Production of scalar $K\overline{K}$ molecules in ϕ radiative decays

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The potentialities of the production of the scalar $K\overline{K}$ molecules in ϕ radiative decays are considered beyond the narrow resonance width approximation. It is shown that $B(\phi\to\gamma f_0(a_0)\to\gamma\pi\pi(\pi\eta))\approx(1-2)\times10^{-5}$, $B(\phi\to\gamma(f_0+a_0)\to\gamma K^+K^-)\lesssim10^{-6}$, and $B(\phi\to\gamma(f_0+a_0)\to\gamma K_SK_S)<5\times10^{-8}$. The mass spectra in the $\pi\pi$, $\pi\eta$, K^+K^- channels are calculated. The imaginary part of the amplitude $\phi\to\gamma f_0(a_0)$ is calculated analytically. The phase of the scalar resonance production amplitude that causes the interference patterns in the reaction $e^+e^-\to\gamma\pi^+\pi^-$ in the ϕ meson mass region is obtained. [S0556-2821(97)04111-8]

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I. INTRODUCTION

The problem of the scalar $f_0(980)$ and $a_0(980)$ mesons has been a central problem of hadron spectroscopy up to the charm quark. The point is that these states possess peculiar properties from the point of view of the naive $(q\overline{q})$ model; see, e.g., the reviews in [1–5]. In time, all their challenging properties have been understood [2,5,6] in the framework of the four-quark $(q^2\overline{q}^2)$ MIT bag model [7,8]. Along with the $q^2\overline{q}^2$ nature of the $a_0(980)$ and $f_0(980)$ mesons the possibility of them being $K\overline{K}$ molecules is discussed [9–12]. Furthermore, probably, the $f_0(980)$ and $a_0(980)$ mesons are witnesses of confinement [13].

Through the efforts of theorists over the years, it has been established [14,15] (see also references quoted in [15]) that research into the decays $\phi\to\gamma f_0\to\gamma\pi\pi$ and $\phi\to\gamma a_0\to\gamma\eta\pi$ could play a crucial role in the elucidation of the nature of the scalar $f_0(980)$ and $a_0(980)$ mesons.

At present, the investigation of the $\phi\to\gamma f_0\to\gamma\pi\pi$ decay by the detector CMD-2 is being carried out at an upgraded e^+e^- collider VEPP-2M in Novosibirsk. Another detector, SND, aiming to study the decays under consideration, has been put into operation at the same facility. Moreover, in the immediate future at Frascati the start is expected of the operation of the ϕ factory DAΦNE which, probably, makes possible studying the scalar $f_0(980)$ and $a_0(980)$ mesons in an exhaustive manner.

It seems clear that the definition of theoretical predictions prior to securing data is a natural prerequisite to clear up the mystery of the scalar mesons.

In all theoretical papers (see [15] and references quoted there) except for [14], where the $q^2\overline{q}^2$ nature of the scalar mesons was considered, the approximation of narrow widths of the $f_0(980)$ and $a_0(980)$ mesons was used, for their visible widths are 25–50 MeV. Moreover, experimenters used this approximation to give upper limits to branching ratios of the decays $\phi\to\gamma a_0$ and $\phi\to\gamma f_0$ [16].

But recently [17,18] it was shown that the narrow resonance approximation in this instance is not valid and predictions of decay branching ratios gained in the narrow resonance approximation are at least 2 times overstated.

In this connection, we study the scope for the production of scalar molecules in the ϕ radiative decays beyond the narrow resonance width approximation.

In Sec. II we introduce the formulas needed for our investigation and discuss corrections for the finite widths to the propagators of scalar mesons.

Section III is devoted to the model of the $\phi\to\gamma a_0(f_0)$ transition amplitude. In Sec. III A we state (more precisely, make clear) the model of the production of the scalar $K\overline{K}$ molecules in the ϕ radiative decays. In Sec. III B we calculate in analytical form the imaginary part of the $\phi\to\gamma a_0(f_0)$ transition amplitude that gives near 90% of the branching ratios in the $\pi\pi$ and $\eta\pi$ channels. In Sec. III C the real part of the $\phi\to\gamma a_0(f_0)$ transition amplitude, dominating in the $K\overline{K}$ channels, is derived partly in analytic form, partly in an integral form suitable for the simulation of the experimental data.

In Sec. IV we give the numerical results of our analysis. We found that in the case of the molecule nature of the $f_0(980)$ and $a_0(980)$ resonances $B(\phi\to\gamma f_0(a_0)\ \to\gamma\pi\pi(\pi\eta))\approx(1-2)\times10^{-5}$, $B(\phi\to\gamma(f_0+a_0)\to\gamma K^+K^-)\lesssim10^{-6}$, and in a Pickwickian sense $B(\phi\to\gamma(f_0+a_0)\to\gamma K_SK_S)<5\times10^{-8}$. The mass spectra for the $\pi\pi$, $\pi\eta$, and K^+K^- are presented. The phase of the scalar resonance production amplitude, causing the interference patterns in the reaction $e^+e^-\to\gamma\pi^+\pi^-$ in the ϕ-meson mass region, is calculated.

Sections V and VI review our results and discuss experimental perspectives.

II. FUNDAMENTAL PHENOMENOLOGY OF ϕ RADIATIVE DECAYS INTO SCALAR MESONS

Let us introduce the amplitudes

$$M(\phi\to\gamma R;m)=g_R(m)[\vec{e}(\phi)\cdot\vec{e}(\gamma)],\quad R=a_0,f_0,\quad(1)$$

where $\vec{e}(\phi)$ and $\vec{e}(\gamma)$ are polarization three-vectors of the ϕ meson and the γ quantum in the ϕ-meson rest frame, and m is an invariant mass of two pseudoscalar mesons a and b produced in the $R\to ab$ decay.

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In e^+e^- collisions the ϕ meson is produced transversely polarized relative to the beam axis in the center-of-mass system. That is why the amplitudes from Eq. (1) lead to the angular distribution

$$W(\theta)=\tfrac{3}{8}\,(1+\cos^2\,\theta) \tag{2}$$

if one is not interested in a polarization of the γ quantum in the reaction $e^+e^-\to\phi\to\gamma R$. In Eq. (2), θ is the angle between the momentum of the γ quantum and the axis of the beams.

According to the gauge invariance condition, the decay amplitudes are proportional to the electromagnetic (electric) field, that is the photon energy (ω) in the low energy region,

$$g_R(m)\to\omega\times\text{const}, \tag{3}$$

if $m\to m_\phi$ and $\omega=(1/2)m_\phi(1-m^2/m_\phi^2)\to 0$.

