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Gauge independence in terms of the functional integral

Taro Kashiwa* and Naoki Tanimura†
Department of Physics, Kyushu University Fukuoka 812-81, Japan

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A gauge-invariant formulation in quantum electrodynamics, characterized by an arbitrary function $\phi_\mu(x)$, is reconsidered. Operators in a covariant case, however, are ill defined because of a $\phi_\mu(x)\sim\partial_\mu/\Box$-type singularity in Minkowski space. We then build up a Euclidean path integral formula, starting with a noncovariant but well-defined canonical operator formalism. The final expression is covariant, free from the pathology, and shows that the model can be interpreted as the ϕ_μ-gauge fixing. Utilizing this formula we prove the gauge independence of the free energy as well as the S matrix. We also clarify the reason why it is so simple and straightforward to perform gauge transformations in the path integral. [S0556-2821(97)04416-0]

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I. INTRODUCTION

Gauge transformations in quantum mechanics may be understood as a unitary transformation so that proving gauge invariance corresponds to proving the unitary equivalence between two theories [1]. However, in quantum field theories, it is well known that canonical commutation relations require gauge fixing; that is, each gauge has its own Hilbert space. Consequently, in order to show that a result is gauge independent, we usually compare results which have been obtained in different gauges [2]. (Some approaches, however, treat gauge transformations even in quantum field theories [3].) Therefore, models with an explicit gauge dependence may be suitable, for instance, α in the Nakanishi-Lautrup formalism [4],

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-A^\mu\partial_\mu B+\frac{\alpha}{2}B^2. \tag{1.1}$$

(Throughout the paper, repeated indices imply summation unless otherwise stated.) Covariance is extremely useful in perturbation theories but requires a negative metric which makes it hard to imagine the dynamics. On the other hand, noncovariant gauges, such as the Coulomb gauge, can be formulated in an ordinary Hilbert space with a positive metric, which is more useful in physical situations [5]. Therefore it is desirable for a model to include both cases. For example, Chan and Halpern [6] invented one which bridges between the temporal and the Coulomb gauge.

Meanwhile, following the idea of Dirac [7], d'Emilio and Mintchev [8] and then Steinmann [9] considered gauge-invariant operators in terms of an arbitrary real function which can specify any gauge whether noncovariant or covariant: In QED, operators ψ, $\overline{\psi}$, and A_μ in the Gupta-Bleuler formalism give

$$\Psi(x)\equiv\exp\left[-ie\int d^4y\,\phi^\mu(x-y)A_\mu(y)\right]\psi(x),\overline{\Psi}(x)\equiv\Psi^\dagger\gamma_0, \tag{1.2}$$

with $\phi^\mu(x)$ being a real function (distribution strictly speaking) satisfying

$$\partial_\mu\phi^\mu(x)=\delta^4(x). \tag{1.3}$$

Therefore, by noting

$$F^{\mu\nu}\equiv\partial^\mu A^\nu-\partial^\nu A^\mu, \tag{1.4}$$

all the equations of motion become gauge invariant:

$$\partial^\nu F_{\mu\nu}(x)=ej_\mu(x), \tag{1.5}$$

$$(i\partial\!\!\!/-m)\Psi(x)=e\gamma^\mu\int d^4y\,\phi^\nu(x-y)F_{\mu\nu}(y)\Psi(x). \tag{1.6}$$

They performed perturbative calculations but did not mention that the method is nothing but ϕ_μ gauge fixing, which can be seen as follows [10]: Consider an n-point function

$$\langle 0|T^*A^{\lambda_1}(x_1;\phi)\cdots A^{\lambda_n}(x_n;\phi)\Psi(y_1)\cdots\Psi(y_m)\times\overline{\Psi}(z_1)\cdots\overline{\Psi}(z_m)|0\rangle, \tag{1.7}$$

with T^* designating the covariant T^* product, where $A^\mu(x;\phi)$ is physical, that is, a gauge-invariant photon field, given by

$$A_\mu(x;\phi)\equiv-\int d^4y\,\phi^\nu(x-y)F_{\mu\nu}(y). \tag{1.8}$$

By a perturbative calculation we can see that the original photon propagator

*Electronic address: taro1scp@mbox.nc.kyushu-u.ac.jp
†Electronic address: tnmr1scp@mbox.nc.kyushu-u.ac.jp

$$D^{\mu\nu}(p) \equiv \frac{-i}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \tag{1.9}$$

must be replaced by a ϕ_μ-dependent one

$$D^{\mu\nu}(p;\phi) \equiv \frac{-i}{p^2}\{g^{\mu\nu} + ip^\mu \tilde{\phi}^\nu(p) - i\tilde{\phi}^{\mu *}(p)p^\nu + p^\mu p^\nu |\tilde{\phi}_\rho(p)|^2\} \tag{1.10}$$

in the ordinary n-point function

$$\langle 0|TA^{\lambda_1}(x_1)\cdots A^{\lambda_n}(x_n)\psi(y_1)\cdots\psi(y_m)\overline{\psi}(z_1)\cdots\overline{\psi}(z_m)|0\rangle. \tag{1.11}$$

Here $\tilde{\phi}_\mu(p)$ is the Fourier transform of $\phi_\mu(x)$:

$$\phi_\mu(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx}\tilde{\phi}_\mu(p). \tag{1.12}$$

Since $\phi_\mu(x)$ is real and obeys Eq. (1.3),

$$\tilde{\phi}^*_\mu(p) = \tilde{\phi}_\mu(-p), \quad p^\mu \tilde{\phi}_\mu(p) = i. \tag{1.13}$$

[The fact that their model can be regarded as the ϕ_μ-gauge fixing will be more clearly seen below at Eq. (2.29).] The choices

$$\phi^\mu(x) = \left(0, \frac{\nabla}{\nabla^2}\delta(x^0)\right) \equiv \left(0, \frac{\boldsymbol{x}}{4\pi|\boldsymbol{x}|^3}\delta(x^0)\right),$$

$$\tilde{\phi}^\mu(p) = \left(0, -i\frac{\boldsymbol{p}}{|\boldsymbol{p}|^2}\right), \tag{1.14}$$

$$\phi^\mu(x) = (0,0,0,\delta(x^0)\delta(x^1)\delta(x^2)\theta(x^3)), \quad \tilde{\phi}^3(p) = \frac{i}{p_3 + i\epsilon}, \tag{1.15}$$

$$\phi^\mu(x) = \frac{\partial^\mu}{\Box}, \quad \tilde{\phi}^\mu(p) = \frac{ip^\mu}{p^2}, \tag{1.16}$$

give us the Coulomb, the axial, and the covariant Landau gauges, respectively.

There are, however, problems.

The axial case: the infrared singularity is severe. In the photon propagator (1.10), $|\tilde{\phi}_\mu|^2$ cannot be handled even if the well-defined $\tilde{\phi}_\mu(p)$, Eq. (1.15), is used. To have a well-defined axial gauge theory, we must rely on a negative metric [11].

