Structure of critical lines in quenched lattice QCD with the Wilson quark action

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The structure of critical lines of a vanishing pion mass for the Wilson quark action is examined in quenched lattice QCD. Numerical evidence is presented that the critical lines spread into five branches beyond $\beta=5.6$–5.7 at zero temperature. It is also shown that the critical lines disappear in the deconfined phase for the case of finite temperatures. [S0556-2821(97)01915-2]

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I. INTRODUCTION

The problem of species doublers in lattice fermion formulations is well known. The Wilson quark action, commonly used in numerical simulations of lattice QCD, avoids this problem at the cost of an explicit breaking of chiral symmetry. This means that the conventional Nambu-Goldstone mechanism associated with the spontaneous breakdown of chiral symmetry cannot be invoked to predict the existence of massless pion for this quark action at a finite lattice spacing. It has been found, however, that massless pion does appear with the Wilson quark action once the bare quark mass m_0, or equivalently the hopping parameter $K=1/(2m_0+8)$, is tuned appropriately. It is now widely accepted that the pion mass vanishes along a line $K=K_c(\beta)$, called the critical line, which runs from $K_c(\beta=0)\approx 1/4$ in the strong-coupling limit $\beta=6/g^2=0$ to $K_c(\beta=\infty)=1/8$ in the weak-coupling limit $\beta=\infty$ on the (β,K) plane [1].

One of us suggested some years ago [2] that the massless pion which appears for the Wilson quark action may be understood as a zero mode of a second-order phase transition spontaneously breaking parity-flavor symmetry. It has also been argued that, while there are only two critical lines in the region of strong coupling, there exist ten critical lines in the weak-coupling region, paired into five branches which reach the weak-coupling limit $\beta=\infty$ at $M\equiv 1/2K=\pm 4$, ± 2, and 0. Some analytic [3] and numerical [4] results supporting this view have been previously reported.

More recently arguments have been presented [5] that the end points of five branches move away from the weak-coupling limit for finite temporal lattice sizes corresponding to finite temperatures, so that the critical lines form five cusps at some $\beta\neq\infty$, above which no critical line exists. The existence of the second critical line paired with the conventional one and a formation of a cusp by the two critical lines have been confirmed numerically in a recent full QCD simulation at finite temperature [5].

In this paper we present further investigation of the structure of critical lines for the Wilson quark action using quenched QCD. One of the motivations of the present work is to find evidence for the existence of five pairs of critical lines toward weak coupling at zero temperature. We expect this structure to remain in the quenched approximation, in which case the structure should be computationally much easier to confirm than in full QCD. Another motivation is to examine how the structure of critical lines changes for a finite temporal lattice size N_t corresponding to finite temperature. In particular we wish to examine how the first-order deconfinement phase transition, which takes place at a fixed value $\beta=\beta_c(N_t)$ independent of K for quenched QCD, affects the critical lines. Our expectation would be that massless pion, and hence also the critical lines, disappears in the high-temperature phase above the deconfinement transition $\beta>\beta_c(N_t)$.

II. RESULTS AT ZERO TEMPERATURE

Finding the location of critical lines throughout the (β,K) plane requires a convenient indicator for their positions. For this purpose we employ the number of conjugate gradient iterations $N_{\rm CG}$ necessary to invert the Wilson fermion matrix to a given value of the residual. For infinite volume this quantity increases toward critical lines and is expected to diverge in between a pair of critical lines, where parity is spontaneously broken, due to the presence of zero modes of the Wilson fermion matrix. On a finite lattice where zero mode eigenvalues should be slightly shifted away from zero, we expect a peak in $N_{\rm CG}$ across each pair of critical lines. Let us note that the phase structure on the (β,K) plane should be symmetric under $K\to -K$. For $K>0$ we thus expect three peaks in $N_{\rm CG}$ as a function of K toward weak coupling.

We have carried out a measurement of the number $N_{\rm CG}$ for a set of values of $K>0$ at $\beta=5.0$, 5.5, 5.6, 5.7, on a 6^4 lattice, and at $\beta=5.8$, 5.9, 6.0, on a 12^4 lattice. Results are obtained by an average over 60 (for 6^4) or 20 (for 12^4) configurations separated by 1000 sweeps of the pseudo-heat-bath algorithm. The stopping condition is taken to be $r=\sqrt{\|b-Dx\|^2/3V}<10^{-6}$ where b represents a wall source of unit strength for all space-time sites and colors, and $V=L^4$ the lattice volume with $L=6$ or 12. Runs are made only for $K>0$ since the structure of critical lines should be symmetric under $K\to -K$.

