

## Mechanisms of the reaction $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$ at high energies

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The main dynamical mechanisms of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  at high energies, currently investigated at Serpukhov and Brookhaven, are considered in detail. It is shown that the observed forward peak in its differential cross section can be explained within the framework of the Regge pole model only by the conspiring  $\rho_2$  Regge pole exchange. The tentative estimates of the absolute  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  reaction cross section at  $P_{\text{lab}}^{\pi^-} = 18$  GeV/c are obtained:  $\sigma \approx 200$  nb and, in the forward direction,  $d\sigma/dt \approx 940$  nb/GeV<sup>2</sup>. The contribution of the one pion exchange, which is forbidden by  $G$  parity and which can arise owing to the  $f_0^0(980) - a_0^0(980)$  mixing, is also estimated. The role of the Regge cuts in the nonflip helicity amplitude is briefly examined and the conclusion is made that the contributions of the cuts have to be inessential in comparison with the conspiring  $\rho_2$  Regge pole exchange. [S0556-2821(97)05711-1]

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### I. INTRODUCTION

In the  $q\bar{q}$  model ( $q$  is a light quark), every rotational excitation with orbital angular momentum  $L$  consists of four nonets: states  $^{2S+1}L_J = ^3L_{L-1}$ ,  $^3L_L$ , and  $^3L_{L+1}$  with charge parity  $C = (-1)^{L+1}$  and state  $^1L_L$  with  $C = (-1)^L$ . However, so far there is a white spot in the lower-lying family with  $L = 2$  [1]. The nonstrange members of the  $^3D_2$  nonet with quantum numbers  $I^G(J^{PC}) = 1^+(2^{--})$  and  $0^-(2^{--})$ , i.e., the  $\rho_2$ ,  $\omega_2$ , and  $\phi_2$  mesons (the masses of which are expected to be near 1.7 and 1.9 GeV [2–4]), are not yet identified as peaks in corresponding multibody mass spectra [1]. Discussions of the possible reasons for this unusual situation are contained, for example, in Refs. [3,4]. However, the mass distributions are not unique keepers of information on the resonances. The resonance spectrum is also reflected in the Regge behavior of the reaction cross sections at high energies. At present detailed investigations of the reaction  $\pi^- p \rightarrow \pi^0 \eta n$  at  $P_{\text{lab}}^{\pi^-} \approx 40$  and 18 GeV/c are carried out, respectively, at Serpukhov [5,6] and Brookhaven [7]. The  $\pi^0 \eta$  mass spectrum in this reaction is dominated by the  $a_0^0(980)$  and  $a_0^0(1320)$  mesons [5–7]. In this connection, we should like to draw special attention to the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  because its differential cross section near the forward direction can be dominated by the Regge pole exchange with quantum numbers of the ‘‘lost’’  $\rho_2$  meson. In general, this reaction is unique in that it involves only unnatural parity exchanges in the  $t$  channel.

The purpose of this paper is to describe in detail the main dynamical mechanisms of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  in the Regge region. The paper is organized as follows. In Sec. II, we present the Reggeization of the  $s$ -channel helicity amplitudes of the reaction  $\pi^- p \rightarrow a_0^0(980)n$  and show that, in the framework of the Regge pole model, the observed forward peak in its cross section [7] can be explained by a very interesting and fine phenomenon such as a ‘‘conspiracy’’ between the  $\rho_2$  trajectory and its daughter one. For the first time, the  $\rho_2$  Regge trajectory was introduced (at that time it was named  $Z$ ) for

an explanation of the absence of a dip near the forward direction in  $\rho_{00}^H d\sigma/dt(\pi^- p \rightarrow \omega n)$  [8–10]. However, the nontrivial reason why the  $\rho_2$  Regge pole contribution in the  $s$ -channel amplitudes without a helicity flip in the nucleon vertex and with zero helicity of the  $\omega$  meson does not vanish at  $t = 0$ , i.e., a conspiracy of the Regge poles in  $\pi^- p \rightarrow \omega n$ , was not discussed at all in Refs. [8–10]. Notice that, for similar cases, the necessary type of conspiracy was known [11–14] well before the works in Refs. [8–10]. Here we make up this omission by the example of the reaction  $\pi^- p \rightarrow a_0^0(980)n$ . We present also a tentative estimate of the  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  reaction cross section<sup>1</sup> at  $P_{\text{lab}}^{\pi^-} = 18$  GeV/c:  $\sigma \approx 200$  nb and in the forward direction  $d\sigma/dt \approx 940$  nb/GeV<sup>2</sup>. In Sec. III, we recall one more interesting feature of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  associated with  $f_0^0(980) - a_0^0(980)$  mixing [15] and estimate the contribution of the one-pion exchange which is possible owing to this mixing. In Sec. IV, the role of the Regge cuts in the non-flip-helicity amplitude is briefly discussed. In the Appendix, the conspiracy phenomenon is explained by the example of the elementary  $\rho_2$  exchange.

### II. REACTION $\pi^- p \rightarrow a_0^0(980)n$ AT HIGH ENERGIES IN THE REGGE POLE MODEL

The  $s$ -channel helicity amplitudes of this reaction can be written as

$$M_{\lambda_n \lambda_p} = \bar{u}_{\lambda_n}(p_2) \gamma_5 [-A - \frac{1}{2} \gamma^\mu (q_1 + q_2)_\mu B] u_{\lambda_p}(p_1), \quad (1)$$

where  $q_1, p_1, q_2$ , and  $p_2$  are four-momenta of  $\pi^-$ ,  $p$ ,  $a_0^0$ , and  $n$ , respectively,  $\lambda_p$  and  $\lambda_n$  are the proton and neutron helicities, and  $A$  and  $B$  are the invariant amplitudes depending on

<sup>1</sup>Often the normalization of the reaction events turns out to be a complicated problem. Probably in this connection, experimental information on the absolute cross section of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  is so far absent.

$s=(p_1+q_1)^2$  and  $t=(q_1-q_2)^2$  and free of kinematical singularities [16]. Using the normalization  $\bar{u}u=2m_N$  and taking the proton and the neutron as ‘‘second particles’’ [17] we obtain that, in the c.m. system,

$$M_{++} = -M_{--} = \cos(\theta/2)[A\sqrt{-t_{\min}} - B\sqrt{-t_{\max}}], \quad (2)$$

$$M_{+-} = +M_{-+} = \sin(\theta/2)[A\sqrt{-t_{\max}} - B\sqrt{-t_{\min}}], \quad (3)$$

where  $\theta$  is the scattering angle,  $t_{\min}$  and  $t_{\max}$  are the values of the variable  $t$  at  $\theta=0^\circ$  and  $180^\circ$ , respectively,  $\sin(\theta/2)=[-(t-t_{\min})/4|\vec{q}_1||\vec{q}_2|]^{1/2}$ , and

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s |\vec{q}_1|^2} (|M_{++}|^2 + |M_{--}|^2). \quad (4)$$

Equations (2) and (3) have the most simple form at high energies. Taking into account that  $A$  and  $B$  at fixed  $t$  and  $s \gg m_N^2$  behave like  $s^\alpha$  and  $s^{\alpha-1}$ , respectively (see below), and also  $t_{\min} \approx -m_N^2(m_{a_0}^2 - m_\pi^2)^2/s^2$  and  $t_{\max} \approx -s$ , we get

$$M_{++} \approx -sB, \quad M_{+-} \approx \sqrt{-(t-t_{\min})}A. \quad (5)$$

The helicity amplitudes in the  $t$ -channel c.m. system,  $F_{\lambda_{\bar{p}}\lambda_n}$ , corresponding to the reaction  $\pi^- a_0^0 \rightarrow \bar{p}n$  have the form

$$F_{++} = -F_{--} = \sqrt{t}A + \frac{m_N(m_{a_0}^2 - m_\pi^2)}{\sqrt{t}}B, \quad (6)$$

$$F_{+-} = +F_{-+} = 2|\vec{q}_t||\vec{p}_t|\sin\theta_t B \equiv \sqrt{\frac{\Phi(s,t,u)}{t}}B. \quad (7)$$

In these equations, the  $a_0^0(980)$  meson and neutron are taken as ‘‘second particles,’’  $\theta_t$  is the scattering angle, and  $|\vec{q}_t|$  and  $|\vec{p}_t|$  are the absolute values of the momenta of the particles in the initial and final states, respectively.  $\cos\theta_t=(s-u)/4|\vec{q}_t||\vec{p}_t|$ , and  $u=2m_N^2+m_{a_0}^2+m_\pi^2-s-t$ ; the equation  $\Phi(s,t,u)=0$  gives the boundary of the physical region. It is obvious, from Eqs. (6) and (7), that the helicity amplitudes

$$G_{++} = \sqrt{t}F_{++} \quad \text{and} \quad G_{+-} = \left(\sqrt{\frac{\Phi(s,t,u)}{t}}\right)^{-1} F_{+-} = B \quad (8)$$

are free of kinematical singularities. Their Reggeization can be performed by the usual way [18,19].