The decay width in narrow scalar resonance width approximation is

$$\Gamma(\phi\to\gamma R;m)=\frac{1}{3}\,\frac{|g_R(m)|^2}{4\pi}\,\frac{1}{2m_\phi}\left(1-\frac{m^2}{m_\phi^2}\right). \tag{4}$$

The physically measured partial widths are

$$\Gamma(\phi\to\gamma R\to\gamma ab)=\frac{2}{\pi}\int_{m_a+m_b}^{m_\phi} m\,dm\;\frac{m\Gamma(R\to ab;m)\Gamma(\phi\to\gamma R;m)}{|D_R(m)|^2} \tag{5}$$

for $ab=\pi\pi,\pi^0\eta$.

For $ab=K^+K^-,K^0\overline{K^0}$,

$$\Gamma(\phi\to\gamma(a_0+f_0)\to\gamma K^+K^-)=\frac{2}{\pi}\int_{2m_{K^+}}^{m_\phi} m^2\Gamma(f_0\to K^+K^-;m)\Gamma(\phi\to\gamma f_0;m)\left|\frac{1}{D_{f_0}(m)}+\frac{g_{a_0}(m)g_{a_0K^+K^-}}{g_{f_0}(m)g_{f_0K^+K^-}}\,\frac{1}{D_{a_0}(m)}\right|^2 dm,$$

$$\Gamma(\phi\to\gamma(a_0+f_0)\to\gamma K^0\overline{K^0})=\frac{2}{\pi}\int_{2m_{K^0}}^{m_\phi} m^2\Gamma(f_0\to K^0\overline{K^0};m)\Gamma(\phi\to\gamma f_0;m)\left|\frac{1}{D_{f_0}(m)}+\frac{g_{a_0}(m)g_{a_0K^0\overline{K^0}}}{g_{f_0}(m)g_{f_0K^0\overline{K^0}}}\,\frac{1}{D_{a_0}(m)}\right|^2 dm, \tag{6}$$

where $1/D_R(m)$ is the propagator of a scalar meson.

The width of the decay of the scalar meson R into two pseudoscalar meson state ab with an invariant mass m is

$$\Gamma(R\to ab;m)=\frac{g_{Rab}^2}{16\pi}\,\frac{1}{m}\;\rho_{ab}(m),$$

$$\rho_{ab}(m)=\sqrt{(1-m_+^2/m^2)(1-m_-^2/m^2)},\quad m_\pm=m_a\pm m_b\,. \tag{7}$$

The final particle identity in the $\pi^0\pi^0$ case is taken into account in the determination of $g_{f_0\pi^0\pi^0}$.

In Eq. (6) it is good to bear in mind isotopic symmetry

$$g_{f_0K^+K^-}=g_{f_0K^0\overline{K^0}},\quad g_{a_0K^+K^-}=-g_{a_0K^0\overline{K^0}}. \tag{8}$$

The use of a narrow scalar resonance width approximation in the case under consideration is not valid for two reasons [17,18].

The first and major reason is connected with soft (according to strong interaction standards) photons. From Eqs. (3), (4), (5), and (6) it follows that the right slope of the resonance is suppressed by at least the factor $(\omega/\omega_0)^3$, where $\omega_0=m_\phi(1-M_R^2/m_\phi^2)/2$ and M_R is the resonance mass. As was shown [18] it leads to the suppressions of the integral contribution from the right slope of the resonance by at least the factor of 5 for decays into $\pi\pi$ and $\pi\eta$ channels and by at least the factor of 50 for decays into the K^+K^- and $K^0\overline{K^0}$ channels. So the physically measured widths, Eq. (5), are caused in practice fully by ‘‘a half’’ of resonance, that is, by its left slope in the channels $\pi\pi$ and $\pi\eta$. Only for this reason are the results $[\Gamma(\phi\to\gamma R;M_R)]$ obtained in the narrow width approximation overstated 2 times.

The second reason is connected with a finite width correction in the propagators of the scalar mesons. Let us take the generally accepted Breit-Wigner formulas.

If $m>2m_{K^+},\ 2m_{K^0}$,

$$\frac{1}{D_R(m)}=\frac{1}{m_R^2-m^2-i[\Gamma_0(m)+\Gamma_{K\overline{K}}(m)]m},$$

$$\Gamma_{K\overline{K}}(m)=\frac{g_{RK^+K^-}^2}{16\pi}\;\left(\sqrt{1-4m_{K^+}^2/m^2}+\sqrt{1-4m_{K^0}^2/m^2}\right)\;\frac{1}{m}. \tag{9}$$

If $2m_{K^+}<m<2m_{K^0}$,

$$\frac{1}{D_R(m)}=\frac{1}{m_R^2-m^2+[(g_{RK^+K^-}^2)/16\pi]\sqrt{4m_{K^0}^2/m^2-1}-i\;[(g_{RK^+K^-}^2)/16\pi]\sqrt{1-4m_{K^+}^2/m^2}-i\Gamma_0(m)m}. \tag{10}$$

When $2m_{K^+},\ 2m_{K^0}>m$,

$$\frac{1}{D_R(m)}=\frac{1}{m_R^2-m^2+[(g_{RK^+K^-}^2)/16\pi](\sqrt{4m_{K^+}^2/m^2-1}+\sqrt{4m_{K^0}^2/m^2-1})-i\Gamma_0(m)m}, \tag{11}$$

where the width of the decay of the scalar R resonance into the $\pi\eta$ or $\pi\pi$ channels $\Gamma_0(m)$ is determined by Eq. (7).

Since the scalar resonances lie under the $K\bar{K}$ thresholds, the position of the peak in the cross section or in the mass spectrum does not coincide with m_R as is easy to see using Eqs. (9)–(11). That is why it is necessary to renormalize the mass in the Breit-Wigner formulas, Eqs. (9)–(11),

$$m_R^2=M_R^2-\frac{g_{RK^+K^-}^2}{16\pi}\left(\sqrt{4m_{K^+}^2/M_R^2-1}+\sqrt{4m_{K^0}^2/M_R^2-1}\right), \tag{12}$$

where M_R^2 is the physical mass squared ($M_{a_0}=980$ MeV and $M_{f_0}=980$ MeV) while m_R^2 is the bare mass squared. So the physical mass is heavier than the bare one. This circumstance is especially important in the case of a strong coupling of the scalar mesons with the $K\bar{K}$ channels as in the four-quark and molecule models. Meanwhile it was not taken into account both in the fitting of the data and theoretical works, except for Refs. [2, 14, 19]. It should be noted that Eqs. (9)–(11) are applied only in the resonance region. They, for example, have incorrect analytic properties at $m^2=0$. The expressions, in which this defect is removed, can be found in [2,14,19].

