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Cosmological and black hole horizon fluctuations

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The quantum fluctuations of horizons in Robertson-Walker universes and in Schwarzschild spacetime are discussed. The source of the metric fluctuations is taken to be quantum linear perturbations of the gravitational field. Light cone fluctuations arise when the retarded Green's function for a massless field is averaged over these metric fluctuations. This averaging replaces the δ function on the classical light cone with a Gaussian function, the width of which is a measure of the scale of the light cone fluctuations. Horizon fluctuations are taken to be measured in the frame of a geodesic observer falling through the horizon. In the case of an expanding universe, this is a comoving observer either entering or leaving the horizon of another observer. In the black hole case, we take this observer to be one who falls freely from rest at infinity. We find that cosmological horizon fluctuations are typically characterized by the Planck length. However, black hole horizon fluctuations in this model are much smaller than Planck dimensions for black holes whose mass exceeds the Planck mass. Furthermore, we find black hole horizon fluctuations which are sufficiently small as not to invalidate the semiclassical derivation of the Hawking process. [S0556-2821(97)00316-0]

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I. INTRODUCTION

One of the characteristics of classical gravitation is the existence of horizons, surfaces which divide spacetime into causally distinct regions. The most striking example is the black hole horizon, the boundary which hides the events within from the outside world. Cosmological models also possess horizons of a different sort; a given observer generally cannot see all of the other observers in the universe at a given time. If the expansion rate in comoving time is less than linear, then previously unseen objects enter the observer's horizon. If it is faster than linear (inflationary expansion), then objects leave the horizon. Horizons are of course light cones, and the notion of an event being within or without a horizon means being at a timelike or a spacelike separation, respectively.

It is expected that quantum metric fluctuations should smear out this precise distinction and, hence, smear out the classical concept of a horizon. Information could presumably leak across the horizon in a way that is not allowed by classical physics. Bekenstein and Mukhanov [1] have suggested that horizon fluctuations could lead to discreteness of the spectrum of black holes. Several other authors [2,3] have recently made proposals for models which describe the horizon fluctuations. In this paper, we will propose a different model, in which quantized linear perturbations of the gravitational field act as the source of the underlying metric fluctuations. Our analysis will be based on the formalism for the study of light cone fluctuations proposed in Ref. [4] and further developed in Ref. [5].

The necessary formalism will be reviewed and extended in Sec. II. It will be applied to the case of cosmological horizons in Sec. III, and to black hole horizons in Sec. IV. Our results will be summarized and discussed in Sec. V. We will also give a critical assessment of the previous attempts [2,3] to estimate the horizon fluctuations.

II. BASIC FORMALISM

In Ref. [4], henceforth I, a model of light cone fluctuations on a flat background was developed. It was assumed that the quantized gravitational field is in a squeezed vacuum state. This is the natural quantum state for gravitons produced by quantum particle creation processes, as, for example, in the early universe. Here we wish to generalize this formalism to the case of curved background spacetimes. Consider an arbitrary background metric $g^{(0)}_{\mu\nu}$ with a linear perturbation $h_{\mu\nu}$, so that the spacetime metric is [6]

$$ds^2=(g^{(0)}_{\mu\nu}+h_{\mu\nu})dx^\mu dx^\nu. \tag{1}$$

For any pair of spacetime points x and x', let $\sigma(x,x')$ be one-half of the squared geodesic separation in the full metric and $\sigma_0(x,x')$ be the corresponding quantity in the background metric. We can expand $\sigma(x,x')$ in powers of $h_{\mu\nu}$ as

$$\sigma=\sigma_0+\sigma_1+\sigma_2+\cdots, \tag{2}$$

where σ_1 is first order in $h_{\mu\nu}$, etc. We now suppose that the linearized perturbation $h_{\mu\nu}$ is quantized and that the quantum state $|\psi\rangle$ is a ''vacuum'' state in the sense that we can decompose $h_{\mu\nu}$ into positive and negative frequency parts $h^+_{\mu\nu}$ and $h^-_{\mu\nu}$, respectively, such that

$$h^+_{\mu\nu}|\psi\rangle=0, \quad \langle\psi|h^-_{\mu\nu}=0. \tag{3}$$

It follows immediately that

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$$\langle h_{\mu\nu}\rangle = 0 \tag{4}$$

in state $|\psi\rangle$. In general, however, $\langle (h_{\mu\nu})^2\rangle \neq 0$, where the expectation value is understood to be suitably renormalized. This reflects the quantum metric fluctuations.

We now wish to average the retarded Green's function $G_{\mathrm{ret}}(x,x')$ for a massless field over the metric fluctuations. In a curved spacetime, $G_{\mathrm{ret}}(x,x')$ can be nonzero inside the future light cone as a result of backscattering off of the spacetime curvature. However, its asymptotic form near the light cone is the same as in flat spacetime:

$$G_{\mathrm{ret}}(x,x') \sim \frac{\theta(t-t')}{4\pi}\delta(\sigma), \quad \sigma \to 0. \tag{5}$$

We will ignore the backscattered portion and average this δ-function term over the fluctuations, following the method of I. The result is

$$\langle G_{\mathrm{ret}}(x,x')\rangle = \frac{\theta(t-t')}{8\pi^2}\sqrt{\frac{\pi}{2\langle\sigma_1^2\rangle}}\exp\left(-\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}\right). \tag{6}$$

The effect of the averaging has been to replace the δ function by a Gaussian with a finite width determined by the magnitude of the quantity $\langle\sigma_1^2\rangle$, which is the measure of the light cone fluctuations.

The operational meaning of the smeared light cone can be understood by considering a source and a detector of photons. If we ignore the finite sizes of photon wavepackets, then in the absence of light cone fluctuations, all photons should traverse the interval between the source and the detector in the same amount of time. The effect of the light cone fluctuations is to cause some photons to travel slower than the classical light speed and others to travel faster. The Gaussian function in Eq. (6) is symmetrical about the classical light cone, $\sigma_0 = 0$, and so the quantum light cone fluctuations are equally likely to produce a time advance as a time delay.

In order to find the magnitude of the light cone fluctuations in a particular situation, it is necessary to calculate σ_0 for the metric in question, as well as $\langle\sigma_1^2\rangle$ in the appropriate quantum state. This enables one to find Δt, the mean time delay or advance (measured in a suitable reference frame). This is an ensemble-averaged quantity, not necessarily the expected variation in flight time of two photons emitted in rapid succession. To find the latter quantity, one must examine a correlation function. This is the topic of Ref. [5]. In the present paper, we will not be concerned with correlation functions and will use Eq. (6) to estimate the magnitude of the horizon fluctuations.

