

# Determination of color-octet matrix elements from $e^+e^-$ processes at low energies

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We present an analysis of the preliminary experimental data of direct  $J/\psi$  production in  $e^+e^-$  processes at low energies. We find that the color-octet contributions are crucially important to the cross section in this energy region, and their inclusion produces a good description of the data. By fitting to the data, we extract the individual values of two color-octet matrix elements:  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle \approx 1.1 \times 10^{-2} \text{ GeV}^3$ ;  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 \approx 7.4 \times 10^{-3} \text{ GeV}^3$ . We discuss the allowed range of the two matrix elements constrained by the theoretical uncertainties. We find that  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  is poorly determined because it is sensitive to the variation of the choice of  $m_c$ ,  $\alpha_s$  and  $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$ . However,  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2$  is quite stable [about  $(6-9) \times 10^{-3} \text{ GeV}^3$ ] when the parameters vary in reasonable ranges. The uncertainties due to large experimental errors are also discussed. [S0556-2821(97)02515-0]

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In recent years, the nonrelativistic QCD (NRQCD) factorization formalism [1] has made impressive progress in the study of heavy quarkonium production [2]. In this approach, the production process is factorized into short and long distance parts, while the latter is associated with the nonperturbative matrix elements of four fermion operators. The cross section for the inclusive production of a quarkonium state  $H$  can be expressed as a sum of products having the form

$$d\sigma(A+B \rightarrow H+X) = \sum_n d\hat{\sigma}(A+B \rightarrow c\bar{c}[n]+X) \langle \mathcal{O}_n^H \rangle. \quad (1)$$

In the above,  $d\hat{\sigma}$  represent the short distance coefficients associated with the perturbative subprocesses in which a  $c\bar{c}$  pair is produced in a configuration denoted by  $n$  (angular momentum  $^{2S+1}L_J$  and color index 1 or 8).  $\langle \mathcal{O}_n^H \rangle$  are the long distance nonperturbative matrix elements demonstrating the probability of a  $c\bar{c}$  pair evolving into the physical state  $H$ .  $d\hat{\sigma}$  can be obtained from perturbative calculations, while  $\langle \mathcal{O}_n^H \rangle$  are nonperturbative parameters which cannot be calculated perturbatively.

$\langle \mathcal{O}_n^H \rangle$  consist of two kinds of matrix elements: i.e., the color-singlet and color-octet matrix elements. Color-singlet matrix elements may be related to the quarkonium radial wave function or its derivatives at the origin, and may be calculated by potential models or estimated by leptonic decay widths of quarkonium states. Whereas the nonperturbative color-octet matrix elements can only be determined from lattice QCD calculations or by fitting the theoretical prediction of quarkonium production rates to the experimental data. The preliminary lattice QCD calculations of quarkonium decay matrix elements have been presented in the literature [3].

For the quarkonium production matrix elements, before lattice QCD produces its result, the NRQCD velocity scaling rules may be used to give a rough (order of magnitude) estimate about size of the color-octet matrix elements. Under this velocity-scaling rules, the matrix elements  $\langle \mathcal{O}_n^H \rangle$  (including both color singlet and color octet) are related to each other by orders of  $m_Q v^2$  or  $v^2$ , where  $m_Q$  is the mass of the heavy quark and  $v$  is the typical relative velocity of the heavy quark in the bound state. For example, in  $J/\psi$  production, the most important NRQCD matrix elements are  $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$ ,  $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$ ,  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ , and  $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$ , and according to the scaling rules, their relative sizes satisfy the relations

$$\begin{aligned} \langle \mathcal{O}_1^\psi(^3S_1) \rangle &\sim m_c^3 v^3, & \langle \mathcal{O}_8^\psi(^3S_1) \rangle &\sim m_c^3 v^7, \\ \langle \mathcal{O}_8^\psi(^1S_0) \rangle &\sim m_c^3 v^7, & \langle \mathcal{O}_8^\psi(^3P_J) \rangle &\sim m_c^5 v^7. \end{aligned} \quad (2)$$

