) connections.

In this paper we extend Verlinde's results to 2D $N=1$ induced supergravity. Namely, starting from Osp(1|2) Chern-Simons gauge theory, we obtain the supercovariant action for 2D $N=1$ induced supergravity in the Wess-Zumino supergauge. We also demonstrate that the moduli space of flat Osp(1|2) connections yields super Teichmüller space of DeWitt super Riemann surfaces with arbitrary spin structures.

Let us begin by briefly reviewing the Beltrami parametrization of supervielbeins in 2D $N=1$ supergravity. A DeWitt super Riemann surface $S\Sigma$ [7] is a fiber bundle over a Riemann surface Σ [with the coordinate reference frame $(z,\bar{z})$] whose fiber is a vector space parametrized by two Grassmann odd coordinates $(\theta,\bar{\theta})$. The supervielbein for the rigid superspace is given by[1] $e^z=dz+\theta d\theta$, $e^\theta=d\theta$, and their complex conjugates. Their dual vectors are $\partial=\partial/\partial z$, $D=\partial/\partial\theta+\theta\partial/\partial z$, and their complex conjugates. According to Refs. [9,10], any supervielbein $\{E^A;A=+,-,\hat{1},\hat{2}\}$ which represents Howe's superspace geometry [11] can be written as

$$E^+=\rho e^Z,\quad E^-=\overline{E^+},$$

$$E^{\hat{1}}=\sqrt{\rho}\{e^\Theta+\tfrac{1}{2}e^Z D_\Theta \ln(\rho\bar{\rho})\},\quad E^{\hat{2}}=\overline{E^{\hat{1}}},\tag{1}$$

where ρ is the superconformal factor. $e^Z\equiv dZ+\Theta d\Theta$, $e^\Theta\equiv d\Theta$, and their complex conjugates constitute a local basis of one-forms on $S\Sigma$. Their dual vectors are given by $\partial_Z=\partial/\partial Z$, $D_\Theta=\partial/\partial\Theta+\Theta\partial/\partial Z$, and their complex conjugates.

Because $(Z,\bar{Z})$ and $(\Theta,\bar{\Theta})$ are smooth scalar functions of coordinates $(z,\bar{z},\theta,\bar{\theta})$ which are Grassmann even and odd, respectively, we can expand e^Z and e^Θ by the rigid basis

$$e^Z=(e^z+e^{\bar{z}}H^z_{\bar{z}}+e^\theta H^z_\theta+e^{\bar{\theta}}H^z_{\bar{\theta}})\Lambda \quad \text{and c.c.,}\tag{2}$$

$$e^\Theta=(e^z+e^{\bar{z}}H^z_{\bar{z}}+e^\theta H^z_\theta+e^{\bar{\theta}}H^z_{\bar{\theta}})\tau$$
$$+(e^\theta H^\theta_\theta+e^{\bar{z}}H^\theta_{\bar{z}}+e^{\bar{\theta}}H^\theta_{\bar{\theta}})\sqrt{\Lambda}\quad \text{and c.c.}$$

Here H^N_M $(M,N=z,\bar{z},\theta,\bar{\theta})$ are called Beltrami coefficients and Λ and τ are called integrating factors. Owing to the structure equations $de^Z=-e^\Theta e^\Theta$ and $de^\Theta=0$, not all of these coefficients are independent. It turns out that the Beltrami coefficients are local functionals of H^z_θ and $H^z_{\bar{\theta}}$ (and their complex conjugates) and that τ is locally expressed by Λ and H^z_θ. Moreover, the integrating factor Λ is subject to the equation

$$\left[\bar{D}-\left(\frac{H^\theta_{\bar{\theta}}}{H^\theta_\theta}\right)D-\left(H^z_{\bar{\theta}}-\frac{H^\theta_{\bar{\theta}}}{H^\theta_\theta}H^z_\theta\right)\partial\right]\ln\Lambda=\partial H^z_{\bar{\theta}}-\frac{H^\theta_{\bar{\theta}}}{H^\theta_\theta}\partial H^z_\theta.\tag{3}$$

The integrating factor Λ is therefore regarded as a nonlocal functional of $(H^z_\theta,H^z_{\bar{\theta}})$ which is uniquely determined up to superconformal redefinitions of (Z,Θ). In consequence, 2D $N=1$ induced supergravity is described by 20 local degrees of freedom which are component fields of $(H^z_\theta,H^z_{\bar{\theta}},H^{\bar{z}}_{\bar{\theta}},H^{\bar{z}}_\theta)$ and of $\rho\bar{\rho}$. Instead of $\rho\bar{\rho}$ we will frequently use the super Liouville field $\Phi\equiv\ln(\rho\bar{\rho}\Lambda\bar{\Lambda})$ $=\phi+\theta\chi+\overline{\theta\chi}+\theta\bar{\theta}iF$. Two-dimensional $N=1$ supergravity is invariant under graded local Lorentz transformations $\rho\to e^{i\sigma}\rho$, $\bar{\rho}\to e^{-i\sigma}\bar{\rho}$ (σ is a real superfield). The associated