We may find a general expression for $\langle\sigma_1^2\rangle$, which is the curved space generalization of the result obtained in I. Let us first consider timelike geodesics. If we adopt a timelike metric, then in Eq. (1) we have that $ds^2 > 0$. Let $u^\mu = dx^\mu/d\tau$ be the tangent to the geodesic and τ be the proper time. We will define $\langle\sigma_1^2\rangle$ by integrating along the unperturbed geodesic, in which case u^μ is normalized to unity in the background metric:

$$g^{(0)}_{\mu\nu}u^\mu u^\nu = 1. \tag{7}$$

The geodesic interval in the unperturbed metric is given by

$$\sigma_0 = \frac{1}{2}(\Delta\tau)^2, \tag{8}$$

where $\Delta\tau$ is the proper time elapsed along the geodesic. We have

$$\frac{ds}{d\tau} = \sqrt{1 + h_{\mu\nu}u^\mu u^\nu} \approx 1 + \frac{1}{2}h_{\mu\nu}u^\mu u^\nu, \tag{9}$$

and hence the geodesic length between a pair of points in the perturbed metric is $\Delta s = \Delta\tau + \Delta s_1$, where

$$\Delta s_1 = \frac{1}{2}\int d\tau\, h_{\mu\nu}u^\mu u^\nu. \tag{10}$$

Thus

$$\sigma = \frac{1}{2}(\Delta s)^2 = \frac{1}{2}(\Delta\tau)^2 + \Delta\tau\Delta s_1 + O(h^2), \tag{11}$$

and hence $\sigma_1 = \Delta\tau\Delta s_1$. If we average over the metric fluctuations, the result is

$$\langle\sigma_1^2\rangle = \frac{1}{2}\sigma_0\int d\tau_1 d\tau_2\, u_1^\mu u_1^\nu u_2^\rho u_2^\sigma \langle h_{\mu\nu}(x_1)h_{\rho\sigma}(x_2)\rangle, \tag{12}$$

where $u_1^\mu = dx^\mu/d\tau_1$ and $u_2^\mu = dx^\mu/d\tau_2$. An analogous expression holds for the case of a spacelike geodesic, in which the integrations are over the proper length parameter of the geodesic:

$$\langle\sigma_1^2\rangle = -\frac{1}{2}\sigma_0\int d\lambda_1 d\lambda_2\, u_1^\mu u_1^\nu u_2^\rho u_2^\sigma \langle h_{\mu\nu}(x_1)h_{\rho\sigma}(x_2)\rangle, \tag{13}$$

where now $u_1^\mu = dx^\mu/d\lambda_1$ is the tangent to the geodesic and λ is the proper length. Here we have $\sigma_0 = -\frac{1}{2}(\Delta\lambda)^2$.

As noted previously, the quantity $\langle\sigma_1^2\rangle$ is formally divergent and needs to be renormalized. This may be done by defining the graviton two-point function $\langle h_{\mu\nu}(x_1)h_{\rho\sigma}(x_2)\rangle$ using, for example, the Hadamard renormalization scheme proposed by Brown and Ottewill [7]. These authors give a detailed prescription for expanding the singular, state-independent parts of the scalar and vector two-point functions in an arbitrary curved spacetime. Hadamard renormalization consists of subtracting this expansion from a given two-point function. This procedure seems not to have been developed in detail for the graviton two-point function, but there seems to be no barrier in principle to doing so. Allen *et al.* [8] have applied the Hadamard renormalization method to the graviton two-point function in the vicinity of a cosmic string. In this paper, we will be content with simple approximations or order of magnitude estimates and will not require the full renormalization machinery.

III. COSMOLOGICAL HORIZONS

Consider a spatially flat Robertson-Walker universe, for which the metric may be written as

$$ds^2=a^2(\eta)(d\eta^2-d\mathbf{x}^2). \tag{14}$$

In general, this spacetime has ''particle horizons'' associated with the comoving observers across which other observers may appear or disappear. In the case of a radiation- or matter-dominated universe with an initial singularity ($a\propto\eta$ or $a\propto\eta^2$, respectively), a given observer at a given time has not yet received any light signals from distant observers, who are said to be outside of the first observer's horizon. In the case of de Sitter space ($a\propto\eta^{-1}$), a given observer eventually ceases to receive signals from other comoving observers and views them as having moved outside of the horizon. Clearly, these cosmological horizons are observer dependent in a way that black hole event horizons are not and are basically the past light cone of a given observer at a given time. Nonetheless, it will be of interest to estimate the magnitude of the quantum fluctuation of these horizons in various models.

We must first study timelike and spacelike geodesics in the limits in which these approach null geodesics over some interval. Because the light cone fluctuations are symmetrical, we may focus our attention on the timelike case. The geodesic equations for a timelike observer moving in the x direction in the metric of Eq. (14) may be expressed as

$$a^2\frac{dx}{d\tau}=\frac{1}{\sqrt{2\alpha}} \tag{15}$$

and

$$\frac{d^2\eta}{d\tau^2}+\frac{a'}{a}\left(\frac{d\eta}{d\tau}\right)+\frac{a'}{2\alpha a^5}=0, \tag{16}$$

where τ is the proper time along the geodesic, $a'=da/d\eta$, and α is a constant. In the flat space limit ($a=1$), we find from Eq. (15) that $\alpha=(1-v^2)/(2v^2)$ where v is the magnitude of the three-velocity. Thus, in the null limit, $\alpha\rightarrow 0$. The geodesic for a particle which starts at $\eta=\eta_0$ at $x=0$ may be expressed as

$$x=\eta-\eta_0-f(\eta,\eta_0), \tag{17}$$

where $f\rightarrow 0$ in the null limit. For nearly null geodesics, we may assume $|f|\ll 1$. To first order in f, the solution of Eqs. (15) and (16), corresponding to a properly normalized four-velocity, is

$$f(\eta,\eta_0)=\alpha\int_{\eta_0}^{\eta}a^2(\eta')d\eta'. \tag{18}$$

From Eq. (14), we can write

$$ds=\sqrt{1-\left(\frac{dx}{d\eta}\right)^2}\,a\,d\eta\approx\sqrt{2\alpha}a^2d\eta, \tag{19}$$

from which we find

$$\Delta\tau=\sqrt{2\alpha}\int_{\eta_0}^{\eta_1}a^2(\eta)d\eta \tag{20}$$

and

$$\sigma_0=\frac{1}{2}(\Delta\tau)^2. \tag{21}$$

Gravitons propagating on a Robertson-Walker background may be quantized in the transverse, trace-free gauge, which eliminates all of the gauge freedom [9]. Only the purely spatial components of h^{μ}_{ν} are nonzero, and they each satisfy the wave equation for a massless, minimally coupled scalar field in this metric. Thus the graviton two-point function can be expressed in terms of the scalar two-point function $\langle\varphi(x)\varphi(x')\rangle$ as

$$\langle h_{ij(x)}h_{kl}(x')\rangle=-\frac{1}{3}a^2(\eta)a^2(\eta')\left(\delta_{ij}\delta_{kl}-\frac{3}{2}\delta_{ik}\delta_{jl}-\frac{3}{2}\delta_{il}\delta_{jk}\right)\langle\varphi(x)\varphi(x')\rangle. \tag{22}$$

We may use this result to write

$$\langle\sigma_1^2\rangle=\frac{\sigma_0}{6\alpha}\int_{\eta_0}^{\eta_1}d\eta\int_{\eta_0}^{\eta_1}d\eta'\langle\varphi(x)\varphi(x')\rangle, \tag{23}$$

Note that in this expression we can replace the unsymmetrized two-point function $\langle\varphi(x)\varphi(x')\rangle$ by the symmetrized form (the Hadamard function)

$$G(x,x')=\frac{1}{2}\langle\varphi(x)\varphi(x')+\varphi(x')\varphi(x)\rangle. \tag{24}$$

The latter function is real and is, hence, more convenient. To proceed further, we must designate the quantum state of the gravitons. In the following two subsections, some particular examples will be considered.