Constructing the helicity amplitudes with definite parity [18],

$$G_{\lambda_{\bar{p}}\lambda_n}^{(\pm)} = G_{\lambda_{\bar{p}}\lambda_n} [1 \pm \eta_\pi \eta_{a_0} (-1)^{\lambda_{\bar{p}} - \lambda_n + |\lambda_{\bar{p}} - \lambda_n|}] / 2, \quad (9)$$

we obtain that, because of the difference of the intrinsic parities of the  $\pi$  and  $a_0$  mesons,  $\eta_\pi$  and  $\eta_{a_0}$ , the amplitudes  $G_{++}^{(+)}$  and  $G_{+-}^{(+)}$  identically vanish and thus  $G_{++} \equiv G_{++}^{(-)}$  and  $G_{+-} \equiv G_{+-}^{(-)}$ . Consequently, both amplitudes  $G_{++}$  and  $G_{+-}$  have unnatural parity, as it must be, since each state of the  $\pi a_0$  system with angular moment  $J$  has parity

$P_{\pi a_0} = (-1)^{J+1}$ . It follows from the parity conservation condition  $P_{\bar{p}n} = (-1)^{L+1} = P_{\pi a_0}$ , where  $L$  is angular momentum of the  $\bar{p}n$  system, that  $L=J$  both for the singlet  $\bar{p}n$  spin state and for the triplet one. The amplitudes  $G_{++}$  and  $G_{+-}$  correspond to the triplet and singlet (because  $G_{++} = -G_{--}$ )  $\bar{p}n$  configurations, respectively. The  $G$ -parity conservation condition  $(-1)^{L+S+I} = (-1)^{J+S+1} = +1$ , where  $S$  and  $I$  are the spin and isospin of  $\bar{p}n$ , gives that in the triplet (singlet) state only even (odd) values of  $J$  are possible. The partial wave expansions of  $G_{++}$  and  $G_{+-}$  are [18]

$$G_{++} = \sum_{J=1,3,\dots} (2J+1) f_{++}^J P_J(\cos\theta_t),$$

$$G_{+-} = \sum_{J=2,4,\dots} (2J+1) f_{+-}^J \frac{P_J'(\cos\theta_t)}{\sqrt{J(J+1)}}. \quad (10)$$

Thus, the amplitude  $G_{++}$  has to contain the Regge pole exchanges with  $I=1$ ,  $G=+1$ , signature  $\tau=-1$ , and ‘‘naturality’’  $\tau P=-1$ . The high-lying Regge trajectory with such quantum numbers is the  $b_1$  trajectory [the well-known  $b_1(1235)$  meson is its lower-lying representative]. The second independent amplitude  $G_{+-}$  has to contain the Regge pole exchanges with  $I=1$ ,  $G=+1$ ,  $\tau=+1$ , and  $\tau P=-1$  and here the  $\rho_2$  Regge trajectory is a leading one. Taking into account Eqs. (10), the contributions of the  $b_1$  and  $\rho_2$  Regge pole exchanges in the physical region of the  $s$ -channel can be written as

$$G_{++}^{b_1} = \beta_{b_1}(t) \left(\frac{s}{s_0}\right)^{\alpha_{b_1}(t)} i e^{-i\pi\alpha_{b_1}(t)/2},$$

$$G_{+-}^{\rho_2} = \beta_{\rho_2}(t) \left(\frac{s}{s_0}\right)^{\alpha_{\rho_2}(t)-1} e^{-i\pi\alpha_{\rho_2}(t)/2}, \quad (11)$$

where  $\beta(t)$ ,  $\alpha(t)$ , and complex factors are residues, trajectories, and signatures of the corresponding Regge poles, and  $s_0=1$  GeV<sup>2</sup>. For compensation of the nonphysical branch points in  $G_{+-}$  connected with  $1/\sqrt{J(J+1)}$  [see Eq. (10)], the factor  $\sqrt{J(J+1)}$  has been extracted from  $f_{+-}^J$  [12].

Let us return to Eqs. (6)–(8) and express the invariant amplitudes  $A$  and  $B$  in terms of  $G_{++}$  and  $G_{+-}$ :

$$A = \frac{1}{t} [G_{++} - m_N(m_{a_0}^2 - m_\pi^2)G_{+-}], \quad (12)$$

$$B = G_{+-}. \quad (13)$$

To avoid the  $1/t$  singularity in the invariant amplitude  $A$  [see Eq. (12)], it is necessary to complement the Reggeization scheme by the conditions on the behavior of the various contributions to  $G_{\lambda_{\bar{p}}\lambda_n}$  as  $t \rightarrow 0$ . Let us attempt to satisfy the analyticity of  $A$  assuming the  $b_1$  and  $\rho_2$  exchanges only and also that the amplitudes  $G_{++}^{b_1}$  and  $G_{+-}^{\rho_2}$  do not vanish as  $t \rightarrow 0$ . Then, substituting Eq. (11) into (12) and going to the limit  $t \rightarrow 0$ , we obtain two relations,

$$\alpha_{b_1}(0) = \alpha_{\rho_2}(0) - 1, \quad \beta_{b_1}(0) = m_N(m_{a_0}^2 - m_\pi^2)\beta_{\rho_2}(0), \quad (14)$$

the first of which is rather silly because, at the usual values of  $\alpha_{b_1}(0) \approx -(0.05-0.3)$  [8–10,20], it requires  $\alpha_{\rho_2}(0) \approx 0.95-0.7$  (also, for the linear  $\rho_2$  trajectory with the slope  $\alpha' \approx 0.8-1$  GeV<sup>-2</sup>, it predicts the  $\rho_2$  mass  $m_{\rho_2} \approx 1.02-1.27$  GeV). For the  $\rho_2$  trajectory having unnatural parity, this is evidently ruled out. In fact, we conclude that there is no way to make it such that the residue of the  $b_1$  exchange in Eq. (11) would be finite at  $t=0$ . Of course, in order for the amplitude  $A$  to be regular for  $t \rightarrow 0$ , one can accept that the amplitudes  $G_{++}^{b_1}$  and  $G_{+-}^{\rho_2}$  are separately proportional to  $t$ .<sup>2</sup> In this case, the amplitude  $B$  in Eq. (13) and amplitude  $M_{++}$  in Eq. (5) caused by the  $\rho_2$  exchange are also proportional to  $t$ . Then, from Eqs. (5) and (4), it follows immediately that, for  $b_1$  and  $\rho_2$  Regge pole exchanges,  $d\sigma/dt \sim |t|$  at small  $|t|$ . Thus this Regge pole model predicts a dip near the forward direction in the  $\pi^- p \rightarrow a_0^0 n$  reaction cross section. On the contrary, the experiment in Ref. [7] shows a clear forward peak. This means that the amplitude  $M_{++}$  with quantum numbers of the  $\rho_2$  exchange in the  $t$  channel does not vanish as  $t \rightarrow 0$ . In the framework of the Regge pole model, this can be attended only in the case of a conspiracy of the  $\rho_2$  Regge trajectory with its daughter one ( $d$ ), which has to have quantum numbers of the  $b_1$  exchange. Let us write down the contribution of such a daughter trajectory near  $t=0$  in the form

$$G_{++}^d = \beta_d(t) \left( \frac{s}{s_0} \right)^{\alpha_d(t)} i e^{-i\pi\alpha_d(t)/2}. \quad (15)$$

