PHYSICAL REVIEW D VOLUME 56, NUMBER 3 1 AUGUST 1997

Constraining almost degenerate three-flavor neutrinos

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(Received 5 December 1996)

We discuss constraints on a scenario of almost degenerate three-flavor neutrinos imposed by the solar and the atmospheric neutrino anomalies, hot dark matter, and in particular by the neutrinoless double β decay experiments. It is found that in the Majorana version of the model the region with relatively large θ_{13} is favored and the model is not compatible with the popular small-θ_{12} Mikheyev-Smirnov-Wolfenstein solution of the solar neutrino problem. A constraint on the CP-violating phases including the one characteristic to Majorana neutrinos is also obtained. The stability of our conclusion against the uncertainty of the nuclear matrix elements of double β decay is briefly addressed. [S0556-2821(97)02015-8]

PACS number(s): 14.60.Pq, 23.40.−s, 26.65.+t, 95.35.+d

There exist several experimental hints which indicate that most probably neutrinos have tiny masses and flavor mixings. The first is the solar neutrino deficit observed in four different experiments: the chlorine, the Kamiokande II-III, GALLEX, and SAGE [1–4]. It became highly unlikely that the data of various experiments can be reconciled with any sensible modifications of the standard solar model. The second is the atmospheric neutrino anomaly, the large deviation in the observed ratio ν_μ/ν_e from the expectation of the Monte Carlo simulations [5,6]. While the anomaly was not observed within the statistics of the NUSEX and the Frejus experiments [7,8], the evidence of the Kamiokande and IMB detectors are so impressive that they force us to seriously consider the anomaly. The presence of the anomaly is also supported by the newest tracking detector, Soudan 2 [9].

The possible third hint for neutrino masses comes from the cosmological model with cold and hot dark matter (CHDM). Neutrinos are the only known candidates for the hot component. They could be responsible for the large-scale structure formation in a way consistent with the Cosmic Background Explorer (COBE) observation of anisotropy of cosmic microwave background [10–12]. While less direct compared with the first and the second hints, it provides a good motivation for examining the possibility of neutrino masses of a few eV range.

It has been pointed out by various authors that if at least one of the neutrino states has mass of the dark matter scale and if there is a hierarchy in two Δm^2, the difference in squared masses, the accelerator, and the reactor experiments put powerful constraints on mixing angles [13–15]. It is very remarkable that the mixing pattern of neutrinos is determined to be essentially unique [13] if one imposes the additional constraints that come from the requirement of solving either the solar neutrino problem or the atmospheric neutrino anomaly, together with that from neutrinoless double β decays [16,17].

The problem with the above framework with only three-flavor neutrinos (i.e., without sterile neutrinos) is that one cannot account for the solar neutrino deficit, the atmospheric neutrino anomaly, and the hot dark matter simultaneously. The only known possibility that can accommodate these two phenomena as well as supplying neutrino masses appropriate for hot dark matter within the standard three-flavor framework is the case of almost degenerate neutrinos (ADN's). An incomplete list of earlier references on ADN's is in [20].

In this paper we discuss the constraints that can be imposed on such an almost degenerate neutrino scenario from the solar and atmospheric neutrino observations as well as the terrestrial neutrino experiments. We will point out that, in the case of Majorana neutrinos, the neutrinoless double β decay experiment is of key importance. In particular, the solar neutrino and the double β decay experiments constrain the mixing angle θ_{13} not to be small.

Let us start by defining more precisely what we mean by the almost degenerate neutrinos. Due to the requirement of solving the solar and the atmospheric neutrino problems the two Δm^2 should have values $\lesssim 10^{-5}$ and $\sim 10^{-2}$ eV ^{2}, respectively. This implies that three neutrino states are degenerate up to the accuracy of 0.1 eV. Then, the requirement from the hot dark matter hypothesis implies that they must have masses of the order of a few to several eV [10–12]. Then, the degeneracy in the masses is better than 0.01 eV, hence the name of almost degenerate neutrinos (ADN's).