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[1]We will use the convention of Ref. [8] for differential forms. As for the complex conjugation of the product of two Grassmann odd variables, we adopt the rule $\overline{(\chi\eta)}=\bar{\chi}\bar{\eta}$.

covariant derivative for a superfield Ξ of Lorentz weight[2] w is given by $\mathcal{D}\Xi = d\Xi + iw\Xi\Omega$, where Ω is the graded spin connection

$$\Omega = -ie^Z\partial_Z \ln\overline{\rho} + ie^{\overline{Z}}\partial_{\overline{Z}}\ln\rho - ie^{\Theta}D_{\Theta}\ln\overline{\rho} + ie^{\overline{\Theta}}D_{\overline{\Theta}}\ln\rho. \tag{4}$$

From this graded spin connection we can calculate the superfield whose $\theta\overline{\theta}$ component yields the scalar curvature [10]. We will henceforth call this superfield the ''supercurvature'' R_3:

$$R_3 = -2(\rho\overline{\rho})^{-1/2}D_{\Theta}D_{\overline{\Theta}}\ln(\rho\overline{\rho}). \tag{5}$$

Because 2D $N=1$ supergravity should have symmetry under reparametrizations of the supercoordinates $(z,\overline{z},\theta,\overline{\theta})$, there are a large number of choices to fix a gauge. From now on we will consider a particular gauge fixing called the Wess-Zumino (WZ) supergauge:

$$H^z_{\theta}=0, \quad H^z_{\overline{\theta}} = \overline{\theta}\mu + \theta\overline{\theta}(-i\alpha). \tag{6}$$

This gauge-fixing condition is equivalent to the condition

$$Z = Z_0(z,\overline{z}) + \theta\Theta_0\sqrt{\partial Z_0 + \Theta_0\partial\Theta_0} \text{ and c.c.,}$$

$$\Theta = \Theta_0(z,\overline{z}) + \theta\sqrt{\partial Z_0 + \Theta_0\partial\Theta_0} \text{ and c.c.} \tag{7}$$

A merit of the WZ supergauge is that all the component fields (except the auxiliary field F) appear in the lowest component of the supervielbein. Thus it is convenient to introduce the zweibein $e^{\pm}(z,\overline{z})$ and the gravitino $\psi^{\alpha}(z,\overline{z})$ $(\alpha=\hat{1},\hat{2})$:

$$e^+ \equiv E^+_m|dx^m = e^{\varphi}(dz+\mu d\overline{z}), \quad e^- = \overline{e^+},$$

$$\psi^{\hat{1}} \equiv 2E^{\hat{1}}_m|dx^m = e^{\varphi/2}\{\chi(dz+\mu d\overline{z}) + i\alpha d\overline{z}\}, \quad \psi^{\hat{2}} = \overline{\psi^{\hat{1}}}. \tag{8}$$

Here we have used the notation $(x^m)=(z,\overline{z})$ and the vertical bar denotes the $\theta=\overline{\theta}=0$ component. We have also used φ to mean the $\theta=\overline{\theta}=0$ component of $\ln(\rho\Lambda)$. We should note the relation $\varphi+\overline{\varphi}=\phi$. The spin connection associated with the local Lorentz symmetry is given by

$$\omega \equiv \Omega| = -i(dz+\mu d\overline{z})\frac{(\partial-\overline{\mu}\overline{\partial})\overline{\varphi} - \overline{\partial}\overline{\mu} + \dfrac{i}{2}\overline{\alpha\chi}}{1-\mu\overline{\mu}} + \text{c.c.} \tag{9}$$

As a consequence of the torsion constraints [11,10] this spin connection satisfies

$$\mathcal{D}e^+ \equiv de^+ + ie^+\omega = -\tfrac{1}{4}\psi^{\hat{1}}\psi^{\hat{1}} \text{ and c.c.} \tag{10}$$

[2]For example, the Lorentz weights of $E^{\pm}$, $E^{\hat{1}}$, and $E^{\hat{2}}$ are ± 1, $+\frac{1}{2}$, and $-\frac{1}{2}$, respectively.

$N=1$ supergravity in the WZ supergauge can be regarded as a minimal superextention of the bosonic gravity in the sense that the theory has residual gauge symmetry under general coordinate transformations and local supersymmetry (SUSY) transformations generated by the spinor field parameter $(\epsilon,\overline{\epsilon})$

$$\delta_{\rm SUSY}e^+ = \tfrac{1}{2}\epsilon\psi^{\hat{1}} \text{ and c.c.,}$$

$$\delta_{\rm SUSY}\psi^{\hat{1}} = \mathcal{D}\epsilon + \tfrac{1}{2}\overline{\epsilon}(iFe^{-\phi/2})e^+ \text{ and c.c.} \tag{11}$$