Practically, the color-octet matrix elements have been determined by fitting the theoretical prediction of quarkonium production to the experimental data at various colliders. According to the NRQCD factorization formalism, the matrix elements  $\langle \mathcal{O}_n^H \rangle$  are universal, so that their values measured from different colliders must be the same. This is an important test to the color-octet production mechanism. In previous studies [4,5], the matrix element  $\langle \mathcal{O}_8^\psi(^3S_1) \rangle$  is determined by fitting to the high  $P_T$  prompt  $J/\psi$  production at the Fermilab Tevatron, and their results are consistent with the velocity-scaling rules. However, the matrix elements  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  and  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$  are not determined individually, but as a linear combination [5]. Another linear combination of these two elements is also obtained in the studies of  $J/\psi$  photoproduction in  $e^-p$  collisions [6] and hadroproduction at fix target experiments [7], and their results are not com-

patible with that of [5]. Recently, some further investigations on prompt  $J/\psi$  production at the Tevatron are performed [8,9]. Their results show that the parton distributions strongly affect the extraction of the values of the color-octet matrix elements, i.e., different sets of parton distributions result in different values of the matrix elements. They also show that the initial- and final-state radiation are important in prompt  $J/\psi$  production at the Tevatron [9]. All those progresses show that the inconsistency between the values measured at the Collider Detector at Fermilab (CDF) and those measured in other processes may be mostly due to the large theoretical uncertainties in those calculations, such as the parton distributions.

In contrast, the mechanism of  $J/\psi$  production in  $e^+e^-$  annihilation process is much clearer than those in hadronic processes discussed above. The parton structure is simpler, and there is no higher twist effects to be considered, so the theoretical uncertainty is much smaller. Because in  $e^+e^-$  processes  $J/\psi$  production has much smaller theoretical uncertainty than those in hadronic  $J/\psi$  production processes, it can be used to extract the color-octet matrix elements.  $J/\psi$  production at  $e^+e^-$  colliders has been investigated by several authors in literature [10–17]. Braaten and Chen have noted that a clean signature of color-octet mechanism may be observed in the angular distribution of  $J/\psi$  production near the end point region at  $e^+e^-$  collider in the low energy region such as at CLEO [16]. In this paper, we make use of the previous results of the calculations on the  $J/\psi$  production in  $e^+e^-$  process to extract the color-octet matrix elements by comparing with the preliminary experimental data at low energies [18,19].

In our previous studies [17], we have calculated color-singlet and color-octet contributions to prompt  $J/\psi$  production in  $e^+e^-$  annihilation at different energy scales under the NRQCD factorization formalism and we have used the non-perturbative matrix elements which are consistent with the NRQCD velocity-scaling rules. We find that the color-octet contributions dominate over the color-singlet contributions in all energy regions. At low energies, as pointed out in [16,17], the dominant production channels are  $e^+e^- \rightarrow J/\psi + g$  via color-octet  $^1S_0$  and  $^3P_J$  processes,

while at high energies (say, above 20 GeV), the dominant channel is  $e^+e^- \rightarrow q\bar{q}g^*$  followed by  $g^*$  fragmentation to color-octet  $J/\psi$  process.

For the color-octet processes

$$e^+e^- \rightarrow \gamma^* \rightarrow g + c\bar{c} [{}^2S+1L_J], \quad (3)$$

where  ${}^{2S+1}L_J$  represent the  $c\bar{c}$  pair states  $^1S_0$  and  $^3P_J$ , from Ref. [16], we readily have

$$\sigma(e^+e^- \rightarrow J/\psi g) = C_s \langle \mathcal{O}_8^\psi(^1S_0) \rangle + C_p \langle \mathcal{O}_8^\psi(^3P_0) \rangle, \quad (4)$$

with

$$C_s = \frac{64\pi^2 e_c^2 \alpha_s^2}{3} \frac{1-r}{s^2 m}, \quad (5)$$

$$C_p = \frac{256\pi^2 e_c^2 \alpha_s^2}{9s^2 m^3} \left[ \frac{(1-3r)^2}{1-r} + \frac{6(1+r)}{1-r} + \frac{2(1+3r+6r^2)}{1-r} \right], \quad (6)$$

where  $r = m^2/s$ ,  $m$  is the mass of  $J/\psi$ , and  $s$  is the  $e^+e^-$  collision c.m. energy squared. Here, we have used the approximate heavy quark spin symmetry relations

$$\langle \mathcal{O}_8^\psi(^3P_J) \rangle \approx (2J+1) \langle \mathcal{O}_8^\psi(^3P_0) \rangle. \quad (7)$$