For definiteness, we assign the smaller Δm^2 to $\Delta m^2_{12} \equiv m_2^2 - m_1^2$ and the larger to Δm^2_{13}. It should be noticed that this can be done without loss of generality. Despite the almost degeneracy in neutrino masses there is a hierarchy in $\Delta m^2; \Delta m^2_{13} \simeq \Delta m^2_{23} \gg \Delta m^2_{12}$. It allows us to simplify greatly formulas for the oscillation probabilities. With neutrino mixing matrix $U_{\alpha i}$ they read

$$P(\nu_\beta \to \nu_\alpha) = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2\left(\frac{\Delta m^2_{13} L}{4E}\right), \tag{1}$$

$$1-P(\nu_\alpha\to\nu_\alpha)=4|U_{\alpha 3}|^2(1-|U_{\alpha 3}|^2)\sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right), \tag{2}$$

where the CP-violating terms have been dropped in the approximation with the mass hierarchy; i.e., they are obtained under the so-called one mass scale dominance approximation.

We use the standard form of the Cabibbo-Kobayashi-Maskawa quark mixing matrix:

$$U=\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \tag{3}$$

for the neutrino mixing matrix. From Eqs. (1) and (2) one can see that the accelerator and the reactor experiments probe the mixing angles θ_{13} and θ_{23}.

We first summarize the constraints from the accelerator and the reactor experiments. Unlike the case of dark-matter-mass neutrinos with hierarchy the constraints from these terrestrial experiments are very mild. With $\Delta m^2 \lesssim 0.01$ eV^2 only the relevant channel is ν_e disappearance experiments whose most extensive runs were done at Bugey [18] and at Krasnoyarsk [19]. We note that there is no constraint on s_{13}^2 for $\Delta m_{13}^2 < 7\times 10^{-3}$ eV^2, where we have made the substitutions of the variables in [18] and [19] $\theta\to\theta_{13}, \Delta m^2\to\Delta m_{13}^2$, which follow from the present approximation with the mass hierarchy. Notice that there is an allowed region at large s_{13}^2, whose dominant part will be excluded by the solar neutrino constraint as we will see below.

Let us now address the constraint from the solar neutrino experiments. While extensive analyses have been done within the two-flavor mixing scheme the full three-flavor analysis of the solar neutrino experiments is very rare. To our knowledge it has been carried out quite recently for the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [21] by Fogli, Lisi, and Montanino [22]. We do not know corresponding analyses for the vacuum oscillation solution, but there was an attempt [23]. For this reason let us focus on the MSW solution in this paper.

It has been known for some time that the solar neutrino deficit is explained by the interplay of averaged oscillations due to the sizable value of s_{13}^2 and the MSW mechanism which operates at small s_{12}^2 [24]. The extensive analysis of Fogli *et al.*, exploring full three-flavor parameter space under the assumption of mass-squared hierarchy, confirmed the existence of such a large s_{13}^2 MSW solution [22]. They observed a number of new features; the well-known small-s_{12} and large-s_{12} solutions fuse into a single one at around $s_{13}^2=0.33$ and this large-s_{13} solution extends up to $s_{13}^2\simeq 0.6$. The large-s_{13} solution is interesting because the "two-flavor" parameters Δm_{12}^2 and s_{12}^2 differ from that obtained by the two-flavor analysis. At the largest value of $s_{13}, s_{13}^2=0.6$, $s_{12}^2\simeq 2\times 10^{-2}$ and $\Delta m_{12}^2\simeq 4\times 10^{-6}$ eV^2. In contrast, the best fit values of the two-flavor analysis are $s_{12}^2\simeq 2\times 10^{-3}$ and $\Delta m_{12}^2\simeq 5.2\times 10^{-6}$ eV^2. We will see below that the constraint from neutrinoless double β decays does indeed prefer the large-s_{13} solution.