We are now ready to demonstrate the relationship between Osp(1|2) Chern-Simons gauge theory and 2D $N=1$ induced supergravity in the WZ supergauge. For this purpose we first introduce the Osp(1|2) connection

$$A \equiv -\omega J_3 - i\lambda e^a J_a + \psi^{\alpha}Q_{\alpha}, \tag{12}$$

where $a=1,2$, and (J_i,Q_{α}) $(i=1,2,3$ and $\alpha=\hat{1},\hat{2})$ are generators of the Osp(1|2) algebra:

$$[J_i,J_j] = \epsilon_{ijk}J_k, \quad [J_i,Q_{\alpha}] = -\left(\frac{\sigma_i}{2i}\right)_{\alpha}^{\beta}Q_{\beta},$$

$$\{Q_{\alpha},Q_{\beta}\} = \frac{i}{2}\lambda\left(\frac{\sigma_i}{2i}\right)_{\alpha}^{\gamma}\epsilon_{\gamma\beta}. \tag{13}$$

It is possible to represent this connection A by a 3×3 matrix-valued one-form. If we define $e^{\pm} = -e^2 \pm ie^1$ (and $J_{\pm} = -J_2 \mp iJ_1$), the connection A is written as

$$A = \begin{pmatrix} \dfrac{i}{2}\omega & -\dfrac{i\lambda}{2}e^- & -\sqrt{\dfrac{i\lambda}{8}}\psi^{\hat{2}} \\ \dfrac{i\lambda}{2}e^+ & -\dfrac{i}{2}\omega & \sqrt{\dfrac{i\lambda}{8}}\psi^{\hat{1}} \\ \sqrt{\dfrac{i\lambda}{8}}\psi^{\hat{1}} & \sqrt{\dfrac{i\lambda}{8}}\psi^{\hat{2}} & 0 \end{pmatrix}. \tag{14}$$

We should remark that the bosonic part of this representation yields the SL(2,**R**) [or SU(2)] connection if the parameter λ is real [or pure imaginary],[3] corresponding to the case of genus $g\geqslant 2$ (or $g=0$).

Next we calculate the curvature of the connection. We find

[3]Note that this is true in the algebraic sense but not literally. Namely, we have to perform the unitary transformation $A \to A' = e^{-\pi J_1/2}Ae^{\pi J_1/2}$ in order to make the bosonic part SL(2,**R**) valued in its original sense. The whole connection A' then becomes Osp(1|2;**R**) valued.

$$\mathcal{F}\equiv dA+AA(\equiv\mathcal{F}^iJ_i+\mathcal{F}^\alpha Q_\alpha)$$
$$=-\left(d\omega+\frac{i\lambda^2}{2}e^+e^--\frac{\lambda}{4}\psi^{\hat{1}}\psi^{\hat{2}}\right)J_3$$
$$-\frac{i\lambda}{2}\left(\mathcal{D}e^++\frac{1}{4}\psi^{\hat{1}}\psi^{\hat{1}}\right)J_+-\frac{i\lambda}{2}\left(\mathcal{D}e^-+\frac{1}{4}\psi^{\hat{2}}\psi^{\hat{2}}\right)J_-$$
$$+\left(\mathcal{D}\psi^{\hat{1}}-\frac{i\lambda}{2}e^+\psi^{\hat{2}}\right)Q_{\hat{1}}+\left(\mathcal{D}\psi^{\hat{2}}+\frac{i\lambda}{2}e^-\psi^{\hat{1}}\right)Q_{\hat{2}}. \quad (15)$$

From this expression we see immediately that $\mathcal{F}^\pm=0$ is equivalent to the torsion condition (10). Moreover, after a somewhat tedious calculation, it turns out that the equations $\mathcal{F}^3=\mathcal{F}^{\hat{1}}=\mathcal{F}^{\hat{2}}=0$ together with the equation $F=\lambda e^{\phi/2}$ yield the condition that the supercurvature in the WZ supergauge is constant: $R_3=2i\lambda$.

Let us now consider the local gauge transformation

$$\delta_\zeta A=-d\zeta+[A,\zeta],\quad \zeta=\zeta^3J_3+\zeta^aJ_a+\epsilon^\alpha Q_\alpha. \quad (16)$$

The result of the computation tells us that ζ^3 generates local Lorentz transformations and that the transformations generated by ϵ^α are nothing but local SUSY transformations (11) if we set $F=\lambda e^{\phi/2}$. As for the transformations generated by ζ^a, it is closely related to diffeomorphisms. This is because diffeomorphisms of the flat connection are generated by gauge parameters of the form $\zeta=\xi^mA_m$ and because we are only interested in the case where the zweibein is nondegenerate.

From the above considerations we expect that Osp(1|2) Chern-Simons gauge theory (CSGT) on $\mathbf{R}\times\Sigma$ describes dynamics of super Teichmüller space of super Riemann surfaces $S\Sigma$ in the WZ supergauge, because any supervielbein is related to a supervielbein with a constant supercurvature by a unique super Weyl transformation [12]. We will see in the following that the dynamics is in fact equivalent to that of 2D $N=1$ induced supergravity. We will use the canonical quantization.