A. Gravitons in a radiation-dominated universe

A radiation-dominated universe, for which

$$a(\eta)=a_0\eta, \tag{25}$$

is presumably a reasonably good description of a significant fraction of the history of our universe. Let us consider a thermal bath of gravitons in such a universe, for which the temperature is always high compared to the scale set by the local radius of curvature; i.e., the thermal wavelength is much less than the horizon size. In this case, the minimally coupled scalar field two-point function is approximately equal to that for the conformally coupled field, $G_{cc}(x,x')$. However, the latter is conformally related to the flat space Hadamard function $G_0(x,x')$, and so we have

$$G(x,x')\approx G_{cc}(x,x')=a^{-1}(\eta)a^{-1}(\eta')G_0(x,x'). \tag{26}$$

We now need the flat space renormalized thermal Green's function on the light cone. This was calculated in Appendix A of Ref. [5], where it was shown that in the high temperature limit this function is given by

$$G_0(x,x')\approx\frac{1}{8\pi\beta\rho},\quad \beta\ll\rho, \tag{27}$$

where β is the inverse temperature and $\rho=|\mathbf{x}-\mathbf{x}'|$. We may now use Eqs. (23)–(27) to write

$$\langle\sigma_1^2\rangle=\frac{\sigma_0}{48\pi\alpha\beta a_0^2}\int_{\eta_0}^{\eta_1}d\eta\int_{\eta_0}^{\eta_1}d\eta'\,\frac{1}{\eta\eta'|\eta-\eta'|}. \quad (28)$$

Unfortunately, this integral diverges because of the singularity of the integrand at $\eta=\eta'$. This is due to the fact that Eq. (27) is not valid for small ρ. We can remedy this by excluding the range $|\eta-\eta'|<\epsilon$, where ϵ is a cutoff which will be taken to be of order β. Thus, the relevant integral is, in the limit of $\eta_1\gg\eta_0$,

$$\langle\sigma_1^2\rangle=\frac{\sigma_0}{48\pi\alpha\beta a_0^2}\int_{\eta_0}^{\eta_1}d\eta\left(\int_{\eta_0}^{\eta-\epsilon}+\int_{\eta+\epsilon}^{\eta_1}\right)d\eta'\,\frac{1}{\eta\eta'|\eta-\eta'|}$$
$$\approx\frac{\sigma_0[\ln(\eta_0/\epsilon)+1]}{24\pi\alpha\beta\eta_0a_0^2}. \quad (29)$$

In this same limit, one finds

$$\sigma_0\approx\frac{1}{9}\alpha a_0^2\eta_1^6. \quad (30)$$

We now wish to define α_c as that value of α for which the argument of the exponential in Eq. (6) is unity; that is,

$$\frac{\sigma_0^2}{2\langle\sigma_1^2\rangle}=1. \quad (31)$$

Thus α_c describes a geodesic whose deviation from the light cone characterizes the fluctuations. From Eqs. (29), (30), and (31), we find that

$$\alpha_c=\frac{\sqrt{3[\ln(\eta_0/\epsilon)+1]}}{\sqrt{4\pi\alpha\beta\eta_0}a_0^3\eta_1^3}. \quad (32)$$

We need to define a physical measure of the magnitude of the light cone fluctuations. This may be taken to be the mean time delay or advance $\Delta\eta$ for a photon emitted at η_0 and detected at η_1. Equivalently, we can think of $\Delta\eta$ as the characteristic interval around η_0 within which photons could be emitted and still reach a detector at a coordinate distance of $\Delta x=\eta_1-\eta_0$ at time η_1. (See Fig. 1.) From Eqs. (17) and (18), we have that $\Delta\eta$ is related to α_c by

$$\Delta\eta=f(\eta_1,\eta_0)=\alpha_c\int_{\eta_0}^{\eta_1}a^2(\eta')d\eta'\approx\frac{1}{3}\alpha_c a_0^2\eta_1^3. \quad (33)$$

It is perhaps more convenient to express this time delay or advance as a coordinate time interval $\Delta t=a(\eta_0)\Delta\eta$, which is given by

$$\Delta t=\left(\frac{2t_0}{a_0}\right)^{1/4}\frac{\sqrt{3[\ln(\eta_0/\epsilon)+1]}}{6\sqrt{\pi\beta}}, \quad (34)$$

where $t_0=\frac{1}{2}a_0\eta_0^2$ is the coordinate time at η_0.

We may interpret this formula by noting that in an expanding universe β^{-1} is a coordinate temperature, not, in general, the physical temperature. It is, however, the physical

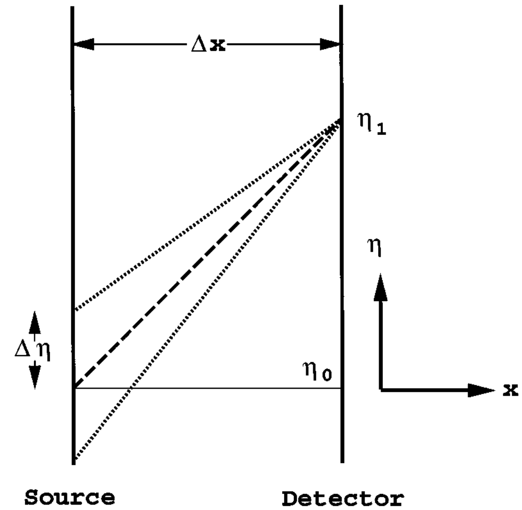

FIG. 1. A photon is received by a detector at conformal time η_1. In the absence of metric fluctuations, it has traveled along the classical light cone (dashed line) from a source at a coordinate distance $|\Delta\mathbf{x}|=\eta_1-\eta_0$ and was emitted at conformal time η_0. In the presence of metric fluctuations, it could have been emitted a characteristic time $\Delta\eta$ before or after η_0 and traveled along a mean trajectory which is either timelike or spacelike, respectively (dotted lines).

temperature at a time at which $a=1$. Let us take that time to be t_0, the time of emission, at which time the physical temperature is T_0. This leads to

$$\frac{\Delta t}{t_0}=\frac{\sqrt{6}}{6}\sqrt{\ln(\eta_0/\epsilon)+1}\left(\frac{T_0}{T_P}\right)\left(\frac{t_P}{t_0}\right), \quad (35)$$

where T_P is the Planck temperature and t_P is the Planck time. The logarithmic factor can be taken to be of order one, and so we see that if $T_0=T_P$ and $t_0=t_P$, then we have $\Delta t/t_0\approx1$. Otherwise, with $T_0\ll T_P$ and $t_0\gg t_P$, we have $\Delta t/t_0\ll1$, and the fractional light cone fluctuations are small. This is what we should perhaps expect; a bath of gravitons with the Planck temperature at the Planck time (which would correspond to a few degrees kelvin today) results in large horizon fluctuations, but otherwise the fluctuations are small at much lower temperatures.