We discuss the constraint from neutrinoless double β decays, which applies only to the Majorana neutrinos. We will see that it gives rise to the strongest constraint. Observation of no neutrinoless double β decay implies the constraint on $\langle m_{\nu e}\rangle$, which can be written in our notation of the mixing matrix as

$$\langle m_{\nu e}\rangle =|c_{12}^2c_{13}^2m_1e^{-i(\beta+\gamma)}+s_{12}^2c_{13}^2m_2e^{i(\beta-\gamma)}+s_{13}^2m_3e^{2i(\gamma-\delta)}|, \tag{4}$$

where β and γ are the extra CP-violating phases characteristic to Majorana neutrinos [25,26].

Let us first discuss the CP-invariant case $(e^{2i\beta},e^{i(\beta+3\gamma-2\delta)}=\pm 1)$ because it is easier to understand. In this case the phase factors in Eq. (4) can be reduced to the CP parities η_j of mass eigenstates j with masses m_j [27]. Under the circumstance of almost degeneracy with which we are working we can approximate the expressions of $\langle m_{\nu e}\rangle$ by ignoring the mass differences. Then, it further simplifies depending upon the pattern of the CP parities of three neutrinos. Let us take the convention that $\eta_1=+$ and denote them collectively as $(\eta_1,\eta_2,\eta_3)\equiv(1,e^{2i\beta},e^{i(\beta+3\gamma-2\delta)})=(++-)$, etc. Then,

$$r\equiv\frac{\langle m_{\nu e}\rangle}{m}=\begin{cases} 1 \quad \text{for } (+++), \\ |1-2s_{13}^2| \quad \text{for } (++-), \\ |1-2s_{12}^2c_{13}^2| \quad \text{for } (+-+), \\ |1-2c_{12}^2c_{13}^2| \quad \text{for } (+--). \end{cases} \tag{5}$$

Let us refer to the ratio $\langle m_{\nu e}\rangle/m$ as r hereafter.

We take the mixed dark matter model with $\Omega_{\text{total}}=1$ to estimate the masses of neutrinos. The cold-hot dark matter (CHDM) model with $\Omega_{\text{total}}=1$ might have problems with age of the universe. The measurement by the Hubble Space Telescope [28] gave a value of $h=0.8\pm 0.17$, where the Hubble constant H_0 is given by h as $H_0=100h$ km/s Mpc. The value of H_0 suggests that the total contribution Ω_{total} by matter to the density parameter should be smaller than 1 in order to have the age of the universe greater than 10 G yr. Our attitude to this problem is that we must take at least two

σ uncertainty in the observed value of the Hubble constant seriously because the systematic errors in various methods of measuring the Hubble constant do not appear to be well understood. (For a recent status of the measurement of the Hubble constant, see, e.g., [29]).

We assume that 20–30 % of the universe is shared by the hot dark matter. We note that the neutrino contribution to the Ω parameter is $\Omega_\nu = (\Sigma m_i / 91.5\ \mathrm{eV}) h^{-2}$ [30]. The CHDM model with three kinds of neutrinos has been analyzed by Pogosyan and Starobinsky [11] and they concluded that the allowed region is given by $0.55 \lesssim h \lesssim 0.7, 0.25 \lesssim \Omega_\nu \lesssim 0.3$. If we take these values, we obtain $2.3\ \mathrm{eV} \lesssim m_j \simeq m \lesssim 4.5$ eV as masses of almost degenerate neutrinos. We will use $m = 2.3$ and 4.5 eV for neutrino masses as reference values in the following analysis.

We impose the experimental bound on $\langle m_{\nu e} \rangle$ obtained by negative results of neutrinoless double β decays. The most stringent one to date is $\langle m_{\nu e} \rangle < 0.6$ eV derived in the ^{76}Ge experiment done by the Heidelberg-Moscow group [17]. The bound is based on the calculation of the nuclear matrix elements by [31]. It implies the bounds on the r parameter $r \leqslant 0.26$ and 0.13, for neutrino masses $m = 2.3$ and $m = 4.5$ eV, respectively.