Then, the amplitude  $A$  should be regular at  $t=0$  [see Eq. (12)] if the following relations for the  $\rho_2$ ,  $d$  and  $b_1$  exchanges are valid:

$$\alpha_d(0) = \alpha_{\rho_2}(0) - 1, \quad \beta_d(0) = m_N(m_{a_0}^2 - m_\pi^2)\beta_{\rho_2}(0), \quad (16)$$

$$\beta_{\rho_2}(0) \neq 0, \quad \beta_{b_1}(t) \sim t. \quad (17)$$

Now neither the amplitude  $B$  [see Eqs. (11), (13), and (17)] nor the amplitude  $M_{++}$  in Eq. (5) vanishes at  $t=0$ . Moreover, asymptotically (at large  $s$ )  $M_{++}$  is dominated by the  $\rho_2$  trajectory [see Eqs. (5), (11), and (13)] and  $M_{+-}$  is dominated by the  $b_1$  trajectory [see Eqs. (5), (11), (12), (15), and (16)]. As for the daughter trajectory contribution and the nonasymptotic contribution of the  $\rho_2$  trajectory (which behaves as  $\sim s^{\alpha_{\rho_2}(t)-1}$ ) to the amplitude  $A$  and consequently to  $M_{+-}$ , they can all be neglected [see Eqs. (5), (11), (12), (15), and (16)]. Thus, on the one hand, a role of the daughter

trajectory, in practice, comes to only the fact that the residue of the leading  $\rho_2$  Regge pole  $\beta_{\rho_2}(t)$ , owing to a conspiracy [see Eqs. (16) and (17)], does not vanish when  $t \rightarrow 0$  and can be parametrized, for example, by the simplest exponential form  $\beta_{\rho_2}(t) = -\gamma_{\rho_2} \exp(b_{\rho_2}^0 t/s_0)$ . At the same time, the residue of the  $b_1$  Regge pole in Eq. (11) has to be proportional to  $t$  [see Eq. (17)] and can be parametrized, for example, as  $\beta_{b_1}(t) = t\gamma_{b_1} \exp(b_{b_1}^0 t/\sqrt{s_0})$ . In our normalization, the constants  $\gamma_{\rho_2}$  and  $\gamma_{b_1}$  are dimensionless. On the other hand, if the daughter trajectory is parallel to the  $\rho_2$  trajectory (as, for example, in the Veneziano model), then, near 1.7 GeV, it should be expected a state with the  $b_1$  meson quantum numbers, which can be searched for in the  $a_0\pi$ ,  $\omega\pi$ , and  $A_2\pi$  channels.

Thus, in the model with the  $b_1$  and conspiring  $\rho_2$  Regge poles, the  $s$ -channel helicity amplitudes given by Eq. (5) can be written in the following form convenient for fitting to the data:

$$M_{++} = \gamma_{\rho_2} e^{b_{\rho_2}(s)t} \left( \frac{s}{s_0} \right)^{\alpha_{\rho_2}(0)} e^{-i\pi\alpha_{\rho_2}(t)/2}, \quad (18)$$

$$M_{+-} = \sqrt{-(t-t_{\min})/s_0} \gamma_{b_1} e^{b_{b_1}(s)t} \left( \frac{s}{s_0} \right)^{\alpha_{b_1}(0)} i e^{-i\pi\alpha_{b_1}(t)/2}, \quad (19)$$

where  $\alpha_R(t) = \alpha_R(0) + \alpha'_R t$ ,  $b_R(s) = b_R^0 + \alpha'_R \ln(s/s_0)$ , and  $R$  designates a Reggeon. Let us point out, as a guide, that  $\alpha_{b_1}(0) \approx -0.22$  and  $\alpha_{\rho_2}(0) \approx -0.3$  for  $\alpha'_{b_1} \approx \alpha'_{\rho_2} \approx 0.8$  GeV<sup>-2</sup>,  $m_{b_1} \approx 1.235$  GeV, and  $m_{\rho_2} \approx 1.7$  GeV. Using Eqs. (4), (18), and (19), we get that, at large  $s$ ,

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \left[ \gamma_{\rho_2}^2 e^{2b_{\rho_2}(s)t} \left( \frac{s}{s_0} \right)^{2\alpha_{\rho_2}(0)} + \left( \frac{t_{\min}-t}{s_0} \right) \gamma_{b_1}^2 e^{2b_{b_1}(s)t} \left( \frac{s}{s_0} \right)^{2\alpha_{b_1}(0)} \right]. \quad (20)$$

According to the Brookhaven data at  $P_{\text{lab}}^{\pi^-} \approx 18$  GeV/c [7] the  $t$  distribution for events of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  is strongly peaked in the forward direction (see Fig. 1). These data are fitted very well for  $-t_{\min} < -t < 0.6$  GeV<sup>2</sup> by the single-exponential form

$$dN/dt(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n) = C e^{\Lambda t}. \quad (21)$$

The best fit (with  $\chi^2 \approx 15.9$  for 22 degrees of freedom) is obtained with  $\Lambda = 4.7$  GeV<sup>-2</sup> and  $C = 129$  events/GeV<sup>2</sup>. It is shown in Fig. 1 by the solid curve. Unfortunately, the Serpukhov data on  $d\sigma/dt(\pi^- p \rightarrow a_0^0(980)n)$  at 40 GeV/c are not yet presented. It is known only that, in the  $\pi^0 \eta$  invariant mass region  $1 \leq m_{\pi^0 \eta} \leq 1.2$  GeV and for  $-t_{\min} < -t < 0.5$  GeV<sup>2</sup>, the differential cross section  $d\sigma/dt(\pi^- p \rightarrow \pi^0 \eta n)$  has a similar peak in the forward di-

<sup>2</sup>Using for  $b_1\pi a_0$  and  $b_1\bar{N}N$  interactions the effective Lagrangians  $L(b_1\pi a_0) \sim j_\mu^M b_1^\mu$  and  $L(b_1\bar{N}N) \sim j_\mu^B b_1^\mu$ , where  $j_\mu^M = (q_1 - q_2)_\mu$  and  $j_\mu^B = \bar{u}(p_2)\gamma_5(p_2 - p_1)_\mu v(p_1)$ , one can easily verify that the contribution of the elementary  $b_1$  exchange to the amplitude  $G_{++}$  for the reaction  $\pi(q_1) + a_0(q_2) \rightarrow \bar{N}(p_1) + N(p_2)$  turns out to be really proportional to  $t$ .

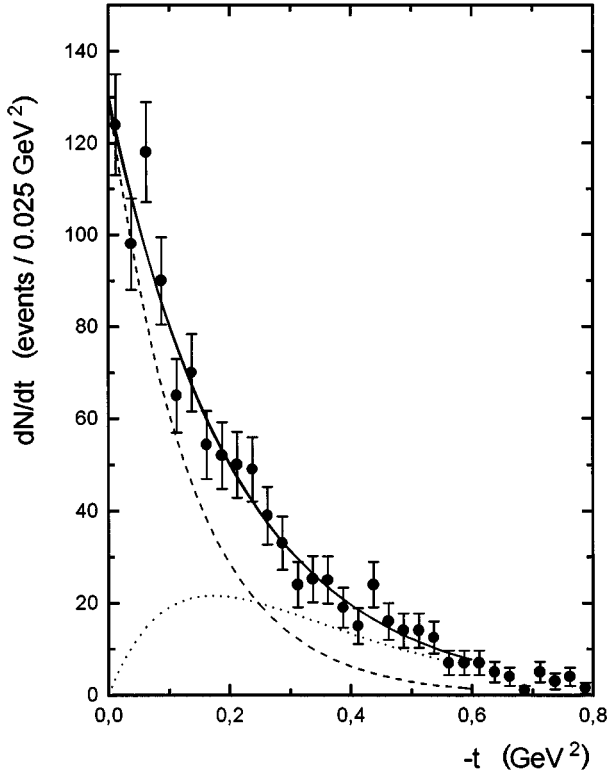


FIG. 1. The  $t$  distribution for the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n \rightarrow 4 \gamma n$  at 18 GeV/c measured at Brookhaven [7]. The fits are described in the text.

rection [5]. Obviously, the Brookhaven data are described formally by the single amplitude  $M_{++}$  with the  $\rho_2$  exchange [see Eqs. (18) and (21)]. However, within  $\pm(10-20)\%$  experimental uncertainties in  $dN/dt$  [7], the form (21) can be effectively reproduced for  $-t_{\min} < -t < 0.6$  by means of Eq. (20) where the  $b_1$  contribution should be also different from zero. The fit to the data [7] to the form  $dN/dt = C_1 \exp(\Lambda_1 t) + (t_{\min} - t) C_2 \exp(\Lambda_2 t)$  with  $C_1 = 131$  events/GeV $^2$ ,  $\Lambda_1 = 7.6$  GeV $^{-2}$ ,  $C_2 = 340$  events/GeV $^2$ , and  $\Lambda_2 = 5.8$  GeV $^{-2}$  gives a  $\chi^2 \approx 15.9$  for 20 degrees of freedom, and the corresponding curve is practically the same as the solid curve in Fig. 1. The dashed and dotted curves in Fig. 1 show the  $\rho_2$  [ $C_1 \exp(\Lambda_1 t)$ ] and  $b_1$  [ $(t_{\min} - t) C_2 \exp(\Lambda_2 t)$ ] contributions separately, with the latter yields approximately 34% of the integrated cross section. In order to determine rather accurately the parameters of the simplest Regge pole model given by Eqs. (18)–(20), the good data on  $d\sigma/dt(\pi^- p \rightarrow a_0^0(980)n)$  at several appreciably different energies are needed. First of all, we have in mind the energies of the  $\pi^-$  beams at Serpukhov ( $\approx 40$  GeV), Brookhaven ( $\approx 18$  GeV), and KEK ( $\approx 10$  GeV). Notice that, according to the estimate  $\sigma \sim (s)^{2\alpha-2}$  with  $\alpha \approx -0.3$ , the  $a_0^0(980)$  production cross section at KEK should be approximately 36 times as large as one at Serpukhov.