B. Gravitons in de Sitter space

If we represent de Sitter space as a spatially flat Robertson-Walker metric, then the metric is of the form of Eq. (14) with $a(\eta)=-H/\eta=H/|\eta|$, where H is a constant and $-\infty<\eta<0$. These coordinates cover one-half of the full de Sitter spacetime, but that is sufficient for our purposes. Gravitons are again represented as a pair of massless, minimally coupled scalar fields. However, in this case there is a subtlety in that there is no de Sitter–invariant vacuum state which is free of infrared divergences [10–12]. The Hadamard function may be represented as [12]

$$G(x,x')=\frac{1}{(2\pi)^3}\mathrm{Re}\int d^3\mathbf{k}\psi_k(\eta)\psi_k^*(\eta')e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}}, \quad (36)$$

where the time part of the mode function is expressible in terms of Hankel functions as

$$\psi_k(\eta)=\frac{\sqrt{\pi}}{2}H|\eta|^{3/2}[c_1H^{(1)}_{3/2}(k\eta)+c_2H^{(2)}_{3/2}(k\eta)]. \quad (37)$$

Here c_1 and c_2 are functions of k which are required to have the properties

$$c_1\to 0, \quad c_2\to 1 \quad \text{as} \quad k\to\infty \quad (38)$$

and

$$|c_1+c_2|\to 0 \quad \text{as} \quad k\to 0. \quad (39)$$

These functions define the quantum state in question. Equation (38) ensures that the very high frequency modes are free of particles, whereas Eq. (39) is the condition that the state be free of infrared divergences.

The Hadamard function given by Eqs. (36) and (37) is the unrenormalized function which is singular on the light cone. In principle, one should evaluate the integral for the given choice of c_1 and c_2, and then extract the state-independent singular terms to obtain the renormalized Hadamard function. Unfortunately, this would be very difficult to do explicitly. However, in the late time limits ($\eta\to 0$ or $\eta'\to 0$) it is possible to give some approximate forms for the renormalized function. It was shown by several authors that the coincidence limit grows logarithmically [11–14]

$$G(x,x)\sim -(H^2/4\pi^2)\ln H|\eta|, \quad (40)$$

as $\eta=\eta'\to 0$. The fact that this asymptotic form is state independent may be understood as a consequence of the exponential expansion having redshifted away any memory of the quantum state.

We may use a similar procedure to investigate the behavior as $\eta\to 0$ with η' fixed. If we insert Eq. (37) into Eq. (36) and then change the variable of integration to $\mathbf{q}=|\eta-\eta'|\,\mathbf{k}$, we have

$$G(x,x')=\frac{H^2|\eta\eta'|^{3/2}}{32\pi^2|\eta-\eta'|^3}\mathrm{Re}\int d^3\mathbf{q}\left[c_1H^{(1)}_{3/2}\left(-\frac{|\eta|}{|\eta-\eta'|}q\right)+c_2H^{(2)}_{3/2}\left(-\frac{|\eta|}{|\eta-\eta'|}q\right)\right]\times\left[c_1^*H^{(2)}_{3/2}\left(-\frac{|\eta'|}{|\eta-\eta'|}q\right)+c_2^*H^{(1)}_{3/2}\left(-\frac{|\eta'|}{|\eta-\eta'|}q\right)\right]e^{i\mathbf{v}\cdot\mathbf{q}}, \quad (41)$$

where $\mathbf{v}=(\mathbf{x}-\mathbf{x}')|\eta-\eta'|^{-1}$. Now assume that η is sufficiently small that the dominant contribution to the integral comes from values of $\mathbf{q}$ for which the magnitude of the arguments of the Hankel functions in the first factor are small compared to unity. In this case, we can use the small argument forms

$$H^{(1)}_{3/2}(-x)\approx H^{(2)}_{3/2}(-x)\approx\sqrt{2/\pi}\;x^{-3/2}, \quad 0<x\ll 1, \quad (42)$$

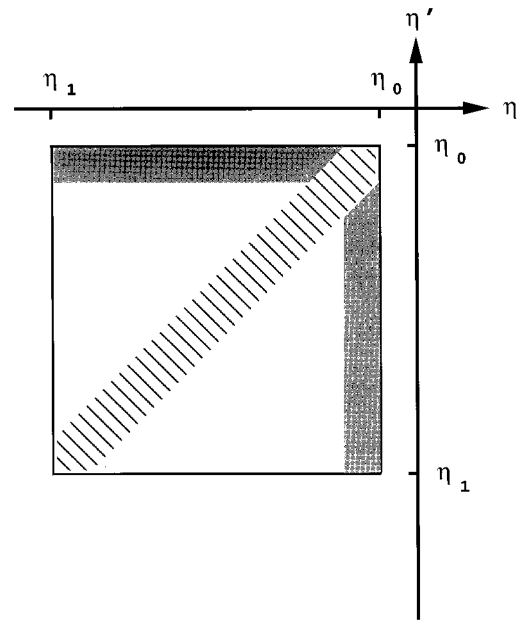

FIG. 2. The domain of integration for $\langle\sigma_1^2\rangle$ is the interior of the square. The integrand $G(x,x')$ is known in the shaded regions to be approximately H^2 and in the crosshatched region to be approximately $-(H^2\ln H|\eta|)/4\pi^2$.

in this factor and find

$$G(x,x')\sim\frac{\sqrt{2}H^2}{32\pi^{5/2}}\mathrm{Re}\int d^3\mathbf{q}\;q^{-3/2}(c_1+c_2)[c_1^*H^{(2)}_{3/2}(-q)+c_2^*H^{(1)}_{3/2}(-q)]e^{i\mathbf{v}\cdot\mathbf{q}}. \quad (43)$$

