PHYSICAL REVIEW D VOLUME 56, NUMBER 4 15 AUGUST 1997

Higgs mechanism in string theory

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(Received 27 March 1997)

In first-quantized string theory, spacetime symmetries are described by inner automorphisms of the underlying conformal field theory. In this paper we use this approach to illustrate the Higgs effect in string theory. We consider string propagation on $M^{24,1}\times S^1$, where the circle has radius R, and study SU(2) symmetry breaking as R moves away from its critical value. We find a gauge-covariant equation of motion for the broken-symmetry gauge bosons and the would-be Goldstone bosons. We show that the Goldstone bosons can be eliminated by an appropriate gauge transformation. In this unitary gauge, the Goldstone bosons become the longitudinal components of massive gauge bosons. [S0556-2821(97)07116-6]

PACS number(s): 11.25.Hf, 11.30.Qc

I. INTRODUCTION

String theory remains the most promising candidate for a unified description of nature. During the past few years, many string dualities have been discovered, but it is fair to say that a deep understanding of string dynamics is still lacking. It is therefore important to understand the role of spacetime symmetries in string dynamics.

At the classical level, every two-dimensional conformal field theory (of the appropriate central charge) is a solution to the string equations of motion [1]. The two-dimensional couplings are the spacetime fields of the string. Conformal invariance determines the couplings, and hence the dynamics of the spacetime fields. During the past decade, a large number of string solutions have been constructed in this way [2].

Fortunately, many of the string solutions are related by symmetries. In ordinary field theory, symmetries are transformations of the spacetime fields which leave the classical action invariant. Barring anomalies, they also hold in the full quantum theory. In string theory, the situation is different. The spacetime fields appear as couplings, so symmetries are not invariances of a spacetime action.

There are good reasons to believe that string theory contains an enormous degree of symmetry, of which gauge and coordinate invariance are but remnants. First, the particle content and interactions of string theory are so tightly constrained that they are presumably fixed by some symmetry. Second, high-energy fixed-angle string scattering obeys a universal behavior which suggests that some large symmetry is being restored [3]. This symmetry mixes massless and massive states, and is spontaneously broken by the vacuum. Other aspects of symmetry breaking in string theory are discussed in [4].

Recently, a simple but powerful approach to string symmetries was developed in [5]. In this work, string symmetries are identified with similarity transformations of the underlying conformal field theory. The key idea is that automorphisms of the operator algebra change the Hamiltonian, but do not affect the physical results.

The approach of Ref. [5] is very general. It gives rise to spacetime symmetries which mix states of different mass. It places unbroken and spontaneously broken spacetime symmetries on exactly the same footing. Unbroken spacetime symmetries are generated by conserved currents of the underlying conformal field theory [6], while spontaneously broken symmetries are generated by currents that are not conserved.

In this paper we will study spontaneously broken symmetries in string theory. We will focus on a simple example: string propagation on $M^{24,1}\times S^1$. For a generic value of the radius of S^1, this string vacuum has an unbroken $U(1)_L\times U(1)_R$ gauge symmetry. At a critical value of R, the symmetry is enhanced to $SU(2)_L\times SU(2)_R$. Away from this critical value, the $SU(2)_L\times SU(2)_R$ is spontaneously broken to $U(1)_L\times U(1)_R$.

In field theory, the spontaneous breaking of a gauge symmetry is associated with the Higgs effect, through which the would-be Goldstone bosons are absorbed by the spontaneously broken gauge bosons. The physical spectrum is manifest in unitary gauge, where one finds a set of massive gauge bosons, one for each spontaneously broken generator.

The formalism of [5] is especially well suited for describing the Higgs effect in string theory. Therefore, in what follows, we will first review the status of symmetries in (perturbative) string theory. We will then restrict our attention to string propagation on $M^{24,1}\times S^1$. We will start at the critical radius and identify the generators of the $SU(2)_L\times SU(2)_R$ gauge symmetry. We will find the full set of massless scalar fields, as well as the massless gauge bosons associated with the unbroken gauge symmetry. We will then shift away from the critical radius by giving a small expectation value to a modulus field. We will see that this vacuum expectation value (VEV) spontaneously breaks the $SU(2)_L\times SU(2)_R$ symmetry to $U(1)_L\times U(1)_R$. We will find that the scalar multiplets split into Goldstone and physical fields, and discover how the gauge bosons absorb the Goldstone modes. We shall see that this model illustrates one particularly simple way in which the full set of string symmetries is broken by the string vacuum.