So far the experimental information on the absolute values of the  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  cross section is absent. Nevertheless, in order to have an idea of this cross section, we shall estimate  $\sigma(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n)$  at  $P_{\text{lab}}^{\pi^-} = 18$  GeV/c using the data on the reaction

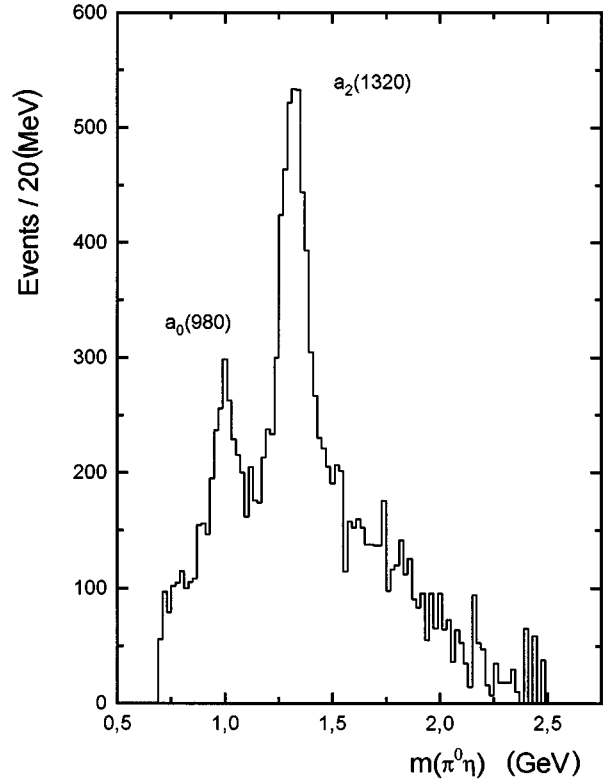


FIG. 2. The  $\pi^0 \eta$  mass spectrum for the reaction  $\pi^- p \rightarrow \pi^0 \eta n \rightarrow 4 \gamma n$  at 18 GeV/c measured at Brookhaven [7].

$\pi^- p \rightarrow a_2^0(1320)n$  and the Brookhaven data on the  $\pi^0 \eta$  mass spectrum in  $\pi^- p \rightarrow \pi^0 \eta n$ . According to Refs. [21,5],

$$\sigma(\pi^- p \rightarrow a_2^0(1320)n) = 18.5 \pm 3.7, \quad 12.3 \pm 2.5, \\ 2.7 \pm 1.0 \quad \text{and} \quad 0.395 \pm 0.080 \quad \mu\text{b} \quad (22)$$

at  $P_{\text{lab}}^{\pi^-} = 12, 15, 40,$  and  $100$  GeV/c, respectively. These data are fitted quite well by the exponential function:

$$\sigma(\pi^- p \rightarrow a_2^0(1320)n) \approx (1.62 \text{ mb}) [P_{\text{lab}}^{\pi^-} / (1 \text{ GeV}/c)]^{-1.8}. \quad (23)$$

Then at 18 GeV/c,  $\sigma(\pi^- p \rightarrow a_2^0(1320)n \rightarrow \pi^0 \eta n) \approx 1.29 \mu\text{b}$  [here we have taken into account that  $B(a_2(1320) \rightarrow \pi \eta) \approx 0.145$  [1]]. Figure 2 shows the Brookhaven data (corrected by the registration efficiency) on the  $\pi^0 \eta$  mass spectrum in the reaction  $\pi^- p \rightarrow \pi^0 \eta n$  at 18 GeV/c [7]. According to our estimate the ratio  $N(a_0(980))/N(a_2(1320)) \approx 1/6-1/7$ , where  $N(a_0(980))$  and  $N(a_2(1320))$  are the numbers of events, respectively, in the  $a_0(980)$  and  $a_2(1320)$  peaks above background. Thus, one can expect that, at  $P_{\text{lab}}^{\pi^-} = 18$  GeV/c,  $\sigma(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n) \approx 200$  nb and also  $[d\sigma/dt(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n)]_{t \approx 0} \approx 940$  nb/GeV $^2$  according to Eq. (21) with  $\Lambda = 4.7$  GeV $^{-2}$ . We emphasize that these estimates are rather tentative.

### III. ONE-PION EXCHANGE IN $\pi^-p \rightarrow a_0^0(980)n \rightarrow \pi^0\eta n$

It is now interesting to estimate the contribution to this reaction of the Reggeized one-pion exchange (OPE), which is forbidden by  $G$  parity. The corresponding cross section has the form

$$\frac{d\sigma^{(\text{OPE})}}{dt dm} = \frac{1}{\pi s^2} \frac{g_{\pi NN}^2}{4\pi} \left[ \frac{-te^{2b_\pi(s)(t-m_\pi^2)}}{(t-m_\pi^2)^2} \right] \times m^3 \rho_{\pi\pi} \sigma(\pi^+\pi^- \rightarrow \pi^0\eta), \quad (24)$$

where  $g_{\pi NN}^2/4\pi \approx 14.6$ ,  $m$  is the invariant mass of the  $\pi^0\eta$  system,  $\rho_{\pi\pi} = (1 - 4m_\pi^2/m^2)^{1/2}$ ,  $b_\pi(s) = b_\pi^0 + \alpha'_\pi \ln(s/s_0)$ , and  $\alpha'_\pi \approx 0.8 \text{ GeV}^{-2}$ . This contribution arises owing to the  $f_0(980)$ - $a_0^0(980)$  mixing violating isotopic invariance. The  $f_0(980)$ - $a_0^0(980)$  mixing phenomenon and its possible manifestations in the various reactions (for example, in  $\pi^\pm N \rightarrow \pi^0\eta N$ ) were considered in detail in Refs. [15]. Therefore, here we give only numerical estimates of the absolute value OPE contribution at the Brookhaven and Serpukhov energies.

Recall that the cross section of the reaction forbidden by  $G$  parity,  $\pi^+\pi^- \rightarrow \pi^0\eta$  [see Eq. (24)], is determined mainly by the transitions  $f_0(980) \rightarrow (K^+K^- + K^0\bar{K}^0) \rightarrow a_0^0(980)$  and, in the region between the  $K^+K^-$  and  $K^0\bar{K}^0$  thresholds, which has a width of 8 MeV, it can be on the average from 0.4 to 1 mb [15]. Outside the region  $2m_{K^+} \leq m \leq 2m_{K^0}$ ,  $\sigma(\pi^+\pi^- \rightarrow \pi^0\eta)$  drops sharply. The mentioned uncertainty in the estimate of  $\sigma(\pi^+\pi^- \rightarrow \pi^0\eta)$  reflects the spectrum of the model assumptions which were made by many authors for the determination of the coupling constants of the  $f_0(980)$  and  $a_0(980)$  resonances with the  $\pi\pi$ ,  $K\bar{K}$ , and  $\pi\eta$  channels (see details in Refs. [15,22]). Note that the value of  $\sigma(\pi^+\pi^- \rightarrow \pi^0\eta)$  between the  $K^+K^-$  and  $K^0\bar{K}^0$  thresholds is controlled mainly by the production of ratios  $(g_{f_0K^+K^-}^2/g_{f_0\pi^+\pi^-}^2)(g_{a_0^0K^+K^-}^2/g_{a_0^0\pi\eta}^2)$  [15], where the coupling constants  $g$  determine the corresponding decay widths of the scalar mesons, for example,  $m\Gamma_{f_0\pi^+\pi^-}(m) = (g_{f_0\pi^+\pi^-}^2/16\pi)\rho_{\pi\pi}$  and so on.