The action of Osp(1|2) CSGT is given by

$$S=\frac{k}{4\pi}\int_{\mathbf{R}\times\Sigma}\mathrm{STr}(\tilde{A}\tilde{d}\tilde{A}+\tfrac{2}{3}\tilde{A}\tilde{A}\tilde{A}), \quad (17)$$

where $\tilde{A}=dtA_t+A$ and $\tilde{d}=dt\partial/\partial t+d$ stand for the connection and the exterior derivative defined on $\mathbf{R}\times\Sigma$, respectively. STr denotes the invariant bilinear form in Osp(1|2):

$$\mathrm{STr}(J_iJ_j)=\delta_{ij},\quad \mathrm{STr}(Q_\alpha Q_\beta)=-\frac{i\lambda}{2}\epsilon_{\alpha\beta},\quad \mathrm{STr}(J_iQ_\alpha)=0.$$

After a (3+1)-decomposition the action becomes

$$S=\frac{k}{4\pi}\int dt\int_\Sigma \mathrm{STr}[A\dot{A}+2A_t\mathcal{F}]\equiv S_{\mathrm{kin}}+S_{\mathrm{con}}, \quad (18)$$

where $\dot{A}\equiv(\partial/\partial t)A$. The second term S_{con} yields the Gauss law constraint $\mathcal{F}=0$ and from the first term S_{kin} we can read off the symplectic structure. In terms of the component fields S_{kin} is written as

$$S_{\mathrm{kin}}=\frac{k}{4\pi}\int dt\int_\Sigma(-2\dot{\omega}_zdz\omega_{\bar{z}}d\bar{z}+\lambda^2\dot{e}^+e^-+i\lambda\dot{\psi}^{\hat{1}}\psi^{\hat{2}}). \quad (19)$$

Quantum commutation relations of the canonical operators are obtained from the operator version of i times the Poisson brackets. We will choose the polarization in which ω $(\equiv\omega_z)$, e^+, and $\psi^{\hat{1}}$ are diagonal. In this polarization the wave functionals are holomorphic with respect to the complex structure equipped with the space of super Riemann surfaces. For our purpose it is more convenient to use the parameters $(\omega,\varphi,\mu,\chi,\alpha)$ as configuration variables. Their canonical conjugate momenta are

$$\pi_\omega=-\frac{k}{2\pi}\omega_{\bar{z}},$$
$$\pi_\varphi=\frac{k}{4\pi}\Big[\lambda^2e^\phi(1-\mu\bar{\mu})+\frac{i\lambda}{2}e^{\phi/2}\{\chi\bar{\chi}(1-\mu\bar{\mu})+i\mu\chi\bar{\alpha}$$
$$+i\overline{\mu\chi}\alpha-\alpha\bar{\alpha}\}\Big],$$
$$\pi_\mu=-\frac{k}{4\pi}[\lambda^2e^\phi\bar{\mu}+i\lambda e^{\phi/2}\chi(\overline{\mu\chi}-i\bar{\alpha})],$$
$$\pi_\chi=\frac{ik\lambda}{4\pi}e^{\phi/2}[\bar{\chi}(1-\mu\bar{\mu})+i\mu\bar{\alpha}],$$
$$\pi_\alpha=-\frac{k\lambda}{4\pi}(-\overline{\mu\chi}+i\bar{\alpha}). \quad (20)$$

On quantization these conjugate momenta are represented by $-i$ times the functional derivatives with respect to the associated configuration variables: $\widehat{\pi_f}=-i\delta/\delta f$ $(f=\omega,\varphi,\mu,\chi,\alpha)$.

The physical wave functional $\Psi[\omega,\varphi,\mu,\chi,\alpha]$ must satisfy the Gauss law constraints $\hat{\mathcal{F}}\cdot\Psi=0$. Instead of solving these constraints directly, we will solve a proper set of their linear combinations which is classically equivalent to the set of Gauss law constraints when the zweibein is nondegenerate. A convenient set is given by

$$d^2z\mathcal{G}^3\equiv-\mathcal{F}^3,\quad d^2z\mathcal{G}^+\equiv\frac{2i}{\lambda}e^{-\varphi}\mathcal{F}^+,$$
$$d^2z\mathcal{G}^{\hat{1}}\equiv e^{-\varphi/2}\mathcal{F}^{\hat{1}}-\frac{\chi}{2}\mathcal{G}^+,$$
$$d^2z\mathcal{G}^-\equiv-\frac{k\lambda}{2\pi}(e^\varphi\mathcal{F}^-+\bar{\mu}e^{\bar{\varphi}}\mathcal{F}^+)+\frac{k\lambda}{4\pi}e^{\phi/2}(i\bar{\alpha}-\overline{\mu\chi})\mathcal{G}^{\hat{1}}$$
$$+\chi\mathcal{G}^{\hat{2}},$$
$$d^2z\mathcal{G}^{\hat{2}}\equiv-\frac{k\lambda}{4\pi}\Big[e^{\varphi/2}\mathcal{F}^{\hat{2}}-\frac{i\bar{\alpha}-\overline{\mu\chi}}{2}e^{\phi/2}\mathcal{G}^+\Big]. \quad (21)$$