An immediate consequence of the constraint from neutrinoless double β decay is that the first pattern of the CP parity $(+++)$ is excluded. Other patterns are not immediately excluded but their parameters are subject to the constraint. In Figs. 1(a) and 1(b) we have plotted the allowed regions of the neutrinoless double β decay constraint for each respective pattern of the CP parity for (a) $r \leqslant 0.13$ and (b) $r \leqslant 0.26$, respectively. The upper rectangular region is for the CP parity $(++-)$, the lower-left band for $(+--)$, and the lower-right band for $(+-+)$. Also plotted in Fig. 1 as darker shaded parts are the allowed regions with 90% C.L. for the three-flavor MSW solution to the solar neutrino problem obtained by Fogli *et al.* [22]. It is actually the superposition of the allowed regions with the mass squared difference Δm^2_{12} from 10^{-6} to 1.0×10^{-4} eV^2.

From these figures one can draw several conclusions. The CP parity pattern of $(+-+)$ is excluded since the two allowed regions do not overlap. The patterns $(++-)$ and $(+--)$ are allowed and they prefer the large-s_{13} solution of the solar neutrino problem. The ''two-flavor'' large angle MSW solution (i.e., large-s_{12} and small-s_{13} solutions) is also marginally allowed. The small angle MSW solutions, which are drawn almost on the axis of $s^2_{12} = 0$ in Fig. 1, are not compatible with the double β decay constraint for $m = 2.3$ and 4.5 eV. In closer detail, with a neutrino mass of 4.5 eV $(r \leqslant 0.13)$ the solution exists for $3.2 \times 10^{-6}\ \mathrm{eV}^2 \lesssim \Delta m^2_{12} \lesssim 6.8 \times 10^{-5}\ \mathrm{eV}^2$ only if $s^2_{13} \gtrsim 0.3$. With $m = 2.3$ eV $(r \leqslant 0.26)$ it exists for $3.2 \times 10^{-6}\ \mathrm{eV}^2 \lesssim \Delta m^2_{12} \lesssim 1.0 \times 10^{-4}\ \mathrm{eV}^2$ only if $s^2_{13} \gtrsim 0.02$.

The reason that θ_{13} has to be large for a solution to exist in the case of $\langle m_{\nu e} \rangle \leqslant 0.6$ eV is because putting $\theta_{13} = 0$ in Eq. (5) gives $r = 1$ for $e^{2i\beta} = 1$ or $r = |\cos 2\theta_{12}|$ for $e^{2i\beta} = -1$, and even the large angle MSW solution with $\theta_{13} = 0$ gives $\cos 2\theta_{12} \gtrsim 0.33$, which does not satisfy the constraint from the neutrinoless double β decay experiments. In other words, the MSW solutions in the two-flavor framework give so large a $\cos 2\theta_{12}$ that sufficient cancellation does not occur between the contributions to m_{ν_e} from the first and the second mass eigenstates.