Taking these remarks into account and integrating Eq. (24) over  $m$  from  $2m_{K^+}$  to  $2m_{K^0}$ , we get

$$\frac{d\sigma^{(\text{OPE})}}{dt} \approx (12 - 30 \text{ nb}) \left[ \frac{-te^{\Lambda_\pi(t-m_\pi^2)}}{(t-m_\pi^2)^2} \right] \quad (25)$$

at  $P_{\text{lab}}^\pi = 18 \text{ GeV}/c$  and approximately 5 times smaller at  $P_{\text{lab}}^\pi = 40 \text{ GeV}/c$ . For the reactions with the one-pion exchange, a typical slope in the considered energy region is  $\Lambda_\pi = 2b_\pi(s) \approx 5 - 7 \text{ GeV}^{-2}$ . Then, the integral of the function confined in brackets in Eq. (25) over  $t$  turns out to be approximately equal to 1. Hence we have  $\sigma^{(\text{OPE})} \approx 12 - 30 \text{ nb}$  at  $18 \text{ GeV}/c$ . This is (6–15)% of our estimate,  $\sigma(\pi^-p \rightarrow a_0^0(980)n \rightarrow \pi^0\eta n) \approx 200 \text{ nb}$ , obtained at the end of Sec. II. Because of the smallness of the  $\pi$  meson mass, the  $d\sigma^{(\text{OPE})}/dt$  is enhanced for small  $|t|$  [about (85–

90)% of the integrated cross section  $\sigma^{(\text{OPE})}$  originate from the region  $0 < -t < 0.2 \text{ GeV}^2$ ]. At the maximum situated near  $t \approx -m_\pi^2$ ,

$$(d\sigma^{(\text{OPE})}/dt)_{t \approx -m_\pi^2} \approx 122 - 305 \text{ nb/GeV}^2. \quad (26)$$

It can make up from 13 to 32.5% of  $[d\sigma/dt(\pi^-p \rightarrow a_0^0(980)n \rightarrow \pi^0\eta n)]_{t \approx 0}$  which has been roughly estimated to be  $940 \text{ nb/GeV}^2$  at  $18 \text{ GeV}/c$  (see the end of Sec. II).

Thus, the  $G$ -parity-violating OPE contribution is able to play a quite appreciable role in the formation of the peak in  $d\sigma/dt(\pi^-p \rightarrow a_0^0(980)n \rightarrow \pi^0\eta n)$  near the forward direction. Note that the features of the interference between the  $\pi$  and  $b_1$  exchanges in the amplitude  $M_{+-}$  were discussed in some detail in Ref. [15]. To extract uniquely the amplitude  $M_{+-}$  which can be dominated in the low  $|t|$  range by the ‘‘forbidden’’  $\pi$  exchange, a polarized target and a measurement of the neutron polarization in the reaction  $\pi^-p \rightarrow a_0^0(980)n \rightarrow \pi^0\eta n$  are necessary. It is also desirable to measure the charge-symmetric reaction  $\pi^-n \rightarrow a_0^0(980)p \rightarrow \pi^0\eta p$  in which the  $f_0^0(980)$ - $a_0^0(980)$  interference has to have opposite sign [15].

### IV. CONTRIBUTIONS OF THE REGGE CUTS

The Regge cuts, just like the conspiring  $\rho_2$  Regge pole, can give a nonvanishing contribution to the amplitude  $M_{++}$  for  $t \rightarrow 0$ . Generally speaking, it is difficult to distinguish the contributions of the conspiring poles and cuts. However, the standard numerical estimates (such as below) show that in the considered reaction the Regge cuts have to be insignificant.

First of all, we carry out a classification of the two-Reggeon cuts contributing to the amplitude  $M_{++}$  of the reaction  $\pi^-p \rightarrow a_0^0(980)n$ . According to Ref. [23], the signature of the cut is given by  $\tau_c = \tau_1\tau_2$ , where  $\tau_1$  and  $\tau_2$  are the signatures of the Regge poles associated with the cut. The signature of the amplitude  $M_{++}$  is positive and therefore the  $\tau_1$  and  $\tau_2$  must be equal. Then, it is found that the two-Reggeon cuts associated with the Regge poles having equal and opposite ‘‘naturalities’’ ( $\tau P$ ) have a principle different behavior as  $t \rightarrow 0$ . Parity conservation gives that the cuts with  $(\tau_1 P_1)(\tau_2 P_2) = -1$  do not vanish as  $t \rightarrow 0$  [24]. Among these are the  $a_2\pi$ ,  $\rho b_1$ , and  $\omega a_1$  cuts and also the  $P\rho_2$  cut, where  $P$  is the Pomeron. The cuts with  $(\tau_1 P_1)(\tau_2 P_2) = +1$  give vanishing contributions to  $M_{++}$  as  $t \rightarrow 0$  (they turn out to be proportional to  $t$ ) [24]. In this group, the  $\rho\rho$  and  $a_2a_2$  cuts are leading at large  $s$ .

The amplitude of the two-Reggeon cut associated with the  $R_1$  and  $R_2$  Regge pole exchanges can be calculated in the absorption model approximation by the formula [25–27]

$$M_{ab \rightarrow cd}^{R_1 R_2}(s, t) = \frac{i}{8\pi^2 s} \int d^2 k_\perp \sum_{e, f} M_{ab \rightarrow ef}^{R_1}(s, \vec{q} - \vec{k}_\perp) \times M_{ef \rightarrow cd}^{R_2}(s, \vec{k}_\perp), \quad (27)$$

that is, considering the  $R_1 R_2$  cut contribution as a process of a double quasielastic rescattering. In Eq. (27),  $\vec{k}_\perp$  and  $\vec{q}$  are

the momenta transferred from the particle  $e$  to the particle  $c$  and from  $a$  to  $c$ , respectively,  $\vec{q}^2 \approx -t$ , and the intermediate states  $e$  and  $f$  represent stable particles or narrow resonances. The accumulated wide experience of the work with the two-Reggeon cuts shows that reasonable estimates can be obtained considering the contributions of the simplest (lowest-lying) intermediate states. The calculation methods of the two-Reggeon cuts are well known (see, for example, Refs. [25–27,19,24]). Therefore, omitting details, we go at once to a discussion of the final results. All these are concerned with  $P_{\text{lab}}^{\pi^-} = 18 \text{ GeV}/c$ .

Begin with the  $a_2\pi$  cut. Taking into account in Eq. (27) the low-lying  $\eta n$  intermediate state, we get the following contributions of the  $a_2\pi$  cut to the  $(d\sigma/dt)_{t \approx 0}$  and integral cross section  $\sigma$  of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$ :

$$\left(\frac{d\sigma^{a_2\pi}}{dt}\right)_{t \approx 0} = \frac{I(m_\pi^2(\tilde{b}_{a_2} + \tilde{b}_\pi))}{4\pi|\tilde{b}_{a_2} + \tilde{b}_\pi|^2} \left(\frac{1}{t} \frac{d\sigma_{\text{hf}}^{a_2}}{dt}\right)_{t \approx 0} \left(\frac{m_\pi^4}{t} \frac{d\sigma^\pi}{dt}\right)_{t \approx 0} \approx 25 \text{ (nb/GeV}^2) B(a_0^0(980) \rightarrow \pi\eta), \quad (28)$$

$$\sigma^{a_2\pi} \approx 3.4 \text{ (nb)} B(a_0^0(980) \rightarrow \pi\eta). \quad (29)$$