Our results are shown in Figs. 1(a) and 1(b). On a 6^4 lattice [Fig. 1(a)] we observe an appearance of three peaks at $M\approx 3.0$, 1.5, and 0 starting at $\beta=5.6$–5.7. On a 12^4 lattice [Fig. 1(b)] the three peaks become increasingly sharper as β increases. For the peaks at $M\approx 3.0$ and 1.5, the peak position shifts toward larger values of M, albeit only slightly over the range $\beta=5.5$–6.0 examined in the simulation. These results are consistent with the expected structure of multiple critical lines appearing at $\beta>5.6$–5.7 [6].

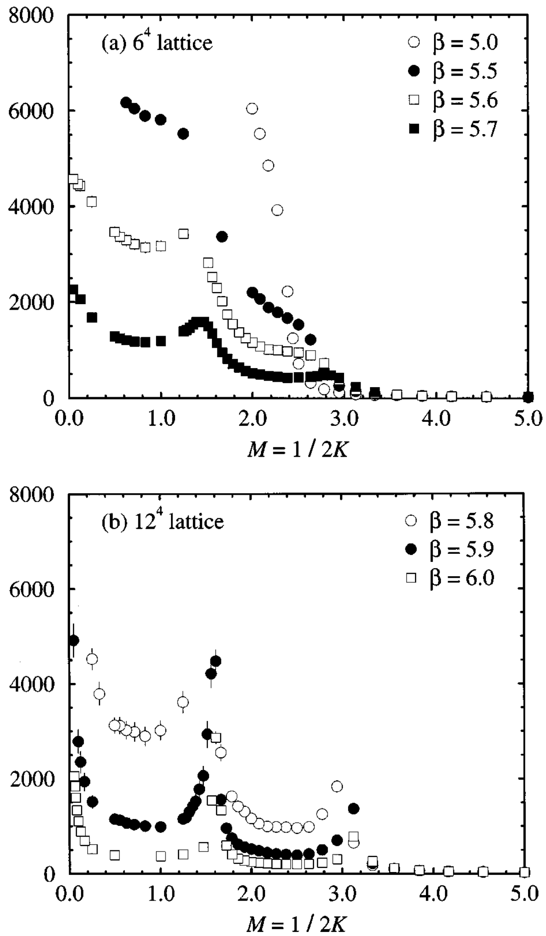

FIG. 1. Number of conjugate gradient iterations N_{CG} necessary to invert the Wilson fermion matrix with a wall source of unit strength for all sites and colors as a function of $M=1/2K$. The stopping condition is chosen to be $r=\sqrt{\|b-Dx\|^2/3V}<10^{-6}$ with $V=L^4$ the lattice volume. (a) 6^4 lattice for $\beta=5.0$–5.7, (b) 12^4 lattice for $\beta=5.8$–6.0.

In order to confirm that the peaks of N_{CG} observed at $\beta>5.6$–5.7 correspond to five critical lines of massless pion, we have calculated the pion mass at $\beta=6.0$ on a $10^3\times20$ and a $16^3\times20$ lattice. The pion propagator is averaged over

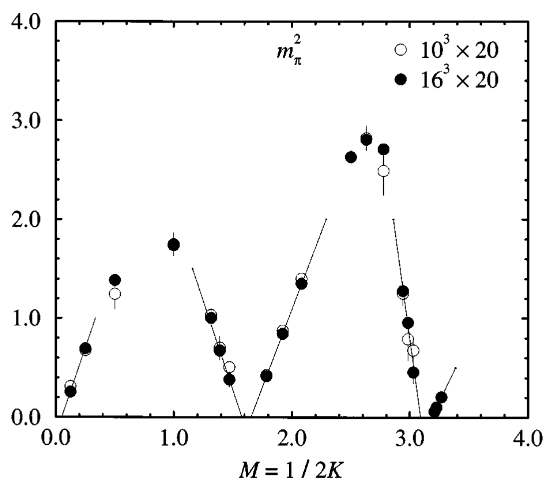

FIG. 2. Pion mass squared m_π^2 at zero temperature as a function of $M=1/2K$ at $\beta=6.0$. Open circles are for a $10^3\times20$ lattice, and filled circles for a $16^3\times20$ lattice. Lines show linear fits of m_π^2 in M.