The covariant case: in Minkowski space it is problematic to divide by p^2. If the $i\epsilon$ prescription is employed, $\tilde{\phi}_\mu(p) = ip_\mu/(p^2 + i\epsilon)$, $\phi_\mu(x)$ becomes complex, $\tilde{\phi}^*_\mu(p) \neq \tilde{\phi}_\mu(-p)$. If then $\tilde{\phi}_\mu(p) = ip_\mu/(p^2 + i\epsilon p_0)$ or $\tilde{\phi}_\mu(p) = ip_\mu/(p^2 - i\epsilon p_0)$ is employed, $p_0 \leftrightarrow -p_0$ symmetry is broken, yielding time-reversal violation. The principal value prescription $\tilde{\phi}_\mu(p) = [ip_\mu/(p^2 + i\epsilon) + ip_\mu/(p^2 - i\epsilon)]/2$, on the other hand, violates microcausality because of the acausal part $ip_\mu/(p^2 - i\epsilon)$. The issue can also be recognized when looking at the propagator (1.10). We must define the dipole $1/p^4$ part, which is, however, impossible without introducing a negative metric. There is another issue in view of the invariant operator (1.2): The support of $\phi_\mu(x)$ must be spacelike. Under the timelike support of $\phi_\mu(x)$, there is an ordering problem for the operators ψ / $\overline{\psi}$ and A_μ which spoils the definition of the invariant operator.

Therefore we are almost forced to adopt a Coulomb-type, spacelike $\phi_\mu(x)$ for a well-defined operator formalism. We follow the canonical procedure finding a well-defined Hamiltonian with which we can build up the Euclidean path integral expression [12] for the trace formula of an imaginary time evolution operator. The expression is of course far from covariant but may be made covariant by reviving the redundant variables into the path integral formula [13]. The final form is free from the difficulty mentioned above. These are the contents of Sec. II. In Sec. III, the proof of gauge independence of the free energy and the S matrix is presented. In Sec. IV, we clarify the reason why we can discuss the gauge invariance more straightforwardly in the functional approach, since despite the many discussions on gauge transformations using the path integral, there seems to have been no close examination of the justification. The final Sec. V is devoted to a discussion.

II. EUCLIDEAN PATH INTEGRAL EXPRESSION

As was mentioned in the Introduction, we must work with a noncovariant ϕ_μ to have a well-defined operator theory. Covariance, however, is useful in perturbation theory. In this section we build up a path integral formula starting with a canonical theory obtained from the noncovariant ϕ_μ, since we expect that path integral expressions would be covariant all the time. We rely on the Euclidean formalism since the path integral becomes well defined and much more effective even for performing nonperturbative treatments, such as the WKB approximation, instantons, etc. We follow a step-by-step procedure to obtain a path integral expression because a rough argument utilizing a continuum representation would spoil the plausibility of our path integral formula.[1]

We first decompose A^μ such that

$$A^\mu(x) = A^\mu(x;\phi) + \partial^\mu \omega(x), \tag{2.1}$$

where

$$A^\mu(x;\phi) \equiv \int d^4y\{\delta^\mu_\nu \delta^4(x-y) - \partial^\mu_x \phi_\nu(x-y)\}A^\nu(y), \tag{2.2}$$

$$\omega(x) \equiv \int d^4y\, \phi^\mu(x-y)A_\mu(y). \tag{2.3}$$

[1]For example, a nonlocal bilinear term, which must be present in the holomorphic representation under a discretized (well-defined) measure, goes away in the continuum (ill-defined) measure [14].

Relation (2.2) is nothing but Eq. (1.8) with the use of Eq. (1.4) as well as Eq. (1.3). In order to discuss the path integral the Schrödinger picture is employed so that the time argument of the fields is omitted, and thus $A^{\mu}(\boldsymbol{x})$ is used for the time being.

Now to avoid the infrared singularity as well as the operator ordering problem, we take a $\phi_{\mu}(x)$ whose support is three dimensional and spacelike:

$$\phi^{\mu}(x)=(0,f^{i}(\boldsymbol{x})\,\delta(x_0)),\quad \tilde{\phi}^{\mu}(p)=(0,\tilde{f}^{i}(\boldsymbol{p})),\qquad (2.4)$$

which converts Eqs. (2.1)–(2.3) into

$$A^{i}(\boldsymbol{x})=A^{i}(\boldsymbol{x};\phi)+\nabla^{i}\omega(\boldsymbol{x}),\qquad (2.5)$$

$$A^{i}(\boldsymbol{x};\phi)=\int d^3y\{\delta^{ij}\delta^3(\boldsymbol{x}-\boldsymbol{y})-\nabla_x^{i}f^{j}(\boldsymbol{x}-\boldsymbol{y})\}A^{j}(\boldsymbol{y}),\qquad (2.6)$$

$$\omega(\boldsymbol{x})=\int d^3y\,f^{j}(\boldsymbol{x}-\boldsymbol{y})A^{j}(\boldsymbol{y}).\qquad (2.7)$$

Since the number of genuine physical components is 2, say, $A^{(\alpha)}$ $(\alpha=1,2)$, we must select them out from $A^{i}(\boldsymbol{x})$ $(i=1,2,3)$. After a brief calculation [15], we find

$$A^{(\alpha)}(\boldsymbol{x})\equiv n^{k}_{(\alpha)}(-i\nabla)A^{k}(\boldsymbol{x})\equiv\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}e^{i\boldsymbol{p}\cdot\boldsymbol{x}}n^{k}_{(\alpha)}(\boldsymbol{p})\tilde{A}^{k}(\boldsymbol{p}),\qquad (2.8)$$

where $n^{k}_{(\alpha)}(\boldsymbol{p})$ $(\alpha=1,2)$ are an orthonormal base with $n^{k}_{(3)}(\boldsymbol{p})$, obeying

$$\sum_{\alpha=1}^{3}n^{j*}_{(\alpha)}(\boldsymbol{p})n^{k}_{(\alpha)}(\boldsymbol{p})=\delta^{jk},\quad n^{k*}_{(\alpha)}(\boldsymbol{p})=n^{k}_{(\alpha)}(-\boldsymbol{p}),\qquad (2.9)$$

and explicitly given by

$$n^{k}_{(1)}(\boldsymbol{p})\equiv\epsilon^{klm}n^{l}_{(2)}(\boldsymbol{p})n^{m}_{(3)}(\boldsymbol{p}),$$

$$n^{k}_{(2)}(\boldsymbol{p})\equiv\{ip^{k}+\boldsymbol{p}^2\tilde{f}^{k}(\boldsymbol{p})\}/\sqrt{\boldsymbol{p}^2(\boldsymbol{p}^2|\tilde{\boldsymbol{f}}(\boldsymbol{p})|^2-1)},$$

$$n^{k}_{(3)}(\boldsymbol{p})\equiv ip^{k}/|\boldsymbol{p}|.\qquad (2.10)$$

Therefore $A^{i}(\boldsymbol{x};\phi)$ in Eq. (2.6) can be given by

$$A^{i}(\boldsymbol{x};\phi)=n^{i*}_{(1)}(-i\nabla)A^{(1)}(\boldsymbol{x})+[\delta^{ij}-\nabla^{i}\tilde{f}^{j}(-i\nabla)]\times n^{j*}_{(2)}(-i\nabla)A^{(2)}(\boldsymbol{x}).\qquad (2.11)$$