To extract the color-octet matrix elements, one needs to subtract the color-singlet contributions from the total cross section. The leading order color-singlet  $J/\psi$  production rates at low energies ( $< 25$  GeV) come from the process

$$e^+e^- \rightarrow \gamma^* \rightarrow J/\psi + gg. \quad (8)$$

The cross section of this process is [12]

$$\frac{d\sigma(e^+e^- \rightarrow J/\psi gg)}{\sigma_{\mu\mu} dz dx_1} = \frac{64e_c^2 \alpha_s^2}{27} \frac{\langle \mathcal{O}_1^\psi(^3S_1) \rangle}{m^3} r^2 f(z, x_1; r), \quad (9)$$

where

$$f(z, x_1; r) = \frac{(2+x_2)x_2}{(2-z)^2(1-x_1-r)^2} + \frac{(2+x_1)x_1}{(2-z)^2(1-x_2-r)^2} + \frac{(z-r)^2-1}{(1-x_2-r)^2(1-x_1-r)^2} + \frac{1}{(2-z)^2} \left( \frac{6(1+r-z)^2}{(1-x_2-r)^2(1-x_1-r)^2} + \frac{2(1-z)(1-r)}{(1-x_2-r)(1-x_1-r)r} + \frac{1}{r} \right), \quad (10)$$

and

$$\sigma_{\mu\mu} = \sigma_{\text{QED}}(e^+e^- \rightarrow \mu^+\mu^-).$$

The variables  $z, x_i$  are defined as

$$z = \frac{2p \cdot k}{s}, \quad x_i = \frac{2p_i \cdot k}{s}, \quad (11)$$

where  $k$ ,  $p$ , and  $p_i$  are the momenta of the virtual photon  $\gamma^*$ ,  $J/\psi$ , and the outgoing gluon, respectively.

In the numerical calculations we use the following parameters as input:

$$m_c = 1.5 \text{ GeV}, \quad \alpha_s(2m_c) = 0.26, \quad \langle \mathcal{O}_1^\psi(^3S_1) \rangle = 1.08 \text{ GeV}^3 [3]. \quad (12)$$

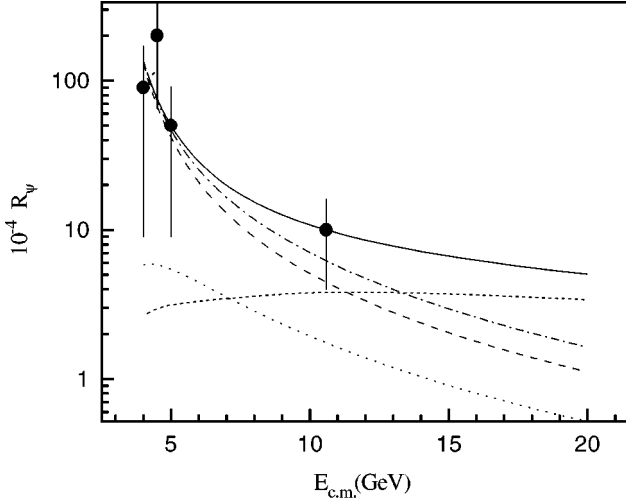


FIG. 1. The total cross section of  $J/\psi$  in  $e^+e^-$  process. Color-octet  $^1S_0$  process contribution is represented by the dotted line, color-octet  $^3P_J$  process by the dashed line, the sum of these two color-octet processes by the dotted-dashed line, and color-singlet contribution by the short dashed line. The experimental data are taken from Refs. [19,18].

In Fig. 1, we show the theoretical prediction of direct  $J/\psi$  production compared with the experimental data (note that contribution from  $\psi'$  feed down has been subtracted from the data sample). The data at  $\sqrt{s}=4-5$  GeV are from PLUTO [18], and those at  $\sqrt{s}=10.6$  GeV are from CLEO II [19,13]. The two color-octet matrix elements  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  and  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$  are treated as free parameters to be determined by fitting the data. We use the data from two energy points to determine these two matrix elements. One is from PLUTO at  $\sqrt{s}=5$  GeV and the other from CLEO at  $\sqrt{s}=10.6$  GeV. At the collision energies below 5 GeV there might exist some  $c\bar{c}$  resonances which could contaminate the prompt  $J/\psi$  production, that is why we neglect the data in the energy region below  $\sqrt{s}=5$  GeV to carry out the extraction. And the extraction result is

$$\begin{aligned} \langle \mathcal{O}_8^\psi(^1S_0) \rangle &= 1.1 \times 10^{-2} \text{ GeV}^3, \\ \frac{\langle \mathcal{O}_8^\psi(^3P_0) \rangle}{m_c^2} &= 0.74 \times 10^{-2} \text{ GeV}^3, \end{aligned} \quad (13)$$