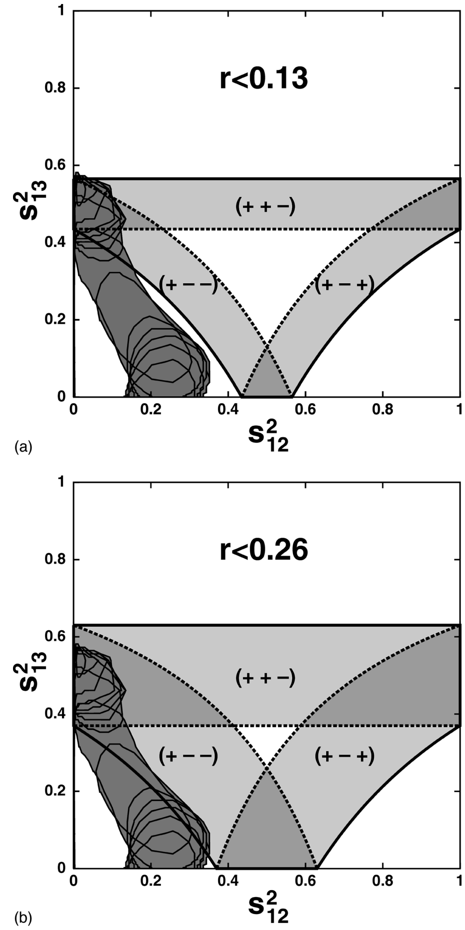

FIG. 1. The lighter shaded strips bounded by the thicker solid and dashed lines are the allowed regions of the neutrinoless double β decay constraints for each pattern of the CP parity in the CP-conserving case with (a) $r < 0.13$ ($\langle m_{\nu e} \rangle < 0.6$ eV, $m = 4.5$ eV) and (b) $r < 0.26$ ($\langle m_{\nu e} \rangle < 0.6$ eV, $m = 2.3$ eV), respectively. The allowed regions in the general CP-violating cases are the whole areas bounded by the thicker solid lines. The darker shaded areas bounded by the thinner solid lines are the allowed regions with 90% C.L. for the three-flavor MSW solution of the solar neutrino problem obtained by Fogli *et al.* [22].

In general CP-noninvariant cases, we have to keep the two CP-violating phases β and γ in Eq. (4). Namely, we have

$$\langle m_{\nu e} \rangle = m | c^2_{13} (e^{-i\beta} c^2_{12} + e^{i\beta} s^2_{12}) + e^{i(3\gamma - 2\delta)} s^2_{13} | \geqslant m | c^2_{13} (1 - \sin^2\beta \sin^2 2\theta_{12})^{1/2} - s^2_{13} |, \tag{6}$$

where we have ignored the mass differences and the equality in the second line holds when

$$\arg(e^{-i\beta} c^2_{12} + e^{i\beta} s^2_{12}) = 3\gamma - 2\delta + (2n+1)\pi, \tag{7}$$

where n is an integer. Note that the constraint from neutrinoless double β decays becomes even more stringent if the CP-violating phases β, γ, δ do not satisfy the relation (7). Then, our task is to look for the region which satisfies $r \leqslant$

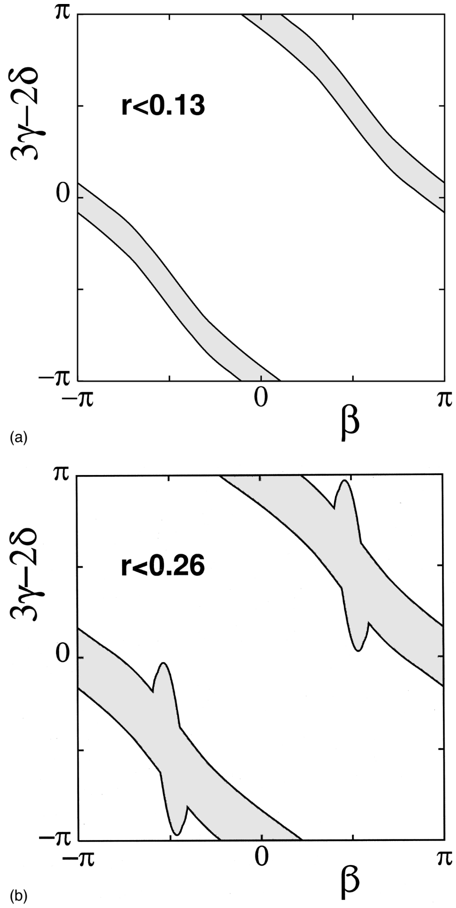

FIG. 2. The shaded areas are the allowed regions for a combination of CP-violating phases obtained from the neutrinoless double β decay experiments and the solar neutrino analysis with 90% C.L. by Fogli *et al.* [22]. (a) and (b) are for $r<0.13$ ($\langle m_{\nu e}\rangle<0.6$ eV,$m=4.5$ eV) and for $r<0.26$ ($\langle m_{\nu e}\rangle<0.6$ eV,$m=2.3$ eV), respectively.