Notice that when the scalar resonances lie between the K^+K^- and $K^0\bar{K}^0$ thresholds it is necessary to renormalize the mass in the Breit-Wigner formulas in the following manner:

$$m_R^2=M_R^2-\frac{g_{RK^+K^-}^2}{16\pi}\sqrt{4m_{K^0}^2/M_R^2-1}. \tag{13}$$

The coupling constants in the molecule model [11,15,20] are

$$g_{f_0K^+K^-}^2/4\pi=g_{a_0K^+K^-}^2/4\pi=0.6\ \text{GeV}^2. \tag{14}$$

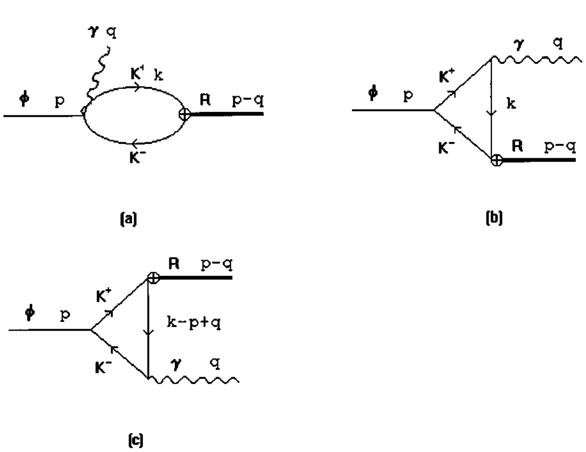

FIG. 1. Diagrams for the model.

Notice that in this model $M_R-m_R=24(10)$ MeV for $M_R=980(2m_{K^+})$ MeV.

III. MODEL OF $K\bar{K}$ SCALAR MOLECULE PRODUCTION IN ϕ RADIATIVE DECAYS

A. $K\bar{K}$ loop production of extended scalar mesons

Unfortunately, at present it is not possible to construct a truly relativistic gauge-invariant model in the case of $K\bar{K}$ scalar molecule production in ϕ radiative decays for the nonrelativistic nature of a wave function of a molecule. But it is possible to ''relativize'' a model constructed in the $K\bar{K}$ molecule rest frame and to place a gauge invariance constraint.

In the rest frame of a scalar $K\bar{K}$ molecule we consider the production mechanism described by the diagrams in Fig. 1, where the cross in the $K^+K^-f_0(a_0)$ vertex points to a coupling of the K^+K^- state with a extended meson.

In the scalar molecule rest frame $\vec{p}=\vec{q}$, the decay amplitude is

$$T(p,q)=M(p,q)-M(p,0),$$
$$M(p,q)=M_1(p,q)+M_2(p,q)+M_3(p,q), \tag{15}$$

where p and q are four-momenta of the ϕ meson and the photon, respectively. The amplitudes

$$M_1(p,q)=ieg_{RK^+K^-}g_{\phi K^+K^-}\epsilon^\mu(\phi)\epsilon^\nu(\gamma)\int\frac{d^4k}{(2\pi)^4}\times\phi(|\vec{k}|)\frac{2g_{\mu\nu}}{D(k)D(k-p+q)}, \tag{16}$$

$$M_2(p,q)=-ieg_{RK^+K^-}g_{\phi K^+K^-}\epsilon^\mu(\phi)\epsilon^\nu(\gamma)\int\frac{d^4k}{(2\pi)^4}\times\phi(|\vec{k}|)\frac{(2k+q)_\nu(2k+2q-p)_\mu}{D(k)D(k-p+q)D(k+q)}, \tag{17}$$

and

$$M_3(p,q)=-ieg_{RK^+K^-}g_{\phi K^+K^-}\times\epsilon^\mu(\phi)\epsilon^\nu(\gamma)\int\frac{d^4k}{(2\pi)^4}\times\phi(|\vec{k}|)\frac{(2k-2p+q)_\nu(2k-p)_\mu}{D(k)D(k-p+q)D(k-p)} \tag{18}$$

correspond to the diagrams in Figs. 1(a), 1(b), and 1(c), respectively. The function $\phi(|\vec{k}|)$ describes a momentum distribution of a $K^+(K^-)$ meson in a molecule, $D(k)=k^2-m^2+i0$, $\epsilon^\mu(\phi)$ and $\epsilon^\nu(\gamma)$ are polarization four-vectors of the ϕ meson and the γ quantum, $e^2/4\pi=\alpha\simeq1/137$, $g_{RK^+K^-}$ is determined by Eq. (7), and $g_{\phi K^+K^-}$ is determined by the following manner:

$$\Gamma(\phi\to K^+K^-)=\frac{1}{3}\frac{g_{\phi K^+K^-}^2}{16\pi}m_\phi\left(\sqrt{1-\frac{4m_{K^+}^2}{m_\phi^2}}\right)^3. \tag{19}$$

The subtraction at $q=0$ is a gauge-invariant regularization that always is necessary in the field theory.

The ''relativization''

$$T(p,q)=[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]g_R(m)$$
$$\equiv g_R(m)\{[\epsilon(\phi)\cdot\epsilon(\gamma)] -[\epsilon(\phi)\cdot q][\epsilon(\gamma)\cdot p]/(p\cdot q)\}|_{\vec{p}=\vec{q}} \tag{20}$$

gives the relativistic and gauge invariant amplitudes. We emphasize that in Eq. (20) $\vec{\epsilon}(\phi)$ and $\vec{\epsilon}(\gamma)$ are polarization three-vectors of the ϕ meson and the γ quantum in the molecule rest frame.

When $\phi(|\vec{k}|)=1$, Eqs. (15)–(18) reproduce well the definite gauge-invariant field theory expression[1] [14].