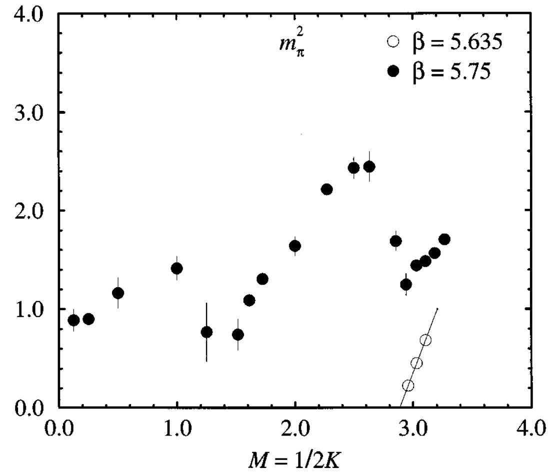

FIG. 3. Pion screening mass squared m_π^2 as a function of $M=1/2K$ on a $12^2\times24\times4$ lattice. Open circles are for $\beta=5.635$ and filled circles for $\beta=5.750$. The line shows a linear fit of m_π^2 in M.

10–20 configurations separated by 1000 pseudo-heat-bath sweeps. The stopping condition for quark propagator calculation is taken to be $r=\|D^\dagger b-D^\dagger Dx\|/\|x\|<10^{-10}$ with b a point source at the origin. We should note that we have en-

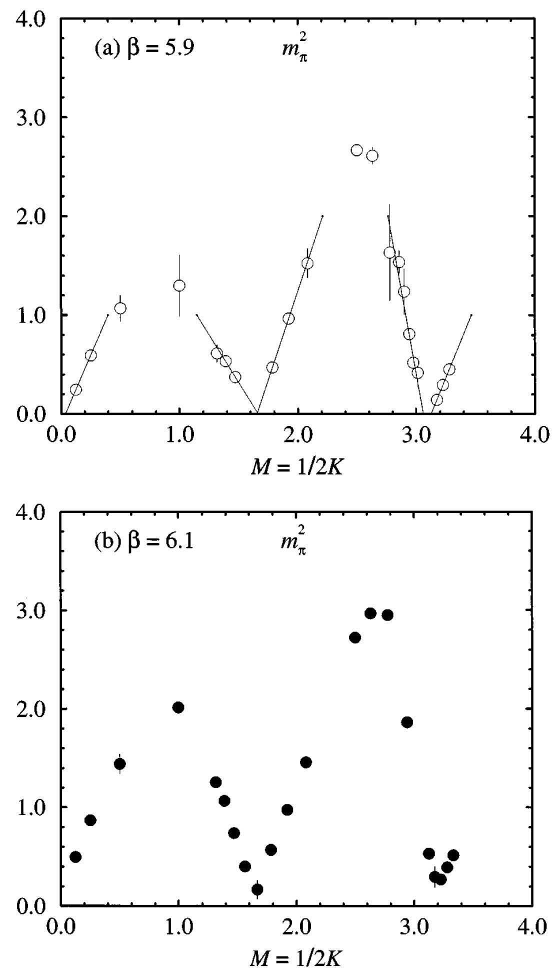

FIG. 4. Pion screening mass squared m_π^2 as a function of $M=1/2K$ on a $16^3\times8$ lattice. (a) $\beta=5.9$, (b) $\beta=6.1$. The lines in (a) show linear fits of m_π^2 in M.

TABLE I. Pion mass m_π as a function of K.