II. SYMMETRIES IN (PERTURBATIVE) STRING THEORY

In ordinary string theory, the classical string solutions are in one-one correspondence with conformal field theories of the appropriate central charge. Given one string solution, a physically equivalent solution can be found by making a similarity transformation on the operator algebra $\mathcal{A}$ of the conformal field theory [5]:

$$\Phi(\sigma)\mapsto e^{ih}\Phi(\sigma)e^{-ih}. \tag{1}$$

This determines an equivalent solution for any operator h.

This automorphism (1) acts on the stress tensor in the obvious way:

$$T_\phi(\sigma)\mapsto e^{ih}T_\phi(\sigma)e^{-ih} \tag{2}$$

[and likewise for $\overline{T_\phi}(\sigma)$], where ϕ denotes a generic spacetime field. In what follows, we shall restrict our attention to automorphisms which change the spacetime fields. Therefore, we require

$$T_{\phi+\delta\phi}(\sigma)-T_\phi(\sigma)=i[h,T_\phi(\sigma)], \tag{3}$$

for some infinitesimal operator h. The transformation $\phi\mapsto\phi+\delta\phi$ is a *symmetry*: it is an infinitesimal change of the spacetime fields which does not change the physics.

From this point of view, symmetries are infinitesimal deformations of the stress tensor, $T_\phi(\sigma)\mapsto T_\phi(\sigma)+\delta T$, where $\delta T=i[h,T_\phi(\sigma)]$. More general deformations are not symmetry transformations, but describe physically distinct solutions. For example, two nearby solutions are flat spacetime $M^{24,1}\times S^1$, and a weak electromagnetic wave propagating through it. These two solutions are not related by any symmetry transformation.

For string propagation on $M^{24,1}\times S^1$, the vacuum stress-energy tensor is given by

$$T(\sigma)=-\tfrac{1}{2}\,\eta_{\mu\nu}\partial X^\mu\partial X^\nu-\tfrac{1}{2}\partial X^{26}\partial X^{26} \tag{4}$$

at the radius $R=R_{\rm cr}=\sqrt{2}$. A weak $U(1)_L$ electromagnetic wave can be obtained by adding

$$\delta T=-A_\mu^{(3)}(X)\,\overline{\partial X}^\mu\partial X^{26}. \tag{5}$$

This deformation preserves conformal invariance provided δT is a primary field of dimension (1,1). This is equivalent to saying that the functions $A_\mu^{(3)}(X)$ satisfy the conditions

$$\Box A_\mu^{(3)}(X)=0,\quad \partial^\mu A_\mu^{(3)}(X)=0. \tag{6}$$

The first is an equation of motion, the second is a Lorentz gauge condition.

Equations (5) and (6) provide an example of a *canonical deformation* [5], defined by $\delta T(\sigma)=V(\sigma)$, where $V(\sigma)$ is a vertex operator, a primary field of dimension (1,1). Canonical deformations take one conformal field theory into another. Furthermore, they induce a variation in the stress tensor which can be expressed as a change in the spacetime fields.

Canonical deformations turn on gauge fields in the Lorentz gauge. To describe the Higgs effect, however, we would like to transform the spacetime fields to an *arbitrary* gauge [7]. This can be achieved by performing an automorphism $\delta T=i[h,T_\phi(\sigma)]$, where h is given by

$$h=\int d\sigma\;\Lambda^{(3)}(X)i\sqrt{2}\partial X^{26} \tag{7}$$

and $\Lambda^{(3)}(X)$ is the parameter of the gauge transformation. Note that if $\Box\Lambda^{(3)}=0$, the integrand is of dimension (1,0), and the transformation (7) preserves the Lorentz gauge.