Our model is practically the same one as in [15]. Nevertheless, it should be noted that in Eqs. (4.21)–(4.23) of Ref. [15] the function $\phi(|\vec{k}|)$ is used to describe a momentum distribution of a $K^+(K^-)$ meson in a molecule whereas a $K^+(K^-)$ meson has the three-momentum equal to $\pm(\vec{k}-\vec{q}/2)$. So Eqs. (4.21)–(4.23) of [15] are not quite correct and our Eqs. (15)–(18) make clear the molecule production model.[2]

Let us integrate Eqs. (16)–(18) over k_0 in the molecule rest frame:

$$M_1(p;q)=-2eg_{RK^+K^-}g_{\phi K^+K^-}[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\int\frac{d^3k}{(2\pi)^3}\,\phi(|\vec{k}|)\,\frac{1}{E_k(m^2-4E_k^2)}, \tag{21}$$

$$M_2(p;q)=eg_{RK^+K^-}g_{\phi K^+K^-}[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\int\frac{d^3k}{(2\pi)^3}\,\phi(|\vec{k}|)\left(\vec{k}^2-\frac{(\vec{k}\vec{q})^2}{\vec{q}^2}\right)\left(\frac{1}{E_k(m^2+2E_km)(2\omega E_k+2\vec{k}\vec{q})}\right.$$
$$\left.+\frac{1}{E_k(m^2-2E_km)(2p_0E_k-m_\phi^2+2\vec{k}\vec{q})}-\frac{1}{E_{k+q}(2\omega^2+2\omega E_{k+q}+2\vec{k}\vec{q})(p_0^2+\omega^2+2p_0E_{k+q}+2\vec{k}\vec{q})}\right), \tag{22}$$

$$M_3(p;q)=-eg_{RK^+K^-}g_{\phi K^+K^-}[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\int\frac{d^3k}{(2\pi)^3}\,\phi(|\vec{k}|)\left(\vec{k}^2-\frac{(\vec{k}\vec{p})^2}{\vec{p}^2}\right)\left(\frac{1}{E_k(m^2+2E_km)(2p_0E_k+m_\phi^2+2\vec{k}\vec{p})}\right.$$
$$\left.+\frac{1}{E_k(m^2-2E_km)(2\omega E_k+2\vec{k}\vec{p})}+\frac{1}{E_{k-q}(\omega^2+\vec{p}^2-2\omega E_{k-q}-2\vec{k}\vec{p})(p_0^2+\vec{p}^2-2p_0E_{k-q}-2\vec{k}\vec{p})}\right), \tag{23}$$

where

$$\vec{p}=\vec{q},\quad p_0-q_0=m,\quad q_0=|\vec{q}|=\omega=(m_\phi^2-m^2)/2m,\quad E_k=\sqrt{\vec{k}^2+m_{K^+}^2-i0},$$
$$E_{k+q}=\sqrt{(\vec{k}+\vec{q})^2+m_{K^+}^2-i0},\quad E_{k-q}=\sqrt{(\vec{k}-\vec{q})^2+m_{K^+}^2-i0}. \tag{24}$$

It is easily seen that the integration over the angle between $\vec{k}$ and $\vec{q}$ (or $\vec{p}$) is performed analytically.

B. Imaginary part of the amplitude of $\phi\to\gamma f_0(a_0)$ decay

Following [15,20] we use

$$\phi(|\vec{k}|)=\frac{\mu^4}{(|\vec{k}|^2+\mu^2)^2}, \tag{25}$$

where $\mu=141$ MeV.

The function $\phi(|\vec{k}|)$ suppresses the contribution of virtual K^+K^- pairs. That is why it is natural to expect that the imaginary part of the $\phi\to\gamma f_0(a_0)$ amplitude is essential in comparison with the real one. The numerical analysis in Sec. IV supports this hope.

So the imaginary part of the $\phi\to\gamma f_0(a_0)$ amplitude,

$$\mathrm{Im}T(p,q)=\mathrm{Im}[M(p,q)-M(p,0)]=[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\mathrm{Im}g_R(m)=[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\mathrm{Im}[\overline{g_R}(m)-\overline{g_R}(m_\phi)], \tag{26}$$

merits individual consideration here. Equation (25) makes possible calculating the imaginary part in analytical form. This calculation is too cumbersome to be presented here. Because of this, we shall only explain the genesis of the imaginary part of the amplitude and shall adduce its analytical form.

The contributors of the imaginary part of the $\phi\to\gamma f_0(a_0)$ amplitude are the regions of the integration where the denominators in Eqs. (21)–(23) vanish. We treat these zeros using an infinitesimal negative imaginary addition to the K^+ mass square $(m_{K^+}^2-i0)$; see Eq. (24).

Our result is

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[1]The resulting at $\phi(|\vec{k}|)=1$ amplitude differs by sign from the respective one in [14].

[2]In contrast to [15] we treat with an amplitude rather than a S-matrix element. The difference is the factor i. In addition, note that in Eq. (4.24) of [15] the sign $(-)$ is lost. Probably, it is a misprint; otherwise, the gauge invariance constraint [see Eq. (3)], is destroyed.

$$\mathrm{Im}g_R(m)=\pi e g_{RK^+K^-}g_{\phi K^+K^-}\frac{\mu^4}{(2\pi)^2}\frac{1}{(m^2-4a^2)^2}\left\{\frac{m_\phi^2}{\omega^3}\left(\ln\frac{(E_1-a)(E_2+a)}{(E_2-a)(E_1+a)}\frac{E_1E_2m^2(12a^2-m^2)-a^2m_\phi^2(m^2+4a^2)}{4a^3m^2}\right)\right.$$
$$+\frac{4m_{K^+}^2}{m\omega}\ln\frac{E_1^2-a^2}{E_2^2-a^2}-\frac{8m_{K^+}^2}{m\omega}\ln\frac{m_\phi-\sqrt{m_\phi^2-4m_{K^+}^2}}{m_\phi+\sqrt{m_\phi^2-4m_{K^+}^2}}+\frac{m_\phi^2(m^2-4a^2)(E_1-E_2)}{2a^2\omega^3}-\frac{32(m_\phi^2-4m_{K^+}^2)^{3/2}(m^2-4a^2)^2}{3m_\phi(m_\phi^2-4a^2)^3}$$
$$\left.+\theta(m-2m_{K^+})\frac{4\sqrt{m^2-4m_{K^+}^2}m_\phi^2}{\omega m^2}+\frac{8m_{K^+}^2}{\omega m}\ln\frac{m-\sqrt{m^2-4m_{K^+}^2}}{m+\sqrt{m^2-4m_{K^+}^2}}\right\},\tag{27}$$

where $a^2=m_{K^+}^2-\mu^2$, $E_1=p_0/2-(\omega/2m_\phi)\sqrt{m_\phi^2-4m_{K^+}^2}$, $E_2=p_0/2+(\omega/2m_\phi)\sqrt{m_\phi^2-4m_{K^+}^2}$ and the step function $\theta(x)$ is

$$\theta(x)=\begin{cases}1, & x>0,\\ 0, & x<0.\end{cases}$$

Notice that

$$\mathrm{Im}\overline{g_R}(m_\phi)=e g_{RK^+K^-}g_{\phi K^+K^-}\frac{8}{3\pi}\frac{\mu^4(m_\phi^2-4m_{K^+}^2)}{(m_\phi^2-4a^2)^3}\sqrt{1-\frac{4m_{K^+}^2}{m_\phi^2}}.\tag{28}$$

C. Real part of amplitude of $\phi\to\gamma f_0(a_0)$ decay

As will be seen from Sec. IV the real part of the $\phi\to\gamma q f_0(a_0)$ amplitude,

$$\mathrm{Re}\ T(p,q)=\mathrm{Re}[M(p,q)-M(p,0)]=[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\mathrm{Re}g_R(m)=[\vec{\epsilon}(\phi)\cdot\vec{\epsilon}(\gamma)]\mathrm{Re}[\overline{g_R}(m)-\overline{g_R}(m_\phi)],\tag{29}$$

dominates in the suppressed K^+K^- and $K^0\overline{K^0}$ channels.