We adopt the ordering in which the momenta are put on the right of the coordinates. Then the operator version of these constraints are written as

$$\hat{\mathcal{G}}^3=\overline{\partial}\omega-\frac{2\pi i}{k}\partial\left(\frac{\delta}{\delta\omega}\right)+\frac{2\pi}{k}\frac{\delta}{\delta\varphi},$$

$$\hat{\mathcal{G}}^+=(\overline{\partial}-\mu\partial)\varphi-\partial\mu-i\omega\mu+\frac{i}{2}\chi\alpha-\frac{2\pi}{k}\frac{\delta}{\delta\omega},$$

$$\hat{\mathcal{G}}^{\hat{1}}=-\frac{i\alpha}{2}(\partial\varphi+i\omega)-i\partial\alpha+\left(\overline{\partial}-\mu\partial-\frac{1}{2}\partial\mu\right)\chi+\frac{2\pi i}{k}\frac{\delta}{\delta\chi},$$

$$\hat{\mathcal{G}}^-=(\partial\varphi+i\omega-\partial)\frac{\delta}{\delta\varphi}-(\overline{\partial}\mu\partial-2\partial\mu)\frac{\delta}{\delta\mu}+\left(\frac{3}{2}\partial\alpha+\frac{1}{2}\alpha\partial\right)\frac{\delta}{\delta\alpha}+\frac{1}{2}(\partial\chi-\chi\partial)\frac{\delta}{\delta\chi},$$

$$\hat{\mathcal{G}}^{\hat{2}}=i\left(\overline{\partial}-\mu\partial-\frac{3}{2}\partial\mu\right)\frac{\delta}{\delta\alpha}+\frac{i\alpha}{2}\frac{\delta}{\delta\mu}+\left(\frac{1}{2}\partial\varphi+\frac{i}{2}\omega-\partial\right)\frac{\delta}{\delta\chi}+\frac{\chi}{2}\frac{\delta}{\delta\varphi}. \tag{22}$$

Now we can solve the constraint equations $\hat{\mathcal{G}}^I\cdot\Psi=0$ $(I=3,+,-,\hat{1},\hat{2})$. The procedure is almost parallel to that in the bosonic case [6]. Namely, we first determine the ω dependence of Ψ by solving $\hat{\mathcal{G}}^+\cdot\Psi=0$ and then determine the (φ,χ) dependence by solving $\hat{\mathcal{G}}^3\cdot\Psi=\hat{\mathcal{G}}^{\hat{1}}\cdot\Psi=0$. The result is

$$\Psi[\omega,\varphi,\mu,\chi,\alpha]=\exp\left\{\frac{ik}{2\pi}(S_O[\omega,\varphi,\mu,\chi,\alpha]+S_L[\varphi,\mu,\chi,\alpha])\right\}\Psi[\mu,\alpha],$$

$$S_O[\omega,\varphi,\mu,\chi,\alpha]=\int_\Sigma d^2z\left[-\frac{\mu}{2}\omega^2-i\omega\left\{(\overline{\partial}-\mu\partial)\varphi-\partial\mu+\frac{i}{2}\chi\alpha\right\}\right],$$

$$S_L[\varphi,\mu,\chi,\alpha]=\int_\Sigma d^2z\left[-\frac{1}{2}\partial\varphi\overline{\partial}\varphi+\mu\left(\frac{1}{2}(\partial\varphi)^2-\partial^2\varphi\right)+\frac{1}{2}\chi(\overline{\partial}-\mu\partial)\chi-\frac{i}{2}\chi\alpha\partial\varphi-i\chi\partial\alpha\right]. \tag{23}$$

By substituting this form of the wave functional, we can reduce the remaining constraints $\hat{\mathcal{G}}^-\cdot\Psi=\hat{\mathcal{G}}^{\hat{2}}\cdot\Psi=0$ to the following equations for the functional $\Psi[\mu,\alpha]$:

$$\hat{\mathcal{V}}\cdot\Psi[\mu,\alpha]=0,$$

$$\hat{\mathcal{V}}\equiv-(\overline{\partial}-\mu\partial-2\partial\mu)\frac{\delta}{\delta\mu}+\left(\frac{3}{2}\partial\alpha+\frac{1}{2}\alpha\partial\right)\frac{\delta}{\delta\alpha}+\frac{ik}{2\pi}\partial^3\mu,$$

$$\hat{\mathcal{S}}\cdot\Psi[\mu,\alpha]=0,$$

$$\hat{\mathcal{S}}\equiv\left(\overline{\partial}-\mu\partial-\frac{3}{2}\partial\mu\right)\frac{\delta}{\delta\alpha}+\frac{1}{2}\alpha\frac{\delta}{\delta\mu}+\frac{ik}{2\pi}\partial^2\alpha. \tag{24}$$

