Dipenguin contribution to D^0-$\bar{D}^0$ mixing

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We study the impact of the four-quark dipenguin operator on D^0-$\bar{D}^0$ mixing. It is shown to contribute to the short distance piece at the same order of magnitude as the box diagram. [S0556-2821(97)03515-7]

The phenomenon of meson mixing has been studied for a long time. Observed in the K^0-$\bar{K}^0$ and B^0-$\bar{B}^0$ systems, it provides an extremely sensitive test of the standard model (SM) and is essential for CP violation in the neutral meson system.

It is well known that because of the Glashow-Iliopoulos-Maiani (GIM) cancellation mechanism and the large mass of the top quark, the short distance box diagram dominates in the B^0-$\bar{B}^0$ system and constitutes a significant fraction of the K^0-$\bar{K}^0$ mixing amplitude. The case of the D^0-$\bar{D}^0$ system is somewhat special: The b-quark contribution to the fermion loop of the box diagram providing a $\Delta C = 2$ transition is diminished by a tiny V_{ub} Cabibbo-Kobayashi-Maskawa (CKM) matrix element. Thus, only the light quark mass difference guarantees that mixing does take place. The effect vanishes in the limit of SU(3) invariance. All of that results in an estimated value for Δm_D being of the order of 10^{-17} GeV if only short distance contributions are taken into account [1,2].

Calculating a box diagram and constructing the effective Hamiltonian one realizes that the smallness of the short distance piece is guaranteed by a factor of $(m_s^2 - m_d^2)^2 / M_W^2 m_c^2$ [2]:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{8\pi \sin^2 \theta_W} \xi_s \xi_d \frac{(m_s^2 - m_d^2)^2}{M_W^2 m_c^2} [\bar{u}\gamma_\mu(1+\gamma_5)c$$
$$\times \bar{u}\gamma_\mu(1+\gamma_5)c + 2\bar{u}(1-\gamma_5)c\,\bar{u}(1-\gamma_5)c], \quad (1)$$

with $\xi_i = V_{ic}^* V_{iu}$. Here, the b-quark contribution is dropped. This leads to the following expression for Δm_D^{box} [1,2]:

$$\Delta m_D^{\text{box}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{4\pi \sin^2 \theta_W} \xi_s \xi_d \frac{4}{3} \frac{(m_s^2 - m_d^2)^2}{M_W^2 m_c^2} f_D^2 m_D (B_D - 2B_D')$$

$$\approx 0.5 \times 10^{-17} \left[\frac{m_s}{0.2}\right]^4 \left[\frac{f_D}{f_\pi}\right]^2 \text{ GeV}, \quad (2)$$

with $f_\pi \simeq 132$ MeV, $f_D \simeq 165$ MeV, and $B_D = B_D' = 1$ in the usual vacuum saturation approximation to

$$\langle D^0 | O_1 | \bar{D}^0 \rangle = \frac{8}{3} \frac{f_D^2 m_D^2}{2m_D} B_D,$$

$$\langle D^0 | O_2 | \bar{D}^0 \rangle = -\frac{5}{3} \frac{m_D^2}{m_c^2} \frac{f_D^2 m_D^2}{2m_D} B_D',$$

$$O_1 = \bar{u}\gamma_\mu(1+\gamma_5)c\,\bar{u}\gamma_\mu(1+\gamma_5)c,$$

$$O_2 = \bar{u}(1-\gamma_5)c\,\bar{u}(1-\gamma_5)c. \quad (3)$$

In contrast with K-meson mixing, the appearance of the second operator can be traced to the fact that the mass of the external c quark provides a large momentum scale. As one can see from Eq. (3.7) of [1], the inclusion of the b quark further decreases the box diagram contribution.

In this paper we would like to address an additional contribution to the short-distance D-meson mixing amplitude which is topologically distinct from the box diagram—the so-called double-penguin or "dipenguin" operator. This operator was initially introduced in [3] for the K^0-$\bar{K}^0$ mixing amplitude and has been subsequently studied in [4] in application to K^0-$\bar{K}^0$ as well as to B^0-$\bar{B}^0$ systems. It was shown to be marginally important in the former and completely negligible in the latter case. Here we will introduce the dipenguin operator for the $\Delta C = 2$ transitions. It will be shown that this operator contributes to the D-meson mass difference at the same order of magnitude as the usual box diagram.