The action, with the source term J^{μ}, can be expressed in terms of $A^{(\alpha)}(\boldsymbol{x})$ such that

$$\begin{aligned}S&=\int d^4x\left\{-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+J^{\mu}A_{\nu}\right\}\\&=\int d^4x\left\{\frac{1}{2}\sum_{\alpha=1}^{2}A^{(\alpha)}(x)\nabla^2A^{(\alpha)}(x)+\frac{1}{2}[\dot{A}^{(1)}(x)]^2-\frac{1}{2}\dot{A}^{(2)}(x)\nabla^2|\tilde{\boldsymbol{f}}(-i\nabla)|^2\dot{A}^{(2)}(x)\right.\\&\left.\quad-A^{0}(x;\phi)\sqrt{\nabla^2(\nabla^2|\tilde{\boldsymbol{f}}(-i\nabla)|^2+1)}\dot{A}^{(2)}(x)+\frac{1}{2}[\nabla A^{0}(x;\phi)]^2+J_0(x)A^{0}(x;\phi)-\boldsymbol{J}(x)\cdot[\boldsymbol{A}(x;\phi)]\right\},\end{aligned}\qquad (2.12)$$

whose last term $[\boldsymbol{A}(x;\phi)]$ implies that $\boldsymbol{A}(x;\phi)$ has been given by $A^{(1)}$ and $A^{(2)}$ through the relation (2.11). We are still in a constrained system because $A^{0}(x;\phi)$ is not a dynamical variable. Solving the constraint, we obtain the Hamiltonian

$$\begin{aligned}H(t)=\int d^3x&\left\{\frac{1}{2}\sum_{\alpha=1}^{2}[[\Pi^{(\alpha)}(\boldsymbol{x})]^2+[\nabla A^{(\alpha)}(\boldsymbol{x})]^2]\right.\\&+J_0(x)\frac{\sqrt{\nabla^2(\nabla^2|\tilde{\boldsymbol{f}}(-i\nabla)|^2+1)}}{\nabla^2}\Pi^{(2)}(\boldsymbol{x})\\&\left.+\frac{1}{2}J_0(x)|\tilde{\boldsymbol{f}}(-i\nabla)|^2J_0(x)+\boldsymbol{J}(x)\cdot[\boldsymbol{A}(\boldsymbol{x};\phi)]\right\},\end{aligned}\qquad (2.13)$$

where $x\equiv(t,\boldsymbol{x})$ and $\Pi^{(\alpha)}$ is the canonical conjugate momentum of $A^{(\alpha)}$. Note that the Hamiltonian explicitly depends on time through the source term.

Now quantization can be carried out:

$$[\hat{A}^{(\alpha)}(\boldsymbol{x}),\hat{\Pi}^{(\beta)}(\boldsymbol{y})]=i\,\delta^{\alpha\beta}\delta^3(\boldsymbol{x}-\boldsymbol{y}),$$

$$[\hat{A}^{(\alpha)}(\boldsymbol{x}),\hat{A}^{(\beta)}(\boldsymbol{y})]=[\hat{\Pi}^{(\alpha)}(\boldsymbol{x}),\hat{\Pi}^{(\beta)}(\boldsymbol{y})]=0,\qquad (2.14)$$

where a caret denotes an operator. Consider the quantity

$$Z_T[J]\equiv\lim_{N\to\infty}\mathrm{Tr}[(\boldsymbol{I}-\Delta\tau\hat{H}_N)(\boldsymbol{I}-\Delta\tau\hat{H}_{N-1})\cdots(\boldsymbol{I}-\Delta\tau\hat{H}_1)],\qquad (2.15)$$

where $\Delta\tau\equiv T/N$, $H_j\equiv H(j\Delta\tau)$, and the source $J_{\mu}(x)$ has been assumed to be analytically continuable. Here Tr can be taken for any complete set, but a functional representation,

$$\hat{A}^{(\alpha)}(\boldsymbol{x})|\{A\}\rangle=A^{(\alpha)}(\boldsymbol{x})|\{A\}\rangle,$$

$$\hat{\Pi}^{(\alpha)}(\boldsymbol{x})|\{\Pi\}\rangle=\Pi^{(\alpha)}(\boldsymbol{x})|\{\Pi\}\rangle,\qquad (2.16)$$

with completeness,

$$\int \mathcal{D}A^{(\alpha)}|\{A\}\rangle\langle\{A\}|=\boldsymbol{I}, \quad \int \mathcal{D}\Pi^{(\alpha)}|\{\Pi\}\rangle\langle\{\Pi\}|=\boldsymbol{I}, \tag{2.17}$$

is employed, yielding the Euclidean path integral representation [13]

$$Z_T[J]=\lim_{N\to\infty}\mathcal{N}^{2N}\int\prod_{k=1}^{N}\mathcal{D}A_k^{(\alpha)}\mathcal{D}\Pi_k^{(\alpha)} \times\exp\left[\Delta\tau\sum_{k=1}^{N}\left\{\int d^3x\, i\sum_{\alpha=1}^{2}\Pi_k^{(\alpha)} \times(\boldsymbol{x})\frac{A_k^{(\alpha)}(\boldsymbol{x})-A_{k-1}^{(\alpha)}(\boldsymbol{x})}{\Delta\tau}-H_k(\Pi_k^{(\alpha)},A_k^{(\alpha)})\right\}\right], \tag{2.18}$$

where $\mathcal{N}$ is the normalization factor defined in

$$\langle\{\Pi\}|\{A\}\rangle\equiv\mathcal{N}\exp\left[-i\int d^3x\sum_{\alpha=1}^{2}\Pi^{(\alpha)}(\boldsymbol{x})A^{(\alpha)}(\boldsymbol{x})\right]:$$

$$\mathcal{N}=\prod_{\boldsymbol{x}}\frac{1}{2\pi}. \tag{2.19}$$

Because of the trace, the boundary condition is periodic $A_0^{(\alpha)}(\boldsymbol{x})=A_N^{(\alpha)}(\boldsymbol{x})$. (This is a formal expression and is ill defined actually. To make $Z_T[J]$ well defined, it is also necessary to discretize the space part and to put the system in a box [13]. But hereafter we use the continuum representation for notational simplicity.)

