Inflaton decay in de Sitter spacetime

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(Received 31 May 1996)

We study the decay of scalar fields, in particular the inflaton, into lighter scalars in a de Sitter spacetime background. After providing a practical definition of the rate, we focus on the case of an inflaton interacting with a massless scalar field either minimally or conformally coupled to the curvature. The evolution equation for the expectation value of the inflaton is obtained to one loop order in perturbation theory and the decay rate is recognized from the solution. We find the remarkable result that this decay rate displays an equilibrium Bose-enhancement factor with an effective temperature given by the Hawking temperature $H/2\pi$, where H is the Hubble constant. This contribution is interpreted as the ''stimulated emission'' of bosons in a thermal bath at the Hawking temperature. In the context of new inflation scenarios, we show that inflaton decay into conformally coupled massless fields slows down the rolling of the expectation value. Decay into Goldstone bosons is also studied. Contact with stochastic inflation is established by deriving the Langevin equation for the coarse-grained expectation value of the inflaton field to one-loop order in this model. We find that the noise is Gaussian and correlated (colored) and its correlations are related to the dissipative (''decay'') kernel via a generalized fluctuation-dissipation relation. [S0556-2821(97)05614-2]

PACS number(s): 98.80.Bp, 98.80.Cq

I. INTRODUCTION

Nonequilibrium processes play a fundamental role in cosmological scenarios motivated by particle physics models. Processes such as thermalization, reheating, and particle decay are important ingredients in most theoretical scenarios that attempt a description of early universe cosmology [1–4]. An important ingredient underlying most attempts at a description of inflationary cosmologies is that of equilibration, which is tacitly assumed whenever quasiequilibrium methods are invoked in their study, such as effective potentials and finite temperature field theory.

The concept of quasiequilibrium during an inflationary stage requires a careful understanding of two different time scales. One is the expansion time scale $t_H = H^{-1}$, where H is the Hubble parameter, while the other is the field relaxation time scale t_r. A quasiequilibrium situation obtains whenever $t_r \ll t_H$ since in this case particle physics processes occur much faster than the universe expands, allowing fields to respond quickly to any changes in the thermodynamic variables.

Can the inflaton of inflationary cosmologies be treated as being in quasiequilibrium? The answer to this question depends crucially on the various processes that contribute to the dynamical relaxation of this field. Some of these processes are the decay of the inflaton into lighter scalars or fermions, as well as collisions that, while they leave the total number of particles fixed, will change the phase space distribution of these particles [5].

In this paper we wish to understand the effects of coupling the inflaton to lighter scalars during the inflationary era. Single-particle decay of the inflaton into lighter scalars or fermions in the post-inflationary era, when spacetime can be well approximated by a Minkowski metric, is by now fairly well understood within the context of the ''old-reheating'' scenario [6–9], but it is much less understood during the *early stages* of inflation.

In saying this, we have to distinguish between the various models of inflation. In ''old'' or ''extended'' inflation [10], the inflaton is trapped in a metastable minimum, from which it exits eventually via the nucleation of bubbles of the true vacuum phase. A critical parameter in these models is the nucleation rate, which is typically calculated assuming that the field is equilibrated [3,4,11]. Such an assumption will only be justified if the relaxation time of the inflaton in the metastable phase, i.e., in de Sitter space, is much smaller than the time scale for nucleation of a critical bubble.

In ''new'' or ''chaotic'' inflationary scenarios [7], the supraluminal expansion of the universe is driven by the so-called ''slow-roll'' dynamics of the inflaton. Typically, this is studied via the zero or finite temperature effective potential. In doing this, the assumption of quasiequilibrium is made, in that one can replace the actual field dynamics by that given by a static quantity, the effective potential. This assumption must be checked, and one way of doing that is by considering the evolution of the inflaton in the presence of couplings to other fields and computing the field relaxation rate. In particular, if dissipative processes associated with the ''decay'' of the inflaton field into lighter scalars are efficient during the slow-roll stage in new or chaotic inflationary scenarios, then these may result in a further slowing down of the inflaton field as it rolls down the potential hill.

Finally, there is the stochastic inflation approach advocated by Starobinskii [12]. In this scenario, inflationary dynamics is studied with a stochastic evolution equation for the inflaton field with a Gaussian white noise that describes the microscopic fluctuations. As pointed out by Hu and collaborators [13] and Habib and Kandrup [14,15], noise and dissi-

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pation are related via a fundamental generalized fluctuation-dissipation theorem and *must* be treated on the same footing.

In this article we study the process of relaxation of the inflaton field via the decay into massless scalar particles in de Sitter spacetime with the goal of understanding the modifications in the decay rates and the relevant time scales as compared to those in Minkowski spacetime. Our study focuses on the description of these processes within ‘‘old’’ or ‘‘extended’’ and ‘‘new’’ inflationary scenarios. Within ‘‘old’’ inflation we study the situation in which the inflaton field oscillates around the metastable minimum (before it eventually tunnels) and decays into massless scalars. In the case of ‘‘new’’ inflation, we address the situation of the inflaton rolling down a potential hill, near the *maximum* of the potential, to understand how the process of ‘‘decay’’ introduces dissipative contributions to the inflaton evolution.

We also establish contact with stochastic inflation by *deriving* the Langevin (stochastic) equation for the inflaton field and analyzing the generalized fluctuation-dissipation relation between the dissipation kernel resulting from the ‘‘decay’’ into massless scalars and the correlations of the stochastic noise.

At this point we would like to make precise the meaning of ‘‘decay’’ in a time-dependent background. The usual concept of the transition rate in Minkowski space, i.e., the transition probability per unit volume per unit time for a particular (in) state far in the past to a (out) state far into the future, is no longer applicable in a time-dependent background. Relaxing this definition to finite time intervals (for example, by obtaining the transition probability from some t_i to some t_f) would yield a time-dependent quantity affected by the ambiguities of the definition of particle states. In Minkowski spacetime the existence of time translational invariance allows a spectral representation of two-point functions and the transition rate (or decay rate) is related to the imaginary part of the self-energy on shell. Such a correspondence is not available in a time-dependent background. Furthermore, the time evolution of a scalar field is damped because of the redshifting of the energy due to the expanding background even in the absence of interactions. In particular in de Sitter space this damping is exponential and any other exponential damping arising from interactions will be mixed with the gravitational redshift.

We propose to *define* the ‘‘decay rate’’ as the contribution to the exponential damping of the inflaton evolution due to the interactions with other fields, which is clearly recognized through the dependence on the coupling to these fields. This practical definition then argues that in order to recognize such a ‘‘decay rate’’ we must study the real-time dynamics of the nonequilibrium expectation value of the inflaton field in interaction with other fields, and the identification of this rate should transpire from the actual evolution of this expectation value.

Although the methods presented in this article can be generalized to arbitrary spatially flat Friedman-Robertson-Walker (FRW) cosmologies and include decay of a heavy inflaton into lighter scalars, we study the simpler but relevant case of inflaton decay into massless scalars in de Sitter spacetime. This case affords an analytic perturbative treatment that illuminates the relevant physical features.

The article is organized as follows. In Sec. II we present a brief pedagogical review of the closed time path formalism and the derivation of the nonequilibrium Green’s functions for free fields in de Sitter spacetime. In Sec. III the model is defined and the methods to obtain the evolution equation are summarized. Here we study several cases, including minimal and conformal coupling and the slow-roll regime. We obtain the remarkable result that the decay rate can be understood from stimulated emission in a bath at the Hawking temperature. In Sec. IV we study the effect of Goldstone bosons on the inflaton evolution, pointing out that Goldstone bosons are necessarily always minimally coupled. In Sec. V we make contact with stochastic inflation, derive the stochastic Langevin equation for the inflaton zero mode, and obtain the dissipative kernel and the noise correlation function, which are related by a generalized fluctuation-dissipation theorem. The resulting noise is Gaussian but colored. In Sec. VI we summarize our results and propose new lines of study. An appendix is devoted to a detailed derivation, establishing the consistency of the flat-space limits of the analytic structure of the dissipative kernel in de Sitter spacetime, with Minkowskian calculations.

II. NONEQUILIBRIUM FIELD THEORY

A. Closed time path formalism

As mentioned in the Introduction we will be studying the real-time evolution of the zero mode of the inflaton in a universe with a time-dependent background metric. Furthermore, we will restrict our investigations to *perturbative* phenomena in theories in which the inflaton interacts with other matter fields in a de Sitter universe. We will be primarily concerned with defining and calculating quantities such as the perturbative ‘‘decay rate’’ of the inflaton. For studies of nonperturbative particle production in nonequilibrium situations see [19] (and references therein). Calculation of particle production in curved spacetime has also been done using the Boguliubov transformations; see, e.g., [23].

The tools required for studying time-dependent phenomena in quantum field theories have been available for quite some time, and were first used by Schwinger [16] and Keldysh [17]. There are many articles in the literature using these techniques to study time-dependent problems [18,19]. However, these techniques have not yet become an integral part of the available methods for studying field theory in extreme environments. We thus present a concise pedagogical introduction to the subject for the nonpractitioner.