The effective operator relevant to dipenguin $\Delta C = 2$ transition can be obtained from the usual $\Delta C = 1$ penguin vertex (we neglect a tiny dipole contribution):

$$\Gamma_\mu^a = -\frac{G_F}{\sqrt{2}} \frac{g_s}{4\pi^2} F_1 \bar{u}\gamma_\mu(1+\gamma_5)\frac{\lambda^a}{2}c(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)A_\nu^a. \quad (4)$$

Here F_1 is a modified Inami-Lim function [5]. Using unitarity of the CKM matrix it reads $F_1 = \Sigma_i \xi_i F_1^i = \xi_s(F_1^s - F_1^d) + \xi_b(F_1^b - F_1^d)$. It is common to discard the b-quark contribution to F_1 as being suppressed by small V_{ub} factors. Note, however, that by keeping it we *enhance* the F_1 by approximately 20–30 %. Also, for the intermediate b quark this vertex (as well as the following Hamiltonian) is *local*. From Eq. (4) we obtain the effective Hamiltonian
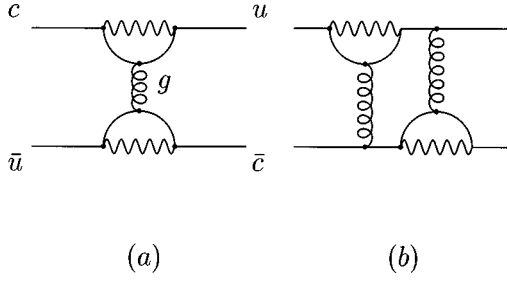
FIG. 1. Dipenguin diagram (a) and a possible long distance contribution (b).

$$\mathcal{H}_{dp} = -\frac{G_F^2}{128\pi^2}\frac{\alpha_s}{\pi}F_1^2\{[\bar{u}\gamma_\mu(1+\gamma_5)\lambda^a c]\partial^\mu\partial^\nu$$

$$\times[\bar{u}\gamma_\nu(1+\gamma_5)\lambda^a c] - [\bar{u}\gamma_\mu(1+\gamma_5)\lambda^a c]$$

$$\times \Box[\bar{u}\gamma_\mu(1+\gamma_5)\lambda^a c]\}. \tag{5}$$

In what follows, we denote two operators entering Eq. (5) as $\tilde{O}_1$ and $\tilde{O}_2$. In order to study the size of the dipenguin effects in charm mixing we derive an estimate of the Δm_D and compare with the usual box diagram contribution. To do that, in addition to the usual vacuum saturation approximation, we use perturbative QCD (PQCD) in order to independently estimate the Δm_D^{dp} by simply calculating the transition amplitude from Feynman diagram that determines the effect [Fig. 1(a)]. We believe that PQCD can provide a reliable order-of-magnitude estimate of the dipenguin contribution since the momentum transferred through the gluon line in Fig. 1(a) is relatively large, $Q^2 \sim -m_c^2$.

Using the equations of motion and neglecting the up-quark mass we obtain, for the first operator in Eq. (5)

$$\tilde{O}_1 = \bar{u}\gamma_\mu(1+\gamma_5)\lambda^a c\partial^\mu\partial^\nu\bar{u}\gamma_\nu(1+\gamma_5)\lambda^a c$$

$$\simeq m_c^2\bar{u}(1-\gamma_5)\lambda^a c\,\bar{u}(1-\gamma_5)\lambda^a c. \tag{6}$$

Before we compare the estimate of the Δm_D^{dp} with the relevant box diagram contribution we would like to note that the dipenguin diagram does not have a power dependence upon the internal quark masses. The leading behavior of the Inami-Lim function is logarithmic in $m_{s(d)}$, and the estimate of the operator brings about a power dependence upon the *external* quark masses, i.e., m_c. This feature distinguishes this contribution from that of the usual box diagram. Recalling the fact that the dominant contribution to K and B mixing amplitudes is proportional to the square of the top quark mass, it is not surprising that this effect is negligible in the K and B sectors. It is the fact of the ''reduced'' heavy quark dependence of the amplitude of D^0-$\bar{D}^0$ mixing which makes the dipenguin operator contribution effectively enhanced.