where the central values of experimental data ( $R_\psi^{\text{expt}}=5.0 \times 10^{-3}$  at  $\sqrt{s}=5.0$  GeV and  $R_\psi^{\text{expt}}=1.0 \times 10^{-3}$  at  $\sqrt{s}=10.6$  GeV) are used and the experimental errors are not included. The dotted line in Fig. 1 represents the contribution from color-octet  $^1S_0$  subprocess, the dashed line is from color-octet  $^3P_J$  process, and their sum is plotted as the dotted-dashed line, and the color-singlet contributions is shown as the short dashed line. From this figure, we can see clearly that the color-singlet contributions alone cannot explain the observed cross sections in these energy regions. After including the contributions from both color-singlet and color-octet production processes (shown as the solid line), a satisfactory agreement between theoretical prediction and the experimental data will be achieved. Especially, the cross section of direct  $J/\psi$  production at  $\sqrt{s}=5$  GeV is much larger

than that at  $\sqrt{s}=10.6$  GeV. This behavior cannot be explained by the color-singlet model with perturbative QCD.

Here, we would like to emphasize that the PLUTO data are due to direct  $J/\psi$  production but not the decays of  $c\bar{c}$  resonances [18]. Furthermore, we assume that in the energy region around  $\sqrt{s}=5$  GeV, perturbative QCD is approximately applicable to describe the  $J/\psi$  production processes. In this connection, we note that Driesen *et al.* have applied perturbative QCD to calculate the  $J/\psi$  production in  $e^+e^-$  process within the color-singlet model [13]. They could explain the experimental data from PLUTO by using a large running coupling constant, i.e., setting  $\alpha_s(Q^2)=1$  if  $\sqrt{Q^2}<1$  GeV. Their results indicate that the main contribution to  $J/\psi$  production in this process comes from the region where the conjugated gluons are soft. It is not entirely clear whether this color-singlet  $c\bar{c}$  plus a soft gluon can be factorized into a higher Fock state for the physical  $J/\psi$ , such as color-octet  $^3P_J$  states and color-octet  $^1S_0$  state. Nevertheless, if the experimental results of these two collaborations (CLEO and PLUTO) are further confirmed, the color-octet production mechanism will probably provide a quite unique explanation.

In the following we estimate the theoretical uncertainties induced by the choice of the charm quark mass, the strong coupling constant, and the color-singlet matrix element  $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$ . First, we consider a variation of the charm quark mass  $m_c$  from 1.4 GeV to 1.6 GeV with other parameters unchanged [as in Eq. (12)]. This results in a variation of the two color-octet matrix elements fitted values,

$$\begin{aligned} \langle \mathcal{O}_8^\psi(^1S_0) \rangle &= -0.25 \text{ to } 2.3 \times 10^{-2} \text{ GeV}^3, \\ \frac{\langle \mathcal{O}_8^\psi(^3P_0) \rangle}{m_c^2} &= 0.91 \text{ to } 0.61 \times 10^{-2} \text{ GeV}^3. \end{aligned} \quad (14)$$

Also, a variation of the strong coupling constant  $\alpha_s$  from 0.24 to 0.30 will result in

$$\begin{aligned} \langle \mathcal{O}_8^\psi(^1S_0) \rangle &= 1.8 \text{ to } -0.009 \times 10^{-2} \text{ GeV}^3, \\ \frac{\langle \mathcal{O}_8^\psi(^3P_0) \rangle}{m_c^2} &= 0.76 \text{ to } 0.71 \times 10^{-2} \text{ GeV}^3. \end{aligned} \quad (15)$$

And a variation of the color-singlet matrix element  $\langle \mathcal{O}_1^\psi(^3S_1) \rangle$  from  $1.0 \text{ GeV}^3$  to  $1.2 \text{ GeV}^3$  will result in

$$\begin{aligned} \langle \mathcal{O}_8^\psi(^1S_0) \rangle &= 1.4 \text{ to } 0.77 \times 10^{-2} \text{ GeV}^3, \\ \frac{\langle \mathcal{O}_8^\psi(^3P_0) \rangle}{m_c^2} &= 0.72 \text{ to } 0.76 \times 10^{-2} \text{ GeV}^3. \end{aligned} \quad (16)$$

From the above results, we can see that the fitted value of the element  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  is sensitive to a variation of the three parameters  $m_c$ ,  $\alpha_s$ , and  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ . This is because the cross section of  $J/\psi$  production is not sensitive to the value of the element  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  [i.e., the coefficient of the  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  term in the expression of cross section is very small, see Eqs. (4) and (5)]. In contrast, the fitted value of  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle/m_c^2$  remains rather stable when the three parameters vary in reasonable ranges. The above rough estimates