Here  $\tilde{b}_R = b_R - i\pi\alpha'_R/2$  (the argument  $s$  of the slope  $b_R$  is omitted from this moment),  $d\sigma_{\text{hf}}^{a_2}/dt$  is the part of the  $\pi^- p \rightarrow \eta n$  differential cross section caused by the  $a_2$  Regge pole exchange with a helicity flip in the nucleon vertex, and  $d\sigma^\pi/dt$  is the differential cross section of the reaction  $\eta n \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  caused by the  $\pi$  Regge pole exchange. According the Fermilab data on  $\pi^- p \rightarrow \eta n$  [28],  $[(1/t)d\sigma_{\text{hf}}^{a_2}/dt]_{t \approx 0} \approx 555 \text{ } \mu\text{b/GeV}^4$ ,  $b_{a_2} \approx 4.18 \text{ GeV}^{-2}$ , and  $\alpha'_{a_2} \approx 0.8 \text{ GeV}^{-2}$  [ $\alpha_{a_2}(0) \approx 0.371$ ]. For the reaction with the  $\pi$  exchange,  $[(m_\pi^4/t)d\sigma^\pi/dt]_{t \approx 0} = g^2_{\pi NN}(g^2_{a_0\pi\eta}/16\pi)\exp(-2b_\pi m_\pi^2)B(a_0^0(980) \rightarrow \pi\eta)$ , where  $g^2_{a_0\pi\eta}/16\pi = \Gamma_{a_0\eta\pi} m_{a_0}/\rho_{\eta\pi}$  and  $\rho_{\eta\pi} = \{[1 - (m_\eta - m_\pi)^2/m_{a_0}^2][1 - (m_\eta + m_\pi)^2/m_{a_0}^2]\}^{1/2}$ . According the Particle Data Group [22], the width  $\Gamma_{a_0\eta\pi}$  can be from 50 to 300 MeV. We use its maximal value. Then,  $g^2_{a_0\pi\eta}/16\pi \approx 0.454 \text{ GeV}^2$ . Also we assume that  $\alpha'_\pi \approx 0.8 \text{ GeV}^{-2}$  and  $b_\pi \approx 3.5 \text{ GeV}^{-2}$ . The factor  $I(m_\pi^2(\tilde{b}_{a_2} + \tilde{b}_\pi))$  in Eq. (28) has the form  $|1 + z\exp(z)\text{Ei}(-z)|^2$ , where  $z = m_\pi^2(\tilde{b}_{a_2} + \tilde{b}_\pi)$  and  $\text{Ei}(-z)$  is the integral exponential function. Here we have  $I(m_\pi^2(\tilde{b}_{a_2} + \tilde{b}_\pi)) \approx 0.55$ .

Because  $B(a_0^0(980) \rightarrow \pi\eta) < 1$ , then Eqs. (28) and (29) give, respectively, less than 2.7% and 1.7% of the expected values  $[d\sigma/dt(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n)]_{t \approx 0} \approx 940 \text{ nb/GeV}^2$  and  $\sigma(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n) \approx 200 \text{ nb}$ .<sup>3</sup> Even though we magnify these numbers by an order of magnitude (attributing the enhancement to the contributions of the other

intermediate states), all the same they would be appreciably smaller than the expected values.

Turn to the  $\rho b_1$  cut. The contribution of the low-lying  $\pi^- p$  intermediate state is conveniently represented in the form

$$\begin{aligned} \left(\frac{d\sigma^{\rho b_1}}{dt}\right)_{t \approx 0} &= \frac{1}{4\pi|\tilde{b}_\rho + \tilde{b}_{b_1}|^4} \left(\frac{1}{t} \frac{d\sigma_{\text{hf}}^\rho}{dt}\right)_{t \approx 0} \left(\frac{1}{t} \frac{d\sigma^{b_1}}{dt}\right)_{t \approx 0} \\ &= \frac{4}{\pi} \frac{b_\rho^2 b_{b_1}^2}{|\tilde{b}_\rho + \tilde{b}_{b_1}|^4} \sigma_{\text{st}}^\rho \sigma^{b_1} \\ &< 0.5 \text{ nb/GeV}^2, \end{aligned} \quad (30)$$

where  $\sigma_{\text{hf}}^\rho$  is the  $\pi^- p \rightarrow \pi^- p$  cross section with the proton helicity flip caused by the  $\rho$  Regge pole exchange, and  $\sigma^{b_1}$  is the cross section of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  associated with the  $b_1$  Regge pole exchange. The limitation (30) has been obtained in terms of the following inequalities:  $\sigma_{\text{hf}}^\rho < \sigma(\pi^- p \rightarrow \pi^0 n)/2 \approx 12.5 \text{ } \mu\text{b}$  [29],  $\sigma^{b_1} < \sigma(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n) \approx 200 \text{ nb}$ , and  $b_\rho^2 b_{b_1}^2/|\tilde{b}_\rho + \tilde{b}_{b_1}|^4 < 1/16$ . Thus, the  $\rho b_1$  cut contribution should be considered as a whole as very small.

The  $\omega a_1$  cut is more difficult to estimate because there appear amplitudes with the  $a_1$  Regge pole exchange which are directly unobservable by experiment. Consider the contributions of the two simplest intermediate states  $\rho^- p$  and  $b_1^- p$ . At the expense of the  $b_1^- p$  intermediate state, we have

$$\begin{aligned} \left(\frac{d\sigma^{\omega a_1}}{dt}\right)_{t \approx 0} &\approx \frac{1}{4\pi|\tilde{b}_\omega + \tilde{b}_{a_1}|^2} \left(\frac{d\sigma^\omega}{dt}\right)_{t \approx 0} \left(\frac{d\sigma^{a_1}}{dt}\right)_{t \approx 0} \\ &\approx \frac{1}{\pi} \frac{b_\omega b_{a_1}}{|\tilde{b}_\omega + \tilde{b}_{a_1}|^2} \sigma^\omega \sigma^{a_1} \\ &< \frac{1}{4\pi} \sigma^\omega \sigma^{a_1} \approx 10 \text{ (nb/GeV}^2) \\ &\quad \times B(a_0^0(980) \rightarrow \pi\eta), \end{aligned} \quad (31)$$

where  $\sigma^\omega$  and  $\sigma^{a_1}$  are cross sections of the reactions  $\pi^- p \rightarrow b_1^- p$  and  $b_1^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  with  $\omega$  and  $a_1$  Regge pole exchanges, respectively (there is no helicity flip in the nucleon vertices and the intermediate  $b_1^-$  meson has in the main helicity zero [30]):  $\sigma^\omega \approx [\sigma(\pi^+ p \rightarrow b_1^+ p) + \sigma(\pi^- p \rightarrow b_1^- p) - \sigma(\pi^- p \rightarrow b_1^0 n)]/2 \approx 20 \text{ } \mu\text{b}$  [29–31]. To estimate  $\sigma^{a_1}$  one can be guided by virtually only the data on the reaction  $\pi^- p \rightarrow \rho^0 n$  at 17.2 GeV/c [32]. The exchanges with the  $a_1$  quantum numbers make up approximately 4% of this reaction cross section ( $\approx 20\%$  in the amplitude), i.e.,  $\approx 2.5 \text{ } \mu\text{b}$  [33]. To obtain the estimate (31), we have used a rough assumption  $\sigma^{a_1}(b_1^- p \rightarrow a_0^0 n) \approx \sigma^{a_1}(\pi^- p \rightarrow \rho^0 n)$ . Similarly, one can obtain for the contribution of the  $\rho^- p$  intermediate state with the transverse polarized  $\rho^-$  meson that  $(d\sigma^{\omega a_1}/dt)_{t \approx 0} < (\sigma^\omega \sigma^{a_1}/4\pi) \approx 7.5 \text{ (nb/GeV}^2) B(a_0^0(980) \rightarrow \pi\eta)$ , where  $\sigma^\omega \approx [\sigma(\pi^+ p \rightarrow \rho^+ p) + \sigma(\pi^- p \rightarrow \rho^- p) - \sigma(\pi^- p \rightarrow \rho^0 n)]/2 \approx 15 \text{ } \mu\text{b}$

<sup>3</sup>Note that the  $a_2\pi$  cut contribution to  $d\sigma/dt$  has a minimum around  $t \approx -0.4 \text{ GeV}^2$  and decreases by approximately 54 times over the range of  $t$  from 0 to  $-0.4 \text{ GeV}^2$ . However, experimentally  $d\sigma/dt$  falls by a factor of 6.5 in this  $t$  range and does not have a minimum [see Eq. (21)].

[29,34] and, for the  $\rho^- p \rightarrow a_0^0(980)n$  reaction cross section with the  $a_1$  exchange, we take simply the same  $2.5 \mu\text{b}$  as just above. The relative sign of the  $\rho^- p$  and  $b_1^- p$  intermediate state contributions is unknown. As a result for the  $\omega a_1$  cut, we have a very rough limitation

$$(d\sigma^{\omega a_1}/dt)_{t \approx 0} < 35 \text{ (nb/GeV}^2) B(a_0^0(980) \rightarrow \pi \eta). \quad (32)$$

As mentioned above, the contributions of the  $\rho\rho$  and  $a_2 a_2$  cuts to  $M_{++}$  vanish as  $t \rightarrow 0$ . However, the absorption corrections to these contributions, i.e., the  $\rho\rho P$  and  $a_2 a_2 P$  cuts, are finite as  $t \rightarrow 0$ .<sup>4</sup> The estimates of the  $\rho\rho P$  and  $a_2 a_2 P$  cut contributions to  $[d\sigma/dt(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n)]_{t \approx 0}$ , quite similar to as done above, show that each does not exceed by itself  $2 \text{ nb/GeV}^2$ , that is, very small. Moreover, a strong compensation between the  $\rho\rho$  and  $a_2 a_2$  cuts (and analogously between  $\rho\rho P$  and  $a_2 a_2 P$  cuts) takes place within the framework of the  $\rho$ - $a_2$  exchange degeneracy hypothesis because the productions of the Regge pole signature factors for these cuts are opposite in sign.