To see this, let us start with the field $A_\mu^{(3)}(X)$ in the Lorentz gauge:

$$T(\sigma)=-\tfrac{1}{2}\,\eta_{\mu\nu}\partial X^\mu\partial X^\nu-\tfrac{1}{2}\partial X^{26}\partial X^{26}-A_\mu^{(3)}(X)\,\overline{\partial X}^\mu\partial X^{26}, \tag{8}$$

where $\Box A_\mu^{(3)}(X)=\partial^\mu A_\mu^{(3)}(X)=0$. Let us then compute the commutator $i[h,T(\sigma)]$. This gives rise to the deformed stress-energy tensor

$$\begin{aligned}T'(\sigma)=T(\sigma)+i[h,T(\sigma)]=&-\tfrac{1}{2}\,\eta_{\mu\nu}\partial X^\mu\partial X^\nu-\tfrac{1}{2}\partial X^{26}\partial X^{26}-[A_\mu^{(3)}(X)+\partial_\mu\Lambda^{(3)}(X)]\,\overline{\partial X}^\mu\partial X^{26}-\tfrac{1}{2}\Box\Lambda^{(3)}(X)\partial^2X^{26}\\&-\tfrac{1}{2}\Box\partial_\mu\Lambda^{(3)}(X)\partial X^\mu\partial X^{26}+\tfrac{1}{2}\Box\partial_\mu\Lambda^{(3)}(X)\,\overline{\partial X}^\mu\partial X^{26}.\end{aligned} \tag{9}$$

Writing

$$A'^{(3)}_\mu(X)=A_\mu^{(3)}(X)+\partial_\mu\Lambda^{(3)}(X) \tag{10}$$

and imposing the conformal condition (6), we find the general stress tensor

$$\begin{aligned}T'(\sigma)=&-\tfrac{1}{2}\,\eta_{\mu\nu}\partial X^\mu\partial X^\nu-\tfrac{1}{2}\partial X^{26}\partial X^{26}-A'^{(3)}_\mu(X)\,\overline{\partial X}^\mu\partial X^{26}-\tfrac{1}{2}\partial^\mu A'^{(3)}_\mu(X)\partial^2X^{26}-\tfrac{1}{2}\Box A'^{(3)}_\mu(X)\partial X^\mu\partial X^{26}\\&+\tfrac{1}{2}\Box A'^{(3)}_\mu(X)\,\overline{\partial X}^\mu\partial X^{26},\end{aligned} \tag{11}$$

and the gauge-covariant equations of motion

$$\Box A'^{(3)}_{\mu}(X)-\partial_{\mu}\partial^{\nu}A'^{(3)}_{\nu}(X)=0. \tag{12}$$

Note that this gauge-invariant equation of motion reduces to Eq. (6) when $\partial^{\mu}A'^{(3)}_{\mu}(X)=0$.

III. STRINGS ON $M^{24,1}\times S^{1}$

In this section we will take a closer look at bosonic string propagation on $M^{24,1}\times S^{1}$. We take X^{26} to be periodic: $X^{26}\sim X^{26}+2\pi R$, where R is the radius of the circle S^{1}. On this space there are two types of excitations: strings with quantized momenta in the compact dimension, and strings that wind around the compact dimension a fixed number of times. The mass formula for the 25-dimensional particle states receives contributions from both:

$$M^{2}=\frac{n^{2}}{R^{2}}+\frac{m^{2}R^{2}}{4}+N_{L}+N_{R}-2, \tag{13}$$

where m and n are integers, and N_L (N_R) denote the oscillator contributions from the left (right) sectors. Physical string states must also satisfy the reparametrization constraint $N_L-N_R=mn$.

The space $M^{24,1}\times S^{1}$ is a consistent string vacuum for arbitrary radius R. The vacuum stress tensor is

$$T_{R}(\sigma)=-\tfrac{1}{2}\eta_{\mu\nu}\hat{\partial}X^{\mu}\hat{\partial}X^{\nu}-\frac{1}{2}\frac{R^{2}}{R_{\rm cr}^{2}}\hat{\partial}X^{26}\hat{\partial}X^{26}, \tag{14}$$

together with its conjugate $\overline{T_{R}}(\sigma)$. The operator $\hat{\partial}$ is the usual light-cone derivative. Note that $\hat{\partial}X^{\mu}$ remains invariant as the radius R is varied, but that the operator $\hat{\partial}X^{26}$ does not. This is easy to see by writing the operators in terms of the string coordinates (X^{μ},X^{26}), together with the conjugate momenta (π_{μ},π_{26}):