Note that the dependence upon both η and η' has dropped out. The integral in Eq. (43) has a logarithmic ultraviolet divergence on the light cone, but the leading quadratic divergence has disappeared. A similar disappearance of divergent parts occurs in the derivation of Eq. (40). (See, for example, Ref. [12].) The renormalization of this logarithmic divergence requires a subtraction of a term proportional to the scalar curvature $R=12H^2$. We expect the result to be a constant which is of order of H^2. By symmetry, we obtain the same result if $\eta'\to 0$ with η fixed, so that

$$G(x,x')\approx H^2, \quad \eta\to 0 \quad \text{or} \quad \eta'\to 0. \quad (44)$$

The integral in Eq. (23) requires us to know $G(x,x')$ in the square illustrated in Fig. 2, which also illustrates the regions in which either Eq. (40) or Eq. (44) is applicable. For the purposes of obtaining an order of magnitude estimate for $\langle\sigma_1^2\rangle$, we will assume that $G(x,x')$ is of the form

$$G(x,x')\approx H^2F(\eta,\eta') \quad (45)$$

throughout this region, where F is either a constant of order unity or else a logarithmic function which will contribute multiplicative constants of order unity to the integral in Eq. (23).

The result of the evaluation of this integral is

$$\langle \sigma_1^2 \rangle = \kappa \frac{H^2 \sigma_0}{\alpha} (\eta_1 - \eta_0)^2, \tag{46}$$

where κ is a constant of order unity. From Eq. (20) we find

$$\sigma_0 = \frac{\alpha(\eta_1 - \eta_0)^2}{H^4 \eta_1^2 \eta_0^2}. \tag{47}$$

The value of α which is characteristic of the light cone fluctuations is

$$\alpha_c \approx \sqrt{2\kappa} H^3 \eta_1 \eta_0. \tag{48}$$

If we follow the line of reasoning in the previous subsection, we find that the characteristic time associated with the de Sitter horizon fluctuations is

$$\Delta t \approx \sqrt{2\kappa}. \tag{49}$$

This is of order unity, and hence again the horizon fluctuations are of Planck dimensions.

We may use Δt to find the frequency fluctuations observed at η_1 from a constant frequency source. Suppose that the source is emitting photons with a constant frequency ν_0. In the absence of metric fluctuations, the photons will be detected at frequency $\nu = \nu_0 a(\eta_0)/a(\eta_1)$. The effect of the metric fluctuations is equivalent to a drift in the source frequency whose magnitude is $\Delta \nu_0 = \nu_0^2 \Delta t$. Consequently, the fractional variation of frequency at the detector is

$$\frac{\Delta \nu}{\nu} = \nu_0 \Delta t. \tag{50}$$

However, this is an ensemble-averaged frequency variation, not necessarily the drift in frequency that would be observed in any one trial. The reason for this is that pulses emitted close together in time tend to have correlated time delays or advances [5]. Thus, Eq. (50) should be interpreted as giving an upper bound in the frequency drift seen by the detector.

IV. BLACK HOLE HORIZONS

Here we wish to discuss the fluctuations of the event horizon of a Schwarzschild black hole, for which the metric is

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{51}$$

Timelike radial geodesics in this metric satisfy [16]

$$\left(\frac{dr}{d\tau}\right)^2 = \tilde{E}^2 - C(r) \tag{52}$$

and

$$\frac{dt}{d\tau} = \tilde{E}/C, \tag{53}$$

where $C(r) = 1 - 2M/r$ and $\tilde{E}$ is a constant of the motion which is equal to the energy per unit rest mass of the particle, as measured at infinity. From these relations, one may show that

$$\left(\frac{dr}{dt}\right)^2 = C^2 \left(1 - \frac{C}{\tilde{E}^2}\right) \tag{54}$$

and that

$$\left(\frac{dr^*}{dt}\right)^2 = 1 - \frac{C}{\tilde{E}^2} \tag{55}$$

where

$$\frac{dr}{dr^*} = C. \tag{56}$$

From Eq. (53), we see that the proper time elapsed along a segment of a geodesic is

$$\Delta\tau = \tilde{E}^{-1} \int_{t_0}^{t_1} C(r) dt, \tag{57}$$

where r is understood to be a function of t along the geodesic. We are primarily interested in the case of nearly null outgoing geodesics, for which $\tilde{E} = \tilde{E}_0 \gg 1$. In this limit

$$\frac{dr}{dt} \approx C(r). \tag{58}$$

Thus, $|\Delta\tau| \approx |\Delta r|/\tilde{E}_0$ and

$$\sigma_0 \approx \frac{(\Delta r)^2}{2\tilde{E}_0^2}, \tag{59}$$

where Δr is the radial coordinate interval traversed. An analogous treatment may be given for spacelike geodesics. The constant of the motion $\tilde{E}$ no longer has a simple physical interpretation, but we can express σ_0 in this case as

$$\sigma_0 \approx -\frac{(\Delta r)^2}{2\tilde{E}_0^2}. \tag{60}$$

We now turn to the task of estimating $\langle \sigma_1^2 \rangle$ near the horizon of a Schwarzschild black hole. From Eq. (12) and the fact that

$$\frac{dr}{d\tau} \approx \tilde{E}_0 \tag{61}$$

for nearly null outgoing timelike geodesics, we have that

$$\langle \sigma_1^2 \rangle \approx \frac{1}{2} \sigma_0 \tilde{E}_0^{-2} \int dr_1 dr_2 u_1^\mu u_1^\nu u_2^\rho u_2^\sigma \langle h_{\mu\nu}(x_1) h_{\rho\sigma}(x_2) \rangle. \tag{62}$$

If we are interested in a black hole radiating into empty space, then the relevant quantum state for the quantized graviton field is the Unruh state. It would be a rather formidable task to explicitly compute the renormalized graviton two-point function in the state. Instead, we will content ourselves with an order of magnitude estimate. First we must choose a convenient gauge. Again we wish to impose the transverse, tracefree gauge, which eliminates all gauge freedom. Because all of the modes of the graviton field are propagating waves which either originate at $\mathcal{I}^-$ or reach $\mathcal{I}^+$, we can impose the requirement that these modes satisfy the flat space transverse, tracefree gauge condition at $r=\infty$.