Therefore we write

$$\begin{aligned}Z_T[J]&=\int\mathcal{D}A^{(\alpha)}\mathcal{D}\Pi^{(\alpha)}\exp\left[\int d^4x_E\left\{i\sum_{\alpha=1}^{2}\Pi^{(\alpha)}\dot{A}^{(\alpha)}-H(\Pi,A)\right\}\right]\\&=\int\mathcal{D}A^{(\alpha)}\mathcal{D}\Pi^{(\alpha)}\exp\left[\int d^4x_E\left\{i\sum_{\alpha=1}^{2}\Pi^{(\alpha)}(\tau,\boldsymbol{x})\dot{A}^{(\alpha)}(\tau,\boldsymbol{x})-\frac{1}{2}\sum_{\alpha=1}^{2}\{[\Pi^{(\alpha)}(\tau,\boldsymbol{x})]^2+[A^{(\alpha)}(\tau,\boldsymbol{x})]^2\}\right.\right.\\&\left.\left.\quad+iJ_4(\tau,\boldsymbol{x})\frac{\sqrt{\nabla^2(\nabla^2|\tilde{\boldsymbol{f}}(-i\nabla)|^2+1)}}{\nabla^2}\Pi^{(2)}(\tau,\boldsymbol{x})+\frac{1}{2}J_4(\tau,\boldsymbol{x})|\tilde{\boldsymbol{f}}(-i\nabla)|^2J_4(\tau,\boldsymbol{x})-\boldsymbol{J}(\tau,\boldsymbol{x})\cdot[\boldsymbol{A}(\tau,\boldsymbol{x};\phi)]\right\}\right],\end{aligned} \tag{2.20}$$

where

$$\int d^4x_E\equiv\int_0^T d\tau\int d^3x, \quad J_4\equiv iJ_0. \tag{2.21}$$

Integrating with respect to $\Pi^{(\alpha)}$, inserting the Gaussian identity

$$\boldsymbol{I}=\int\mathcal{D}A^4(\tau,\boldsymbol{x})[\det(-\nabla^2)]^{1/2} \times\exp\left[-\int d^4x_E\frac{1}{2}A^4(\tau,\boldsymbol{x})(-\nabla^2)A^4(\tau,\boldsymbol{x})\right], \tag{2.22}$$

and then introducing the new integration variable such that

$$A^4(\tau,\boldsymbol{x};\phi)\equiv A^4(\tau,\boldsymbol{x})+\frac{\sqrt{\nabla^2(\nabla^2|\tilde{\boldsymbol{f}}(-i\nabla)|^2+1)}}{\nabla^2}\dot{A}^{(2)}(\tau,\boldsymbol{x}) +\frac{1}{\nabla^2}J_4(\tau,\boldsymbol{x}), \tag{2.23}$$

we finally arrive at

$$Z_T[J]=\int\mathcal{D}A^{(\alpha)}\mathcal{D}A^4(\tau,\boldsymbol{x};\phi)[\det(-\nabla^2)]^{1/2} \times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}(\tau,\boldsymbol{x};\phi)F_{\mu\nu}(\tau,\boldsymbol{x};\phi) +J_\mu(\tau,\boldsymbol{x})A_\mu(\tau,\boldsymbol{x};\phi)\right\}\right], \tag{2.24}$$

where $F_{\mu\nu}(\tau,\boldsymbol{x};\phi)\equiv\partial_\mu A_\nu(\tau,\boldsymbol{x};\phi)-\partial_\nu A_\mu(\tau,\boldsymbol{x};\phi)$ $(\mu,\nu=1,2,3,4)$. $A^i(\tau,\boldsymbol{x};\phi)$ is now defined [in view of Eq. (2.11)] by

$$A^i(\tau,\boldsymbol{x};\phi)\equiv n_{(1)}^{i*}(-i\nabla)A^{(1)}(\tau,\boldsymbol{x})+[\delta^{ij}-\nabla^i\tilde{f}^j(-i\nabla)] \times n_{(2)}^{j*}(-i\nabla)A^{(2)}(\tau,\boldsymbol{x}). \tag{2.25}$$

In Eq. (2.24), almost everything has been recovered but the functional measure which still consists of three components. To cure this, the gauge degree of freedom ω, Eq. (2.7),

$$\omega(\tau,\boldsymbol{x})=\int d^3y\boldsymbol{f}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{A}(\tau,\boldsymbol{y})$$
$$=\tilde{\boldsymbol{f}}(-i\nabla)\cdot\boldsymbol{A}(\tau,\boldsymbol{x}), \quad (2.26)$$

is revived by means of the δ function, giving

$$Z_T[J]=\int \mathcal{D}A_\mu\delta(\tilde{\boldsymbol{f}}(-i\nabla)\cdot\boldsymbol{A}(\tau,\boldsymbol{x}))\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+J_\mu A_\mu\right\}\right], \quad (2.27)$$

where use has been made of the relation of the functional measure

$$\mathcal{D}A^i=\mathcal{D}A^{(\alpha)}\mathcal{D}\omega[\det(-\nabla^2)]^{1/2}, \quad (2.28)$$

obtained from Eqs. (2.25) and (2.26).

Going back to the original notation (2.4) we find the covariant expression

$$Z_T[J]=\int \mathcal{D}A_\mu\delta\left(\int d^4y_E\phi_\mu(x-y)A_\mu(y)\right)\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+J_\mu A_\mu\right\}\right]. \quad (2.29)$$

In view of Eq. (2.29), we can recognize that the d'Emilio-Mintchev-Steinmann model is nothing but a ϕ_μ-gauge fixing. It is a straightforward task to check that the propagator is correctly given by Eq. (1.10).

In the Coulomb case, Eq. (1.14), that is, $\tilde{\boldsymbol{f}}(-i\nabla)\equiv\nabla/\nabla^2$ in Eq. (2.27), a familiar expression,

$$Z_T^{\mathrm{Coul}}[J]=\int \mathcal{D}A_\mu\delta(\nabla\cdot\boldsymbol{A})|\det(-\nabla^2)|\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+J_\mu A_\mu\right\}\right] \quad (2.30)$$

is obtained. Furthermore, the troubles of the covariant case are avoided, since the expression has no singularity at all in the Landau gauge:

$$\phi_\mu(x)=\frac{\partial_\mu}{\Box_E}, \quad \Box_E\equiv\partial_\mu\partial_\mu, \quad (2.31)$$

$$Z_T^{\mathrm{land}}[J]=\int \mathcal{D}A_\mu\delta(\partial_\mu A_\mu)|\det(-\Box_E)|\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+J_\mu A_\mu\right\}\right]. \quad (2.32)$$

(However, there still remains the infrared singularity in the axial gauge. We therefore do not take the case into account.)