As for the  $P\rho_2$  cut, the absorption correction of this type accompanies any Regge pole exchange. In the cases that the pole contributions do not vanish as  $t \rightarrow 0$  (beyond general kinematic and factorization requirements), these corrections are effectively unimportant at least for the description of the differential cross sections. The reactions  $\pi^- p \rightarrow \pi^0 n$  and  $\pi^- p \rightarrow \eta n$  give classical examples of this situation. At small  $|t|$  and in a wide energy region, the differential cross sections of these reactions are described remarkably well by a simple Regge pole model with the linear  $\rho$  and  $a_2$  trajectories [28,35].

## V. CONCLUSION

We have considered the main dynamical mechanisms of the reaction  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  at high energies and shown that the observed peak in its differential cross section in the forward direction can be explained within the framework of the Regge pole model only by a ‘‘conspiracy’’ of the  $\rho_2$  trajectory with its daughter one. Notice that there is realized another type of conspiracy in the well-known cases of the reactions  $\gamma p \rightarrow \pi^+ n$  and  $p n \rightarrow n p$  (in which the corresponding peaks in the forward direction are usually described in terms of the Regge cuts [19,25,27]) than in the reaction  $\pi^- p \rightarrow a_0^0(980)n$  [12]. We have also obtained the estimates of the  $\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n$  reaction cross section at  $P_{\text{lab}}^{\pi^-} = 18 \text{ GeV}/c$  and of the OPE contribution which can be caused by the  $f_0^0(980)$ - $a_0^0(980)$  mixing. Examining the Regge cut contributions to the non-flip-helicity amplitude, we have come to the conclusion that they have to be inessential in comparison with the conspiring  $\rho_2$  Regge pole exchange.

Certainly, it would be very interesting to find some signs of the  $\rho_2$  state and its daughter state with  $b_1$  meson quantum numbers, for example, in the  $a_0 \pi$ ,  $\omega \pi$ , and  $A_2 \pi$  mass spec-

tra around  $1.7 \text{ GeV}$  in the reactions induced by  $\pi^{\mp}$  mesons or in  $\bar{N}N$  annihilation.

Let us note that there is the early estimate of the  $a_0(980)$  production cross section made in Ref. [36] which assumed  $b_1$  exchange dominance. Using the result of Ref. [36], one can obtain that  $\sigma(\pi^- p \rightarrow a_0^0(980)n \rightarrow \pi^0 \eta n)$  should be approximately equal to  $620 \text{ nb}$  at  $18 \text{ GeV}/c$ . Now, it is obvious that it is excluded. First, this large cross section contradicts the Brookhaven data on the  $a_0^0(980)$  and  $a_2^0(1320)$  production [7] and the data [5,21] on  $a_2^0(1320)$  production extrapolated to  $18 \text{ GeV}/c$ . Second, the pure  $b_1$  exchange mechanism contradicts the new Brookhaven data [7] on the  $t$  distribution for  $a_0^0(980)$  production in the low  $|t|$  region.

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## APPENDIX

Let us explain the conspiracy phenomenon by the example of the elementary  $\rho_2$  exchange in the reaction  $\pi(q_1) + a_0(q_2) \rightarrow \rho_2(q) \rightarrow \bar{N}(p_1) + N(p_2)$ . The effective Lagrangians for the  $\rho_2 \pi a_0$  and  $\rho_2 \bar{N}N$  interactions can be written as (we omit the coupling constants)

$$L(\rho_2 \pi a_0) = j_{\mu\nu}^M \rho_2^{\mu\nu}, \quad j_{\mu\nu}^M = Q_\mu Q_\nu, \quad (A1)$$

$$L(\rho_2 \bar{N}N) = j_{\mu\nu}^B \rho_2^{\mu\nu},$$

$$j_{\mu\nu}^B = \bar{u}(p_2) \gamma_5 \frac{1}{4} (\gamma_\mu P_\nu + \gamma_\nu P_\mu) v(p_1), \quad (A2)$$

where  $P = p_2 - p_1$ ,  $Q = q_1 - q_2$ , and  $q = q_1 + q_2 = p_1 + p_2$  ( $PQ = s - u$ ,  $Pq = 0$ ,  $Qq = m_\pi^2 - m_{a_0}^2$ , and  $q^2 = t$ ). The helicity amplitudes of the process  $\pi^- a_0^0 \rightarrow \rho_2^- \rightarrow \bar{p} n$  are then

$$F_{\lambda \bar{p} \lambda_n} = V_{\lambda \bar{p} \lambda_n}^{\mu\nu} \frac{\Pi_{\mu\nu\mu'\nu'}}{q^2 - m_{\rho_2}^2} Q^{\mu'} Q^{\nu'}, \quad (A3)$$

where

$$V_{\lambda \bar{p} \lambda_n}^{\mu\nu} = \bar{u}_{\lambda_n}(p_2) \gamma_5 \frac{1}{4} (\gamma^\mu P^\nu + \gamma^\nu P^\mu) v_{\lambda \bar{p}}(p_1), \quad (A4)$$

$$\Pi_{\mu\nu\mu'\nu'} = \frac{1}{2} \pi_{\mu\mu'} \pi_{\nu\nu'} + \frac{1}{2} \pi_{\mu\nu'} \pi_{\nu\mu'} - \frac{1}{3} \pi_{\mu\nu} \pi_{\mu'\nu'},$$

$$\pi_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / m_{\rho_2}^2. \quad (A5)$$

Off mass shell ( $q^2 \neq m_{\rho_2}^2$ ), the tensor  $\Pi_{\mu\nu\mu'\nu'}$  is not the spin-2 projection operator but contains the contributions of the lower (daughter) spins. In the case of coupling to non-conserved tensor currents, these daughter contributions appear in the physical amplitudes. In the case under consideration, the elementary  $\rho_2$  exchange with spin 2 is accompanied by the spin-1 contribution only [the spin-0 contribution is uncoupled to the  $\bar{N}N$  system because it has the

<sup>4</sup>Formulas for the such type cuts were obtained in Ref. [24].

exotic quantum numbers  $J^G(J^{PC}) = 1^+(0^{--})$ . Using the relations  $j_{\mu\nu}^B g^{\mu\nu} = 0$  and  $j_{\mu\nu}^B q^\mu q^\nu = 0$ , Eq. (A3) can be rewritten in the form

$$F_{\lambda_{\bar{p}}\lambda_n} = V_{\lambda_{\bar{p}}\lambda_n}^{\mu\nu} \frac{P_{\mu\nu\mu'\nu'}^{(2)} + [(q^2 - m_{\rho_2}^2)/m_{\rho_2}^2] P_{\mu\nu\mu'\nu'}^{(1)}}{q^2 - m_{\rho_2}^2} \times Q^{\mu'} Q^{\nu'}, \quad (\text{A6})$$

where the tensors

$$P_{\mu\nu\mu'\nu'}^{(2)} = \frac{1}{2} \theta_{\mu\mu'} \theta_{\nu\nu'} + \frac{1}{2} \theta_{\mu\nu'} \theta_{\nu\mu'} - \frac{1}{3} \theta_{\mu\nu} \theta_{-\mu'\nu'} \\ (\theta_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2), \quad (\text{A7})$$

and

$$P_{\mu\nu\mu'\nu'}^{(1)} = \frac{1}{2q^2} [\theta_{\mu\mu'} q_\nu q_{\nu'} + \theta_{\nu\nu'} q_\mu q_{\mu'} + \theta_{\mu\nu'} q_\nu q_{\mu'} \\ + \theta_{\nu\mu'} q_\mu q_{\nu'}] \quad (\text{A8})$$

are the spin-2 and spin-1 projection operators, respectively [37] [ $(P^{(2)})^2 = P^{(2)}$ ,  $(P^{(1)})^2 = P^{(1)}$ , and  $P^{(2)\mu\nu} = 5$ ,  $P^{(1)\mu\nu} = 3$ ]. The spin-2 and spin-1 parts of the elementary  $\rho_2$  exchanges give the contributions to the amplitudes  $F_{+-}$  and  $F_{++}$ , respectively. This immediately follows from the explicit form of the angular and threshold behaviors of these amplitudes:

$$F_{+-} = -8 |\vec{q}_t|^2 |\vec{p}_t|^2 \cos\theta_t \sin\theta_t \frac{1}{t - m_{\rho_2}^2} \\ \equiv - \sqrt{\frac{\Phi(s, t, u)}{t}} \frac{s - u}{t - m_{\rho_2}^2}, \quad (\text{A9}) \\ F_{++} = 4 |\vec{q}_t| |\vec{p}_t| \cos\theta_t \frac{m_N(m_{a_0}^2 - m_\pi^2)}{\sqrt{tm_{\rho_2}^2}}$$

$$\equiv \frac{(s - u)m_N(m_{a_0}^2 - m_\pi^2)}{\sqrt{tm_{\rho_2}^2}}. \quad (\text{A10})$$