The nonequilibrium or time-dependent description of a system is determined by the time evolution of the density matrix that describes it. This is in turn described by the Liouville equation (in the Schrödinger picture)

$$i\hbar \frac{\partial \rho(t)}{\partial t} = [H(t), \rho(t)]. \tag{2.1}$$

Here we have allowed for an explicitly time-dependent Hamiltonian which is in fact the case in an expanding universe. The formal solution of the Liouville equation is

$$\rho(t) = U(t)\rho_i U^{-1}(t), \tag{2.2}$$

where ρ_i is the initial density matrix specified at some initial time $t=0$. The time dependent expectation value of an operator is then easily seen to be

$$\langle \mathcal{O}\rangle(t)=\mathrm{Tr}[\rho_i U^{-1}(t)\mathcal{O}U(t)]/\mathrm{Tr}[\rho_i]. \tag{2.3}$$

In most cases of interest the initial density matrix is either a thermal one or a pure state corresponding to the ground state of some initial Hamiltonian. In either case,

$$\rho_i=e^{-\beta H_i}, \tag{2.4}$$

$$H_i=H(t<0). \tag{2.5}$$

The ground state of H_i can be projected out by taking the $\beta\to\infty$ limit. To study strongly out-of-equilibrium situations it is usually convenient to introduce a time-dependent Hamiltonian $H(t)$ such that $H(t)=H_i$ for $-\infty\leqslant t\leqslant 0$ and $H(t)=H_{\mathrm{evol}}(t)$ for $t>0$ where $H_{\mathrm{evol}}(t)$ is the evolution Hamiltonian that determines the dynamics of the theory.

For thermal initial conditions, Eq. (2.3) can be written in a more illuminating form by choosing an arbitrary time $T<0$ for which $U(T)=\exp[-iTH_i]$ so that we may write $\exp[-\beta H_i]=\exp[-iH_i(T-i\beta-T)]=U(T-i\beta,T)$. Inserting the identity $U^{-1}(T)U(T)=1$, commuting $U^{-1}(T)$ with ρ_i, and using the composition property of the evolution operator, we get

$$\langle \mathcal{O}\rangle(t)=\mathrm{Tr}[U(T-i\beta,t)\mathcal{O}U(t,T)]/\mathrm{Tr}[U(T-i\beta,T)] \tag{2.6}$$

$$=\mathrm{Tr}[U(T-i\beta,T)U(T,T')U(T',t)\times\mathcal{O}U(t,T)]/\mathrm{Tr}[U(T-i\beta,T)], \tag{2.7}$$

where we have introduced an arbitrary large positive time T'. The numerator now represents the process of evolving from $T<0$ to t, inserting the operator $\mathcal{O}$, and evolving to a large positive time T', backwards from T' to T and down the imaginary axis to $T-i\beta_i$. Eventually one takes $T\to-\infty$, $T'\to\infty$. This method can be easily generalized to obtain real-time correlation functions of a string of operators.

As usual, an operator insertion may be achieved by introducing sources into the time evolution operators and taking a functional derivative with respect to the source. Thus we are led to the following generating functional, in terms of time evolution operators in the presence of sources:

$$Z[J^+,J^-,J^\beta]=\mathrm{Tr}[U(T-i\beta,T;J^\beta)U(T,T';J^-)\times U(T',T;J^+)], \tag{2.8}$$

with $T\to-\infty$, $T'\to\infty$. One can obtain a path integral representation for the generating functional [16–21] by inserting a complete set of field eigenstates between the time evolution operators,

$$Z[J^+,J^-,J^\beta]=\int D\Sigma D\Sigma_1 D\Sigma_2\int \mathcal{D}\Sigma^+\mathcal{D}\Sigma^-\mathcal{D}\Sigma^\beta \times\exp\left(i\int_T^{T'}\{\mathcal{L}[\Sigma^+,J^+]-\mathcal{L}[\Sigma^-,J^-]\}\right)\times\exp\left(i\int_T^{T-i\beta}\mathcal{L}[\Sigma^\beta,J^\beta]\right). \tag{2.9}$$

The above expression for the generating functional is a path integral defined on a complex time contour. Since this path integral actually represents a trace, the field operators at the boundary points have to satisfy the conditions

$$\Sigma^+(T)=\Sigma^\beta(T-i\beta)=\Sigma,$$
$$\Sigma^+(T')=\Sigma^-(T')=\Sigma_2,$$
$$\Sigma^-(T)=\Sigma^\beta(T)=\Sigma_1. \tag{2.10}$$

The superscripts $(+)$ and $(-)$ refer to the forward and backward parts of the time contour, while the superscript (β) refers to the imaginary-time part of the contour. In the limit $T\to-\infty$ it can be shown that cross correlations between fields defined at real times and those defined on the imaginary-time contour vanish due to the *Riemann-Lebesgue* lemma. The fact that the density matrix is thermal for $t<0$ shows up in the boundary condition on the fields and the Green's functions. Thus the generating functional for calculating *real-time* correlation functions simplifies to

$$Z[J^+,J^-]=\exp\left\{i\int_T^{T'}dt[\mathcal{L}_{\mathrm{int}}(-i\delta/\delta J^+)-\mathcal{L}_{\mathrm{int}}(i\delta/\delta J^-)]\right\}\times\exp\left\{\frac{i}{2}\int_T^{T'}dt_1\int_T^{T'}dt_2 J_a(t_1)J_b(t_2)\times G_{ab}(t_1,t_2)\right\}, \tag{2.11}$$

with $a,b=+,-$. We will now proceed to a calculation of the nonequilibrium Green's functions in de Sitter spacetime.

B. Nonequilibrium Green's functions in de Sitter spacetime

To obtain the nonequilibrium Green's functions we focus on a free scalar field Σ in a FRW background metric [20]. The generating functional of nonequilibrium Green's functions is written in terms of a path integral along the closed time path (CTP), Eq. (2.11), with a free Lagrangian density $\mathcal{L}_0(\Sigma^\pm)$ given by

$$\mathcal{L}_0(\Sigma^\pm)=\tfrac{1}{2}[a^3(t)\dot{\Sigma}^{+2}-a(t)(\vec{\nabla}\Sigma^+)^2-a^3(t)(M^2+\xi\mathcal{R})\Sigma^{+2}]-[+\to-], \tag{2.12}$$

where $\mathcal{R}=6(\ddot{a}/a+\dot{a}^2/a^2)$.

We prepare the system such that it is thermal for $t<0$ with temperature $1/\beta$. At $t=0$, we switch the interactions on and follow the resulting time evolution of the coupled fields.

As seen in the previous section, the CTP formulation of nonequilibrium field theory imposes certain boundary conditions on the fields (2.10).

Rewriting the Lagrangian as a quadratic form in $\Sigma^{\pm}$ we obtain the Green's function equation

$$\left[\frac{\partial^2}{\partial t^2}+3\frac{\dot{a}}{a}\frac{\partial}{\partial t}-\frac{\nabla^2}{a^2}+(M^2+\xi\mathcal{R})\right]G(x,t;x',t') =\frac{\delta^4(x-x')}{a^{3/2}(t)a^{3/2}(t')}. \tag{2.13}$$

We use spatial translational invariance to define

$$G_k(t,t')=\int d^3x e^{i\vec{k}\cdot\vec{x}}G(x,t;0,t'), \tag{2.14}$$

so that the spatial Fourier transform of the Green's function obeys

$$\left[\frac{d^2}{dt^2}+3\frac{\dot{a}}{a}\frac{d}{dt}+\frac{k^2}{a^2}+(M^2+\xi\mathcal{R})\right]G_k(t,t')=\frac{\delta(t-t')}{a^{3/2}(t)a^{3/2}(t')}. \tag{2.15}$$

The solution can be cast in a more familiar form by writing

$$G_k(t,t')=\frac{f_k(t,t')}{a^{3/2}(t)a^{3/2}(t')}, \tag{2.16}$$

so that the function $f_k(t,t')$ obeys the second-order differential equation

$$\left[\frac{d^2}{dt^2}-\left(\frac{3\ddot{a}}{2a}+\frac{3\dot{a}^2}{4a^2}\right)+\frac{k^2}{a^2}+(M^2+\xi\mathcal{R})\right]f_k(t,t')=\delta(t-t'). \tag{2.17}$$