So far we have concentrated only on the photon sector but it is a rather simple task to include fermions in the path integral [16,17]. The total path integral expression is, therefore,

$$Z[J,\bar{\eta},\eta]=\int \mathcal{D}A_\mu\mathcal{D}\psi\mathcal{D}\bar{\psi}\delta\left(\int d^4y_E\phi_\mu(x-y)A_\mu(y)\right)\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+\bar{\psi}(\not{D}+m)\psi+J_\mu A_\mu+\bar{\eta}\psi+\bar{\psi}\eta\right\}\right], \quad (2.33)$$

where antiperiodic boundary conditions for fermions must be understood.[2] We call Eq. (2.33) the generating functional.

III. PROOF OF GAUGE INDEPENDENCE

In this section, gauge independence of Eq. (2.33) is proved by showing that any choice of ϕ_μ leads to the same result in the case of the free energy as well as the S matrix. To this end, let us first study how a gauge transformation affects the expression (2.33): The gauge transformation from A_μ to A'_μ is given by

$$A_\mu(x)\mapsto A'_\mu(x)=A_\mu(x)+\partial_\mu\theta(x). \quad (3.1)$$

The gauge conditions are taken to be

$$\int d^4y_E\phi_\mu(x-y)A_\mu(y)=0 \quad \text{and}$$
$$\int d^4y_E\phi'_\mu(x-y)A'_\mu(y)=0, \quad (3.2)$$

respectively. The second relation can be rewritten as

$$0=\int d^4y_E\phi'_\mu(x-y)A'_\mu(y)$$
$$=\int d^4y_E\phi_\mu(x-y)A_\mu(y)$$
$$+\int d^4y_E\Delta\phi_\mu(x-y)A_\mu(y)+\theta(x), \quad (3.3)$$

by use of Eqs. (3.1) and (1.3), where

[2] Again, for notational simplicity, we have employed a continuum representation which is, however, valid only in perturbation theory: For example, the so-called Wilson term is necessary under the well-defined fermionic measure [17,18].

$$\Delta\phi_\mu(x)\equiv\phi'_\mu(x)-\phi_\mu(x),\qquad \partial^\mu\Delta\phi_\mu(x)=0. \tag{3.4}$$

Therefore, under the gauge conditions (3.2), $\theta(x)$ is given by

$$\theta(x)=-\int d^4y_E\Delta\phi_\mu(x-y)A_\mu(y). \tag{3.5}$$

From Eq. (2.33), the generating functional in the ϕ'_μ gauge is

$$Z'[J,\bar\eta,\eta]=\int \mathcal{D}A'_\mu\mathcal{D}\psi'\mathcal{D}\bar\psi'\,\delta\left(\int d^4y_E\phi'_\mu(x-y)A'_\mu(y)\right)\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F'_{\mu\nu}F'_{\mu\nu}+\bar\psi'(\not{D}'+m)\psi'+J_\mu A'_\mu+\bar\eta\psi'+\bar\psi'\eta\right\}\right], \tag{3.6}$$

where A'_μ, ψ', and $\bar\psi'$ are related to their analogous quantities in the ϕ_μ gauge, A_μ, ψ, and $\bar\psi$, through $\psi'(x)=e^{i\theta(x)}\psi(x)$, $\bar\psi'(x)=\bar\psi(x)e^{-i\theta(x)}$, and Eq. (3.1). Then a simple change of variables with a trivial Jacobian[3] leads to

$$Z'[J,\bar\eta,\eta]=\int \mathcal{D}A_\mu\mathcal{D}\psi\mathcal{D}\bar\psi\delta\left(\int d^4y_E\phi_\mu(x-y)A_\mu(y)\right)\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+\bar\psi(\not{D}+m)\psi+J_\mu(A_\mu+\partial_\mu\theta)+\bar\eta e^{ie\theta}\psi+\bar\psi e^{-ie\theta}\eta\right\}\right]. \tag{3.7}$$

Therefore, if the sources J, $\bar\eta$, and η are absent, the relation

$$Z'=Z(=\mathrm{Tr}e^{-TH}) \tag{3.8}$$

implies that the free energy of QED is gauge independent. Moreover, it can be recognized that expectation values of a gauge invariant operator, such as the Belinfante energy-momentum tensor $\langle\{n\}|\Theta_{\mu\nu}(x)|\{n\}\rangle$, are gauge invariant,[4] where $|\{n\}\rangle$ denotes states of fermions and photons. In this way, the path integral gives us a quick and intuitive derivation of gauge independence, whose meaning is clarified in the next section.

In order to discuss gauge independence of the S matrix, however, we need a further consideration: Suppose $\theta(x)$ is infinitesimal so that the difference between $Z'[J,\bar\eta,\eta]$ and $Z[J,\bar\eta,\eta]$ is

$$\begin{aligned}\Delta Z[J,\bar\eta,\eta]&=Z'[J,\bar\eta,\eta]-Z[J,\bar\eta,\eta]\\&=\int \mathcal{D}A_\mu\mathcal{D}\psi\mathcal{D}\bar\psi\delta\left(\int d^4y_E\phi_\mu(x-y)A_\mu(y)\right)\int d^4x_E[\theta\partial_\mu J_\mu+ie\theta(\bar\psi\eta-\bar\eta\psi)]\\&\quad\times\exp\left[-\int d^4x_E\left\{\frac{1}{4}F_{\mu\nu}F_{\mu\nu}+\bar\psi(\not{D}+m)\psi+J_\mu A_\mu+\bar\eta\psi+\bar\psi\eta\right\}\right].\end{aligned} \tag{3.9}$$

The S matrix is given, after rotating back to the Minkowski space, by cutting external legs and multiplying the wave functions of fermions and photons, that is, multiplying the Green's function by[5]

$$\frac{\not{p}-m}{i\sqrt{z_2}}\frac{u(\boldsymbol{p},s)}{\sqrt{(2\pi)^3 2p_0}},\quad \frac{\overline{u(\boldsymbol{p},s)}}{\sqrt{(2\pi)^3 2p_0}}\frac{\not{p}-m}{i\sqrt{z_2}}\quad\text{for fermions,}\qquad \frac{-q^2}{i\sqrt{z_3}}\frac{\xi^{(i)}_\mu(\boldsymbol{q})}{\sqrt{(2\pi)^3 2q_0}}\quad\text{for photons.} \tag{3.10}$$

---

[3]It reads $|\delta A'_\mu/\delta A_\nu|=|\det(\delta_{\mu\nu}-\partial_\mu\Delta\phi_\nu)|$, according to Eqs. (3.1) and (3.5), and is unity: Consider the determinant of the matrix, $M_{ij}=\delta_{ij}+A_iB_j$, with $\Sigma_iA_iB_i=0$, to find $\det M=\exp[\mathrm{Tr}\ln(1+AB)]=1$, since $\mathrm{Tr}(AB)^n=0$, $\forall n$.