Our result is

$$\mathrm{Re}\ g_R(m)=e g_{RK^+K^-}g_{\phi K^+K^-}\frac{\mu^4}{(2\pi)^2}\left(\frac{8m_\phi^2}{\omega m(m^2-4a^2)^2}\left\{\frac{\mu}{a}\arctan\frac{a}{\mu}+\left(\arctan\frac{a}{\mu}\right)^2\right.\right.$$
$$-\theta(2m_{K^+}-m)\left[\frac{\sqrt{4m_{K^+}^2-m^2}}{m}\arctan\frac{m}{\sqrt{4m_{K^+}^2-m^2}}+\left(\arctan\frac{m}{\sqrt{4m_{K^+}^2-m^2}}\right)^2\right]$$
$$\left.-\theta(m-2m_{K^+})\left[\frac{\sqrt{m^2-4m_{K^+}^2}}{2m}\ln\frac{m+\sqrt{m^2-4m_{K^+}^2}}{m-\sqrt{m^2-4m_{K^+}^2}}-\frac{1}{4}\ln^2\frac{m+\sqrt{m^2-4m_{K^+}^2}}{m-\sqrt{m^2-4m_{K^+}^2}}+\frac{\pi^2}{4}\right]\right\}$$
$$+\frac{1}{(m^2-4a^2)a^2}\left(2+\frac{m}{2\omega}+\frac{p_0(m^2-4a^2)}{2\omega^2m}\right)\left(1-\frac{m_{K^+}^2}{\mu a}\arctan\frac{a}{\mu}\right)+\frac{2}{\omega m(m^2-4a^2)}$$
$$-\theta(m-2m_{K^+})\frac{m_{K^+}^2}{2\omega m}\left(\frac{4}{(m^2-4a^2)^2}\left[\frac{m}{2a}\ln\frac{m_{K^+}-a}{m_{K^+}+a}+\ln\frac{4\mu^2}{(2m_{K^+}^2-m)^2}\right]\right.$$
$$\left.-\frac{1}{2(m^2-4a^2)}\left[\frac{m}{2a^3}\ln\frac{m_{K^+}-a}{m_{K^+}+a}+\frac{mm_{K^+}+2a^2}{a^2\mu^2}\right]\right)\ln\frac{m+\sqrt{m^2-4m_{K^+}^2}}{m-\sqrt{m^2-4m_{K^+}^2}}$$
$$+\int_0^\infty\frac{|\vec{k}|d|\vec{k}|}{(\vec{k}^2+\mu^2)^2 2\omega^3}\left(E_+-E_-+\frac{\omega^2m_{K^+}^2}{E_km(-2E_k)}\ln\frac{(E_k+\omega+E_-)(E_k+\omega-E_+)}{(E_k+\omega+E_+)(E_k+\omega-E_-)}\right.$$
$$-\frac{\omega^2m_{K^+}^2}{E_km(2E_k+m)}\ln\frac{(E_--E_k+\omega)(E_k-\omega+E_+)}{(E_+-E_k+\omega)(E_-+E_k-\omega)}+\frac{m_\phi^2(E_k+E_1)(E_k+E_2)}{E_km(2E_k+m)}$$
$$\times\ln\frac{(E_k+p_0+E_-)^2}{(E_k+p_0+E_+)^2}+\frac{m_\phi^2(E_k-E_1)(E_k-E_2)}{E_km(2E_k-m)}\ln\frac{(E_k-p_0+E_+)^2}{(E_k-p_0+E_-)^2}$$

$$+\theta(m-2m_{K^+})\frac{m_{K^+}^2\omega^2}{E_k(m^2-2E_km)}\ln\frac{m-\sqrt{m^2-4m_{K^+}^2}}{m+\sqrt{m^2-4m_{K^+}^2}}\Bigg)\Bigg)-\mathrm{Re}\,\overline{g_R}(m_\phi), \tag{30}$$

where $E_+=\sqrt{\vec{k}^2+\omega^2+2|\vec{k}|\omega+m_{K^+}^2}$, $E_-=\sqrt{\vec{k}^2+\omega^2-2|\vec{k}|\omega+m_{K^+}^2}$, and

$$\mathrm{Re}\,\overline{g_R}(m_\phi)=eg_{RK^+K^-}g_{\phi K^+K^-}\frac{\mu^4}{(2\pi)^2}\Bigg\{\Bigg[\frac{\mu}{a}\arctan\frac{a}{\mu}-\frac{\sqrt{m_\phi^2-4m_{K^+}^2}}{2m_\phi}\ln\frac{m_\phi+\sqrt{m_\phi^2-4m_{K^+}^2}}{m_\phi-\sqrt{m_\phi^2-4m_{K^+}^2}}\Bigg]$$

$$\times\left(\frac{64(m_\phi^4-3m_\phi^2m_{K^+}^2-4m_{K^+}^2a^2)}{3m_\phi^2(m_\phi^2-4a^2)^3}\right)-\arctan\left(\frac{a}{\mu}\right)\left(\frac{2m_{K^+}^2}{\mu a^3(m_\phi^2-4a^2)}+\frac{8\mu m_{K^+}^2}{3a^3m_\phi^2(m_\phi^2-4a^2)}\right.$$

$$\left.-\frac{16m_{K^+}^2\mu}{3a^3(m_\phi^2-4a^2)^2}+\frac{2m_{K^+}^2\mu}{m_\phi^2a^5}\right)+\frac{2(5m_\phi^2m_{K^+}^2-20m_{K^+}^4-2m_\phi^2\mu^2+12m_{K^+}^2\mu^2+8\mu^4)}{3a^4(m_\phi^2-4a^2)^2}$$

$$+\frac{32m_{K^+}^2\sqrt{m_\phi^2-4m_{K^+}^2}}{3m_\phi^3(m_\phi^2-4a^2)^2}\ln\frac{m_\phi+\sqrt{m_\phi^2-4m_{K^+}^2}}{m_\phi-\sqrt{m_\phi^2-4m_{K^+}^2}}\Bigg\}. \tag{31}$$

It is easy to verify that the integrand in Eq. (30) is nonsingular at $2E_k=m$ for the principal value of the integral was calculated analytically. That is why Eq. (30) is suitable for a simulation of experimental data.