0.13 and 0.26, respectively, with β, γ, and δ unconstrained. In the procedure s_{12}^2 and s_{13}^2 take all possible values within the constraint of the solar neutrino solution of [22]. The resulting allowed region has the same boundary, as depicted by the thicker solid lines in Figs. 1(a) and 1(b), as that of the CP-conserving cases but covers the whole region inside the boundary. For $r\leqslant 0.13$ which is obtained from $h=0.7$ and $\Omega_\nu=0.3$, the solution exists only if $s_{13}^2 \gtrsim 0.3$; for $r\leqslant 0.26$, only if $s_{13}^2\gtrsim 0.02$.

It should be emphasized that, irrespective of whether CP is violated or not, the small-s_{12} MSW solution, which is favored by theorists most, is disfavored in the CHDM model with almost degenerate neutrino masses.

On the other hand, we can get a condition for the CP-violating phases β, γ, and δ by imposing both constraints from neutrinoless double β decays and from the solar neutrino deficits with s_{12}^2 and s_{13}^2 constrained by the three-flavor analysis of [22]. The results are shown in Figs. 2(a) and 2(b), where the allowed regions are located in the neighborhood of the line $\beta+3\gamma-2\delta=\pm\pi$. In these plots the CP-conserving cases with the patterns $(+++)$,$(++-)$, $(+-+)$, and $(+--)$ correspond to the points $(\beta,3\gamma-2\delta)=(0,0),\pm(\pi,\pi),\pm(\pi,-\pi);(0,\pm\pi),(\pm\pi,0);$ $\pm(\pi/2,(\beta,3\gamma-2\delta)=(0,0),\pm(\pi,\pi),\pm(\pi,-\pi);(0,\pm\pi),$ $(\pm\pi,0);\pm(\pi/2,-\pi/2);\pm(\pi/2,\pi/2)$, respectively. From this we can verify in Figs. 2(a) and 2(b) that the CP-conserving cases with the CP-parity patterns $(+++)$ and $(+-+)$ are indeed excluded both for $r\leqslant 0.13$ and $r\leqslant 0.26$.

Now let us address the question of theoretical uncertainties of the bound on $\langle m_{\nu e}\rangle$ on which our discussion has heavily relied. We do not know any precise way of estimating the theoretical uncertainties in the calculation of the nuclear matrix elements. Therefore, we simply discuss to what extent the bound on $\langle m_{\nu e}\rangle$ is stable against the varying computation of the nuclear matrix elements. This issue is addressed by the experimental group itself [17]. They quote seven calculations of the nuclear matrix elements. Among them four calculations [31–35] result in a tighter bound than $\langle m_{\nu e}\rangle<0.6$ eV. One leads to a somewhat looser bound $\langle m_{\nu e}\rangle<0.78$ eV [37] but the computation does not contain the effect of the pairing correlations. The remaining two appear to be problematical. The somewhat old calculation by Engel, Vogel, and Zirnbauer [36] gives the bound $\langle m_{\nu e}\rangle<1.38$ eV, but it does not use a realistic nucleon-nucleon force. The weakest bound $\langle m_{\nu e}\rangle<1.76$ eV is obtained by using the nuclear matrix elements computed by Pantis *et al.* [37]. However, the computation of the paper is based on a different quasiparticle random phase approximation (QRPA) than that exploited by the other authors. Namely, the authors take the BCS-like state which contains Cooper pairs not only of proton-proton and neutron-neutron (as usual) but also of proton-neutron. We do not know if such a QRPA calculation is able to describe consistently overall features of the nuclear β decays. Therefore, it is difficult, at least at the moment, to judge the reliability of the resultant bound on $\langle m_{\nu e}\rangle$ derived by using their matrix elements. A similar remark is made by the experimental group [17].

Thus, the bound on $\langle m_{\nu e}\rangle$ appears to be rather stable within the usual QRPA computations of the nuclear matrix elements which take into account a realistic nucleon-nucleon force. Nevertheless, one may ask to what extent our conclusion is stable against the change in the upper limit of $\langle m_{\nu e}\rangle$. Since we do not know exactly the theoretical uncertainties of the nuclear matrix elements, we try to illuminate how the constraints can be affected by simply taking the weakest bound by Pantis *et al.* [37], or by just assuming a factor of $\sim$ 3 uncertainty in $\langle m_{\nu e}\rangle$.