Employing vacuum saturation method to estimate matrix elements we obtain

$$\langle D^0|\tilde{O}_1|\bar{D}^0\rangle = \frac{16}{9}\frac{f_D^2 m_D^4}{2m_D}B,$$

$$\langle D^0|\tilde{O}_2|\bar{D}^0\rangle = -\frac{32}{9}\frac{f_D^2 m_D^2(2m_c^2-m_D^2)}{2m_D}B', \tag{7}$$

with B and B' being the bag parameters. In addition to the vacuum saturation we assumed that each derivative acting on the quark field involves an average momentum of the quark. This yields

$$\Delta m_D^{dp} = 2\langle D^0|\mathcal{H}_{dp}|\bar{D}^0\rangle$$

$$= \frac{G_F^2}{72\pi^2}\frac{\alpha_s}{\pi}F_1^2(m_b^2,m_s^2,m_d^2)f_D^2 m_D(m_D^2-4m_c^2). \tag{8}$$

This formula deserves some additional discussion. In the case of D^0-$\bar{D}^0$ mixing the chief effect comes from the light quark sector. This is true for both the box and dipenguin diagrams, and makes the calculation a little more involved—one cannot simply discard the external quark momenta (masses). That is why one must use a modified expression for the Inami-Lim function [5]. The leading contribution to F_1^i comes from the integral

$$F_1^i(m_i^2,Q^2) = -4\int_0^1 dx\,x(1-x)\ln\left[\frac{m_i^2}{M_W^2}-\frac{Q^2}{M_W^2}x(1-x)\right]$$

$$= -4\left[\frac{1}{6}\ln\frac{m_i^2}{M_W^2}+\Pi\left(\frac{Q^2}{m_i^2}\right)\right], \tag{9}$$

where $\Pi(Q^2/m_i^2)$ was defined in [5] for $Q^2>0$. In our case, of course, $Q^2<0\simeq -m_c^2$. The necessity of the second term becomes obvious if one looks at the chiral limit, wherein the first logarithm in Eq. (9) blows up. The final result for F_1 involves a delicate cancellation among these contributions, yielding a final result $|F_1|=0.01$–0.02 (whereas $F_1\sim 1$ if the momentum flow is discarded). Comparing Eqs. (2) and (8) we find

$$\left|\frac{\Delta m_D^{dp}}{\Delta m_D^{box}}\right| \approx \frac{\alpha_s}{8\pi}\frac{F_1^2(m_b^2,m_s^2,m_d^2)m_D^4}{|\xi_s\xi_d|(m_s^2-m_d^2)^2}, \tag{10}$$

where we have put $m_c\approx m_D$ and $\alpha_s\simeq 0.4$. The relative size of the box and dipenguin contribution shows that the latter is of the same order of magnitude as the box diagram (our estimate gives ~ 20–50% depending on the choice of quark masses). This is not surprising if one recalls that higher order QCD corrections tend to ''smooth out'' a power-like GIM suppression, just as in the case of B-meson decays. This is not so relevant in the bottom (or strange) meson sector since a large mass of the top quark actually converts GIM suppression to ''GIM enhancement,'' thus making higher order corrections relatively unimportant.

Another interesting observation is the fact that the dipenguin diagram actually contributes to the Δm_D with a sign *opposite* to the box diagram [compare Eqs. (2) and (8)]. This implies that the short distance piece is even smaller than was claimed in previous estimates based solely on the box dia-

gram contribution. Note that this operator gives rise to a whole family of the diagrams [e.g., Fig. 1(b)] that by no means can be calculated in perturbative QCD but might be potentially important for D^0-$\bar{D}^0$ mixing.