The general solution of this equation is of the form

$$f_k(t,t')=f_k^{>}(t,t')\Theta(t-t')+f_k^{<}(t,t')\Theta(t'-t), \tag{2.18}$$

where $f_k^{>}$ and $f_k^{<}$ are solutions to the homogeneous equation obeying the appropriate boundary conditions. They can be expanded in terms of normal mode solutions to the homogeneous equation:

$$f_k^{>}(t,t')=A^{>}U_k(t)U_k^{*}(t')+B^{>}U_k(t')U_k^{*}(t),$$

$$f_k^{<}(t,t')=A^{<}U_k(t)U_k^{*}(t')+B^{<}U_k(t')U_k^{*}(t). \tag{2.19}$$

The choice of mode functions can be made unique for a given ξ and M by requiring that

$$\lim_{k\to\infty}U_k(t)\sim e^{-ikt}$$

or

$$\lim_{H\to 0}U_k(t)\sim e^{-i\omega_k t},$$

$$\omega_k=\sqrt{k^2+M^2}. \tag{2.20}$$

Here $H\equiv a(\dot{t})/a(t)$ is the Hubble parameter. Stated simply, these conditions require that we must necessarily recover the rules of field theory in Minkowski space when we take the flat-space or the short-distance ($k\to\infty$) limits. These boundary conditions are similar to those invoked by Bunch and Davies [22,23]. For the de Sitter space case, with $a(t)=\exp(Ht)$, the mode functions are the Bessel functions [23], $J_{\pm\nu}(ke^{-Ht}/H)$, provided ν is not an integer, where

$$\nu=i\sqrt{\frac{M^2}{H^2}+12\xi-\frac{9}{4}}. \tag{2.21}$$

For the values of ξ and M for which ν becomes real these Bessel functions become manifestly real and the solutions that fulfill the boundary conditions (2.20) are obtained as linear combinations of these functions. In order to obtain the full Green's functions the constants $A^{>},B^{>},A^{<},B^{<}$ Eq. (2.19), are determined by implementing the boundary conditions [18–20]

$$f_k^{>}(t,t)=f_k^{<}(t,t),$$

$$\dot{f}_k^{>}(t,t)-\dot{f}_k^{<}(t,t)=1,$$

$$f_k^{>}(T-i\beta,t)=f_k^{<}(T,t). \tag{2.22}$$

The first two conditions are obtained from the continuity of the Green's function and the jump discontinuity in its first derivative. The third boundary condition is just a consequence of the periodicity of the fields in imaginary time, which followed from the assumption that the density matrix is that of a system in thermal equilibrium at time $T<0$. Although we allowed for a thermal initial density matrix, we will focus only on the zero temperature case, since during the de Sitter era the temperature is rapidly redshifted to zero. The most general case for the Green's function can be found in [20]. Implementing these conditions for the case of imaginary ν and zero temperature ($\beta\to\infty$) we find

$$G_k^{>}(t,t')=\frac{-\pi J_{\nu}(z)J_{-\nu}(z')}{2H\sin(\pi\nu)e^{3Ht/2}e^{3Ht'/2}},$$

$$G_k^{<}(t,t')=\frac{-\pi J_{-\nu}(z)J_{\nu}(z')}{2H\sin(\pi\nu)e^{3Ht/2}e^{3Ht'/2}}, \tag{2.23}$$

$$z=k\frac{e^{-Ht}}{H},\quad z'=k\frac{e^{-Ht'}}{H}, \tag{2.24}$$

and ν is given by Eq. (2.21).

In the case of interest for this article, that of a massless scalar field either conformally or minimally coupled to the curvature, the Bessel functions in the above expressions must be replaced by the corresponding Hankel functions. These two cases correspond to

$$\text{(i)}\quad M=0,\xi=0\Rightarrow U_k(t)=H^{(1)}{}_{3/2}(ke^{-Ht}/H), \tag{2.25}$$

$$\text{(ii)}\quad M=0,\xi=\tfrac{1}{6}\Rightarrow U_k(t)=H^{(1)}{}_{1/2}(ke^{-Ht}/H). \tag{2.26}$$

Finally, the Green's functions for these two cases of a massless scalar field, in terms of z and z', are

$$G_k^{>}(t,t')=\frac{-i\pi H_\nu^{(1)}(z)H_\nu^{(2)}(z')}{4He^{3Ht/2}e^{3Ht'/2}}, \tag{2.27}$$

$$G_k^{<}(t,t')=\frac{-i\pi H_\nu^{(2)}(z)H_\nu^{(1)}(z')}{4He^{3Ht/2}e^{3Ht'/2}}, \tag{2.28}$$

with $\nu=3/2$ for minimally coupled or $\nu=1/2$ for conformally coupled. From the expressions for $G_k^{>}$ and $G_k^{<}$ we can compute the required two-point functions

$$i\langle T\Sigma^+(\vec{r},t)\Sigma^+(\vec{r}\,',t')\rangle=G^{>}(\vec{r},t;\vec{r}\,',t')\Theta(t-t')+G^{<}(\vec{r},t;\vec{r}\,',t')\Theta(t'-t), \tag{2.29}$$

$$i\langle T\Sigma^-(\vec{r},t)\Sigma^-(\vec{r}\,',t')\rangle=G^{>}(\vec{r},t;\vec{r}\,',t')\Theta(t'-t)+G^{<}(\vec{r},t;\vec{r}\,',t')\Theta(t-t'), \tag{2.30}$$

$$-i\langle T\Sigma^{+(-)}(\vec{r},t)\Sigma^{-(+)}(\vec{r}\,',t')\rangle=G^{<(>)}(\vec{r},t;\vec{r}\,',t'). \tag{2.31}$$

In our subsequent analysis we will always work in the $\beta\to\infty$ limit (zero initial temperature) unless stated otherwise.

III. MODEL AND THE METHOD

We now turn to the main topic of this work, the study of an inflaton, described by a field Φ coupled to a massless scalar field σ with the nonequilibrium Lagrangian density

$$\mathcal{L}=\tfrac{1}{2}[e^{3Ht}\dot{\Phi}^{+2}-e^{Ht}(\vec{\nabla}\Phi^+)^2-e^{3Ht}(m_\Phi^2+12\xi_\Phi H^2)\Phi^{+2}+e^{3Ht}\dot{\sigma}^{+2}-e^{Ht}(\vec{\nabla}\sigma^+)^2-12e^{3Ht}\xi_\sigma H^2\sigma^{+2}]+\tfrac{1}{2}e^{3Ht}[g\Phi^+\sigma^{+2}+h(t)\Phi^+] \tag{3.1}$$

$$-[+\to-]. \tag{3.2}$$

We have introduced a ''magnetic'' (source) term $h(t)$ for two reasons.

(1) Such a term will be generated by renormalization from tadpole graphs; therefore, we introduce $h(t)$ as a counterterm to cancel these contributions,

(2) $h(t)$ will serve as a Lagrange multiplier to define our problem as an initial condition problem. That is, we will fix $h(t)$ such that the divergences from tadpole graphs contributing an inhomogeneous term to the equation of motion are canceled and the initial value of the inflaton field is fixed for $t<0$. Such a procedure will be implemented below when we obtain the equation of motion for the expectation value of the inflaton.

Clearly, in de Sitter spacetime the coupling to the curvature only serves to redefine the mass of the fields. For the inflaton, we just absorb this term in a redefinition of the mass, and set $\xi_\Phi=0$ without any loss of generality.

To study the time evolution of the expectation value of the inflaton field we invoke the tadpole method (see Ref. [19] and references therein) to obtain its equation of motion. This is implemented by first shifting Φ by its expectation value in the nonequilibrium state as follows:

$$\Phi^\pm(\vec{x},t)=\phi(t)+\psi^\pm(\vec{x},t),\quad \phi(t)=\langle\Phi^\pm(\vec{x},t)\rangle, \tag{3.3}$$

and then enforcing the tadpole condition which is simply a consequence of Eq. (3.3),

$$\langle\psi^\pm(\vec{x},t)\rangle=0, \tag{3.4}$$

$\phi(t)$ being the spatially homogeneous inflaton zero mode that drives inflation. We also require that $\langle\sigma^\pm(\vec{x},t)\rangle=0$ to all orders, which means that the σ field does not acquire an expectation value. After the shift of the inflaton field (3.3), the nonequilibrium action reads

$$S=\int d^3x dt\Big\{\mathcal{L}_0(\psi^+)+\mathcal{L}_0(\sigma^+)+\psi^+e^{3Ht}[-\ddot{\phi}-3H\dot{\phi}-m_\Phi^2\phi]+e^{3Ht}\frac{g}{2}\phi(\sigma^+)^2+e^{3Ht}\frac{g}{2}\psi^+(\sigma^+)^2+e^{3Ht}\frac{h}{2}\psi^+-(+\to-)\Big\}, \tag{3.5}$$

where $\mathcal{L}_0$ represents the free theories for the ψ and σ fields. The tadpole condition (3.4) is implemented by calculating the expectation value of the field operator $\psi^\pm$ by inserting it into the nonequilibrium path integral and expanding in powers of g about the free theory and setting the resulting expression to zero. In particular we evaluate the following expression to order g^2 in perturbation theory:

$$\langle\psi^+(\vec{x},t)\rangle=\int\mathcal{D}[\psi^\pm]\mathcal{D}[\sigma^\pm]e^{i(S_0[+]-S_0[-])}\psi^+(\vec{x},t)\times\exp\Big\{\int d^3x'dt'\Big[e^{3Ht'}[-\ddot{\phi}-3H\dot{\phi}-m_\Phi^2\phi]\psi^++e^{3Ht'}\frac{g}{2}\phi(\sigma^+)^2+e^{3Ht'}\frac{g}{2}\psi^+(\sigma^+)^2+e^{3Ht'}\frac{h}{2}\psi^+-(+\to-)\Big]\Big\}=0. \tag{3.6}$$

The equation of motion is obtained by independently setting the coefficients of $\langle\psi^+(x)\psi^+(x')\rangle$ and $\langle\psi^+(x)\psi^-(x')\rangle$ to zero. This is justified because these two correlators are independent functions. It is easily seen that the condition $\langle\psi^-\rangle=0$ yields exactly the same equations.

This procedure gives rise to an equation of motion for the zero mode $\phi(t)$ and is in fact equivalent to extremizing the one-loop effective action which incorporates the equilibrium boundary conditions at $t<0$.