[4]Putting $\Theta_{\mu\nu}(x)$ into the right-hand side of Eq. (2.33) and letting all sources zero, we obtain a gauge invariant quantity $\mathrm{Tr}[\Theta_{\mu\nu}(x)e^{-TH}]=\Sigma_{\{n\}}\langle\{n\}|\Theta_{\mu\nu}(x)|\{n\}\rangle e^{-TE_{\{n\}}}$. Then, $T\to\infty$ picks up the expectation value between the vacuum $|0\rangle$, which is gauge invariant. Next, put $T\to\infty$ in the quantity $\mathrm{Tr}[(\Theta_{\mu\nu}(x)-\langle 0|\Theta_{\mu\nu}(x)|0\rangle)e^{-TH}]$ to give the expectation value between the first excited state, which is again gauge invariant. Repeating these steps, we find the expectation value between any state is gauge invariant. Q.E.D.

[5]It is troublesome to write out the Lehmann-Symanzik-Zimmerman-(LSZ)-asymptotic state for fermions in this way; since in a noncovariant gauge z_2 is matrix valued acting differently on each spinor index [19]. However, there are additional renormalization conditions, since the self-energy is not merely a function of $\not{p}$: It depends on $p_0\gamma_0$ as well as $p_k\gamma_k$ in the Coulomb gauge, for instance. Here we assume that z_2 has already been diagonalized by utilizing these additional degrees.

Here the photon polarization $\xi_\mu^{(i)}(\boldsymbol{q})$ fulfills the transversal condition $q^\mu \xi_\mu^{(i)}(\boldsymbol{q})=0$. Because of this, the $\theta\partial_\mu J_\mu$ term in Eq. (3.9) (now $\theta\partial^\mu J_\mu$) drops out so that it is enough to concentrate only on the fermion legs. The S-matrix element of the fermion sectors, S_g, reads

$$S_g=S_g(z_2,G^{(2n)})=\prod_{j=1}^{n}\frac{\overline{u(\boldsymbol{p}_j,s_j')}}{\sqrt{(2\pi)^3 2(p_j)_0}}\frac{\not{p}_j-m}{i\sqrt{z_2}}G^{(2n)}(p_1,\cdots,p_n;k_1,\cdots,k_n)\prod_{j=1}^{n}\frac{\not{k}_j-m}{i\sqrt{z_2}}\frac{u(\boldsymbol{k}_j,s_j)}{\sqrt{(2\pi)^3 2(k_j)_0}} \tag{3.11}$$

and whose difference under the gauge transformation is

$$\Delta S_g=S_g'-S_g=\Delta G^{(2n)}\frac{\partial S_g}{\partial G^{(2n)}}+\Delta z_2\frac{\partial S_g}{\partial z_2}, \tag{3.12}$$

where

$$\begin{aligned}
&\Delta G^{(2n)}(p_1,\ldots,p_n;k_1,\ldots,k_n)\Big|_{k_n=p_n+\sum_{j=1}^{n}(p_j-k_j)}\\
&\equiv\int\prod_{j=1}^{n-1}(d^4x_j d^4y_j)d^4x_n\exp\left[\sum_{j=1}^{n-1}(ip_jx_j-ik_jy_j)+ip_nx_n\right]\\
&\times\frac{\delta^{2n}}{\delta\overline{\eta(x_1)}\cdots\delta\overline{\eta(x_n)}\,\delta\eta(y_1)\cdots\delta\eta(y_{n-1})\,\delta\eta(0)}\Delta Z[J,\overline{\eta},\eta]\Big|_{J=\overline{\eta}=\eta=0}\\
&=\int\prod_{j=1}^{n-1}(d^4x_j d^4y_j)d^4x_n\exp\left[\sum_{j=1}^{n-1}(ip_jx_j-ik_jy_j)+ip_nx_n\right]\\
&\times ie\left\langle 0\left|T\left\{\sum_{j=1}^{n-1}[\theta(x_j)-\theta(y_j)]+[\theta(x_n)-\theta(0)]\right\}\psi(x_1)\cdots\psi(x_n)\overline{\psi}(y_1)\cdots\overline{\psi}(y_{n-1})\overline{\psi}(0)\right|0\right\rangle
\end{aligned} \tag{3.13}$$

and Δz_2 is the change of z_2. In order to find Δz_2, take $n=1$ in Eq. (3.13),

$$\Delta G^{(2)}(x,y)=ie\langle 0|T(\theta(x)-\theta(y))\psi(x)\overline{\psi}(y)|0\rangle, \tag{3.14}$$

which is depicted in Fig. 1. By noting that

$$G^{(2)}(p)|_{\text{pole}}=\frac{iz_2}{\not{p}-m}, \tag{3.15}$$

$$\Delta G^{(2)}(p)|_{\text{pole}}=\overline{A(p)}\Big|_{\not{p}=m}\frac{iz_2}{\not{p}-m}-\frac{iz_2}{\not{p}-m}A(p)\Big|_{\not{p}=m}\equiv\frac{i\Delta z_2}{\not{p}-m}, \tag{3.16}$$

where

$$\overline{A(p)}\overset{\text{FT}}{=}\int d^4z\;\; ie\langle 0|T\theta(x)\psi(x)\overline{\psi}(z)|0\rangle(G^{(2)})^{-1}(z,y),\quad A(p)\overset{\text{FT}}{=}\int d^4z ie(G^{(2)})^{-1}(x,z)\langle 0|T\theta(y)\psi(z)\overline{\psi}(y)|0\rangle, \tag{3.17}$$

since there are no poles in $\overline{A(p)}$ and $A(p)$ at $\not{p}=m$.

As a result of $(\not{p}_i-m)$ or $(\not{k}_i-m)$ in Eq. (3.11), the surviving part of $\Delta G^{(2n)}$ must have $2n$ one-particle poles. Graphs (see Fig. 2), in which the photon in θ, Eq. (3.5), is attached to its original fermion line, that is, graphs including $\overline{A(p)}$ and $A(p)$, have the same pole structure as $G^{(2n)}$ and do contribute, but those in which the photon goes somewhere other than its original fermion line change the pole structure and do not contribute to Eq. (3.18). Therefore we write the surviving part as $\Delta\overline{G^{(2n)}}$ to find

$$\Delta\overline{G^{(2n)}}(p_1,\ldots,p_n;k_1,\ldots,k_n)$$
$$=\Delta G^{(2n)}(p_1,\ldots,p_n;k_1,\ldots,k_n)|_{2n\text{ poles}}$$
$$=\sum_{j=1}^{n}(\overline{A(p_j)}|_{\not p_j=m}G^{(2n)}(p_1,\ldots,p_n;k_1,\ldots,k_n)$$
$$-G^{(2n)}(p_1,\ldots,p_n;k_1,\ldots,k_n)A(k_j)|_{\not k_j=m})$$
$$=\sum_{j=1}^{n}\frac{\Delta z_2}{z_2}G^{(2n)}(p_1,\ldots,p_n;k_1,\ldots,k_n)=\frac{n\Delta z_2}{z_2}G^{(2n)}(p_1,\ldots,p_n;k_1,\ldots,k_n). \tag{3.18}$$