$$\hat{\partial}X^{\mu}=\frac{1}{\sqrt{2}}(\eta^{\mu\nu}\pi_{\nu}+X'^{\mu}),\quad \hat{\overline{\partial}}X^{\mu}=\frac{1}{\sqrt{2}}(\eta^{\mu\nu}\pi_{\nu}-X'^{\mu}),$$

$$\hat{\partial}X^{26}=\frac{1}{\sqrt{2}}\left(\frac{R_{\rm cr}^{2}}{R^{2}}\pi_{26}+X'^{26}\right),\quad \hat{\overline{\partial}}X^{26}=\frac{1}{\sqrt{2}}\left(\frac{R_{\rm cr}^{2}}{R^{2}}\pi_{26}-X'^{26}\right). \tag{15}$$

To exhibit the Higgs effect, we will need to compare conformal field theories at different radii—that is, at different values of the background fields. It is therefore essential to express the stress tensors in terms of fixed, background-independent operators, such as π_{26} and X^{26}. Therefore, in what follows, we *define* the symbols ∂X^{μ}, ∂X^{26}, $\overline{\partial}X^{\mu}$, and $\overline{\partial}X^{26}$ (without the hats) to be

$$\partial X^{\mu}=\frac{1}{\sqrt{2}}(\eta^{\mu\nu}\pi_{\nu}+X'^{\mu}),\quad \overline{\partial}X^{\mu}=\frac{1}{\sqrt{2}}(\eta^{\mu\nu}\pi_{\nu}-X'^{\mu}),$$

$$\partial X^{26}=\frac{1}{\sqrt{2}}(\pi_{26}+X'^{26}),\quad \overline{\partial}X^{26}=\frac{1}{\sqrt{2}}(\pi_{26}-X'^{26}). \tag{16}$$

These operators have fixed, radius-independent commutation relations. At the critical radius, $R=R_{\rm cr}=\sqrt{2}$, they reduce to the light-cone derivatives.

The light-cone derivatives $\hat{\partial}X^{\mu}$ and $\hat{\partial}X^{26}$ can be expressed in terms of the fixed operators (16). The result is

$$\hat{\partial}X^{\mu}=\partial X^{\mu},\quad \hat{\partial}X^{26}=\frac{1}{2}\left[\frac{R_{\rm cr}^{2}}{R^{2}}(\partial X^{26}+\overline{\partial}X^{26})+(\partial X^{26}-\overline{\partial}X^{26})\right]. \tag{17}$$

When substituted into Eq. (14), they give the stress tensor in the fixed basis:

$$T_{R}=-\tfrac{1}{2}\eta_{\mu\nu}\partial X^{\mu}\partial X^{\nu}-\frac{1}{8}\Bigg[\left(\frac{R_{\rm cr}}{R}+\frac{R}{R_{\rm cr}}\right)^{2}\partial X^{26}\partial X^{26}-\left(\frac{R_{\rm cr}}{R}-\frac{R}{R_{\rm cr}}\right)^{2}\overline{\partial}X^{26}\overline{\partial}X^{26}-2\left(\frac{R_{\rm cr}{}^{2}}{R^{2}}-\frac{R^{2}}{R_{\rm cr}{}^{2}}\right)\partial X^{26}\overline{\partial}X^{26}\Bigg]. \tag{18}$$

For small variations $R=R_{\rm cr}+\delta R$, one finds

$$\hat{\partial}X^{26}=\partial X^{26}\left(1-\frac{\delta R}{R_{\rm cr}}+3\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\right)+\overline{\partial}X^{26}\left(-\frac{\delta R}{R_{\rm cr}}+3\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\right),$$

$$\hat{\overline{\partial}}X^{26}=\partial X^{26}\left(-\frac{\delta R}{R_{\rm cr}}+3\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\right)+\overline{\partial}X^{26}\left(1-\frac{\delta R}{R_{\rm cr}}+3\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\right), \tag{19}$$

and

$$T_{R}=-\tfrac{1}{2}\eta_{\mu\nu}\partial X^{\mu}\partial X^{\nu}-\left(\frac{1}{2}+\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\right)\partial X^{26}\partial X^{26}-\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\overline{\partial}X^{26}\overline{\partial}X^{26}-\left(-\frac{\delta R}{R_{\rm cr}}+\frac{(\delta R)^{2}}{2R_{\rm cr}^{2}}\right)\partial X^{26}\overline{\partial}X^{26}. \tag{20}$$