We now make the assumption that in this gauge the renormalized two-point function measured in the frame of an infalling observer who starts from infinity at rest can be estimated by dimensional considerations. Near the horizon at $r=2M$, the geometry and the quantum state are characterized by a single scale M. If we were to reinstate explicit factors of Newton's constant G, then $h_{\mu\nu}\propto G^{-1/2}\propto m_P$, where m_P is the Planck mass. However, $h_{\mu\nu}$ is dimensionless in any set of units, and so our assumption tells us that the two-point function should be proportional to m_P^2/M^2. However, the actual values of the components of this bitensor depend upon the choice of frame. Our assumption is that infalling observers with $\tilde{E}=\tilde{E}_i\approx 1$ should be regarded as preferred in the sense that they do not introduce any very large or very small dimensionless redshift or blueshift factors. Let v^μ be the four-velocity of such an observer. Our assumption may be expressed as

$$v_1^\mu v_1^\nu v_2^\rho v_2^\sigma \langle h_{\mu\nu}(x_1) h_{\rho\sigma}(x_2)\rangle \approx \frac{m_P^2}{M^2} \tag{63}$$

near the horizon. The components of the infalling observer's four-velocity are

$$v^t=C^{-1}, \quad v^r=-\sqrt{1-C}\approx -1, \tag{64}$$

and those for an outgoing observer are

$$u^t=\tilde{E}_0 C^{-1}, \quad u^r=\sqrt{\tilde{E}_0^2-C}\approx \tilde{E}_0. \tag{65}$$

Thus $|u^\mu|=\tilde{E}_0|v^\mu|$ and we can write

$$u_1^\mu u_1^\nu u_2^\rho u_2^\sigma \langle h_{\mu\nu}(x_1) h_{\rho\sigma}(x_2)\rangle approx \tilde{E}_0^4 \frac{m_P^2}{M^2}. \tag{66}$$

Our basic assumption receives some support from the work of York [15] who estimates the magnitude of the quantum fluctuations of the lowest modes of vibration of a Schwarzschild black hole. He treats these modes as quantum-mechanical harmonic oscillators and calculates their root-mean-square fluctuation amplitudes. The amplitudes of the first few modes yield a result consistent with Eq. (63) or (66). Of course, this is heuristic support and by no means a proof of our assumption. A full proof would require one to sum over an infinite number of degrees of freedom and then extract any ultraviolet-divergent parts.

The graviton two-point function in this approximation is a constant in the vicinity of the horizon. It must also fall off to zero at large distances from the black hole. Thus the integral in Eq. (62) gets its dominant contribution over an interval in r of the order of M, regardless of the upper limit of the integration. In any case, we can stop the integration at a maximum value of r which is just a few times M. Whether the outgoing photons emitted in the vicinity of the horizon are detected at $r=4M$ or at a much larger value of r has little effect on the discussion of the horizon fluctuations. Thus we may let

$$\langle \sigma_1^2\rangle \approx \sigma_0 \tilde{E}_0^2 \frac{(\Delta r)^2}{M^2}. \tag{67}$$

In analogy to the discussion in the previous section, we wish to define a characteristic value $\tilde{E}_c$, which is the value of $\tilde{E}_0$ at which Eq. (31) holds. From Eqs. (59) and (67) we find

$$\tilde{E}_c \approx \sqrt{M}. \tag{68}$$

We may find the associated time delay or advance Δt, from Eq. (55), which tells us that, when $\tilde{E}_0 \gg 1$,

$$dt \approx dr^* + \frac{C}{2\tilde{E}_0^2} dr^* = dr^* + \frac{1}{2\tilde{E}_0^2} dr. \tag{69}$$

A radial null geodesic in the classical background geometry covers an r^* distance of Δr^* in a coordinate time $\Delta t=\Delta r^*$. The second term on the right-hand side of the above equation tells us the extra amount of time required by a timelike particle. Analogous expressions hold for spacelike geodesics and yield the same magnitude of time variation. Thus we are led to an expression for the characteristic time delay or advance due to horizon fluctuations:

$$\Delta t \approx \frac{\Delta r}{M}. \tag{70}$$

As discussed above, we can take Δr to be of order M, although we might also want to consider the possibility of taking it to be much smaller. Thus let

$$\Delta r = \gamma M, \tag{71}$$

where γ is a constant of the order of or less than unity. Now we have

$$\Delta t \approx \gamma, \tag{72}$$

and so the time delay, measured in coordinate time, is of Planck dimensions. However, a more physical measure is obtained by expressing this time interval in terms of the proper time of a local observer. Let the photons be emitted at $r=r_0=2M(1+\epsilon)$, with $\epsilon \ll 1$, and let $C_0=C(r_0)\approx\epsilon$. The time interval in the frame of a static (nongeodesic) observer at rest at $r=r_0$ is

$$\Delta\tau_s \approx \gamma\sqrt{C_0}, \tag{73}$$

that in the frame of an infalling observer with $\tilde{E}=1$ is

$$\Delta\tau_i \approx \gamma C_0 \approx \gamma\epsilon, \tag{74}$$

and that in the frame of an outgoing geodesic observer with $\tilde{E}=1$ is

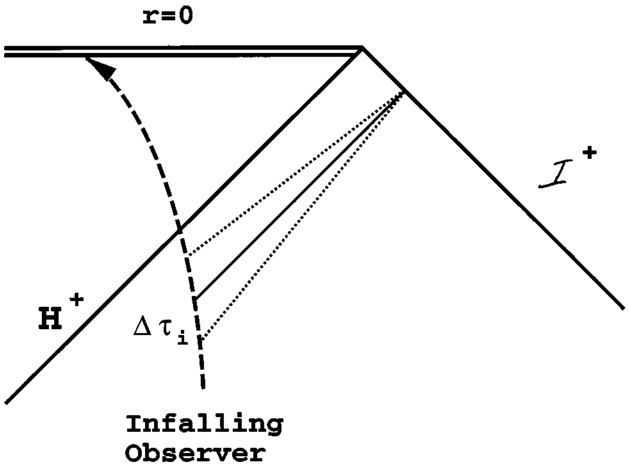

FIG. 3. An observer falling across the future horizon H^+ of a black hole emits photons which reach $\mathcal{I}^+$. In the presence of metric fluctuations, these photons need not follow the classical light cone (solid line), but rather may follow timelike or spacelike paths in the background geometry (dotted lines). The characteristic variation in emission time, as measured in the frame of the infalling observer, of photons which reach $\mathcal{I}^+$ at the same point is $\Delta\tau_i$.

$$\Delta\tau_0 \approx \gamma. \tag{75}$$

One might regard $\Delta\tau_i$, the characteristic time as measured by an infalling observer, to be the best measure of the magnitude of the horizon fluctuations. Such an observer can cross and continue beyond the classical event horizon at $r=2M$. Suppose that an outgoing photon emitted by this observer reaches infinity. An observer at infinity who detects this photon and who is unaware of the light cone fluctuations might trace the history of this photon backwards in the classical Schwarzschild geometry and infer that it was emitted at a proper time of τ_0 on the infalling observer's world line. In fact, it could have been emitted anywhere in a band of width $\Delta\tau_i$ centered around τ_0. (See Fig. 3.) The remarkable feature of the result (74) is that $\Delta\tau_i \to 0$ as $\tau_0 \to \tau_H$, the proper time at which the infalling observer reaches $r=2M$. In the cosmological models discussed in Sec. III, the fluctuation in emission time was typically of the order of the Planck time. In the black hole case, the horizon fluctuations are more strongly suppressed. Note that the proper time required for the infalling observer to pass from $r=r_0=2M(1+\epsilon)$ to $r=2M$ is $T\approx 2\epsilon M$. This is always large compared to $\Delta\tau_i$ for large black holes:

$$\frac{\Delta\tau_i}{T} \approx \gamma \frac{m_p}{M}. \tag{76}$$

Thus the only outgoing photons which manage to cross the classical horizon are part of an extreme tail of a Gaussian distribution.