K	m_π		K	m_π		K	m_π	
	$10^3\times 20$ $\beta=6.0$	$16^3\times 20$ $\beta=6.0$		$12^3\times 4$ $\beta=5.635$	$12^3\times 4$ $\beta=5.750$		$16^3\times 8$ $\beta=5.9$	$16^3\times 8$ $\beta=6.1$
0.153		0.454(9)	0.153		1.306(12)	0.150		0.718(16)
0.155		0.320(13)	0.157		1.251(13)	0.1525	0.673(09)	0.627(24)
0.1558		0.245(22)	0.161	0.829(14)	1.218(15)	0.155	0.543(10)	0.520(51)
0.165	0.823(60)	0.674(84)	0.165	0.672(16)	1.201(22)	0.1575	0.384(13)	0.544(100)
0.1675	0.890(123)	0.978(27)	0.169	0.475(41)		0.160		0.730(36)
0.170	1.118(53)	1.130(30)	0.170		1.117(51)	0.166	0.647(110)	
0.180	1.578(78)	1.646(10)	0.175		1.300(40)	0.168	0.723(46)	
0.190	1.679(38)	1.675(27)	0.190		1.563(50)	0.170	0.900(28)	1.365(20)
0.200	1.622(22)	1.622(7)	0.200		1.559(36)	0.1725	1.114(105)	
0.240	1.184(11)	1.162(15)	0.220		1.488(17)	0.175	1.240(48)	
0.260	0.936(16)	0.918(8)	0.250		1.281(38)	0.180	1.278(190)	1.717(09)
0.280	0.654(32)	0.644(7)	0.290		1.143(23)	0.190	1.615(26)	1.723(06)
0.340	0.712(28)	0.619(62)	0.310		1.043(31)	0.200	1.633(13)	1.650(09)
0.360	0.838(69)	0.821(58)	0.330		0.862(93)	0.240	1.235(60)	1.208(05)
0.380	1.016(26)	1.002(43)	0.400		0.876(170)	0.260	0.984(08)	0.988(07)
0.500	1.322(45)	1.318(30)	0.500		1.189(50)	0.280	0.687(18)	0.755(14)
1.000	1.116(68)	1.176(10)	1.000		1.079(72)	0.300		0.411(114)
2.000	0.823(27)	0.833(17)	2.000		0.948(32)	0.320		0.633(36)
4.000	0.558(21)	0.505(31)	4.000		0.942(58)	0.340	0.614(49)	0.859(13)
						0.360	0.731(40)	1.032(10)
						0.380	0.784(58)	1.121(26)
						0.500	1.139(137)	1.420(09)
						1.000	1.034(65)	1.201(42)
						2.000	0.769(37)	0.931(14)
						4.000	0.495(42)	0.704(22)

countered ‘‘exceptional’’ configurations near the peaks of $N_{\rm CG}$, on which the pion propagator takes a W shape as a function of time. Since such exceptional configurations are never encountered in our full QCD simulations [5], we think that they are an artifact of the quenched approximation in which configurations with small fermionic eigenvalues are not suppressed contrary to full QCD. We have therefore excluded the exceptional configurations from our propagator average.

We plot our results for the pion mass squared in Fig. 2 as a function of $M=1/2K$ [see Table I for numerical values]. A reasonable agreement of results for 10^3 and 16^3 spatial sizes shows that finite spatial size effects are not severe in our data. We observe that the results are consistent with the existence of four more critical values of vanishing pion mass in addition to the conventional one located on the right most side of the figure. Making a linear extrapolation of m_π^2 in $1/K$ using two or three data points, we find $1/2K_c=0.052(14)$, $1.577(63)$, $1.653(5)$, $3.091(11)$, and $3.1848(44)$ for the five critical values from left to right.

III. RESULTS AT FINITE TEMPERATURE

For a finite temporal lattice size N_t corresponding to finite temperature, the critical line should be defined by vanishing of the pion screening mass extracted from the pion propagator for large spatial separations. In quenched QCD an interesting question is how the structure of critical lines changes across the deconfinement transition which takes place at $\beta=\beta_c(N_t)$ independent of the hopping parameter K of valence quark. In the confining phase $\beta<\beta_c(N_t)$, it is natural to expect that the structure of critical lines remains qualitatively the same as at zero temperature except for a possible shift of their location by a magnitude depending on N_t. On the other hand, the pion screening mass would not vanish in the deconfined phase $\beta>\beta_c(N_t)$, and hence critical lines would disappear. For values of K close to the conventional critical value, such a behavior has been seen in a previous work [7]. Our aim here is to carry out a systematic study over the entire interval of the hopping parameter.