It is worth noting that this scheme, referred to as the *amplitude expansion*, also assumes that the amplitude of the zero mode $\phi(t)$ is ''small'' so that terms such as $g\phi(t)\sigma^2$ may be treated perturbatively.

To one-loop order $[O(g^2)]$ we obtain the following form of the equation of motion:

$$\ddot{\phi}+3H\dot{\phi}+m_\Phi^2\phi-\frac{g}{2}\langle\sigma^2\rangle(t)-i\frac{g^2}{4\pi^2}\int_{-\infty}^{t}d\tau K(t,\tau)\phi(\tau)$$

$$-\frac{h(t)}{2}=0, \tag{3.7}$$

$$K(t,\tau)=\int_0^\infty k^2dk[G_k^{>2}(\tau,t)-G_k^{<2}(\tau,t)]e^{3H\tau}. \tag{3.8}$$

Spatial translational invariance guarantees that $\langle\sigma^2\rangle$ is only time dependent. We can combine the ϕ-independent contributions to the equation of motion into a single function

$$\tilde{h}(t)=\frac{h(t)}{2}+\frac{g}{2}\langle\sigma^2\rangle(t). \tag{3.9}$$

If the σ is conformally coupled to the curvature, $\langle\sigma^2\rangle$ represents a quadratically divergent piece which must be subtracted away by a renormalization of the ''magnetic field'' h. On the other hand, if σ is minimally coupled, one obtains the well-known linearly growing contribution [3] after subtracting away the divergence leading to a modified inhomogeneous term in the zero mode equation of motion:

$$\tilde{h}(t)=\frac{gH^3t}{8\pi^2}+\frac{h(t)}{2}. \tag{3.10}$$

We will, however, show explicitly in the next section that this term does *not* enter in the calculation of the perturbative ''decay rate'' in this theory.

We use $\tilde{h}(t)$ as a Lagrange multiplier to enforce the constraint that for $t<0$, $\phi(t)=\phi_i$, $\dot{\phi}(t)=0$ is a solution of the effective equation of motion for the expectation value. In this manner, we define an initial value problem with Cauchy data on a spacelike surface for the dynamics in which the expectation value of the inflaton field is ''released'' from some initial value at time $t=0$. For $t>0$ the equation of motion becomes

$$\ddot{\phi}(t)+3H\dot{\phi}(t)+m_\Phi^2\phi(t)-i\frac{g^2}{4\pi^2}\int_0^t d\tau K(t,\tau)\phi(\tau)=\tilde{h}(t), \tag{3.11}$$

$$\phi(t=0)=\phi_i,\quad \dot{\phi}(t=0)=0, \tag{3.12}$$

with the kernel given by Eq. (3.8).

We now proceed to study the solution to this equation of motion in the cases of conformal and minimal coupling.

A. Conformally coupled massless σ

For a conformally coupled massless scalar, the Green's functions are given by Eqs. (2.27) and (2.28) with the Hankel functions as in Eq. (2.26).

The kernel has a particularly simple form in this case:

$$K(t,\tau)=-i\int_0^\infty dk\frac{\sin[2k(e^{-H\tau}-e^{-Ht})/H]}{2e^{2Ht}e^{-H\tau}}. \tag{3.13}$$

The k integral can be done by introducing a small convergence factor as follows:

$$\int_0^\infty \sin(\alpha k)dk=\lim_{\epsilon\to 0}\mathrm{Im}\left[\int_0^\infty e^{i(\alpha+i\epsilon)k}dk\right]=\lim_{\epsilon\to 0}\frac{\alpha}{\alpha^2+\epsilon^2}. \tag{3.14}$$

Implementing the above prescription and absorbing a positive finite piece into ϵ yields

$$K(t,\tau)=-i\frac{H}{4}\frac{[e^{H(t-\tau)}-1]}{(e^{H(t-\tau)})[(e^{H(t-\tau)}-1)^2+\epsilon^2]}. \tag{3.15}$$

The equation of motion then becomes

$$\ddot{\phi}+3H\dot{\phi}+m_\Phi^2\phi$$
$$-\frac{g^2H}{16\pi^2}\int_0^t d\tau\frac{[e^{H(t-\tau)}-1]}{(e^{H(t-\tau)})[(e^{H(t-\tau)}-1)^2+\epsilon^2]}\phi(\tau)$$
$$=\tilde{h}(t). \tag{3.16}$$

A remarkable point to note is that although time translation is *not* a symmetry in de Sitter spacetime, nonetheless the term inside the integral is in the form of a convolution, and the equation, being linear in ϕ, can be solved via Laplace transforms. This is presumably due to the O(4,1) symmetry of de Sitter space.

Notice that ϵ serves as a short-distance regulator, and the $\epsilon\to 0$ limit cannot be taken inside the integral because doing so results in a logarithmic short-distance singularity in the equation of motion. This divergence is isolated by performing an integration by parts, keeping the ϵ dependence, resulting in the equation of motion

$$\ddot{\phi}(t)+3H\dot{\phi}(t)+m_\Phi^2\phi(t)-\frac{g^2H}{16\pi^2}\left\{-\frac{\phi(t)}{H}\ln\epsilon-\frac{\phi(t)}{H}\right.$$
$$\left.+\frac{\phi_i}{H}[\ln(1-e^{-Ht})+e^{-Ht}]+\int d\tau\frac{\dot{\phi}(\tau)}{H}\xi(t-\tau)\right\}$$
$$=\tilde{h}(t), \tag{3.17}$$

$$\xi(t-\tau)=[\ln(1-e^{-H(t-\tau)})+e^{-H(t-\tau)}]. \tag{3.18}$$

This expression makes it clear that the divergence can be absorbed in a mass renormalization,

$$m_{\Phi,R}^2=m_\Phi^2+\frac{g^2}{16\pi^2}(\ln\epsilon+1). \tag{3.19}$$

In what follows we will refer to m_Φ as the renormalized mass to avoid cluttering the notation.

Taking the Laplace transform of the equations of motion, defining

$$\tilde{\phi}(s)=\int_0^\infty e^{-st}\phi(t)dt, \tag{3.20}$$

$$\tilde{\xi}(s)=\int_0^\infty e^{-st}\xi(t)dt, \tag{3.21}$$

$$\tilde{h}(s)=\int_0^\infty e^{-st}\tilde{h}(t)dt, \tag{3.22}$$

and using the initial condition $\dot{\phi}(t=0)=0$, we find

$$s^2\tilde{\phi}(s)-s\phi_i+3H(s\tilde{\phi}(s)-\phi_i)+m^2_{\Phi,R}\tilde{\phi}(s)+\frac{g^2}{16\pi^2}\tilde{\Sigma}(s)\tilde{\phi}(s)=\tilde{h}(s), \tag{3.23}$$

$$\tilde{\Sigma}(s)=-s\int_0^\infty dte^{-st}[\ln(1-e^{-Ht})+e^{-Ht}]=-s\tilde{\xi}(s). \tag{3.24}$$

B. Gibbons-Hawking temperature

It is well known that fields in de Sitter space can be thought of, in some contexts, as being embedded in a thermal bath at a temperature $H/2\pi$, the so-called Gibbons-Hawking temperature [24]. How do we see these effects in the context of our calculation?

First we notice that the Green's functions (2.23), (2.24), (2.27), and (2.28) and the kernels (3.15) and (3.18) in Eq. (3.17) obey the periodicity condition, in *imaginary time*,

$$K(t-\tau)=K\left(t-\tau+\frac{2\pi i}{H}\right). \tag{3.25}$$

These kernels can be analytically continued to imaginary time $t-\tau=-i\eta$ and expanded in a discrete series in terms of the Matsubara frequencies

$$\omega_n=\frac{2\pi n}{\beta_H},\quad \frac{1}{\beta_H}=T_H=\frac{H}{2\pi}. \tag{3.26}$$

T_H is recognized as the Gibbons-Hawking temperature in this spacetime [23]. Then as a function of imaginary time Eq. (3.18) can be written as

$$\xi(\eta)=\frac{1}{\beta_H}\sum_{n=-\infty}^{\infty}e^{i\omega_n\eta}\Xi(\omega_n), \tag{3.27}$$

$$\Xi(\omega_n)=\int_0^{\beta_H}d\eta e^{-i\omega_n\eta}\xi(\eta). \tag{3.28}$$

We find

$$\Xi(\omega_n)=-\beta_H\left(\frac{H}{\omega_n}-\delta_{n,1}\right),\quad n\geqslant 1. \tag{3.29}$$

From this expression, $\tilde{\Sigma}(s)$ can be obtained at once:

$$\tilde{\Sigma}(s)=-s\tilde{\xi}(s)=\sum_1^\infty H\left(\frac{1}{nH}-\frac{1}{s+nH}\right)-\frac{s}{s+H}. \tag{3.30}$$

This Laplace transform has simple poles at minus the Matsubara frequencies corresponding to the Hawking temperature.