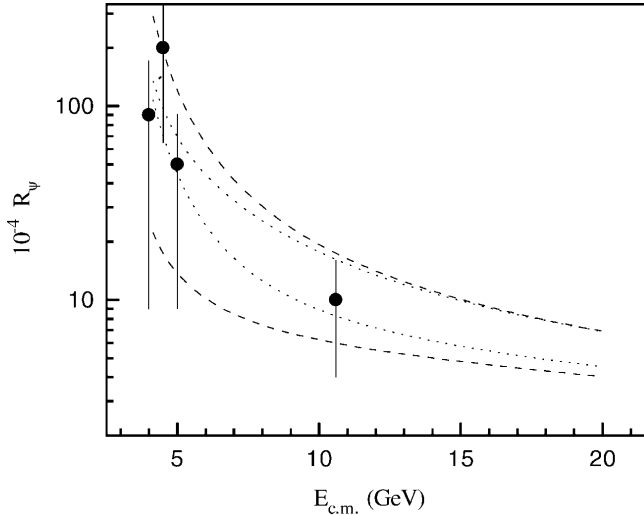


FIG. 2. Description of the allowed values of the color-octet matrix elements constrained by the experimental data. The dotted lines correspond to  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = 7.4 \times 10^{-3} \text{ GeV}^3$ ,  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle = 5.0 \times 10^{-2} \text{ GeV}^3$  (up), and  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle = 10^{-4} \text{ GeV}^3$  (down), respectively. The dash lines correspond to  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle = 1.0 \times 10^{-2} \text{ GeV}^3$ ,  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = 2.0 \times 10^{-2} \text{ GeV}^3$  (up), and  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = 2.0 \times 10^{-3} \text{ GeV}^3$  (down), respectively.

may show the value of the element  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = (0.6 - 0.9) \times 10^{-2} \text{ GeV}^3$ .

Finally, we discuss how the present experimental errors can affect the fitted values of the two matrix elements. Obviously, the large errors in experimental data will result in a larger error for the fitted values of the two color-octet matrix elements. To see this, in Fig. 2 we show four curves for the  $J/\psi$  production cross section, where the input parameters  $\{\alpha_s, m_c, \langle \mathcal{O}_1^\psi(^3S_1) \rangle\}$  are same as those in Fig. 1. The two dotted lines correspond to  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = 7.4 \times 10^{-3} \text{ GeV}^3$ ,  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle = 5.0 \times 10^{-2} \text{ GeV}^3$  (up), and  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle = 1.0 \times 10^{-4} \text{ GeV}^3$  (down), respectively. The two dashed lines correspond to  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle = 1.0 \times 10^{-2} \text{ GeV}^3$ ,  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = 2.0 \times 10^{-2} \text{ GeV}^3$  (up), and  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle / m_c^2 = 2.0 \times 10^{-3} \text{ GeV}^3$  (down), respectively. We can see that the total cross section is not sensitive to the value of  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ . When it increases from  $1.0 \times 10^{-4} \text{ GeV}^3$  to  $5.0 \times 10^{-2} \text{ GeV}^3$ , the total cross section only changes a little. But the value of  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$  varies rapidly as the data value changes. Therefore, in order to determine the color-octet matrix elements more rigorously, more precise measurement for the  $J/\psi$  cross section at low energies is apparently needed.

Meanwhile, we note that the color-octet matrix elements discussed in this paper have also been determined from other experiments. At the Tevatron, prompt  $J/\psi$  production at high  $P_T$  has been compared with theoretical predictions, and a global fit to all  $P_T$  region shows that at low  $P_T$  the theoretical prediction is dominated by the contributions from  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  and  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$ , and the fitted result is [5]

$$\langle \mathcal{O}_8^\psi(^1S_0) \rangle + \frac{3}{m_c^2} \langle \mathcal{O}_8^\psi(^3P_0) \rangle = 6.6 \times 10^{-2} \text{ GeV}^3. \quad (17)$$

Recently, Beneke and Krämer [8] updated the extraction of color-octet matrix elements by comparing the  $P_T$  distribution for unpolarized direct  $J/\psi$  production with the most recent CDF data [20]. By using the CTEQ4L parton distribution function (PDF) set (in their paper, they have used three PDF sets, which resulted in different extractions [8]), they obtained a different linear combination of the two color-octet matrix elements,

$$\langle \mathcal{O}_8^\psi(^1S_0) \rangle + \frac{3.5}{m_c^2} \langle \mathcal{O}_8^\psi(^3P_0) \rangle = 4.38 \times 10^{-2} \text{ GeV}^3. \quad (18)$$

