QCD corrections to $B \rightarrow J/\psi +$ anything

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We calculate the branching ratio for $B \rightarrow J/\psi +$ anything, within the color-singlet approximation for J/ψ production, but including perturbative QCD corrections beyond the leading logarithm approximation. Such higher order corrections are necessary, in order to obtain a result that is not strongly dependent on the renormalization scale. As in the earlier work of Bergström and Ernström, we use a double expansion in α_s and in the small ratio of Wilson coefficients L_0/L_2, to identify the dominant terms in the decay amplitude. We complete their work by calculating all the leading order terms in this double expansion. The predicted branching ratio is then $B(B \rightarrow J/\psi + \text{anything}) = 0.9^{+1.1}_{-0.3} \times 10^{-3}$, which is well below the experimental value $B_{\text{expt}} = (0.80 \pm 0.08)\%$. This confirms the suspicion that nonperturbative corrections to the color-singlet approximation for J/ψ production in B decays are important. [S0556-2821(97)05415-5]

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I. INTRODUCTION

The hadronic B decays into charmonium originate in the weak transition $b \rightarrow qc\bar{c}$ (with $q = s,d$), followed by the hadronization of the $c\bar{c}$ pair into the charmonium bound state. They are examples of color-suppressed hadronic B decays: at the weak vertex, the $c\bar{c}$ pair is not created automatically in a color singlet, and so it will be harder for it to hadronize into a charmonium state, rather than into a pair of D-$\bar{D}$ mesons. The exclusive decays, such as $B \rightarrow K^{(*)}J/\psi$, and the inclusive decay $B \rightarrow J/\psi +$ anything, as well as analogous decays into the charmonium states $\psi(2S)$ and χ_{c1}, are well studied experimentally. However, color-suppressed hadronic B decays are still not well understood theoretically.

We will concentrate on the inclusive decay $B \rightarrow J/\psi +$ anything. In order for the $c\bar{c}$ pair to hadronize into a J/ψ, we require that it form a color singlet, with spin $S = 1$ and no relative velocity—this is the color-singlet mechanism for J/ψ production. Other $c\bar{c}$ configurations may also hadronize into J/ψ; they appear at higher orders in an expansion in the small relative velocity of the J/ψ constituents [1,2]. Since they contribute incoherently to the $B \rightarrow J/\psi +$ anything decay rate, they can be studied separately. We are interested in obtaining a reliable prediction for the leading color-singlet contribution. Comparing our prediction to the data will determine whether other $c\bar{c}$ configurations are indeed important.

At first sight, the decay rate in the color-singlet mechanism, with QCD corrections included in the leading logarithm approximation, appears to be well below the experimental value. However, the result is not satisfactory, as it retains a strong dependence on the renormalization scale. To obtain a reliable prediction, it is necessary to include higher order QCD corrections. This was the subject of the work of Bergström and Ernström in Ref. [3]. These authors have shown how a clever reorganization of the higher order corrections can be used to identify the relevant contributions to the decay rate, and how this will eliminate the strong dependence on the renormalization scale. Here, we complete their calculation and derive the prediction for the $B \rightarrow J/\psi +$ anything decay rate, in the color-singlet mechanism.

Up to small corrections of higher order in Λ_{QCD}/m_b, the inclusive decay $B \rightarrow J/\psi +$ anything is described by the corresponding parton decay. In the next section, we give the effective weak Hamiltonian for the b-quark decay, with the next-to-leading order Wilson coefficients of Ref. [4]. In the following section, we adopt the program outlined in Ref. [3], and obtain the leading terms in the $b \rightarrow qJ/\psi$ decay amplitude. Finally, we give our numerical results and discuss their significance.