Both amplitudes are singular as  $t \rightarrow 0$  as  $1/\sqrt{t}$ . Now we go from the amplitudes  $F_{+-}$  and  $F_{++}$  given by Eqs. (A9) and (A10) to the amplitudes  $G_{++}$  and  $G_{+-}$  [see Eq. (8)]. Substituting  $G_{++}$  and  $G_{+-}$  into Eq. (12), we see that, owing to the compensation between these helicity amplitudes having different quantum numbers, the  $1/t$  singularity in the invariant amplitude  $A$  is canceled out:

$$A = \frac{1}{t} [G_{++} - m_N(m_{a_0}^2 - m_\pi^2)G_{+-}] \\ = 4 |\vec{q}_t| |\vec{p}_t| \cos\theta_t \frac{m_N(m_{a_0}^2 - m_\pi^2)}{t} \left( \frac{1}{m_{\rho_2}^2} + \frac{1}{t - m_{\rho_2}^2} \right) \\ = \frac{s - u}{t - m_{\rho_2}^2} \frac{m_N(m_{a_0}^2 - m_\pi^2)}{m_{\rho_2}^2}. \quad (\text{A11})$$

It takes place automatically since the conspiracy condition

$$G_{++} = m_N(m_{a_0}^2 - m_\pi^2)G_{+-} \quad \text{at } t=0, \quad (\text{A12})$$

for the elementary  $\rho_2$  exchange, is exactly satisfied. The forward peak in  $d\sigma/dt$  is provided by the second invariant amplitude  $B = G_{+-} = -(s - u)/(t - m_{\rho_2}^2)$ , which, as seen, does not vanish at  $t=0$  [see Eqs. (4) and (5)]. As for the contribution of the amplitude  $A$  to  $d\sigma/dt$ , it is  $s$  times smaller at large  $s$  and therefore it can be neglected [see Eqs. (A11), (5), and (4)].

- [1] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 99 (1996).  
[2] R. H. Dalitz, in *The Quark Model*, edited by J. J. J. Kokkedee (W. A. Benjamin, Inc., New York, 1969), Appendix.  
[3] H. Burkhardt and A. J. G. Hey, Nucl. Phys. **B108**, 514 (1976).  
[4] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).  
[5] V. D. Appel *et al.*, Yad. Fiz. **41**, 126 (1985) [Sov. J. Nucl. Phys. **41**, 80 (1985)]; D. Alde *et al.*, Phys. Lett. B **205**, 397 (1988).  
[6] D. Alde *et al.*, Yad. Fiz. **59**, 1027 (1996) [Phys. At. Nucl. **59**, 982 (1996)]; S. Sadovskiy, in *Hadron '95*, Proceedings of the 6th International Conference on Hadron Spectroscopy, University of Manchester, 1995, edited by M.C. Birse, G. D. Lafferty, and J. A. McGovern (World Scientific, Singapore, 1996), p. 445.  
[7] A. R. Dzierba, in *Proceedings of the Second Workshop on Physics and Detectors for DAΦNE'95*, Frascati, 1995, edited

- by R. Baldini, F. Bossi, G. Capon, and G. Pancheri, Frascati Physics Series Vol. 4 (INFN, Frascati, 1996), p. 99.  
[8] A. C. Irving and C. Michael, Nucl. Phys. **B82**, 282 (1974); A. C. Irving, Phys. Lett. B **59**, 451 (1975).  
[9] A. C. Irving, Nucl. Phys. **B105**, 491 (1976).  
[10] M. H. Shaevitz *et al.*, Phys. Rev. Lett. **36**, 8 (1976).  
[11] L. Durand, Phys. Rev. Lett. **18**, 59 (1967).  
[12] L. Bertocchi, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, Heidelberg, 1967, edited by H. Filthuth (North-Holland, Amsterdam, 1968), p. 197.  
[13] B. Diu and M. Le Bellac, Nuovo Cimento A **53**, 158 (1968).  
[14] L. Leader, Phys. Rev. **166**, 1599 (1968).  
[15] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Phys. Lett. **88B**, 367 (1979); Yad. Fiz. **33**, 1337 (1981) [Sov. J. Nucl. Phys. **33**, 715 (1981)]; Usp. Fiz. Nauk **142**, 361 (1984) [Sov. Phys. Usp. **27**, 161 (1984)].  
[16] M. Goldberger *et al.*, Phys. Rev. **120**, 2250 (1960).



- [17] M. Jacob and G. C. Wick, *Ann. Phys. (N.Y.)* **7**, 404 (1959).
- [18] M. Gell-Mann *et al.*, *Phys. Rev.* **133**, B145 (1964).
- [19] P. D. B. Collins, *An Introduction to Regge Theory and High Energy Physics* (Cambridge University, Cambridge, England, 1977).
- [20] O. I. Dahl *et al.*, *Phys. Rev. Lett.* **38**, 54 (1977).
- [21] M. J. Corden *et al.*, *Nucl. Phys.* **B138**, 235 (1978).
- [22] Particle Data Group, L. Montanet *et al.*, *Phys. Rev. D* **50**, 1197 (1994); **50**, 1478 (1994); **50**, 1478 (1994).
- [23] V. N. Gribov, *Zh. Eksp. Teor. Fiz.* **53**, 654 (1967) [*Sov. Phys. JETP* **26**, 414 (1968)].
- [24] N. N. Achasov, A. A. Kozhevnikov, and G. N. Shestakov, *Yad. Fiz.* **31**, 468 (1980) [*Sov. J. Nucl. Phys.* **31**, 244 (1980)]; **25**, 1058 (1978).
- [25] A. B. Kaidalov and B. M. Karnakov, *Phys. Lett.* **29B**, 372 (1969); **29B**, 376 (1969); *Yad. Fiz.* **11**, 216 (1970) [*Sov. J. Nucl. Phys.* **11**, 121 (1970)].
- [26] C. Michael, *Phys. Lett.* **29B**, 230 (1969); *Nucl. Phys.* **B13**, 644 (1969).
- [27] F. Henyey *et al.*, *Phys. Rev.* **182**, 1579 (1969); G. L. Kane *et al.*, *Phys. Rev. Lett.* **25**, 1519 (1970); R. A. Miller, *Phys. Rev. D* **2**, 598 (1970).
- [28] O. I. Dahl *et al.*, *Phys. Rev. Lett.* **37**, 80 (1976).
- [29] V. Flamino *et al.*, *Compilation of Cross-Section* (CERN, Geneva, 1983).
- [30] A. C. Irving and V. Chaloupka, *Nucl. Phys.* **B89**, 345 (1975).
- [31] D. Alde *et al.*, *Z. Phys. C* **54**, 553 (1992).
- [32] G. Grauer *et al.*, *Nucl. Phys.* **B75**, 189 (1974); H. Becker *et al.*, *ibid.* **B150**, 301 (1979); **B151**, 46 (1979).
- [33] J. D. Kimel and J. F. Owens, *Nucl. Phys.* **B122**, 464 (1977).
- [34] H. A. Gordon *et al.*, *Phys. Rev. D* **8**, 779 (1973).
- [35] A. V. Barnes *et al.*, *Phys. Rev. Lett.* **37**, 76 (1976).
- [36] A. C. Irving, *Phys. Lett.* **70B**, 217 (1977).
- [37] K. J. Barnes, *J. Math. Phys. (N.Y.)* **6**, 788 (1965).