The vertex operators for the emission or absorption of particle states should also be written in terms of the fixed basis. At the critical radius, the vertex operators

$$V^{(3)}(\sigma)=A_{\mu}^{(3)}(X)\,\overline{\partial}X^{\mu}\partial X^{26},\quad \widetilde{V}^{(3)}(\sigma)=\widetilde{A}_{\mu}^{(3)}(X)\,\partial X^{\mu}\,\overline{\partial}X^{26},$$

$$V^{(33)}(\sigma)=\phi^{(33)}(X)\,\partial X^{26}\overline{\partial}X^{26} \tag{21}$$

are subject to the conditions

$$\Box A_{\mu}^{(3)}(X)=\Box\widetilde{A}_{\mu}^{(3)}(X)=\Box\phi^{(33)}(X)=0,$$

$$\partial^{\nu}A_{\nu}^{(3)}(X)=\partial^{\nu}\widetilde{A}_{\nu}^{(3)}(X)=0. \tag{22}$$

They describe the emission or absorption of massless gauge and scalar bosons. As in Sec. II, the vertex operators create gauge bosons in Lorentz gauge.

For $R=R_{\rm cr}$, there are other massless particles in the spectrum. They include four massless gauge bosons, whose vertex operators are given by

$$V^{(\pm)}(\sigma)=A_\mu^{(\pm)}(X)\,\overline{\partial X}^\mu \exp(\pm i\sqrt{2}X_L^{26}),$$

$$\widetilde{V}^{(\pm)}(\sigma)=\widetilde{A}_\mu^{(\pm)}(X)\,\partial X^\mu \exp(\pm i\sqrt{2}X_R^{26}), \tag{23}$$

together with eight massless scalars, whose vertex operators take the form

$$V^{(3\pm)}(\sigma)=\phi^{(3\pm)}(X)\,\partial X^{26}\exp(\pm i\sqrt{2}X_R^{26}),$$

$$\widetilde{V}^{(\pm 3)}(\sigma)=\phi^{(\pm 3)}(X)\,\overline{\partial X}^{26}\exp(\pm i\sqrt{2}X_L^{26}),$$

$$V^{(\pm\pm)}(\sigma)=\phi^{(\pm\pm)}(X)\exp(\pm i\sqrt{2}X_L^{26})\exp(\pm i\sqrt{2}X_R^{26}),$$

$$V^{(\pm\mp)}(\sigma)=\phi^{(\pm\mp)}(X)\exp(\pm i\sqrt{2}X_L^{26})\exp(\mp i\sqrt{2}X_R^{26}). \tag{24}$$

The fields $A_\mu^{(\alpha)}(X)$, $\widetilde{A}_\mu^{(\alpha)}(X)$, and $\phi^{(\alpha\beta)}(X)$ satisfy the conformal conditions

$$\Box A_\mu^{(\alpha)}(X)=\Box\widetilde{A}_\mu^{(\alpha)}(X)=\Box\,\phi^{(\alpha\beta)}(X)=0,$$

$$\partial^\nu A_\nu^{(\alpha)}(X)=\partial^\nu\widetilde{A}_\nu^{(\alpha)}(X)=0, \tag{25}$$

for $\alpha,\beta=3,\pm$. As above, they are massless equations of motion and Lorentz gauge conditions for the gauge and scalar bosons. The full set of massless gauge bosons fills out the adjoint representation of $SU(2)_L\times SU(2)_R$; the massless scalars transform in the (3,3) representation of the gauge group.

If one deforms the conformal field theory by varying the radius of the circle, the vertex operators change continuously. The new vertex operators are as above, with ∂X^{26} and $\overline{\partial X}^{26}$ replaced by the operators $\hat{\partial}X^{26}$ and $\widehat{\overline{\partial X}}^{26}$. (The operators ∂X^μ and $\overline{\partial X}^\mu$ do not depend on R.)