As in Sec. III B, we may express the time delay or advance in terms of the variation in frequency seen by the observer at infinity. In the black hole case, the analogue of Eq. (50) is

$$\frac{\Delta\nu}{\nu} = \nu_0 \Delta\tau_i \approx \nu_0 \gamma \epsilon. \tag{77}$$

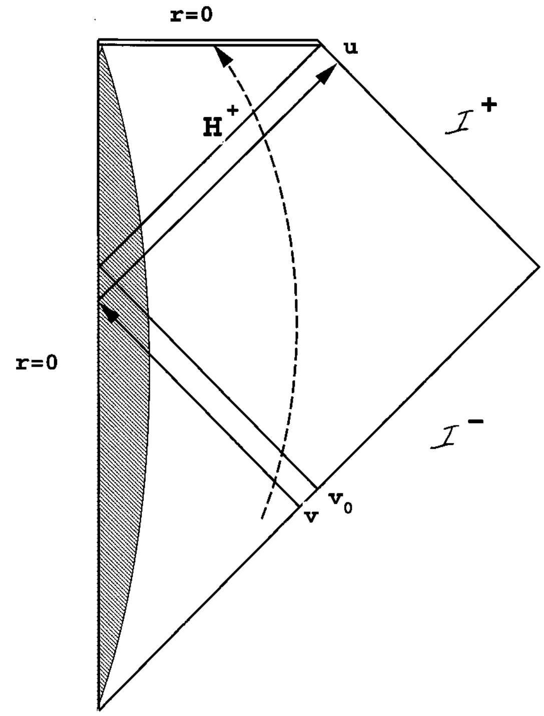

FIG. 4. The spacetime for a black hole formed by gravitational collapse. The shaded region is the interior of the collapsing star. A null ray which leaves $\mathcal{I}^-$ with advanced time v_0 becomes the future horizon H^+. A ray which leaves at an earlier time v passes through the collapsing body and reaches $\mathcal{I}^+$ at retarded time u. The dashed line is the world line of a observer who falls into the black hole after its formation.

Thus, as the source approaches $r=2M$, the fractional variation in frequency observed at infinity goes to zero, and the observed frequency approaches that predicted by classical relativity.

Let us now turn to the question of whether horizon fluctuations are capable of invalidating the semiclassical derivation of the Hawking effect. First let us recall the essential features of this derivation, as given in Hawking's original paper [17]. Consider the spacetime of a black hole formed by gravitational collapse (Fig. 4). The null ray which forms the future horizon leaves $\mathcal{I}^-$ at advanced time $v=v_0$. The modes into which the outgoing thermal radiation will be created leave $\mathcal{I}^-$ at values of v slightly less than v_0, pass through the collapsing body, and reach $\mathcal{I}^+$ as outgoing rays, on which the retarded time u is constant. Hawking shows that the relation between the values of v and of u is

$$u = -4M \ln\left(\frac{v_0 - v}{A}\right), \tag{78}$$

where A is a constant. Thus $u \to \infty$ as $v \to v_0$. As seen by an observer at infinity, these outgoing rays must hover extremely close to the horizon for a very long time. If one starts with a black hole with a mass M large compared to the Planck mass, the semiclassical description should hold for the time required for the black hole to lose most of its original mass. Let

$$t_{\rm evap}=M^3=M\left(\frac{M}{m_P}\right)^2 \tag{79}$$

be this characteristic evaporation time. The basic problem posed by the horizon fluctuations is that they may cause an outgoing ray to either fall back into the black hole or else to prematurely escape. In either case, the semiclassical picture of black hole radiance would need to be modified at times less than $t_{\rm evap}$.

At a large distance from the black hole, $u=t-r^*\approx t-r$. If the observer at ''infinity'' is at a fixed value of r (e.g., $100M$), then $u\approx t$ for most of the black hole's lifetime. Thus, in order not to invalidate the semiclassical treatment, outgoing rays with $u<u_{\rm max}=t_{\rm evap}$ need to be uninfluenced by the horizon fluctuations. In order to investigate this question, let us consider an infalling observer with $\tilde{E}=\tilde{E}_i=1$. From Eq. (54), we have that, near $r=2M$,

$$\frac{dt}{dr}\approx -C^{-1} \tag{80}$$

and, hence,

$$\frac{du}{dr}=\frac{dt}{dr}-\frac{dr^*}{dr}\approx -2C^{-1}. \tag{81}$$

This equation may be integrated to yield

$$u(r)=-4M\,\ln\left(\frac{r-2M}{A'}\right), \tag{82}$$

where A' is a constant. This relation tells us the value of r at which the infalling observer crosses a given constant u line. The constant A' is determined by which infalling observer we consider. Here we are interested in observers who fall into the black hole not long after its formation, and we can set $A'\approx M$. Let r_c be the value of r at which this observer crosses the $u=u_{\rm max}$ line, given by

$$r_c-2M=Me^{-u_{\rm max}/4M}. \tag{83}$$

Near the horizon, Eq. (52) tells us that $dr/d\tau\approx -1$ along the world line of the infalling observer. Thus the proper time required for this observer to cross from $u=u_{\rm max}$ to the classical horizon at $r=2M$ is

$$\delta\tau\approx r_c-2M=Me^{-u_{\rm max}/4M}\approx Me^{-M^2/m_P^2}. \tag{84}$$

We should compare this quantity with $\Delta\tau_i$, where C_0 is evaluated at $r=r_c$, and so $C_0=\frac{1}{2}e^{-u_{\rm max}/4M}$. Thus,

$$\Delta\tau_i=\frac{\gamma m_P}{2M}\,\delta\tau, \tag{85}$$

and hence, as long as $M\gg m_P$, $\Delta\tau_i\ll\delta\tau$. From this result, we conclude that the horizon fluctuations do not invalidate the semiclassical derivation of the Hawking effect until the black hole's mass approaches the Planck mass. This is the point at which we would expect the semiclassical treatment to fail.

The presence of frequencies far above the Planck scale, in the form of the modes leaving $\mathcal{I}^-$, has concerned numerous authors. There have been suggestions that one might be able derive the Hawking effect in a way that transplanckian frequencies do not arise, using some form of ''mode regeneration'' [18,19]. So far, it has not been possible to implement these suggestions in detail. As seen from the our analysis of horizon fluctuations, the semiclassical treatment is remarkably robust.