IV. QUANTITATIVE RESULTS

Below we consider two variants: (i) $\Gamma_0(M_R)=50$ MeV, which corresponds to the visible width $\simeq 25$ MeV and the partial intensity of the decay into the $K\bar{K}$ channels $B(R\to K\bar{K})\simeq 0.35$ in the molecule model, Eq. (14), and (ii) $\Gamma_0(M_R)=100$ MeV, which corresponds to the visible width $\simeq 75$ MeV and $B(R\to K\bar{K})\simeq 0.3$ in the molecule model, Eq. (14). See the definition of $\Gamma_0(m)$ in Eqs. (9)–(11), and (7).

We present $B(\phi\to\gamma R;m)=\Gamma(\phi\to\gamma R;m)/\Gamma(\phi)$, where $\Gamma(\phi)$ is the ϕ meson full width, in Fig. 2, the phase of $-g_R(m)$ ($\delta=\arctan[\mathrm{Im}\,g_R(m)/\mathrm{Re}\,g_R(m)]$) in Fig. 3, and the spectra

$$\frac{dBR(\phi\to\gamma R\to\gamma ab)}{dm}=\frac{d\Gamma(\phi\to\gamma R\to\gamma ab)}{\Gamma(\phi)dm}=\frac{2}{\pi}\frac{m^2\Gamma(R\to ab;m)\Gamma(\phi\to\gamma R;m)}{\Gamma(\phi)|D_R(m)|^2} \tag{32}$$

for $ab=\pi\pi$[3] and $\pi\eta$ for the different $\Gamma_0(M_R)$ in Figs. 4 and 5, respectively. The spectra

$$dB(\phi\to\gamma(a_0+f_0)\to\gamma K^+K^-)/dm$$
$$=d\Gamma(\phi\to\gamma(a_0+f_0)\to\gamma K^+K^-)/\Gamma(\phi)dm$$
$$=\frac{2}{\pi}m^2\Gamma(f_0\to K^+K^-;m)\frac{\Gamma(\phi\to\gamma f_0;m)}{\Gamma(\phi)}\times\left|\frac{1}{D_{f_0}(m)}+\frac{1}{D_{a_0}(m)}\right|^2dm \tag{33}$$

is presented in Fig. 6.

As with the four-quark model [14] in the molecule case the interference is constructive in the K^+K^- channel and

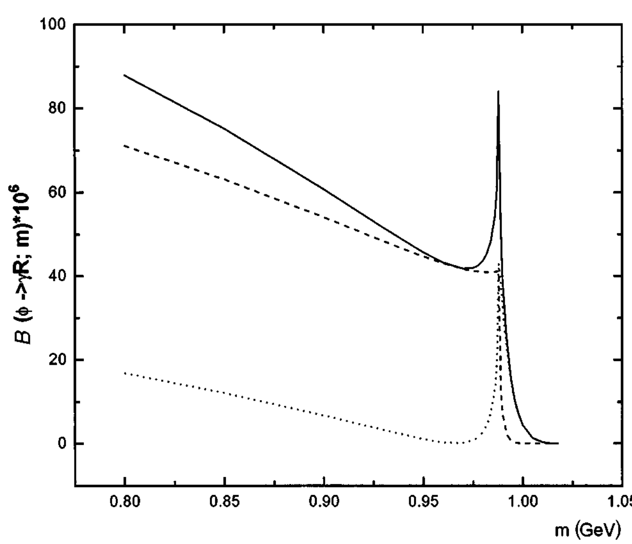

FIG. 2. Branching ratio $B(\phi\to\gamma R;m)$. The dotted line is the real part contribution. The dashed line is the imaginary part contribution. The solid line is the full branching ratio.

[3]Notice that $d\Gamma(\phi\to\gamma R\to\gamma\pi\pi)/dm=d\Gamma(\phi\to\gamma R\to\gamma\pi^+\pi^+)/dm+d\Gamma(\phi\to\gamma R\to\gamma\pi^0\pi^0)/dm=1.5d\Gamma(\phi\to\gamma R\to\gamma\pi+\pi^+)/dm$.

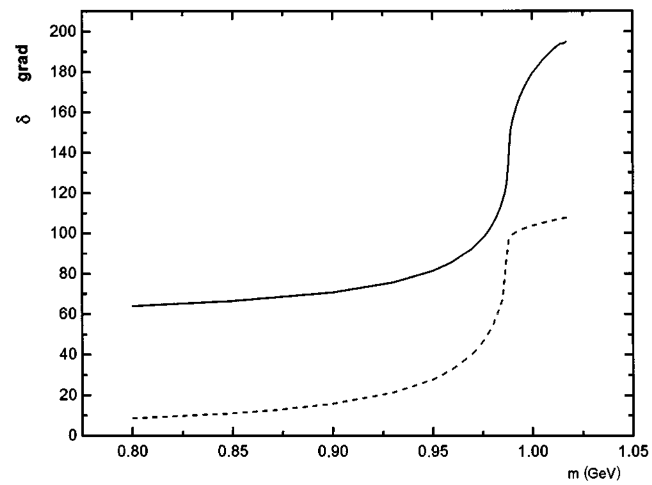

FIG. 3. The phase $\delta = \arctan[\mathrm{Im}\,g_R(m)/\mathrm{Re}\,g_R(m)]$. The solid line presents the m dependence in the $K\bar{K}$ molecular case and the dashed one in the $q^2\bar{q}^2$ case.

destructive in the $K^0\bar{K^0}$ channel [see Eqs. (6) and (8)] for the K^+K^- loop diagram production model.

We study the dependence of the branching ratios under consideration on the resonance mass. Our results are listed in Tables I and Tables II. Calculating Table I we take into account only the imaginary part of the scalar resonance production amplitudes.