For arbitrary radius R, the condition that the deformed vertex operators be (1,1) primary fields with respect to the deformed stress tensor, Eq. (20), gives rise to massless equations of motion and Lorentz gauge conditions for the following spacetime fields:

$$\Box A_\mu^{(3)}(X)=\Box\widetilde{A}_\mu^{(3)}(X)=\Box\,\phi^{(33)}(X)=0,$$

$$\partial^\nu A_\nu^{(3)}(X)=\partial^\nu\widetilde{A}_\nu^{(3)}(X)=0. \tag{26}$$

In contrast, $A_\mu^{(\pm)}(X)$, $\widetilde{A}_\mu^{(\pm)}(X)$, $\phi^{(3\pm)}(X)$, and $\phi^{(\pm 3)}(X)$ obey massive equations of motion and modified, R_ξ-like gauge conditions

$$\Box A_\mu^{(\pm)}(X)+\frac{(\delta R)^2}{2}A_\mu^{(\pm)}(X)=0,$$

$$\Box\,\phi^{(\pm 3)}(X)+\frac{(\delta R)^2}{2}\,\phi^{(\pm 3)}(X)=0,$$

$$\partial^\nu A_\nu^{(\pm)}(X)=-\left(\frac{\delta R}{R_{\rm cr}^2}-\frac{(\delta R)^2}{2R_{\rm cr}^3}\right)\phi^{(\pm 3)}(X). \tag{27}$$

IV. HIGGS MECHANISM IN STRING THEORY

We are now ready to exhibit the string theory Higgs effect. We first need to relax the R_ξ-like gauge condition. As in the previous section, we can do this by carrying out a general $SU(2)_L$ [or $SU(2)_R$] gauge transformation on the spacetime fields.

The $SU(2)_L$ gauge transformation is most easily specified at the critical radius. It is generated by an operator h,

$$h=\int d\sigma\;\Lambda^{(a)}(X)J^{(a)}(\sigma), \tag{28}$$

where the dimension (1,0) currents $J^{(a)}$, $a=1,2,3$, are conserved. For the case at hand, the $SU(2)_L$ currents are simply

$$J^{(3)}(\sigma)=i\sqrt{2}\partial X,\quad J^{(\pm)}(\sigma)=\exp(\pm i\sqrt{2}X_L). \tag{29}$$

Therefore, the operator h can be written as

$$h=\int d\sigma\;[\Lambda^{(3)}i\sqrt{2}\partial X^{26}+\Lambda^{(+)}\exp(i\sqrt{2}X_L^{26}) +\Lambda^{(-)}\exp(-i\sqrt{2}X_L^{26})], \tag{30}$$

where the functions $\Lambda^{(\alpha)}$ are functions of X^μ only. [There are similar currents and transformations for $SU(2)_R$.]

Away from the critical radius, the current $J^{(3)}$ deforms, but the $SU(2)_L$ symmetry algebra continues to hold. For $R\neq R_{\rm cr}$, however, the currents $J^{(\pm)}(\sigma)$ are not of dimension (1,0) with respect to the deformed stress-energy tensor. This implies that the currents are not conserved, and the spacetime symmetry is spontaneously broken.

In this section we will study the Higgs effect by implementing an arbitrary $SU(2)_L$ gauge transformation for $R\neq R_{\rm cr}$. We shall start at the critical radius, and turn on a constant value for the field $\phi^{(33)}$. We shall see that this defines a new stress tensor $T'(\sigma)$ which is equivalent to the stress tensor $T_R(\sigma)$ at a radius $R=R_{\rm cr}+\delta R=R_{\rm cr}(1-\langle\phi^{(33)}\rangle)$. We will then turn on fields $\phi^{(\pm 3)}(X)$ and $A_\mu^{(\pm)}(X)$. This defines a new stress tensor $T''(\sigma)$ which describes infinitesimal fluctuations of the would-be Goldstone bosons $\phi^{(\pm 3)}(X)$ and the broken-symmetry gauge fields $A_\mu^{(\pm)}(X)$ about the string vacuum at radius R. Once we have the stress tensor $T''(\sigma)$, we will compute an arbitrary broken-symmetry gauge transformation. We will see that the would-be Goldstone fields transform by a shift. This will permit us to pass to unitary gauge and identify the physical fields.