It must be stressed that D^0-$\bar{D}^0$ mixing is not dominated by the short distance box diagram contribution [1] but rather by long distance pieces [2,6]. This effect has been estimated in [2] using dispersive techniques for a class of two-body pseudoscalar intermediate states and was shown to boost the value of Δm_D to $\Delta m_D \sim 10^{-16}$ GeV. Indeed, it is not excluded that additional contributions could conspire in a way that they cancel the two-body piece [7].

In conclusion, we have estimated the contribution of the dipenguin diagram to the short distance amplitude for D^0-$\bar{D}^0$ mixing. It is shown to contribute at the same order of magnitude as the box diagram.

APPENDIX

Here we will provide an independent PQCD estimate of the dipenguin matrix element $\langle D^0 | O^{dp} | \bar{D}^0 \rangle$. To do that we employ a Brodsky-Lepage exclusive QCD description [8] for calculating the Feynman diagram in Fig. 1(a). In this formalism, the amplitude of interest can be expressed as a convolution of the mesons' distribution function with the hard scattering amplitude $T(x,y)$. We assume that the momentum fraction carried by the c quark in D meson is x and that the momentum fraction carried by the $\bar{c}$ quark in the $\bar{D}$ meson is $1 - y$. As is well known, the heavy quark carries nearly all the momentum of the heavy-light bound state which makes the distribution functions strongly peaked at $x \sim 1$ and $y \sim 0$. This fact simplifies the choice of the form for the distribution amplitudes. In order to obtain the estimate of the effect we use the simplest form for the distribution amplitudes:

$$\phi(x) = \frac{f_D}{2\sqrt{3}} \delta(1 - x - \epsilon), \quad \phi(y) = \frac{f_D}{2\sqrt{3}} \delta(\epsilon - y).$$
(A1)

These amplitudes are normalized such that

$$\int_0^1 \phi(x_1) dx_1 = \frac{f_D}{2\sqrt{3}}, \quad \langle 0 | A_\mu | \bar{D}^0 \rangle = i f_D p_{D\mu}, \quad \text{(A2)}$$

with $x_1 = 1 - x$. The desired mass difference is then given by

$$\Delta m_D^{dp} = 2\langle D^0 | \mathcal{H}_{dp} | \bar{D}^0 \rangle = -\frac{1}{m_D} \int dx dy \, \phi_D^*(y) T(x,y) \phi_D(y),$$
(A3)

where a ''scattering amplitude'' $T(x,y)$ can be read off the diagram shown in Fig. 1(a) and m_D in the denominator comes from the standard normalization of the meson states. A computation of Eq. (A3) involves the effective vertex $V_{8\mu}$:

$$V_{8\mu} = \frac{G_F}{\sqrt{2}} \frac{\lambda^a}{2} \frac{g_s}{8\pi^2} \{ F_1(Q^2)[Q^2\gamma_\mu - Q_\mu \slashed{Q}](1 + \gamma_5)$$
$$- m_c F_2(Q^2) i \sigma_{\mu\nu} Q^\nu (1 - \gamma_5) \},$$
(A4)

where $F_i(Q^2) = \Sigma_k \xi_k F_{ki}(x_k, Q^2)$, $k = b, s, d$, $x_k = m_k^2/M_W^2$, $F_{ki}(x_k, Q^2)$ are the corresponding modified Inami-Lim functions, and Q^2 is a momentum transferred through the gluon. Numerically $F_1 = -0.015$ for $m_b \simeq 5$ GeV, $m_s \simeq 0.2$ GeV, $m_d \simeq 0.01$ GeV.

The calculation of Eq. (A3) is relatively straightforward and yields

$$|\Delta m_D^{dp}| = \frac{G_F^2}{2\pi^2} m_D^3 F_D^2 \left\{ \frac{C_F}{16} \frac{\alpha_s}{\pi} (F_1 - F_2)^2 \right\}.$$
(A5)

Here $C_F = 4/3$, $\alpha_s \simeq 0.4$, and we put $m_c \approx m_D = 1.87$ GeV. The calculation amounts to $\Delta m_D^{dp} \simeq -0.2 \times 10^{-17} (f_D/f_\pi)^2$ GeV. Note that the PQCD estimate and the vacuum saturation estimate give reasonably close results.

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