These equations are nothing but the Virasoro-Ward identities for a superconformal field theory. These identities specify the transformation laws of superconformal blocks under diffeomorphisms and local SUSY transformations [13]. A solution to these identities is known to be of the form [13]

$$\Psi[\mu,\alpha]=\exp\left(\frac{ik}{2\pi}S_V[\mu,\alpha]\right)\widetilde{\Psi}[\mu,\alpha],$$

$$S_V[\mu,\alpha]=-\frac{1}{2}\int_\Sigma d^2z\int d^2\theta H_{\bar\theta}^{z}\partial D\ln\Lambda$$
$$=-\frac{1}{2}\int_\Sigma d^2z\int d^2\theta\frac{\overline{D}Z-\Theta\overline{D}\Theta}{\partial Z+\Theta\partial\Theta}D\left(\frac{\partial^2Z+\Theta\partial^2\Theta}{\partial Z+\Theta\partial\Theta}\right), \tag{25}$$

where (Z,Θ) are given by Eq. (7) and $\widetilde{\Psi}[\mu,\alpha]$ is a functional of Beltrami differentials which is invariant under diffeomorphisms and local SUSY transformations. Thus we obtain the physical state of Osp(1|2) CSGT in our polarization:

$$\Psi[\omega,\varphi,\mu,\chi,\alpha]=\exp\left\{\frac{ik}{2\pi}(S_O[\omega,\varphi,\mu,\chi,\alpha]+S_L[\varphi,\mu,\chi,\alpha]+S_V[\mu,\alpha])\right\}\widetilde{\Psi}[\mu,\alpha]. \tag{26}$$

In order to discuss the dynamics of CSGT we have to consider the inner product. Because the Hamiltonian in CSGT is a linear combination of Gauss law constraints, the transition amplitudes in CSGT reduce to the inner products evaluated at a fixed time. In our polarization, the physically relevant inner product which yields the correct Hermitian conjugate condition is

$$\langle\Psi_1|\Psi_2\rangle=\int[d\omega de^+de^-d\psi^{\hat{1}}d\psi^{\hat{2}}]\times\exp\left(-\frac{ik}{4\pi}\int(-2\omega_zdz\omega_{\bar z}d\bar z+\lambda^2e^+e^-+i\lambda\psi^{\hat{1}}\psi^{\hat{2}})\right)\overline{\Psi_1[\omega_z,e^+,\psi^{\hat{1}}]}\Psi_2[\omega_z,e^+,\psi^{\hat{1}}]. \tag{27}$$

If we substitute Eq. (26) for Ψ_1 and Ψ_2, then the ω integration is easily performed and we are left with

$$\langle\Psi_1|\Psi_2\rangle=\int[de^+de^-d\psi^{\hat 1}d\psi^{\hat 2}]\times\exp\left(\frac{ik}{2\pi}S_{\rm GSC}^{\rm (WZ)}\right)\overline{\tilde\Psi_1[\mu,\alpha]}\tilde\Psi_2[\mu,\alpha], \tag{28}$$

where $S_{\rm GSC}^{\rm (WZ)}\equiv S_L+K[\mu,\bar\mu]+S_V[\mu,\alpha]-\overline{S_V[\mu,\alpha]}+S_\lambda$. Here S_L, $K[\mu,\bar\mu]$, and S_λ stand for the super Liouville action, the superextension of Verlinde's local counterterm [14], and the cosmological term in the WZ supergauge, respectively:

$$S_L=\int d^2z\Bigg[\frac{-1}{2(1-\mu\bar\mu)}\Big\{(\partial\phi-\overline{\mu\,\partial\phi}+i\overline{\alpha\chi})\times(\bar\partial\phi-\mu\partial\phi-i\alpha\chi)-\frac{1}{2}(\alpha\chi)(\overline{\alpha\chi})\Big\}+\frac{1}{2}\chi(\bar\partial-\mu\partial)\chi+\frac{1}{2}\bar\chi(\partial-\overline{\mu\,\partial})\bar\chi-i\chi\partial\alpha+i\overline{\chi\,\partial\alpha}+\frac{\partial\mu-\mu\overline{\partial\mu}}{1-\mu\bar\mu}\left(\partial\phi+\frac{i}{2}\overline{\alpha\chi}\right)+\frac{\overline{\partial\mu}-\bar\mu\partial\mu}{1-\mu\bar\mu}\left(\bar\partial\phi-\frac{i}{2}\alpha\chi\right)\Bigg],$$

$$K[\mu,\bar\mu]=\int d^2z\frac{-1}{1-\mu\bar\mu}\left(\partial\mu\,\overline{\partial\mu}-\frac{\bar\mu}{2}(\partial\mu)^2-\frac{\mu}{2}(\overline{\partial\mu})^2\right),$$

$$S_\lambda=-\frac{\lambda^2}{2}\int e^+e^--\frac{i\lambda}{2}\int\psi^{\hat 1}\psi^{\hat 2}. \tag{29}$$

After a lengthy and tedious calculation we see that the integrand in Eq. (28) is indeed invariant under diffeomorphisms and local SUSY transformations (11). The exponent $S_{\rm GSC}^{\rm (WZ)}$ therefore yields a natural superextension of the generally covariant action for 2D induced bosonic gravity. Thus we can conclude that the inner product of the physical states in Osp(1|2) CSGT, Eq. (28), gives rise to the partition function (or transition amplitudes) of 2D $N=1$ supergravity in the WZ supergauge.

Next we consider the observables. In CSGT on a manifold with topology $\mathbf{R}\times\Sigma$, the connection A on Σ is always constrained to be flat, and only global objects such as Wilson loops or holonomies are relevant. In the following we will demonstrate that the space of holonomies constructed from the flat Osp(1|2) connections A is closely related to super Teichmüller space of DeWitt super Riemann surfaces. We restrict our discussion to the case of genus $g\geqslant 2$ (i.e., λ real).