Therefore, let us start at the radius $R_{\rm cr}$ and turn on a constant value for the field $\phi^{(33)}$. To first order, the stress tensor is just

$$T'(\sigma)=-\tfrac{1}{2}\eta_{\mu\nu}\partial X^{\mu}\partial X^{\nu}-\tfrac{1}{2}\partial X^{26}\partial X^{26}-\langle\phi^{(33)}\rangle\partial X^{26}\overline{\partial X}^{26}+\cdots. \quad (31)$$

Conformal invariance is satisfied because $\langle\phi^{(33)}\rangle$ is constant. Comparing Eq. (31) with Eq. (20), we see that δR can be identified with $-\langle\phi^{(33)}\rangle R_{\rm cr}$.

Let us now deform this stress tensor by turning on the fields $\phi^{(\pm 3)}(X)$ and $A_{\mu}^{(\pm)}(\mathrm{X})$, following the techniques of Sec. II. Therefore, we add to $T'(\sigma)$ a deformation of the form

$$\delta T'=-\phi^{(+3)}(X)\,\hat{\overline{\partial X}}{}^{26}\exp(i\sqrt{2}X_L^{26})-\phi^{(-3)}(X)\,\hat{\overline{\partial X}}{}^{26}\exp(-i\sqrt{2}X_L^{26})-A_{\mu}^{(+)}(X)\,\overline{\partial X}{}^{\mu}\exp(i\sqrt{2}X_L^{26})$$
$$-A_{\mu}^{(-)}(X)\,\overline{\partial X}{}^{\mu}\exp(-i\sqrt{2}X_L^{26}), \quad (32)$$

where the hatted derivatives are given by Eq. (15), with $\delta R=-\langle\phi^{(33)}\rangle R_{\rm cr}$. This deformation is conformal if the functions $\phi^{(\pm 3)}(X)$ and $A_{\mu}^{(\pm)}(X)$ obey the conditions (27). The resulting stress tensor $T''(\sigma)=T'(\sigma)+\delta T'$ describes a consistent string background with excitations of the broken-symmetry gauge bosons and the would-be Goldstone bosons around a vacuum with arbitrary radius R—that is, a nonzero vacuum expectation value for the spacetime field $\phi^{(33)}(X)$.

We now perform a local gauge transformation generated by

$$h=\int d\sigma\,[\Lambda^{(+)}\exp(i\sqrt{2}X_L^{26})+\Lambda^{(-)}\exp(-i\sqrt{2}X_L^{26})]. \quad (33)$$

The operator h contains the $SU(2)_L$ gauge transformations which do not respect the string vacuum. As in Sec. II, we compute the commutator $i[h,T''(\sigma)]$ and find

$$T''(\sigma)+i[h_1,T''(\sigma)]=-[\phi^{(+3)}(X)+\delta R\Lambda^{(+)}(X)]\,\hat{\overline{\partial X}}{}^{26}\exp(i\sqrt{2}X_L^{26})-[\phi^{(-3)}(X)+\delta R\Lambda^{(-)}(X)]\,\hat{\overline{\partial X}}{}^{26}\exp(-i\sqrt{2}X_L^{26})$$
$$-[A_{\mu}^{(+)}(X)+\partial_{\mu}\Lambda^{(+)}(X)]\,\overline{\partial X}{}^{\mu}\exp(i\sqrt{2}X_L^{26})-[A_{\mu}^{(-)}(X)+\partial_{\mu}\Lambda^{(-)}(X)]\,\overline{\partial X}{}^{\mu}\exp(-i\sqrt{2}X_L^{26}). \quad (34)$$

Defining

$$\phi'^{(\pm 3)}(X)=\phi^{(\pm 3)}(X)+\delta R\Lambda^{(\pm)}(X),\quad A'^{(\pm)}_{\mu}(X)=A_{\mu}^{(\pm)}(X)+\partial_{\mu}\Lambda^{(\pm)}(X), \quad (35)$$

we see we can write $T'''(\sigma)=T''(\sigma)+i[h_1,T''(\sigma)]$ as

$$T'''(\sigma)=T'(\sigma)-\phi'^{(+3)}(X)\,\hat{\overline{\partial X}}{}^{26}\exp(i\sqrt{2}X_L^{26})-\phi'^{(-3)}(X)\,\hat{\overline{\partial X}}{}^{26}\exp(-i\sqrt{2}X_L^{26})-A'^{(+)}_{\mu}(X)\,\overline{\partial X}{}^{\mu}\exp(i\sqrt{2}X_L^{26})$$
$$-A'^{(-)}_{\mu}(X)\,\overline{\partial X}{}^{\mu}\exp(-i\sqrt{2}X_L^{26}), \quad (36)$$

where the conformal condition (27) is the gauge-covariant equation of motion

$$\Box A'^{(\pm)}_{\mu}(X)-\partial_{\mu}\partial^{\nu}A'^{(\pm)}_{\nu}(X)+\frac{(\delta R)^2}{2}A'^{(\pm)}_{\mu}(X)=\left(\frac{\delta R}{R_{\rm cr}^2}-\frac{(\delta R)^2}{2R_{\rm cr}^3}\right)\partial_{\mu}\phi'^{(\pm 3)}(X). \quad (37)$$