We take the bound $\langle m_{\nu e}\rangle<1.76$ eV and reexamine the constraints obtained before. In Figs. 3(a) and 3(b) we present the parameter regions allowed by the neutrinoless double β decay constraint and the three-flavor MSW solution of Fogli *et al.* Again the figures summarize them for the general CP-violating cases as well as for each respective pattern of the CP parities for (a) $r\leqslant 0.39$ and $r\leqslant 0.77$ (b) corresponding to $m = 4.5$ and $m=2.3$ eV, respectively. With such a weak bound, there is a slight change in our conclusion; a part ($m = 4.5$ eV) or the whole region ($m = 2.3$ eV) of the ''two-flavor'' large-angle MSW solution with $\theta_{13}=0$ becomes allowed. The CP parity pattern $(+-+)$ which was excluded with previous discussion with $\langle m_{\nu e}\rangle<0.6$ eV is now partially allowed. Nonetheless, we should stress that even if we take a factor of 3 looser bound the ''two-flavor'' small-angle MSW solution is still disfavored by the ADN

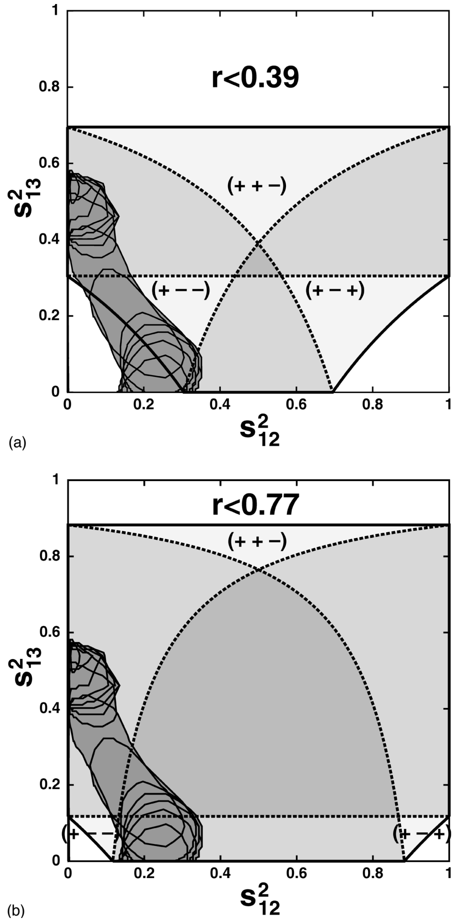

FIG. 3. The same as in Fig. 1 but with a factor of 3 weaker bound $\langle m_{\nu e}\rangle < 1.76$ eV; (a) $r<0.39$ ($m=4.5$ eV) and (b) $r<0.77$ ($m=2.3$ eV).