V. SUMMARY AND CONCLUSIONS

In the preceding sections, we have analyzed the horizon fluctuation problem using a formalism which takes account of the effects of quantized linear perturbations of the gravitational field upon light cones. In the case of the cosmological models treated in Sec. III, the resulting horizon fluctuations were found to be of Planck dimensions for both de Sitter space and a radiation-filled universe with a Planck density of gravitons at the Planck time. These fluctuations are measured as fluctuations in the time of emission of a photon as measured in the frame of a comoving observer. The order of magnitude of the results is what one might have guessed before doing the calculation.

In the case of black hole horizon fluctuations, the results are somewhat more subtle. Whether the time scale which characterizes the horizon fluctuations (the time delay or advance) is of Planck dimensions or not depends crucially upon the frame of reference. It is indeed of Planck dimensions as measured by an observer at infinity. However, as measured by an infalling observer, this time is much less than the Planck scale and vanishes as the infalling observer approaches the classical event horizon at $r=2M$. We further found that this suppression of the horizon fluctuations is exactly what is needed to preserve Hawking's semiclassical derivation of black hole radiance for black holes of mass large compared to the Planck mass.

Our result seems to conflict with the arguments of Sorkin [2] and of Casher *et al.* [3]. These authors claim that the horizon fluctuations are much larger than found in the present paper. It should be noted, however, that the physical mechanisms being postulated in Refs. [2] and [3] are quite different from that of the present paper. Furthermore, in our opinion, the physical basis of both of these calculations seems to be open to question. Casher *et al.* obtain large gravitational perturbations of the horizon by postulating an ''atmosphere'' of particles near the horizon in large angular momentum modes. This arises by decomposing the physical quantum state of an evaporating black hole (the Unruh vacuum) into two pieces which separately have divergent stress tensors on the horizon, the contribution from the Boulware vacuum state, and a term which these authors call the ''atmosphere'' of particles. The large stress tensor fluctuations arise in the analysis of Casher *et al.* when this ''atmosphere'' undergoes thermal fluctuations. Our objection to this procedure is that the fluctuations of the Boulware vacuum energy density are not being considered. The splitting of the finite Unruh vacuum energy density into two singular parts seems rather artificial. If one chooses such a splitting, then care must be taken to prove that fluctuations in one part are not canceled by correlated fluctuations in the other part. Casher *et al.* have not done this.

Sorkin [2] uses a Newtonian treatment to estimate the gravitational field of a mass fluctuation near the horizon and its effects on the Schwarzschild geometry. One can certainly question whether a Newtonian analysis can be trusted in black hole physics. However, our primary objection to Sorkin's treatment is that the dominant contribution to the horizon fluctuations comes from modes whose wavelength is very small compared to the size of the black hole. The same line of reasoning would seem to lead to large stress tensor fluctuations and, hence, large light cone fluctuations, in all spacetimes including flat spacetime. In our view, a more reasonable result is one in which significant fluctuations arise only on scales characterized either by the spacetime geometry or else the chosen quantum state. An approach to defining stress tensor fluctuations on a flat background which has this property was given in Ref. [20]. Here the stress tensor fluctuations are defined in terms of products of operators which are normal ordered with respect to the Minkowski vacuum state.

Recently, the fluctuations of the Hawking flux, as measured in the asymptotic region, have been computed [21] by a similar approach. It was found that this flux undergoes fluctuations of the same order as its average value over time scales of the order of M. This average flux is of order M^{-2}, and so the characteristic associated black hole mass fluctuation is of order M^{-1}. The corresponding metric fluctuation near the horizon is then of order $\delta h \approx M^{-2}$. For macroscopic black holes, this is much smaller than the metric fluctuations due to the quantized linear perturbation, estimated in Eq. (63) to be of order M^{-1}. This analysis does not rule out the possibility of much larger stress tensor fluctuations in the vacuum energy near the horizon. However, the diagonal and off-diagonal components of the expectation value of the stress tensor in the Unruh state near the horizon are of the same order [22]. It is thus plausible that the fluctuations in these various components near the horizon are also of the same order. If so, then the effects of quantized linear perturbations of the gravitational field dominate over those of stress tensor fluctuations.

It must be emphasized that all of the conclusions obtained in the present manuscript are in the context of a model of linearized quantum gravity. Furthermore, much of our discussion is of a heuristic, order of magnitude nature. If the basic picture of horizon fluctuations which we have drawn is correct, much work remains to be done to make the picture more precise.

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[1] J. D. Bekenstein and V. F. Mukhanov, Phys. Lett. B **360**, 7 (1995).

[2] R. D. Sorkin, ''Two Topics concerning Black Holes: Extremality of the Energy, Fractality of the Horizon,'' Report No. gr-qc/9508002 (unpublished); ''How Wrinkled is the Surface of a Black Hole?,'' Report No. gr-qc/9701056 (unpublished).

[3] A. Casher, F. Englert, N. Itzhaki, and R. Parentani, Nucl. Phys. **B484**, 419 (1997).

[4] L. H. Ford, Phys. Rev. D **51**, 1692 (1995).

[5] L. H. Ford and N. F. Svaiter, Phys. Rev. D **54**, 2640 (1996).

[6] Units in which $\hbar = c = 16\pi G = 1$ will be used in this paper. Thus the units of mass, length, and time differ by factors of $\sqrt{16\pi}$ from the usual definitions of the Planck mass, Planck length, and Planck time. As most of the discussion in this paper deals with order of magnitude estimates, this should not cause any confusion. The metric signature will be $(1,-1,-1,-1)$.

[7] M. R. Brown and A. C. Ottewill, Phys. Rev. D **34**, 1776 (1986).

[8] B. Allen, J. G. McLaughlin, and A. C. Ottewill, Phys. Rev. D **45**, 4486 (1992).

[9] L. H. Ford and L. Parker, Phys. Rev. D **16**, 1601 (1977).

[10] L. H. Ford and L. Parker, Phys. Rev. D **16**, 245 (1977).

[11] A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982).

[12] L. H. Ford and A. Vilenkin, Phys. Rev. D **33**, 2833 (1986).

[13] A. Linde, Phys. Lett. **116B**, 335 (1982).

[14] A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).

[15] J. W. York, Phys. Rev. D **28**, 2929 (1983).

[16] C. W. Misner, K. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973), Sec. 25.5.

[17] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[18] T. Jacobson, Phys. Rev. D **48**, 728 (1993); **53**, 7028 (1996).

[19] W. G. Unruh, ''Dumb Holes and the Effects of High Frequencies on Black Hole Evaporation,'' Report No. gr-qc/9409008 (unpublished).

[20] C.-I. Kuo and L. H. Ford, Phys. Rev. D **47**, 4510 (1993).

[21] C. H. Wu and L. H. Ford (in preparation).

[22] B. P. Jensen, J. G. McLaughlin, and A. C. Ottewill, Phys. Rev. D **43**, 4142 (1991).