From this we see that the broken-symmetry gauge bosons and the would-be Goldstone bosons obey coupled equations of motion.

The stress-energy tensor $T'''(\sigma)$ describes a string vacuum that is physically equivalent to that of $T''(\sigma)$. Note that under the automorphism (33), the would-be Goldstone bosons transform by a shift:

$$\phi^{(\pm 3)}(X)\mapsto\phi^{(\pm 3)}(X)+\delta R\Lambda^{(\pm)}(X). \quad (38)$$

This confirms that the would-be Goldstone bosons are gauge artifacts, and that they can be transformed away by a suitable gauge transformation.

To exhibit the Higgs effect explicitly, let us choose the transformation parameters to eliminate the would-be Goldstone bosons from the spectrum:

$$\delta R\Lambda^{(\pm)}(X)=-\phi^{(\pm 3)}(X). \quad (39)$$

In this unitary gauge, the stress tensor reduces to

$$T'''(\sigma)\to T'(\sigma)-A'^{(+)}_{\mu}(X)\,\overline{\partial X}{}^{\mu}\exp(i\sqrt{2}X_L^{26})$$
$$-A'^{(-)}_{\mu}(X)\,\overline{\partial X}{}^{\mu}\exp(-i\sqrt{2}X_L^{26}). \quad (40)$$

The conformal condition (37) reduces to a set of massive equations of motion for the vectors $A'^{(\pm)}_{\mu}(X)$:

$$\Box A'^{(\pm)}_{\mu}(X)-\partial_{\mu}\partial^{\nu}A'^{(\pm)}_{\nu}(X)+\frac{(\delta R)^2}{2}A'^{(\pm)}_{\mu}(X)=0. \tag{41}$$

The would-be Goldstone bosons have become the longitudinal components of the massive gauge bosons.

It is straightforward to verify that the number of physical states does not change as the radius is varied. Indeed, at the critical point, the spectrum includes the massless gauge bosons $A^{(\pm)}_{\mu}$, with 23 polarizations each, as well as the real scalar fields $\phi^{(\pm 3)}$. Away from the critical point, the scalars are gone, but the gauge bosons are massive, with 24 polarizations each, so the total number of degrees of freedom remains the same.

V. CONCLUSIONS

In this paper we illustrated the Higgs mechanism in string theory. We considered the simple example of string propagation on $M^{24,1}\times S^1$, but our procedure may be readily generalized to other string backgrounds. We started with the operator algebra $\mathcal{A}$, at the critical radius of S^1, where the symmetry algebra is $SU(2)_L\times SU(2)_R$. Using a fixed basis of operators, we constructed the stress tensor and vertex operators for the gauge and scalar bosons, as well as the generators of the $SU(2)_L\times SU(2)_R$ symmetry algebra.

We then deformed the conformal field theory by varying the radius of the circle away from its critical value. We studied the change in the vertex operators along the deformation class. We found the $SU(2)_L\times SU(2)_R$ gauge symmetry to be spontaneously broken to $U(1)_L\times U(1)_R$, and the corresponding world-sheet currents to be no longer conserved.

In the final section of the paper, we derived the gauge-covariant equation of motion for the broken-symmetry gauge bosons and the would-be Goldstone bosons. We eliminated the scalars from the spectrum by performing a suitable gauge transformation. In this unitary gauge, the would-be Goldstone bosons became the longitudinal components of the massive gauge bosons.

ACKNOWLEDGMENTS

We would like to thank M. Evans and J. Liu for useful discussions. This work was supported by the U.S. National Science Foundation, Grant No. NSF-PHY-9404057, and the U.S. Department of Energy, Contract No. DE-FG02-91ER40651-TASKB.

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