Finally we obtain the solution for the Laplace transform

$$\tilde{\phi}(s)=\frac{\phi_i(s+3H)+\tilde{h}(s)}{s^2+3Hs+m^2_\Phi+(g^2/16\pi^2)\tilde{\Sigma}(s)}. \tag{3.31}$$

This expression clearly shows that the information on the "decay rate" is obtained from the imaginary part of the poles of the denominator and that the "effective magnetic field" $\tilde{h}(t)$ whose Laplace transform $\tilde{h}(s)$ is explicit in Eq. (3.31) is not relevant to understand this "decay rate." In fact as we will show shortly, the "decay rate" is determined only by the "self-energy" $\Sigma(s)$. Since our goal is to understand the "decay rate," we set $\tilde{h}(s)=0$ without loss of generality.

The denominator in the above expression is the inverse propagator for the massive scalar, evaluated at zero spatial momentum and $\tilde{\Sigma}(s)$ is the one-loop correction to the self-energy. However, from Eq. (3.30) we see that $\tilde{\Sigma}(s)$ is analytic in the s plane with simple poles at $s=-nH$, with n an integer$\neq 1$. This is a surprising result, because we expect that, as in Minkowski space, there would be a cut in the s plane, which since the particles in the loop are massless would run along the entire imaginary axis. Obviously, this is not what happens—the analytic structure is *very different* in de Sitter spacetime from that in Minkowski spacetime. In fact we find that $\tilde{\phi}(s)$ has two simple poles, perturbed from their original positions by $O(g^2)$ corrections. We will understand this result by taking the flat-space limit in a later section.

The real-time dynamics of the zero mode of the inflaton is now found by inverting the Laplace transform by complex integration:

$$\phi(t)=\int_{c-i\infty}^{c+i\infty}e^{st}\tilde{\phi}(s)\frac{ds}{2\pi i}, \tag{3.32}$$

where the Bromwich contour is taken to the right of all the singularities.

The poles of $\tilde{\phi}(s)$ can be obtained by setting,

$$s^2+3Hs+m^2_\Phi+\frac{g^2}{16\pi^2}\tilde{\Sigma}(s)=0. \tag{3.33}$$

We analyze the $4m^2_\Phi>9H^2$ and $4m^2_\Phi<9H^2$ cases separately. In the first case, with $4m^2_\Phi>9H^2$, we have that in the absence of interactions the inflaton undergoes damped oscillations redshifted by the expansion. At zeroth order the poles (3.33) are at

$$s_0^\pm=\frac{-3H}{2}\pm i\frac{\sqrt{4m^2_\Phi-9H^2}}{2}. \tag{3.34}$$

We absorb the real part of the self-energy *on shell* in a further (finite) redefinition of the renormalized mass. Since $\tilde{\Sigma}(s)$ is a meromorphic function of s, $\mathrm{Re}[\tilde{\Sigma}(s_0^+)]=\mathrm{Re}[\tilde{\Sigma}(s_0^-)]$. Then to this order the "pole mass" is given by

$$m^{*2}_\Phi=m^2_\Phi+\frac{g^2\mathrm{Re}[\Sigma(s_0^\pm)]}{16\pi^2}. \tag{3.35}$$

Obviously this mass is independent of the renormalization scheme. We also define the subtracted self-energy

$\tilde{\Sigma}^*(s)=\tilde{\Sigma}(s)-\mathrm{Re}[\tilde{\Sigma}(s_0^+)]$, and at this stage it is convenient to introduce the ''effective mass''

$$M_\Phi^2=m_\Phi^{*2}-\frac{9H^2}{4}, \tag{3.36}$$

in terms of which, to order g^2, we find

$$s^\pm=\frac{-3H}{2}\pm i\sqrt{M_\Phi^2+\frac{g^2}{16\pi^2}\tilde{\Sigma}^*(s^+)}$$
$$=\frac{-3H}{2}\pm iM_\Phi\pm i\frac{g^2\mathrm{Im}\tilde{\Sigma}(s_0^\pm)}{32\pi^2M_\Phi}+O(g^4). \tag{3.37}$$

The Bromwich countour for the inverse Laplace transform can now be deformed by wrapping around the imaginary axis and picking up the poles [19]. The resulting expression has the Breit-Wigner form of a sharp resonance centered at the ''pole mass'' that is very narrow in the weak coupling limit, leading to the time evolution of the inflaton given by

$$\phi(t)=Z\phi_ie^{-3Ht/2}e^{-\Gamma t/2}\cos(M_\Phi t+\alpha), \tag{3.38}$$

where

$$Z^2=1+\frac{9H^2}{4M_\Phi^2}-\frac{2g^2B'}{32\pi^2M_\Phi}-\frac{12g^2HB}{128\pi^2M_\Phi^3}-\frac{18g^2H^2B'}{128\pi^2M_\Phi^3},$$

$$\alpha=\arctan\left[-\frac{3H}{2M_\Phi}+\frac{g^2}{16\pi^2}\left(\frac{A'}{2M_\Phi}+\frac{9H^2A'}{8M_\Phi^3}-\frac{18H^2B}{16M_\Phi^4}\right)\right],$$

$$B=\mathrm{Im}[\tilde{\Sigma}^*(s)]_{s=-3H/2+iM_\Phi},$$

$$A'=\mathrm{Re}\left[\frac{\partial\tilde{\Sigma}^*(s)}{\partial s}\right]_{s=-3H/2+iM_\Phi},$$

$$B'=\mathrm{Im}\left[\frac{\partial\tilde{\Sigma}^*(s)}{\partial s}\right]_{s=-3H/2+iM_\Phi}. \tag{3.39}$$

The most interesting of these quantities is the damping rate, given by

$$\Gamma=\frac{g^2\mathrm{Im}\tilde{\Sigma}^*(s_0^+)}{16\pi^2M_\Phi}. \tag{3.40}$$

The imaginary part of the self-energy in the above expression can be evaluated using standard sum formulas (these sums also appear finite temperature field theory [25,26]) and is found to be

$$\Gamma=\frac{g^2\tanh\left(\frac{\beta_H\sqrt{m_\Phi^{*2}-9H^2/4}}{2}\right)}{32\pi\sqrt{m_\Phi^{*2}-9H^2/4}}=\frac{g^2\tanh(\beta_HM_\Phi/2)}{32\pi M_\Phi}. \tag{3.41}$$

This expression for Γ is a monotonically increasing function of H. When $H=0$, it matches with the flat-space decay rate

$$\Gamma_{\mathrm{Minkowski}}=\frac{g^2}{32\pi m_\Phi}. \tag{3.42}$$

We thus obtain the result that the inflaton decay proceeds more rapidly in a de Sitter background than in Minkowski spacetime.

The rate (3.41) can be written in a more illuminating manner as

$$\Gamma=\frac{g^2\coth(\beta_H\omega_0/2)}{32\pi M_\Phi}=\frac{g^2[1+2n_b(\omega_0)]}{32\pi M_\Phi}, \tag{3.43}$$

$$\omega_0=-is_0^+,\quad n_b(\omega_0)=\frac{1}{e^{\beta_H\omega_0}-1}. \tag{3.44}$$

This is a remarkable result—the decay rate is almost the same as that in Minkowski space, but in a thermal bath at the Hawking temperature [19,27]. Habib [15] has also found an intriguing relationship with the Hawking temperature in the probability distribution functional in his studies of stochastic inflation.

If we now look at the $4m_\Phi^2<9H^2$ case, we see that the inflaton ceases to propagate; i.e., its Compton wavelength approaches the horizon size and there is no oscillatory behavior in the classical evolution of the zero mode:

$$\phi(t)=\left(\frac{9H^2}{4m_\Phi^2}-1\right)^{-1/2}\phi_ie^{-3Ht/2}\sinh(|M_\Phi|t+\beta),$$

$$\tanh\beta=\frac{2|M_\Phi|}{3H}. \tag{3.45}$$

The one-loop contribution is the same as in the previous case but now the poles of $\tilde{\phi}(s)$ lie on the real axis:

$$s_0^\pm=\frac{-3H}{2}\pm\frac{\sqrt{9H^2-4m_\Phi^2}}{2},$$

$$s^\pm=\frac{-3H}{2}\pm\frac{\sqrt{9H^2-4m_\Phi^2-4(g^2/16\pi^2)\tilde{\Sigma}(s^\pm)}}{2}. \tag{3.46}$$

We define

$$\Sigma(s_0^\pm)=C\pm D \tag{3.47}$$

and absorb C into a finite mass renormalization

$$m_{\Phi}^{*2}=m_{\Phi}^{2}+\frac{g^{2}C}{16\pi^{2}}. \tag{3.48}$$

Inverting the transform we obtain

$$\phi(t)=Z\phi_{i}e^{-3Ht/2}e^{-\Gamma t/2}\sinh\left(\frac{K}{2}t+\beta\right), \tag{3.49}$$

with Z being the (finite) wave function renormalization, i.e., the residue at the poles, and

$$K=\sqrt{9H^{2}-4m_{\Phi}^{*2}}, \tag{3.50}$$

$$\Gamma=\frac{g^{2}\tan[(\pi/2)\sqrt{9-(4m_{\Phi}^{*2}/H^{2})}]}{16\pi\sqrt{9H^{2}-4m_{\Phi}^{*2}}} \quad \text{for} \quad \frac{m_{\Phi}^{*}}{\sqrt{2}}>H>\frac{2m_{\Phi}^{*}}{3}. \tag{3.51}$$

We also see that the numerator of the damping rate in Eq. (3.46) diverges at $H=m_{\Phi}^{*}/\sqrt{2}>2m_{\Phi}^{*}/3$, which indicates the breakdown of perturbation theory.