According to the uniformization theorem for super Riemann surfaces [15], any genus $g\geqslant 2$ super Riemann surface of Dewitt type [7] (with a constant supercurvature) is represented as a quotient of the super-upper-half plane SH equipped with the super Poincaré geometry by a discrete hyperbolic subgroup of Osp(1|2;**R**). This implies that super Teichmüller space is represented as the set of homomorphisms of $\pi_1(\Sigma)$ into Osp(1|2;**R**) modulo overall conjugations. What we have to examine is whether the holonomy of a flat Osp(1|2;**R**) connection directly yields such a homomorphism. This is achieved by extending the result of the bosonic case [6] to the case of supergravity.

We first note that the canonical supervielbein with the constant supercurvature $R_3=2i\lambda$, i.e., the super Poincaré geometry, is characterized by the conformal factor

$$\rho=\frac{2}{\lambda}\frac{e^{i\gamma(z,\bar z,\theta,\bar\theta)}}{Z-\bar Z-\Theta\bar\Theta}, \tag{30}$$

where $\gamma(z,\bar z,\theta,\bar\theta)$ is an arbitrary phase factor which is real. In the WZ supergauge the corresponding zweibein, gravitino, and spin connection are

$$e^+=\frac{2}{\lambda}\frac{e^{i\gamma_0(z,\bar z)}}{\Xi_0}e^{Z_0}\ \text{and c.c.},$$

$$\psi^{\hat 1}=2\left(\frac{2}{\lambda}\frac{e^{i\gamma_0(z,\bar z)}}{\Xi_0}\right)^{1/2}\left[e^{\Theta_0}-e^{Z_0}\frac{\Theta_0-\bar\Theta_0}{\Xi_0}\right]\ \text{and c.c.},$$

$$\omega=\frac{i}{\Xi_0}(dZ_0+d\bar Z_0+\bar\Theta_0 d\Theta_0+\Theta_0 d\bar\Theta_0)-d\gamma_0(z,\bar z), \tag{31}$$

where we have used the notation $\Xi_0=Z_0-\bar Z_0-\Theta_0\bar\Theta_0$, $e^{Z_0}=dZ_0+\Theta_0 d\Theta_0$, $e^{\Theta_0}=d\Theta_0$, and $\gamma_0\equiv\gamma|$. Substituting these into Eq. (14), we find that the Osp(1|2;**R**) connection A is locally expressed as a pure gauge $A=-g^{-1}dg$ with

$$g=\begin{pmatrix} e^{\frac{i\gamma_0}{2}+\frac{\pi i}{4}}Z_0\sqrt{\Xi_0}^{-1} & e^{-\frac{i\gamma_0}{2}-\frac{\pi i}{4}}\bar Z_0\sqrt{\Xi_0}^{-1} & (\bar\Theta_0 Z_0-\Theta_0\bar Z_0)(\Xi_0)^{-1} \\ e^{\frac{i\gamma_0}{2}+\frac{\pi i}{4}}\sqrt{\Xi_0}^{-1} & e^{-\frac{i\gamma_0}{2}-\frac{\pi i}{4}}\sqrt{\Xi_0}^{-1} & (\bar\Theta_0-\Theta_0)(\Xi_0)^{-1} \\ -e^{\frac{i\gamma_0}{2}+\frac{\pi i}{4}}\Theta_0\sqrt{\Xi_0}^{-1} & -e^{-\frac{i\gamma_0}{2}-\frac{\pi i}{4}}\bar\Theta_0\sqrt{\Xi_0}^{-1} & 1-\Theta_0\bar\Theta_0(\Xi_0)^{-1}\end{pmatrix}. \tag{32}$$

Because the connection A does not have to be pure gauge globally, the Osp(1|2)-valued field $g(z,\overline{z})$ is not necessarily single valued on Σ. When one goes around any noncontractible loop β on the surface Σ, $g(z,\overline{z})$ in general transforms as

$$g\to h[\beta]\cdot g, h[\beta]=\begin{pmatrix} a & b & b\delta+a\epsilon \\ c & d & d\delta+c\epsilon \\ \delta & -\epsilon & 1+\delta\epsilon \end{pmatrix}, \tag{33}$$

where $a,b,c,d\in\mathbf{R}$ are Grassmann even constants, $\overline{\delta}=\delta$ and $\overline{\epsilon}=\epsilon$ are Grassmann odd constants, and the relation $ad-bc=1-\delta\epsilon$ holds. This $h[\beta]$ is nothing but the holonomy of the Osp(1|2;**R**) connection A around the loop β. This left multiplication induces the following transformation of $(Z_0,\overline{Z_0},\Theta_0,\overline{\Theta_0},\gamma_0)$:

$$Z_0\to\frac{aZ_0+b}{cZ_0+d}+\frac{\Theta_0(-\delta Z_0+\epsilon)}{(cZ_0+d)^2}\ \text{and c.c.},$$

$$\Theta_0\to\frac{-\delta Z_0+\epsilon}{cZ_0+d}+\frac{\Theta_0}{cZ_0+d}\ \text{and c.c.},$$

$$\gamma_0\to\gamma_0-i\ln\left(\frac{cZ_0+d+\Theta_0(d\delta+c\epsilon)}{c\overline{Z_0}+d+\overline{\Theta_0}(d\delta+c\epsilon)}\right). \tag{34}$$