Our simulation is made for two temporal lattice sizes $N_t=4$ and 8. The critical coupling is known to be $\beta_c(N_t=4)=5.69226(41)$ [8] and $\beta_c(N_t=8)=6.0609(9)$ [9]. For $N_t=4$, runs are made at $\beta=5.635$ and 5.75 on a $12^3\times 4$ lattice, and for $N_t=8$ at $\beta=5.90$ and 6.10 on a $16^3\times 8$ lattice. Quark propagators are computed on a periodically doubled lattice $12^2\times 24\times 4$ in the former case and on the same lattice $16^3\times 8$ for the latter. Numerical results for the pion screening mass are listed in Table I.

In Fig. 3 we plot by open circles the pion mass squared on a $12^2\times 24\times 4$ lattice as a function of $M=1/2K$ at $\beta=5.635$, which is in the confining phase. We find only a single critical point located at $1/2K_c=2.886(17)$. Exploring the region $K > K_c$ at $K = 0.2$–0.4, we found that the gauge configurations are dominated by exceptional ones for which the conjugate gradient solver for quark propagator takes over 5000–10 000 iterations to converge, while for $K < K_c$ a thousand iterations or less are sufficient. We take this as an indication that the parity-broken phase extends over $\infty > K > K_c$ at $\beta = 5.635$ for $N_t = 4$, i.e., the system is still in the strong-coupling region where only one critical line exists for $K > 0$.

For $\beta = 5.75$, which is in the deconfined phase, results for the pion screening mass squared are shown by filled circles in Fig. 3. While an overall pattern of m_π^2 as a function of $M = 1/2K$ is similar to the zero-temperature case shown in Fig. 2, the pion mass does not vanish for any value of K for the present case. Thus the critical lines are absent in the deconfined phase as expected.

Our results for $N_t = 8$ at $\beta = 5.9$ (confined phase) is plotted in Fig. 4(a). We observe two critical values at $1/2K \approx 3.0$. The pion mass also appears to vanish at $1/2K \approx 1.65 - 1.66$ and ≈ 0.04, although our data is not sufficiently precise to resolve if there are two critical values separated by a narrow gap or m_π^2 has a small but nonvanishing value. In Fig. 4(b) we show how the behavior changes at $\beta = 6.1$ (deconfined phase). We clearly observe nonvanishing of pion mass in the deconfined phase.

IV. CONCLUSION

In this paper we have presented numerical results which are consistent with the existence of five pairs of critical lines beyond $\beta = 5.6$–5.7 in quenched QCD at zero temperature.

It has recently been suggested that the two critical lines forming each pair merge at a finite value of β, turning into a single line [10]. Our results show that this possibility is not realized up to $\beta = 6.0$ in quenched QCD.

We have also shown that the critical lines are absent above the deconfinement phase transition $\beta > \beta_c(N_t)$. Since the deconfinement transition is of first order, we expect the critical lines to be sharply cutoff at the critical value $\beta = \beta_c(N_t)$. Our data, however, are not precise enough to examine this point in detail.

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[1] N. Kawamoto, Nucl. Phys. **B190** [**FS3**], 617 (1981).
[2] S. Aoki, Phys. Rev. D **30**, 2653 (1984).
[3] S. Aoki, Phys. Rev. D **33**, 2399 (1986); **34**, 3170 (1986); Phys. Rev. Lett. **57**, 3136 (1986); Nucl. Phys. **B314**, 79 (1989).
[4] S. Aoki and A. Gocksch, Phys. Lett. B **231**, 449 (1989); **243**, 1092 (1990); Phys. Rev. D **45**, 3845 (1992).
[5] S. Aoki, A. Ukawa, and T. Umemura, Phys. Rev. Lett. **76**, 873 (1996).
[6] S. Aoki, Prog. Theor. Phys. Suppl. **122**, 179 (1996).
[7] Y. Iwasaki, T. Tsuboi, and Y. Yoshié, Phys. Lett. B **222**, 269 (1989).
[8] M. Fukugita, M. Okawa, and A. Ukawa, Nucl. Phys. **B337**, 181 (1990).
[9] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lütgemeier, and B. Petersson, Nucl. Phys. **B469**, 419 (1996).
[10] M. Creutz, Report No. hep-lat/9608024 (unpublished).