Thus Eq. (3.12) becomes

$$\Delta S_g=\prod_{j=1}^{n}\frac{\overline{u(\boldsymbol{p}_j,s_j')}}{\sqrt{(2\pi)^3 2(p_j)_0}}\frac{\not p_j-m}{i\sqrt{z_2}}\Delta\overline{G^{(2n)}}(p_1,\ldots,p_n;k_1,\ldots,k_n)\prod_{j=1}^{n}\frac{\not k_j-m}{i\sqrt{z_2}}\frac{u(\boldsymbol{k}_j,s_j)}{\sqrt{(2\pi)^3 2(k_j)_0}}+\Delta z_2\left(\frac{\partial}{\partial z_2}z_2^{-n}\right)(z_2^n S_g)=0. \tag{3.19}$$

There is thus no gauge dependence in the S matrix.

IV. FUNCTIONAL METHOD AS AN EFFICIENT TOOL FOR HANDLING GAUGE THEORIES

In this section we give a detailed discussion of why we can perform a gauge transformation so easily and intuitively in the functional representation. As was mentioned in the Introduction, gauge transformations cannot be allowed at all in the canonical operator formalism. In this sense, it is instructive to study the $A_0=0$ gauge in the conventional treatment [20], since there we need a supplementary condition, a so-called physical state condition, implying that *physical states must be gauge invariant.* This statement apparently contradicts the above situation.

In the $A_0=0$ gauge, all three components $\boldsymbol{A}$ are assumed to be dynamical and to obey the commutation relations

$$[\hat{A}_j(\boldsymbol{x}),\hat{\Pi}_k(\boldsymbol{y})]=i\delta_{jk}\delta(\boldsymbol{x}-\boldsymbol{y}),$$
$$[\hat{A}_j(\boldsymbol{x}),\hat{A}_k(\boldsymbol{y})]=[\hat{\Pi}_j(\boldsymbol{x}),\hat{\Pi}_k(\boldsymbol{y})]=0,\quad (j,k=1,2,3). \tag{4.1}$$

Again the caret signifies operators. The physical state condition is given as

$$\hat{\Phi}(\boldsymbol{x})|\text{phys}\rangle\equiv\left[\sum_{k=1}^{3}(\partial_k\hat{\boldsymbol{\Pi}}_k(\boldsymbol{x}))+J_0(\boldsymbol{x})\right]|\text{phys}\rangle=0\quad, \tag{4.2}$$

where $J_\mu(x)$ is supposed to be a c-number current. First this should be read such that *there is no gauge transformation in the physical space* since $\hat{\Phi}$ is the generator of gauge trans-

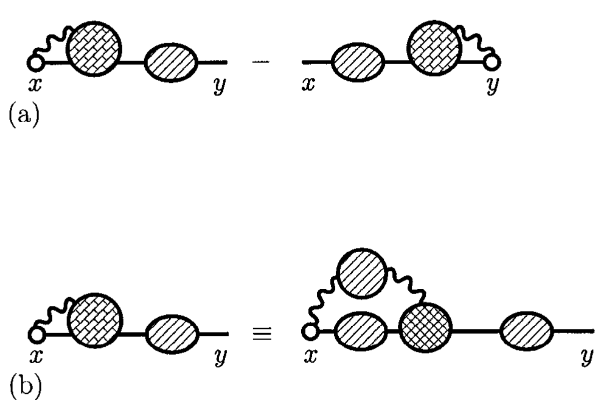

FIG. 1. (a) The two-point function in Eq. (3.14): the circle denotes the θ insertion. The blob in (a) graphs is collections of the full propagators and the vertex, seen in (b).

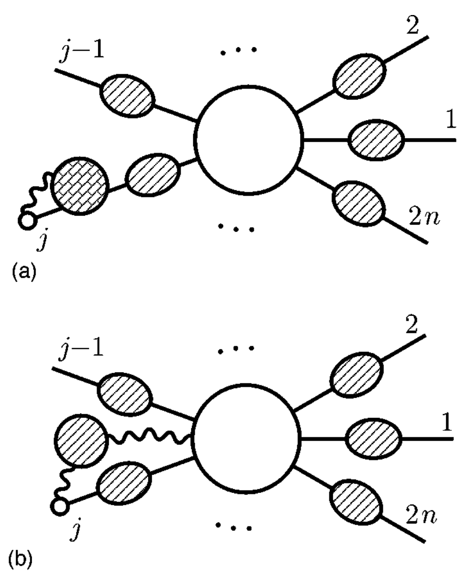

FIG. 2. (a) Graphs that do contribute. (b) Those do not contribute. Large circles at the center of each graph denote the amputated Green's functions.

formations. However, the usual Hilbert space consisting only of normalizable states does not allow $\hat{\Phi}(\boldsymbol{x})$ to exist, since $\hat{\Phi}(\boldsymbol{x})$ is a local operator to result in $\hat{\Phi}(\boldsymbol{x})=0$ due to the theorem of Federbush and Johnson [21]. Therefore we cannot obtain the physical state. Nevertheless, the state can be expressed in the functional (Schrödinger) representation [22]:

$$\hat{\boldsymbol{A}}(\boldsymbol{x})|\{\boldsymbol{A}\}\rangle=\boldsymbol{A}(\boldsymbol{x})|\{\boldsymbol{A}\}\rangle,\quad \hat{\boldsymbol{\Pi}}(\boldsymbol{x})|\{\boldsymbol{\Pi}\}\rangle=\boldsymbol{\Pi}(\boldsymbol{x})|\{\boldsymbol{\Pi}\}\rangle,$$

$$\langle\{\boldsymbol{A}\}|\hat{\boldsymbol{\Pi}}(\boldsymbol{x})=-i\frac{\delta}{\delta\boldsymbol{A}(\boldsymbol{x})}\langle\{\boldsymbol{A}\}|,\cdots. \tag{4.3}$$

To see why this is true, consider the state $|\{\boldsymbol{A}\}\rangle$, which can be constructed in terms of the Fock states as follows: The creation and annihilation operators are given by

$$\hat{\boldsymbol{A}}(\boldsymbol{x})=\int\frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}\sqrt{2|\boldsymbol{k}|}}[\boldsymbol{a}(\boldsymbol{k})e^{i\boldsymbol{k},\cdot\boldsymbol{x}}+\boldsymbol{a}^{\dagger}(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}]$$

$$[a_i(\boldsymbol{k}),a_j^{\dagger}(\boldsymbol{k}')]=\delta_{ij}\delta(\boldsymbol{k}-\boldsymbol{k}'),\quad [a_i(\boldsymbol{k}),a_j(\boldsymbol{k}')]=0, \tag{4.4}$$

and the vacuum $|0\rangle$ obeys $\boldsymbol{a}(\boldsymbol{k})|0\rangle=0$. Now recall the quantum-mechanical case [23]

$$\hat{q}|q\rangle=q|q\rangle,\quad \hat{q}=\frac{1}{\sqrt{2}}(a+a^{\dagger}),\quad a|0\rangle=0; \tag{4.5}$$

then,

$$|q\rangle=\frac{1}{\pi^{1/4}}\exp\left(-\frac{q^2}{2}+\sqrt{2}qa^{\dagger}-\frac{(a^{\dagger})^2}{2}\right)|0\rangle. \tag{4.6}$$