In the WZ supergauge, the transformation of $(Z,\overline{Z},\Theta,\overline{\Theta})$ can be read off from that of $(Z_0,\overline{Z_0},\Theta_0,\overline{\Theta_0})$. In the present case both transformations coincide. Namely, we find

$$Z\to\frac{aZ+b}{cZ+d}+\frac{\Theta(-\delta Z+\epsilon)}{(cZ+d)^2},$$

$$\Theta\to\frac{-\delta Z+\epsilon}{cZ+d}+\frac{\Theta}{cZ+d}\ \text{and c.c.} \tag{35}$$

This is the super Möbius transformation which plays the essential role in the uniformization theorem. If we regard $(Z,\overline{Z},\Theta,\overline{\Theta})$ as a complex coordinate system which maps the super Riemann surface $S\Sigma$ into the super-upper-half plane SH, the holonomy group of the flat Osp(1|2;**R**) connection A turns out to be identical to the discrete group by which SH is divided out. Thus we have explicitly shown the close relationship between the moduli space of flat Osp(1|2;**R**) connections and super Teichmüller space.

Here we comment on spin structures. A spin structure specifies whether or not the fermionic coordinate Θ flips its sign when one goes around each cycle β on Σ. This is determined by the signature of d (or c if $d=0$) in the Osp(1|2;**R**) holonomy $h[\beta]$. In the bosonic case, we cannot distinguish the difference between the identity ($a=d=1$, $b=c=\delta=\epsilon=0$) and the inversion ($a=d=-1$ and $b=c=\delta=\epsilon=0$) and thus a Möbius transformation is isomorphic to an element of PSL(2,**R**). In the super case, on the other hand, the inversion is essentially different from the identity because the former yields the spin structure which is distinct from that of the latter. Owing to this property, a choice of Osp(1|2;**R**) holonomy naturally specifies a unique spin structure.

To summarize the results we have seen that Osp(1|2) Chern-Simons gauge theory describes the dynamics of 2D $N=1$ induced supergravity in the Wess-Zumino supergauge. Physical inner products of Osp(1|2) CSGT yield the partition function (or transition amplitudes) of 2D $N=1$ quantum supergravity. From the holonomy of the Osp(1|2) connection we can extract the spin structure and the super Teichmüller parameters of the super Riemann surface $S\Sigma$.

Is there any possibility of exploiting these consequences? In this paper we have used the polarization in which $(\omega_z,e^+,\psi^{\hat{1}})$ are diagonal and obtained the partition function of 2D $N=1$ induced supergravity. On the other hand, in the holomorphic polarization in which A_z is diagonal, it is well known that the physical wave functional is given by the exponential of the Osp(1|2) WZ-Novikov-Witten (WZNW) action [16] in the case of a trivial topology.[4] Using the Polyakov-Wiegmann identity [17], the inner product in the holomorphic polarization reduces to the partition function of the twisted Osp(1|2;**C**)/Osp(1|2;**R**) WZNW model. Because the $G^{\mathbf{C}}/G$ WZNW model is a conformal field theory which is well studied [19], this may help the description of 2D induced (super)gravity. Before using such a description, however, we will have to establish the direct relationship between the $G^{\mathbf{C}}/G$ WZNW model [with $G=\mathrm{SL}(2,\mathbf{R})$ or Osp(1|2;**R**)] and 2D induced (super)gravity. This is not so easy and is left to the future investigation.

As a by-product, we have obtained the generally super covariant action $S_{\mathrm{GSC}}^{(\mathrm{WZ})}$ in the WZ supergauge. Actually this action can be obtained from the expression

$$S_{\mathrm{GSC}}=\int d^2Zd^2\Theta\left[-\frac{1}{2}D_\Theta\ln(\rho\overline{\rho})D_{\overline{\Theta}}\ln(\rho\overline{\rho})+2i\lambda\sqrt{\rho\overline{\rho}}\right], \tag{36}$$

which is manifestly invariant under reparametrizations of supercoordinates: $(z,\overline{z},\theta,\overline{\theta})\to(z',\overline{z'},\theta',\overline{\theta'})$. This S_{GSC} is therefore regarded as a local expression of the generally supercovariant action for 2D $N=1$ induced supergravity. Using this S_{GSC} as a starting point, we can obtain the action in an arbitrary gauge simply by fixing the gauge. This may open a way to make a further breakthrough in 2D quantum supergravity.

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[4]Exploiting this fact we can show that the light-cone gauge action for $N=\frac{1}{2}$ supergravity ($S_V[\mu,\alpha]$ in Eq. (25)) is related with the *Borel gauged* Osp(1|2) WZNW action through a Legendre transform. This establishes the result obtained in Ref. [18] from a different viewpoint.

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