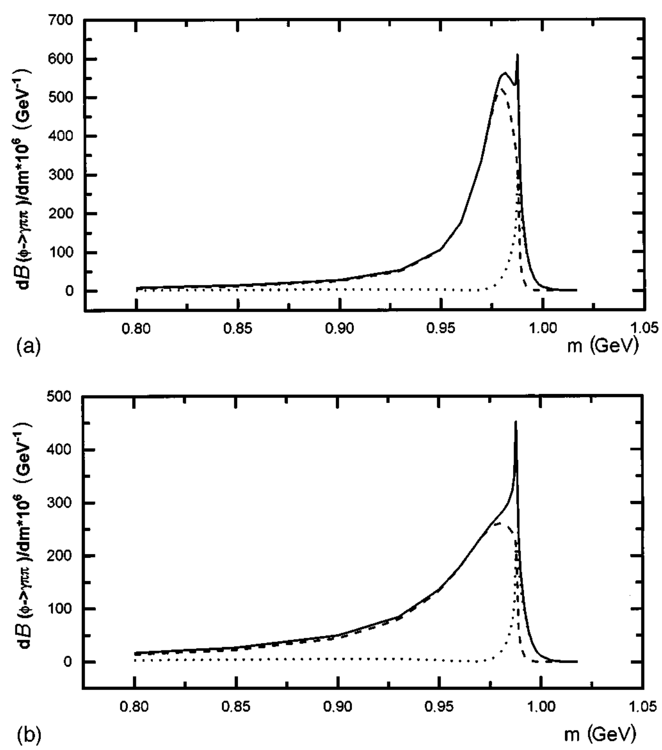

FIG. 4. Mass spectra of the $\pi\pi$ channel. The solid line is the full spectra. The dashed and dotted lines are the imaginary and real part contributions, respectively. (a) $\Gamma_0 = 50$ MeV. The full branching ratio is 1.9×10^{-5}. The branching ratio with an account only of an imaginary part is 1.7×10^{-5}. (b) $\Gamma_0 = 100$ MeV. The full branching ratio is 2.2×10^{-5}. The branching ratio with an account only of the imaginary part is 2.0×10^{-5}.

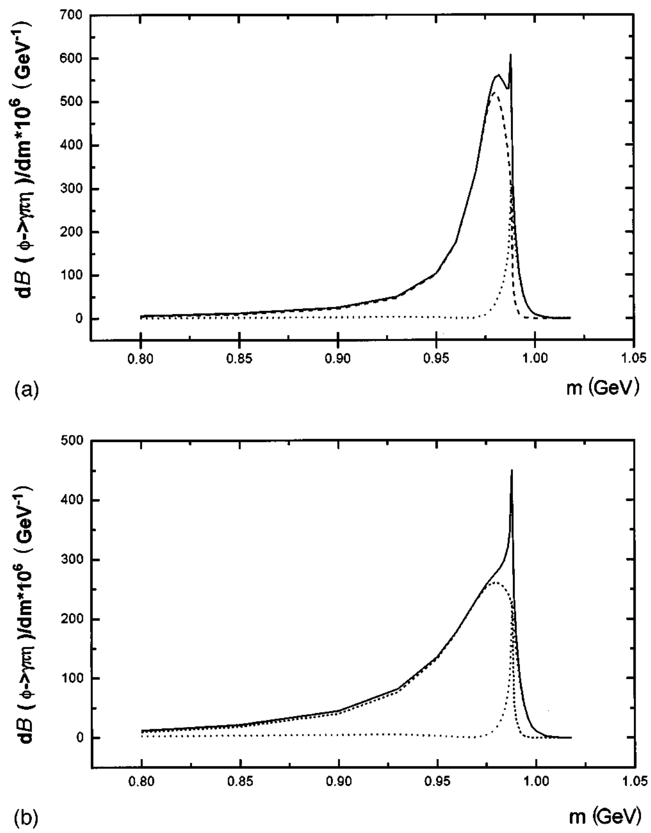

FIG. 5. Mass spectra of the $\pi\eta$ channel. The solid line is the full spectra. The dashed and dotted lines are the imaginary and real part contributions, respectively. (a) $\Gamma_0 = 50$ MeV. The full branching ratio is 1.9×10^{-5}. The branching ratio with an account only of the imaginary part is 1.6×10^{-5}. (b) $\Gamma_0 = 100$ MeV. The full branching ratio is 2.0×10^{-5}. The branching ratio with an account only of the imaginary part is 1.75×10^{-5}.

V. DISCUSSION

At $m_{f_0} = 980$ MeV and $\Gamma_0(m_{f_0}) = 50(100)$ MeV, $B(\phi \to \gamma f_0 \to \gamma\pi\pi) = 1.9(2.2)\times10^{-5}$. At $m_{a_0} = 980$ MeV

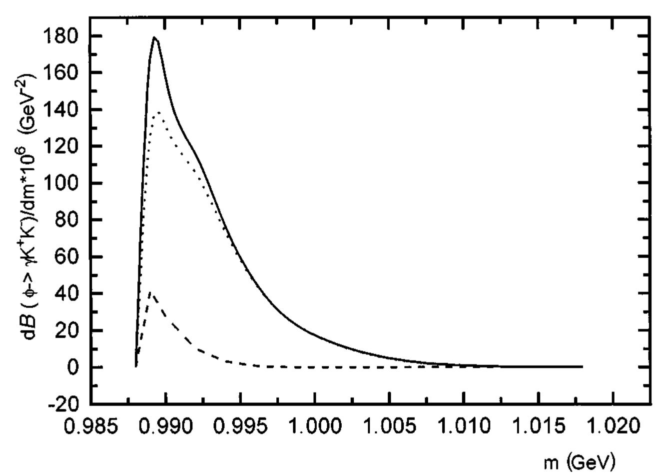

FIG. 6. Mass spectra of K^+K^- channel. $\Gamma_0 = 50$ MeV. The solid line is the full spectra. The dashed and dotted lines are the imaginary and real part contributions, respectively. The full branching ratio is 1.0×10^{-6}. The branching ratio with an account only of the imaginary part is 0.9×10^{-6}.

TABLE I. Branching ratios $B(\phi\to\gamma a_0(f_0)\to\gamma\pi\eta(\pi\pi))$ depending on parameters of the model; see Sec. IV.