In Ref. [9], Sanchis-Lozano and Cano-Coloma used a Monte Carlo event generator to treat the high order initial- and final-states radiation. By fitting to the experimental data of prompt  $J/\psi$  production at the Tevatron, the authors obtained another extraction of these two color-octet matrix elements,

$$\langle \mathcal{O}_8^\psi(^1S_0) \rangle + \frac{3}{m_c^2} \langle \mathcal{O}_8^\psi(^3P_0) \rangle = 1.44 \times 10^{-2} \text{ GeV}^3, \quad (19)$$

where the CTEQ2L PDF set is used. The value in Eq. (19) is much smaller than those in Eqs. (17) and (18). This shows that the initial-states radiation effects are crucially important to the extraction of these two color-octet matrix elements at the Tevatron.

The studies of photoproduction at  $e^-p$  collisions show that the matrix element values in Eq. (17) may be overestimated [6], and the authors obtained another linear combination of these two elements from the forward  $J/\psi$  photoproduction cross-section measurements,

$$\langle \mathcal{O}_8^\psi(^1S_0) \rangle + \frac{7}{m_c^2} \langle \mathcal{O}_8^\psi(^3P_0) \rangle = 2.0 \times 10^{-2} \text{ GeV}^3. \quad (20)$$

The fixed-target hadroproduction result [7] gives the same argument against the matrix elements values in Eq. (17), and gives

$$\langle \mathcal{O}_8^\psi(^1S_0) \rangle + \frac{7}{m_c^2} \langle \mathcal{O}_8^\psi(^3P_0) \rangle = 3.0 \times 10^{-2} \text{ GeV}^3. \quad (21)$$

Our results (13) are not compatible with Eqs. (20) and (21). The inconsistency may be mostly due to theoretical uncertainties and large experimental errors.

However, it should be noted that there are large theoretical uncertainties in our extraction of the color-octet matrix element  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$ , and also the large experimental errors may further affect the extraction of the two matrix elements  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  and  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$ . So, the compatibility of our results with those from Tevatron and the incompatibility of our results with those from fixed-target hadroproduction and photoproduction should not be taken seriously.

In Ref. [21], Fleming *et al.* make an analysis of the present extraction values of the color-octet matrix elements. After considering the constraints by the  $J/\psi$  production rate in  $b$  decays, they obtained a rough range of the linear combination of the two color-octet matrix elements,

$$\langle \mathcal{O}_8^\psi(^1S_0) \rangle + \frac{3}{m_c^2} \langle \mathcal{O}_8^\psi(^3P_0) \rangle = 1.0 \sim 6.0 \times 10^{-2} \text{ GeV}^3. \quad (22)$$

This is compatible with all the above results [Eqs. (13) and (17)–(21)].

When this work was in progress, we received a preprint of Fleming *et al.* [22], in which they calculate the  $J/\psi$  leptoproduction. They find that a negative value for the color-octet matrix element  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$  is still possible, which is inconsistent with our conclusions. We would like to point out that if this matrix element is negative, the color-octet  $^3P_J$  processes would provide a large negative contribution to  $J/\psi$  production in  $e^+e^-$  process at low energies due to a large positive coefficient for the matrix element  $\langle \mathcal{O}_8^\psi(^3P_0) \rangle$  at low energies [see Eqs. (4)–(6)]. If so, the experimental data in Fig. 1 cannot be explained. The conflict between our results with those of [22] about the color-octet matrix elements puts forward a question to the universality of the

color-octet matrix elements in these two situations.

In conclusion, in this paper we have calculated the direct  $J/\psi$  production rates in  $e^+e^-$  process at low energies. Our results show that the color-octet production mechanism is significant in the production cross section in this energy region, and this may provide another positive test for the validity of the color-octet production mechanism. The extraction of elements  $\langle \mathcal{O}_8^\psi(^1S_0) \rangle$  and  $\langle \mathcal{O}_8^\psi(^3P_J) \rangle$  are performed by fitting to the experimental data. But the large errors of the preliminary experimental data cannot give a precise estimate of the color-octet matrix elements. We hope that more precise experimental data of direct  $J/\psi$  production in  $e^+e^-$  annihilation at CLEO II and Beijing Electron-Positron Collider (BEPC) will soon be obtained.

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