The decay rate may again be written in the form that makes explicit the effect of the Hawking temperature and the ''stimulated decay'' with the Bose-Einstein distribution function at the Hawking temperature:

$$\Gamma=\frac{g^{2}[1+2n_{b}(\omega_{0})]}{32\pi\sqrt{\frac{9}{4}H^{2}-m_{\Phi}^{*2}}}, \quad \omega_{0}=-is_{0}^{+}. \tag{3.52}$$

We can also study how the ''decay'' of the inflaton into lighter scalars modifies the ''slow-roll'' evolution such as would occur in new inflationary models. Since we are assuming a small field amplitude for our pertubation theory to make sense, we cannot treat chaotic inflationary scenarios in the same way.

For this we now set $m_{\Phi}^{2}<0$ and study the situation in which the inflaton is ''rolling'' from the top of the potential hill, during the stage of quasiexponential expansion. In this situation the amplitude expansion is valid when the zero mode is close to the origin and at early times.

In this case it is convenient to write $m_{\Phi}^{2}=-\mu^{2}$. Now the poles are on the real axis at the positions

$$s^{\pm}=\frac{-3H}{2}\pm\sqrt{\left(\frac{3H}{2}\right)^{2}+\mu^{2}-\frac{g^{2}}{16\pi^{2}}\widetilde{\Sigma}(s^{\pm})} \tag{3.53}$$

$$\approx s_{0}^{\pm}\mp\frac{g^{2}}{16\pi^{2}}\frac{\widetilde{\Sigma}(s^{\pm})}{\sqrt{(3H/2)^{2}+\mu^{2}}}, \tag{3.54}$$

$$s_{0}^{\pm}=\frac{-3H}{2}\pm\sqrt{\left(\frac{3H}{2}\right)^{2}+\mu^{2}}. \tag{3.55}$$

For $g=0$ the pole that is on the positive real axis, s_{0}^{+}, is the one that dominates the evolution of the inflaton down the potential hill (the growing mode). Since $s_{0}^{+}>0$ we find that

$$\widetilde{\Sigma}(s_{0}^{+})=\sum_{2}^{\infty}\frac{s_{0}^{+}}{n(s_{0}^{+}+nH)}>0, \tag{3.56}$$

and we conclude that, to this order, the pole on the positive real axis is shifted towards the origin. Therefore the rate of growth of the growing mode is diminished by the decay into lighter scalars and the ''rolling'' is slowed down. This is physically reasonable, since the ''decay'' results in a transfer of energy from the inflaton to the massless scalars and the rolling of the field down the potential hill is therefore slowed down by this decay process.

C. Minimally coupled massless σ

Now we turn to the $\xi_{\sigma}=0$ case. The Green's functions are given by Eqs. (2.27) and (2.28) with the Hankel functions (2.25). There are many features in common with the previous case: The kernel is translationally invariant in time and periodic in imaginary time with periodicity β_{H}. Thus it can again be expanded in terms of Matsubara frequencies and the Laplace transform carried out in a straightforward manner. However, for minimal coupling we find a new logarithmic infrared divergence in the kernel along with the ultraviolet logarithmic singularity of the conformally coupled case. Therefore, the k integral in the kernel must be performed with an infrared cutoff μ. The logarithmic ultraviolet divergence is handled in as before, and after this subtraction we find the self-energy to be

$$\begin{aligned}\tilde{\Sigma}(s)=&\overbrace{\sum_{1}^{\infty}H\left(\frac{1}{nH}-\frac{1}{s+nH}\right)-\frac{s}{s+H}}^{\xi_{\sigma}=1/6}-\frac{4H}{3}\sum_{1}^{\infty}\frac{1}{n}\left(\frac{1}{s+nH}-\frac{1}{s+3H+nH}\right)\\&+\left(\frac{1}{s}-\frac{1}{s+3H}\right)\left\{\frac{4H}{3}\ln 2-2H\left[\frac{2}{3}\ln(H/\mu)+\frac{14}{9}\right]\right\}\\&+\frac{4H}{3}\left(\frac{1}{s+H}-\frac{1}{s+2H}\right)+\frac{4H^{2}}{3}\left[\frac{1}{s^{2}}-\frac{1}{(s+3H)^{2}}\right].\end{aligned} \tag{3.57}$$

We notice that the self-energy for the conformally coupled case is contained in the above expression. It is also interesting to note that the imaginary part of the self-energy, on shell, receives no contribution from the infrared-divergent piece, which only contributes to a further renormalization of the ''pole mass'' m_Φ^{*2}. We now focus on the case $4m_\Phi^2-9H^2>0$.

The expression for the decay rate is obtained by evaluating the imaginary part of the self-energy at the pole s_0^+ [see Eq. (3.40)] and found to be

$$\Gamma=\overbrace{\frac{g^2}{32\pi}\left[\frac{[1+2n_b(\omega_0)]}{M_\Phi}\right.}^{\xi_\sigma=1/6}\left.\left(1+\frac{4H^2}{m_\Phi^{*2}}\right)+\frac{8H^3}{\pi m_\Phi^{*4}}\right], \tag{3.58}$$

with Eq. (3.44). The qualitative behavior of the decay rate as a function of H is similar to that in the conformally coupled case. We see that $\Gamma(\xi_\sigma=0)>\Gamma(\xi_\sigma=1/6)>\Gamma(\text{Minkowski})$.

D. Analytic structure of the self-energy

As mentioned earlier, the analytic structure of the self-energy in the de Sitter background is drastically different from what one would expect in flat spacetime. We find that the self-energy for the case where Φ is unstable does *not* display cuts in the s plane. Though the reason for this strange behavior is not clear at present, we can try to understand it by taking the flat-space limit, i.e., the $H\to 0$ limit. In this limit, it suffices to look at $\Sigma(s)$ for $\xi_\sigma=1/6$. It is easily shown that the additional terms in Eq. (3.25) are vanishingly small in the $H\to 0$ limit. We thus obtain

$$\lim_{H\to 0}\Sigma(s)=\lim_{H\to 0}\left[\sum_1^\infty H\left(\frac{1}{nH}-\frac{1}{s+nH}\right)-\frac{s}{s+H}\right]$$

$$=\int_H^\infty\left(\frac{1}{x}-\frac{1}{s+x}\right)dx-1 \tag{3.59}$$

$$=\ln(s/m)+\ln(m/H)-1, \tag{3.60}$$

where m is a mass scale that serves as an infrared regulator. Hence,

$$\tilde{\phi}(s)=\frac{s\phi_i}{s^2+[m_\Phi^2+(g^2/16\pi^2)\ln(m/H)-1]+(g^2/16\pi^2)\ln(s/m)}. \tag{3.61}$$

The above expression has a cut which can be chosen to run from 0 to $-\infty$, so that it is to the left of the Bromwich contour. The discrete singularities of the self-energy at $s=-nH$ have merged into a continuum in the small H limit to give a cut along the negative real axis in the s plane. In addition, there are two simple poles in the first Riemann sheet at $\pm im_\Phi+O(g^2)$. Inverting the transform yields

$$\phi(t)=Z\phi_i e^{-\Gamma t/2}\cos(m_\Phi t+\alpha)-\phi_i\frac{g^2}{16\pi^2}\int_0^\infty d\omega$$
$$\times\frac{\omega e^{-\omega t}}{[\omega^2+m_\Phi^2+(g^2/16\pi^2)\ln(\omega/m)]^2+(g^2/16\pi)^2}. \tag{3.62}$$

We see here a Breit-Wigner form plus contributions from across the cut.

A flat-space analysis with $m_\sigma\neq 0$, carried out in a previous work [19], reveals a very different picture. In particular, we find two cuts extending from $s=\pm 2im_\sigma$ to $\pm i\infty$ and two simple poles which move off into the second Riemann sheet above the two-particle threshold when $m_\Phi>2m_\sigma$. However, upon inverting the transform and taking $m_\sigma\to 0$, we obtain

$$\phi(t)=\phi_i\frac{g^2}{16\pi^2}\int_0^\infty d\omega$$
$$\times\frac{\omega\cos\omega t}{[\omega^2-m_\Phi^2-(g^2/16\pi^2)\ln(\omega/m)]^2+(g^2/32\pi)^2}. \tag{3.63}$$

A deformation of the contour of integration shows that Eqs. (3.62) and (3.63) are in fact identical and hence we have consistency (see the Appendix). Although the limits $H\to 0$ and $m_\sigma\to 0$ do not commute insofar as the analytic structure in the s plane is concerned, the time evolution obtained from the inverse Laplace transform is unambiguously the same.

IV. O(2) MODEL AND GOLDSTONE BOSONS

While the inflaton is typically taken to be a singlet of any gauge or global symmetry, there is no reason that it could not transform nontrivially under a continuous global symmetry. If this occurs and the symmetry is broken spontaneously, the interesting possibility arises of dissipation of energy into Goldstone bosons. Even if the inflaton is a singlet, one could ask about the evolution of fields that belong to multiplets of a global continuous symmetry, during inflation. Our primary motivation, however, is to study dissipative processes of the inflaton via the decay into Goldstone bosons in de Sitter spacetime as new nonequilibrium mechanisms.