	$B\times10^5$				
	$\phi\to\gamma a_0\to\gamma\pi\eta$			$\phi\to\gamma f_0\to\gamma\pi\pi$	
m_{a_0} (GeV)	$\Gamma_{a_0}=50$ (MeV)	$\Gamma_{a_0}=100$ (MeV)	m_{f_0} (GeV)	$\Gamma_{f_0}=50$ (MeV)	$\Gamma_{f_0}=100$ (MeV)
0.960	2.2	2.3	0.960	2.4	2.6
0.965	2.1	2.2	0.965	2.2	2.4
0.970	2.0	2.0	0.970	2.1	2.3
0.975	1.8	1.9	0.975	1.9	2.1
0.980	1.6	1.8	0.980	1.7	2.0
0.985	1.3	1.6	0.985	1.4	1.8
0.990	0.9	1.0	0.990	1.0	1.2
0.995	0.7	0.9	0.995	0.8	1.1

and $\Gamma_0(m_{a_0})=50(100)$ MeV, $B(\phi\to\gamma a_0\to\gamma\pi\eta)=1.9(2)\times10^{-5}$.

As seen from Tables $B(\phi\to\gamma f_0(a_0)\to\gamma\pi\pi(\pi\eta))\approx(1-2)\times10^{-5}$ and $B(\phi\to\gamma(f_0+a_0)\to\gamma K^+K^-)\lesssim10^{-6}$. As for the decays into the $K^0\overline{K^0}$ channels, the destructive interference leads to $B(\phi\to\gamma(f_0+a_0)\to\gamma K^0\overline{K^0})\leqslant6.5\times10^{-9}$ for the set of the input parameters from Table II.

When the branching ratios from Table I is compared with the ones in the narrow resonance approximation in Fig. 2, it is apparent that these latter are at least 2 times overstated. Notice that when the parameter in the momentum distribution $\phi(|\vec{k}|)$ [see Eq. (25)], $\mu=100$ MeV the branching ratios decrease 3 times (the μ^3 low is natural for a three-dimensional integral if an integrand has the range of the order of μ). The value $\mu=141$ MeV corresponds to the bounding energy of the molecule, $E=-10$ MeV [13], that is, $m_{f_0(a_0)}\simeq980$ MeV.

The imaginary part of the $\phi\to\gamma f_0(a_0)$ amplitude gives 90% of the branching ratios in the $\pi\pi(\pi\eta)$ channel; see also Figs. 4 and 5.

The real part of the $\phi\to\gamma f_0(a_0)$ amplitude dominates in the $K\overline{K}$ channel branching ratios; see also Fig. 6.

Over the years, it was believed by some that the decay $\phi\to\gamma K_SK_S$ might prove an obstacle to research on the violation of CP invariance in the decay $\phi\to K_LK_S$ despite the fact that it was shown even in 1987 [14] that $B(\phi\to\gamma(f_0+a_0)\to\gamma K_SK_S)\simeq6.5\times10^{-9}$ in the case which is the worst case from the standpoint of research on the violation of CP invariance, i.e., the case in which the $f_0(980)$ and $a_0(980)$ resonances are of a four-quark nature ($q^2\overline{q}^2$). An upper limit in a Pickwickian sense, $B(\phi\to\gamma(f_0+a_0)\to\gamma K_SK_S)\simeq3.6\times10^{-7}$, was given in [21]. The following investigations (see, for example, [22]) only confirm these results.

TABLE II. Branching ratio $B(\phi\to\gamma K^+K^-)$ depending on parameters of the model; see Sec. IV.

m_{f_0} (GeV)	m_{a_0} (GeV)	Γ_{f_0} (MeV)	Γ_{a_0} (MeV)	$B\times10^6$
0.980	0.980	50	50	1.0
0.980	0.980	100	100	0.5
0.980	0.980	50	100	0.7
0.990	0.990	100	100	0.6
0.980	0.990	50	100	0.7
0.990	0.980	50	100	1.0
0.990	0.990	50	100	1.1
0.990	0.990	50	50	1.8

Here we also confirm that the branching ratio for the decay $\phi\to\gamma K_SK_S$ is low. But its extreme smallness demands to consider final state interaction effects.

Can final state interactions essentially destroy our interference picture in the $K^0\overline{K^0}$ channel for the difference of the S-wave phase shifts in the isotopical vector ($I=1$) and isotopical scalar ($I=0$) channels of $K\overline{K}$ scattering? There is no reason to expect this effect. The point is that the these phase shifts vanish at the $K\overline{K}$ threshold. In our case the maximal momentum of the K^0 and $\overline{K^0}$ mesons in the scalar resonance rest frame is small (110 MeV). In fact, more small momenta are essential in our consideration. The K^0 and $\overline{K^0}$ mesons with momenta less than 60 MeV in the scalar resonance rest frame give 90% of the branching ratios under consideration. Nevertheless, strictly speaking, we do not know anything about the influence of final state interactions on amplitude magnitudes. Let us assume that a final state interaction suppresses the contribution of one of two resonances, for example, a_0. Then we have $B(\phi\to\gamma f_0\to\gamma K^0\overline{K^0})\lesssim2.5\times10^{-8}$ for the set of input parameters from Table II. At last one assumes an extreme case that final state interactions transfer the destructive interference in the $K^0\overline{K^0}$ channel into the constructive one (it is hard to image something more radical now). Then $B(\phi\to\gamma(f_0+a_0)\to\gamma K^0\overline{K^0})=2B(\phi\to\gamma(f_0+a_0)\to\gamma K_SK_S)\simeq1/10B(\phi\to\gamma(f_0+a_0)\to\gamma K^+K^-)$ from Table II for the $K^0\overline{K^0}$ threshold is 8 MeV higher than the K^+K^- one. So $B(\phi\to\gamma(f_0+a_0)\to\gamma K_SK_S)\simeq5\times10^{-8}$ can be considered as an upper limit in a Pickwickian sense for the molecule model.

The phase of $g_R(m)$ is the very important characteristic of the model production because it causes the interference patterns in the reaction $e^+e^-\to\gamma\pi^+\pi^-$ in the ϕ meson mass region. We emphasize that just these interference patterns are used to identify the $\phi\to\gamma f_0$ decay. As seen from Fig. 3 the phases in the molecule case (the solid curve) and in the four-quark (pointlike) case [14] (the dashed curve) differ considerably. The calculation of the interference patterns under discussion is a rather complex problem that is a valid one for further investigation.

VI. CONCLUSION

We have considered the potentialities of the production of the scalar $K\bar{K}$ molecules in the ϕ radiative decays beyond the narrow resonance width approximation. It was found that $B(\phi\to\gamma f_0(a_0)\to\gamma\pi\pi(\pi\eta))\sim 10^{-5}$. Taking into account that in the four-quark model $B(\phi\to\gamma f_0(a_0)\to\gamma\pi\pi(\pi\eta))\sim 10^{-4}$, (see [14,17]), we conclude that radiative ϕ decays give a good possibility for discrimination between the two models experimentally.

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