These bring us to the expression

$$|\{\boldsymbol{A}\}\rangle\simeq\exp\left[-\frac{1}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}\boldsymbol{A}(\boldsymbol{x})K(\boldsymbol{x}-\boldsymbol{y})\boldsymbol{A}(\boldsymbol{y})+\int d^3\boldsymbol{x}\int d^3\boldsymbol{k}\sqrt{\frac{2|\boldsymbol{k}|}{(2\pi)^3}}\boldsymbol{A}(\boldsymbol{x})\cdot\boldsymbol{a}^{\dagger}(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}-\frac{1}{2}\int d^3\boldsymbol{k}\boldsymbol{a}^{\dagger}(\boldsymbol{k})\cdot\boldsymbol{a}^{\dagger}(-\boldsymbol{k})\right]|0\rangle, \tag{4.7}$$

where

$$K(\boldsymbol{x})\equiv\int\frac{d^3\boldsymbol{k}}{(2\pi)^3}|\boldsymbol{k}|e^{i\boldsymbol{k}\cdot\boldsymbol{x}}, \tag{4.8}$$

which is apparently divergent:

$$K(\boldsymbol{x})=O\left(\frac{\Lambda^2}{|\boldsymbol{x}|^2}\right), \tag{4.9}$$

where Λ is some cutoff. The physical state in the functional representation is thus found as

$$\langle\{\boldsymbol{A}\}|\hat{\Phi}(\boldsymbol{x})|\text{phys}\rangle=\left(-i\nabla\frac{\delta}{\delta\boldsymbol{A}(\boldsymbol{x})}-J_0(\boldsymbol{x})\right)\Psi_{\text{phys}}[\boldsymbol{A}]=0, \tag{4.10}$$

where $\Psi_{\text{phys}}[\boldsymbol{A}]\equiv\langle\{\boldsymbol{A}\}|\text{phys}\rangle$.

Therefore physical states can be obtained under the functional representation, implying that gauge transformations are permissible. Now we can see the reason: Within a single Fock state the physical state condition (4.2) merely implies $\hat{\Phi}(\boldsymbol{x})=0$. However, we should bear the following fact in mind: *The functional representation consists of infinitely many collections of inequivalent Fock spaces*, since the inner product of $|\{\boldsymbol{A}\}\rangle$, Eq. (4.7), to the Fock vacuum is found to be

$$\langle\{\boldsymbol{A}\}|0\rangle\sim\exp\left[-\frac{1}{2}\int d^3\boldsymbol{x}d^3\boldsymbol{y}\boldsymbol{A}(\boldsymbol{x})K(\boldsymbol{x}-\boldsymbol{y})\boldsymbol{A}(\boldsymbol{y})\right]$$

$$=\exp\left[-\frac{1}{2}\int_{|\boldsymbol{k}|<\Lambda}\frac{d^3\boldsymbol{k}}{(2\pi)^3}|\boldsymbol{k}|\boldsymbol{A}(\boldsymbol{k})\boldsymbol{A}(-\boldsymbol{k})\right]\overset{\Lambda\to\infty}{\to}0. \tag{4.11}$$

This happens for any value of $\boldsymbol{A}(\boldsymbol{x})$: $\boldsymbol{A}(\boldsymbol{k})$, giving a finite integral in Eq. (4.11), is $O(|\boldsymbol{k}|^{-3/2})$. While the whole functional space contains a huge number of classes such as non-Fourier integrable one or nondifferentiable stochastic one, letting the above class measure zero. Thus the functional representation for *any* $\boldsymbol{A}(\boldsymbol{x})$ is orthogonal to the Fock state, that is, inequivalent to the Fock state.

This fact, that the functional representation contains an infinite set of Fock states, enables us to perform an explicit gauge transformation and prove gauge independence without recourse to any physical state conditions in the path integral. [Recall that Eq. (2.16) and (2.17) are essential in obtaining the path integral representation.]

V. DISCUSSION

In this paper, we have built up the path integral formula of the model with an arbitrary function ϕ_μ which has been introduced to obtain gauge-invariant operators. In the operator formalism the support of the function must be spacelike, and thus generality is lost, but there is no restriction in the Euclidean path integral expression, and so we are able to move, for instance, from the Coulomb to the Landau gauge (but the axial case has a severe infrared singularity). In view of the formula, the model can be recognized as a ϕ_μ-gauge fixing and gauge independence of the free energy is quickly understood (although that of the S matrix needed closer consideration). Furthermore, a closer inspection reveals the reason why gauge transformations are so easily managed in the path integral.

As was seen in the discussion of the S matrix, multiplying by wave functions, that is, the on-shell condition, is indispensable for the proof of gauge independence. The on-shell condition belongs to one of the physical state conditions. Hence, in scattering theories or in perturbation theories, the usual (LSZ-)asymptotic states [24], Eq. (3.10), are known to behave like the physical states. However, it is not so easy to find out the form of the physical state in a nonperturbative manner. The initial intention of d'Emilio-Mintchev and Steinmann seems to explore this: Indeed, the physical fermion (1.2) upon taking the Coulomb ϕ_μ, Eq. (1.14),

$$\Psi^{\mathrm{D}}(x)=\exp\left[ie\frac{\nabla\cdot\boldsymbol{A}}{\nabla^2}\right]\psi(x), \tag{5.1}$$

is the one introduced by Dirac [7,25], which is locally gauge invariant as well as globally charged. The existence of such a state implies an evidence of electron as a real particle [26].

The issue should then be generalized to the non-Abelian gauge case (QCD). In order to study the dynamics of quark confinement, it is important to examine whether physical charged state can be constructed or not. The key to this problem comes from noticing the Gribov ambiguity [27]: In a small region, the Coulomb gauge is well defined; that is, no gauge degrees of freedom are left, owing to the asymptotic freedom. In a larger region, however, more nontrivial degrees of freedom come into play [28]. Since gauge invariance is essential to comprehend quark confinement, the path integral must be useful. Therefore in order for theory to be well defined the integration region of the gauge fields must be connected with that of the Lagrangian, which would finally give us a compact integration of gauge fields given by the lattice QCD [29]. Work in this direction is in progress.

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