The effect may be studied in the O(2) linear σ model, in which the inflaton is part of the O(2) doublet, with Lagrangian density

$$\mathcal{L}=\frac{1}{2}[e^{3Ht}\dot{\Phi}^2-e^{Ht}(\vec{\nabla}\Phi)^2+e^{3Ht}\dot{\pi}^2-e^{Ht}(\vec{\nabla}\pi)^2]$$

$$+\frac{1}{2}e^{3Ht}(\mu^2-12\xi H^2)(\Phi^2+\pi^2)-e^{3Ht}\frac{\lambda}{4!}(\Phi^2+\pi^2)^2$$

$$+h(t)\Phi. \tag{4.1}$$

Assuming that $\mu^2>12\xi H^2$, in the symmetry broken phase the field Φ acquires a vacuum expectation value (VEV)

$$\langle\Phi\rangle=\sqrt{\frac{6(\mu^2-12\xi H^2)}{\lambda}}, \tag{4.2}$$

and the Goldstone bosons are left with no mass terms, and hence are massless, minimally coupled fields in de Sitter spacetime [28]; this is a fairly well-known result.

We now write

$$\Phi(x,t)=\sqrt{\frac{6(\mu^2-12\xi H^2)}{\lambda}}+\phi(t)+\psi(x,t) \tag{4.3}$$

and use the tadpole method to impose

$$\langle\psi(x,t)\rangle=0,$$

$$\langle\pi(x,t)\rangle=0. \tag{4.4}$$

Using Eq. (4.3) it is easy to see that couplings of the form $\psi\pi^2$ are automatically induced in the Lagrangian. Carrying out the same analysis as before and keeping only terms up to first order in $\phi(t)$, we find the equation of motion

$$\ddot{\phi}+3H\dot{\phi}+m^2(t)\phi-i\frac{3\lambda\mu^2}{2\pi^2}\int_0^t d\tau\bigg[K_\phi(t,\tau)$$

$$+\frac{1}{9}K_\pi(t,\tau)\bigg]\phi(\tau)=\tilde{h}(t), \tag{4.5}$$

where

$$K(t,\tau)=\int_0^\infty k^2dk[G_k^{>2}(\tau,t)-G_k^{<2}(\tau,t)]e^{3H\tau}.$$

Here $m^2(t)$ is given by the tree-level mass term $2\mu^2$ plus contributions proportional to the tadpole $\langle\pi^2\rangle(t)$. Since the Goldstone boson is always minimally coupled to the curvature, $\langle\pi^2\rangle(t)$ will grow linearly with time and result in a nontrivial time dependence for the mass of the massive mode ϕ. In this case the methods of the earlier section are not applicable, and the definition of a ''decay rate'' will be plagued by time-dependent ambiguities.

Such a time dependence will, however, be absent in a de Sitter space that exists forever, in which case the contribution from the tadpole will simply be a divergence that is renormalized through a redefinition of the mass. In such a situation, the contribution to the decay rate from the massive mode would be subdominant due to the kinematics and the Goldstone modes would provide the dominant contribution to the decay rate. Thus, neglecting $K_\phi(t,\tau)$ the equation of motion has exactly the same form as Eq. (3.5). The subsequent analysis and results would be identical to those obtained above in the section on minimally coupled fields.

V. LANGEVIN EQUATION

Our results allow us to make contact with the issue of decoherence and the stochastic description of inflationary cosmology [12–15].

Decoherence is a fundamental aspect of dissipational dynamics and in the description of nonequilibrium processes in the early universe [13–15]. The relationship between fluctuation and dissipation as well as a stochastic approach to the dynamics of inflation can be explored by means of the Langevin equation.

This section is devoted to obtaining the corresponding Langevin equation for the nonequilibrium expectation value of the inflaton field (zero mode) in the one-loop approximation within the model addressed in this article.

The first step in deriving a Langevin equation is to determine the ''system'' and ''bath'' variables, and subsequently integrate out the ''bath'' variables to obtain an influence functional for the ''system'' degrees of freedom [13–15].

We first separate out the expectation value and impose the tadpole conditions as follows:

$$\Phi^\pm=\phi^\pm+\psi^\pm,\quad \langle\Phi^\pm(\vec{x},t)\rangle=\phi^\pm(t),\quad \langle\sigma^\pm(\vec{x},t)\rangle=0. \tag{5.1}$$

The nonequilibrium effective action is defined in terms of the Lagrangian given by Eq. (3.2) as

$$e^{iS_{\rm eff}}=\int\mathcal{D}\psi^+\mathcal{D}\psi^-\mathcal{D}\sigma^+\mathcal{D}\sigma^-e^{i\{S_0[\phi^+]-S_0[\phi^-]\}}$$

$$\times e^{i\{S_0[\psi^+]-S_0[\psi^-]\}}e^{i\{S_0[\sigma^+]-S_0[\sigma^-]\}}$$

$$\times e^{i\{S_{\rm int}[\psi^+,\phi^+,\sigma^+]-S_{\rm int}[\psi^-,\phi^-,\sigma^-]\}},$$

where S_0 represents the action for free fields, and $S_{\rm int}$ contains all the remaining parts of the action. The effective action for the zero modes is obtained by tracing out all the degrees of freedom corresponding to the nonzero modes (or the ''bath'' variables), in a consistent loop expansion. It is useful to introduce the center-of-mass $[\phi(t)]$ and relative $[R(t)]$ coordinates as

$$\phi^\pm(t)=\phi(t)\pm\frac{R(t)}{2}. \tag{5.2}$$

Remembering the definitions of the Green's functions in Eqs. (2.29)–(2.31), we expand exp($iS_{\rm int}$) up to order g^2 and consistently impose the tadpole condition to obtain the effective action per unit volume (because the expectation value is taken to be translationally invariant),

$$\frac{S_{\rm eff}}{\Omega}=\int dt[\mathcal{L}_0(\phi^+)-\mathcal{L}_0(\phi^-)]-\frac{ig^2}{16\pi^3}\int_{-\infty}^{\infty}dt\int_{-\infty}^{t}dt'\int d^3k a^3(t)a^3(t')R(t)\phi(t')[G_k^{>2}(t,t')-G_k^{<2}(t,t')]$$
$$-\frac{ig^2}{32\pi^3}\int_{-\infty}^{\infty}dt\int_{-\infty}^{\infty}dt'\int d^3k a^3(t)a^3(t')R(t)R(t')[G_k^{>2}(t,t')+G_k^{<2}(t,t')]+\tilde{h}(t)R(t)+\text{terms independent of }\phi. \tag{5.3}$$

As before $\tilde{h}(t)$ results from subtracting the ϕ-independent contribution of the tadpole from $h(t)$.

Now, it is not difficult to verify that the nonequilibrium Green's functions given by Eqs. (2.16) and (2.19) and obeying the boundary conditions (2.22) have the property,

$$G_k^{>}(t,t')=[G_k^{<}(t,t')]^*. \tag{5.4}$$

This property guarantees that the second term in the effective action is real, while the third term is pure imaginary. The imaginary, nonlocal, acausal part of the effective action gives a contribution to the path integral that may be written in terms of a stochastic field as

$$\exp\left[-\frac{1}{2}\int dt\int dt' R(t)\mathcal{K}(t,t')R(t')\right]$$
$$\propto\int \mathcal{D}\xi\mathcal{P}[\xi]\exp\left[i\int dt\xi(t)R(t)\right],$$
$$\mathcal{P}[\xi]=\exp\left[-\frac{1}{2}\int dt\int dt'\xi(t)\mathcal{K}^{-1}(t,t')\xi(t')\right], \tag{5.5}$$

with

$$\mathcal{K}(t,t')=-\frac{g^2}{16\pi^3}\int d^3k a^3(t)a^3(t')[G_k^{>2}(t,t')+G_k^{<2}(t,t')]. \tag{5.6}$$

The nonequilibrium path integral now becomes

$$Z\propto\int \mathcal{D}\phi\mathcal{D}R\mathcal{D}\xi\mathcal{P}[\xi]\exp\left\{i\left[S_{\rm reff}(\phi,R)+\int dt\xi(t)R(t)\right]\right\}, \tag{5.7}$$

with $S_{\rm reff}[\phi,R]$ being the real part of the effective action and with $\mathcal{P}[\xi]$ the Gaussian probability distribution for the stochastic noise variable.

The Langevin equation is obtained via the saddle point condition [13]

$$\left.\frac{\delta S_{\rm reff}}{\delta R(t)}\right|_{R=0}=\xi(t), \tag{5.8}$$

leading to

$$\ddot{\phi}+3H\dot{\phi}+m_\Phi^2\phi+i\frac{g^2}{16\pi^3}\int_{-\infty}^{t}dt'\int d^3k a^3(t')[G_k^{>2}(t,t')-G_k^{<2}(t,t')]\phi(t')-\tilde{h}(t)=\frac{\xi(t)}{a^3(t)}, \tag{5.9}$$

where the stochastic noise variable $\xi(t)$ has Gaussian, but colored correlations

$$\langle\langle\xi(t)\rangle\rangle=0,\quad \langle\langle\xi(t)\xi(t')\rangle\rangle=\mathcal{K}(t,t'). \tag{5.10}$$

The double brackets stand for averages with respect to the Gaussian probability distribution $\mathcal{P}[\xi]$. The noise is colored and additive. Because of the properties of the Green's functions (5.4), the noise kernel (which contributes to the imaginary part of the effective action) is simply the *real part* of $G_k^{>2}(t,t')$. The dissipative kernel (which gives rise to the nonlocal term in the Langevin equation) on the other hand is given by the *imaginary part* of $G_k^{>2}(t,t')$. Consequently, the *fluctuation-dissipation theorem* is revealed in the *Hilbert transform* relationship between the real and imaginary parts of the analytic function $G_k^{>2}(t,t')$. The equation of motion (3.11) is now recognized to result from Eq. (5.9) by taking the average over the noise.