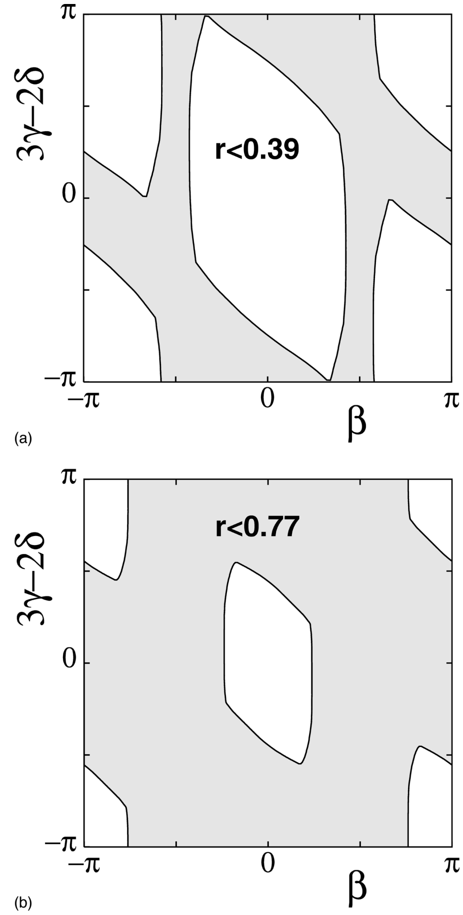

FIG. 4. The same as in Fig. 2 but with a factor of 3 weaker bound $\langle m_{\nu e}\rangle < 1.76$ eV; (a) $r<0.39$ ($m=4.5$ eV) and (b) $r<0.77$ ($m=2.3$ eV).

scenario. In Figs. 4(a) and 4(b) the similar constraints on CP-violating phases with the bound $\langle m_{\nu e}\rangle < 1.76$ eV are obtained for (a) $m = 4.5$ eV and (b) $m = 2.3$ eV.

Finally, let us briefly discuss the constraints from the atmospheric neutrino anomaly. There have been several three-flavor analyses of the atmospheric neutrino anomaly [38–40]. Among them, the most recent and the most detailed are the ones done by one of the authors [39], and by Fogli, Lisi, Montanino, and Scioscia [40]. The analyses by these two groups give rise to slightly different 2σ allowed regions on the $s^2_{13}-s^2_{23}$ parameter plane. The difference stems from their treatments of the data of the NUSEX [7] and Frejus [8] experiments which are included in [40] and are not in [39]. As far as the constraint for θ_{23} is concerned, it is concluded in either analysis [39,40] that the allowed region with 90% C.L. for $\Delta m^2_{13} \sim 5\times 10^{-3}$ eV^2 has to satisfy $s^2_{23} \gtrsim 1/4$. However, there is a difference between the two analyses on the allowed region for θ_{13}. If one includes the data of all the experiments of atmospheric neutrinos [40], then the solution with small-s_{13} is allowed. On the other hand, if one considers only the multi-GeV Kamiokande data [39], the solution with $s^2_{13} \lesssim 0.1$ is excluded at 90% confidence level. As we have seen above, the allowed region for $r \leqslant 0.13$ exists for rather large values of s_{13}, so the difference of the two analyses [39,40] turns out to be irrelevant for $m \gtrsim 3$ eV.

To summarize, we have discussed the almost degenerate three-flavor neutrino scenario as a simultaneous solution to the solar, atmospheric, and dark matter problems. We have shown, using the constraints from neutrinoless double β decays as well as these observational data of the solar and atmospheric neutrinos, that a large value of s^2_{13} is favored, leaving a little room for solutions with small s^2_{13} and large s^2_{12}. The neutrinoless double β decay constraint imposed in ADN makes the small-angle MSW solution untenable in this scenario. If three neutrinos turn out to be degenerate in masses and if precise values of s^2_{12} and s^2_{13} are both determined experimentally, then we get information on the relation among the CP-violating phases β, γ, and δ.

We also briefly addressed the question of uncertainty of the nuclear matrix elements in double β decay and the issue of stability of our conclusion against the uncertainty. We have shown that even if we allow a factor of 3 uncertainty in $\langle m_{\nu e}\rangle$ our conclusion, disfavor of the small-angle MSW solution in the ADN scenario, remain unaltered, leaving a minor change of partial allowance of the CP-parity pattern $(+-+)$.

The authors would like to thank Professor K. Muto for informative discussions on theoretical status of the calculation of the nuclear matrix elements in double β decay, and N. Okada and S. Sasaki for discussions on astrophysical aspects of dark matter scenarios. One of us (H.M.) is grateful to George Fuller for illuminating conversations on CHDM cosmology, and to the Institute for Nuclear Theory, University of Washington for partial support during the completion of this work. We have been supported in part by Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture under Grant No. 0560355, and H.M. was also supported by Grant-in-Aid for Scientific Research on Priority Areas under Grant No. 08237214.

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