One could now obtain the corresponding Fokker-Planck equation that describes the evolution of the probability distribution function for ϕ [12,29].

We must emphasize here that all the results discussed so far in this section are independent of the specifics of the FRW background spacetime, the masses of the fields, and the temperature of the initial thermal state. The importance of the Langevin equation resides at the fundamental level in that it provides a direct link between fluctuation and dissipation including all the memory effects and multiplicative aspects of the noise correlation functions.

In particular in stochastic inflationary models it is typically assumed [12] that the noise term is Gaussian and white (uncorrelated). This simplified stochastic description, in terms of Gaussian white noise, leads to a scale invariant spectrum of scalar density perturbations [12]. Although this description is rather compelling, within the approximations made in our analysis we see that for a σ field with an arbitrary mass and arbitrary couplings to the curvature there is no regime in which the correlations of the noise term (5.10) can be described by a Markovian δ function in time [13]. One could speculate that some other couplings or higher-order effects or, perhaps, some peculiar initial states could lead to a Gaussian white noise, but then one concludes that Gaussian white noise correlations are by no means a generic feature of the microscopic field theory.

We can speculate that our result of Gaussian but correlated noise could have implications for stochastic inflation. In particular it *may* lead to departures from a scale-invariant (Harrison-Zel'dovich) spectrum of primordial scalar density perturbations, which can now be calculated within a particular microscopic model using the nonequilibrium field-theoretical tools described in this work. Of course this requires further and deeper study, which is beyond the scope of this article.

VI. CONCLUSIONS AND FURTHER QUESTIONS

In this article we have studied the decay of inflatons into massless scalars in de Sitter space. The motivation was to understand the nonequilibrium mechanisms of inflaton relaxation during the stage of quasi-inflationary expansion in early universe cosmology, in old, new, and stochastic inflationary scenarios. Chaotic inflation models cannot be treated within our approximation scheme since the field will typically start at values that are large enough so that the amplitude expansion is not valid. Having recognized the inherent difficulties in defining a decay rate in a time-dependent background, we have given a practical definition that requires understanding the real-time evolution of the inflaton interacting with lighter fields. This led us to a model in which an inflaton interacts with massless scalars via a trilinear coupling, allowing for both minimal and conformal coupling to the curvature.

We obtained a decay rate at one loop order that displays a remarkable property. With some minor modifications it can be interpreted as the stimulated decay of the inflaton in a *thermal bath* at the Hawking temperature. This decay rate is larger than that of Minkowski space because of the Bose-enhancement factors associated with the Hawking temperature. The decay rate of the minimally coupled case is larger than that of the conformal case which in turn is larger than the Minkowski rate. We have shown explicitly that in the case of new inflation, the dissipation of the inflaton energy associated with the decay into conformally coupled massless scalars slows down further the rolling of the inflaton down the potential hill. We have also studied decay into Goldstone bosons.

To establish contact with the stochastic inflationary scenario, we derived the Langevin equation for the coarse-grained expectation value of the inflaton field to one-loop order. We find that this stochastic equation has a Gaussian but correlated (colored) noise. The two-point correlation function of the noise and the dissipative kernel fulfill a generalized fluctuation-dissipation relation.

There are several potentially relevant implications of our results for old, new, and stochastic inflation. In the case of old inflation we see that dissipative processes in the metastable maximum can contribute substantially to the ''equilibration'' of the inflaton oscillations, and more so because of the enhanced stimulated decay for a large Hubble constant. In the case of new inflation, we have seen that these dissipative effects help slow the rolling of the inflaton field down the potential hill, possibly extending the stage of exponential expansion.

Within the context of stochastic inflation, our results point to the possibility of incorporating deviations from a scale-invariant spectrum of primordial scalar density perturbations by the noise correlations, which manifest the underlying microscopic correlations of the field theory. This is a possibility that is worth exploring further.

Our results also point to further interesting questions.

(1) Is it possible to understand at a more fundamental level the connection between the decay rate and the Hawking temperature of a bath *in equilibrium*? In particular, is this connection maintained at higher orders?

(2) Is it possible to relate the ''decay rate'' to the rate of particle production via the interaction? Such a relation in Minkowski spacetime is a consequence of the existence of a spectral representation for the self-energy but such a representation is not available in a time-dependent background.

Answers to these questions will undoubtedly offer a much needed deeper understanding of nonequilibrium processes in inflationary cosmology.

ACKNOWLEDGMENTS

D.B. would like to thank H. J. de Vega for useful references and illuminating discussions, and NSF for support through Grant No. PHY-9302534. D.B. and S.P.K. would like to thank Salman Habib for illuminating conversations and comments. S.P.K. and R.H. were partially supported by the U.S. Department of Energy under Contract No. DE-FG02-91-ER40682.

APPENDIX

In this appendix, we show explicitly that Eqs. (3.62) and (3.63) are in fact identical expressions. The behavior of $\phi(t)$ in Minkowski [given by Eq. (3.57)] was obtained in Ref. [19] where m_σ was allowed to have nonzero values. Now, the results from this reference must obviously coincide with the flat-space limits ($H\to 0$) derived in this paper. To see this, consider Eq. (3.63):

$$\phi(t)=\phi_i\frac{g^2}{16\pi^2}\int_0^\infty d\omega \times\frac{\omega\cos\omega t}{[\omega^2-m_\Phi^2-(g^2/16\pi^2)\ln(\omega/m)]^2+(g^2/32\pi)^2}$$
$$=\frac{\phi_i}{\pi}\mathrm{Re}\Bigg[\int_0^\infty -i\,d\omega(\omega e^{i\omega t}) \times\Bigg(\frac{1}{\omega^2-m_\Phi^2-(g^2/16\pi^2)\ln(\omega/m)-ig^2/32\pi} - \frac{1}{\omega^2-m_\Phi^2-(g^2/16\pi^2)\ln(\omega/m)+ig^2/32\pi}\Bigg)\Bigg].$$

The branch cut due to the logarithm in the denominator runs from 0 to $+\infty$, and the contour of integration may be chosen to run from 0 to $+\infty$ on the first sheet. It is easy to see that the integrand has four simple poles, one in each quadrant. In particular, the pole in the first quadrant is at

$$\omega_1=m_\Phi+\frac{g^2}{32\pi^2 m_\Phi}\ln(m_\Phi/m)+i\frac{g^2}{64\pi m_\Phi}. \tag{A1}$$

Furthermore, the exponential is well behaved at infinity in the first quadrant. We can therefore deform the contour of integration so that it runs from 0 to $+i\infty$, picking up the residue from one simple pole. Writing $\omega=iz$, and noticing that the logarithm picks up an imaginary part $i\pi/2$ we now have

$$\phi(t)=\frac{\phi_i}{\pi}\mathrm{Re}\Bigg[\int_0^\infty dz(ize^{-zt})\left(\frac{1}{-z^2-m_\Phi^2-(g^2/16\pi^2)\ln(z/m)-ig^2/16\pi}-\frac{1}{-z^2-m_\Phi^2-(g^2/16\pi^2)\ln(z/m)}\right)$$

$$+\mathrm{Res}\left(\frac{-i\omega e^{i\omega t}}{\omega^2-m_\Phi^2-(g^2/16\pi^2)\ln(\omega/m)-ig^2/32\pi}\right)\Bigg|_{\omega=\omega_1}\Bigg]$$

$$=\frac{\phi_i}{\pi}\Bigg\{-\frac{g^2}{16\pi}\int_0^\infty dz\frac{ze^{-zt}}{[z^2+m_\Phi^2+(g^2/16\pi^2)\ln(z/m)]^2+(g^2/16\pi)^2}$$

$$+2\pi\mathrm{Re}\left[\lim_{\omega\to\omega_1}\left(\frac{\omega e^{i\omega t}(\omega-\omega_1)}{\omega^2-m_\Phi^2-(g^2/16\pi^2)\ln(\omega/m)-ig^2/32\pi}\right)\right]\Bigg\}.$$

Absorbing the real part of the self-energy on shell as an additional mass renormalization, and going through the subsequent algebra we obtain our result

$$\phi(t)=\phi(0)\left(1+\frac{g^2}{32\pi^2 m_\Phi{}^2}\right)e^{-g^2t/64\pi m_\Phi}\cos(m_\Phi t)-\phi_i\frac{g^2}{16\pi^2}\int_0^\infty dz\frac{ze^{-zt}}{[z^2+m_\Phi^2+(g^2/16\pi^2)\ln(z/m)]^2+(g^2/16\pi)^2}.$$

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