II. EFFECTIVE WEAK HAMILTONIAN FOR THE b-QUARK DECAY

The terms of interest in the $\Delta B = 1$ effective weak Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=s,d} V_{cb}V_{cq}^* \left[\frac{1}{3} C_0(\mu)\mathcal{O}_1 + 2C_2(\mu)\mathcal{O}_8 \right] \quad (1)$$

contain the operators

$$\mathcal{O}_1 \equiv \bar{c}\gamma_\mu(1-\gamma_5)c\,\bar{q}\gamma^\mu(1-\gamma_5)b, \quad (2)$$

$$\mathcal{O}_8 \equiv \tfrac{1}{4}\bar{c}\lambda^a\gamma_\mu(1-\gamma_5)c\,\bar{q}\lambda^a\gamma^\mu(1-\gamma_5)b, \quad (3)$$

and they correspond to the $b \rightarrow qc\bar{c}$ ($q = s$ or d) transition, which occurs at the tree level in the weak interaction. The subscripts in $\mathcal{O}_{1,8}$ designate the singlet (1) or octet (8) color structure of the V-A currents in those operators. The Wilson coefficients $C_{0,2}(\mu)$ include perturbative QCD corrections to the weak vertex. They depend on the renormalization scale μ, which effectively separates those QCD corrections, from the QCD effects that appear in the matrix elements $\langle qJ/\psi|\mathcal{O}_{1,8}|b\rangle$. The μ dependence of the matrix elements should cancel that in the Wilson coefficients, so that the final result for the decay amplitude is independent of the renormalization scale.

In the leading logarithm approximation (LLA), the Wilson coefficients are [4]

$$C_0(\mu) = L_0(\mu) = 2L_+(\mu) - L_-(\mu), \tag{4}$$

$$C_2(\mu) = L_2(\mu) = \tfrac{1}{2}[L_+(\mu) + L_-(\mu)], \tag{5}$$

with

$$L_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{\gamma_\pm^{(0)}/2\beta_0} \tag{6}$$

and

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\text{QCD}}^2)}, \tag{7}$$

whereas at next-to-leading order (NLO), they are [4]

$$C_0(\mu) = 2C_+(\mu) - C_-(\mu), \tag{8}$$

$$C_2(\mu) = \tfrac{1}{2}[C_+(\mu) + C_-(\mu)], \tag{9}$$

with

$$C_\pm(\mu) = L_\pm(\mu)\left[1 \pm \frac{\alpha_s(M_W)}{4\pi}\frac{3\mp 1}{6}(11\pm\kappa_\pm)\right.$$
$$\left. + \frac{\alpha_s(\mu) - \alpha_s(M_W)}{4\pi}\left(\gamma_\pm^{(0)}\frac{\beta_1}{2\beta_0^2} - \gamma_\pm^{(1)}\frac{1}{2\beta_0}\right)\right] \tag{10}$$

and

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{\text{QCD}}^2)}\left[1 - \frac{\beta_1 \ln[\ln(\mu^2/\Lambda_{\text{QCD}}^2)]}{\beta_0^2 \ln(\mu^2/\Lambda_{\text{QCD}}^2)}\right]. \tag{11}$$

The anomalous dimensions and β-function coefficients are

$$\gamma_\pm^{(0)} = \pm 2(3 \mp 1), \quad \gamma_\pm^{(1)} = \frac{3\mp 1}{6}\left(-21 \pm \frac{4}{3}n_f - 2\beta_0\kappa_\pm\right), \tag{12}$$

$$\beta_0 = 11 - \tfrac{2}{3}n_f, \quad \beta_1 = 102 - \tfrac{38}{3}n_f, \tag{13}$$

and $n_f = 5$ is the number of active flavors. In the modified minimal subtraction ($\overline{\text{MS}}$) renormalization scheme, $\Lambda_{\text{QCD}} = \Lambda_{\overline{\text{MS}}}^{(5)} = 209^{+39}_{-33}$ MeV, which corresponds to $\alpha_s(M_Z) = 0.118 \pm 0.003$ [5]. The quantity $\kappa_\pm = 0$ [naive dimensional regularization (NDR)], ∓ 4 ['t Hooft–Veltman (HV)] or $\mp 6 - 3$ [dimensional reduction (DRED)] is regularization scheme dependent. A similar scheme dependence appears in the calculation of the matrix elements of $\mathcal{O}_{1,8}$, such that the final result for the decay amplitude is regularization scheme independent.

III. $b \to q J/\psi$ DECAY AMPLITUDE IN THE α_s-L_0/L_2 DOUBLE EXPANSION

The amplitude for $b \to q J/\psi$ ($q = s$ or d), when the mass of the q quark is neglected, can be parametrized in terms of the coefficients g_1 and g_2, which multiply the two possible Lorentz structures of the amplitude [6]
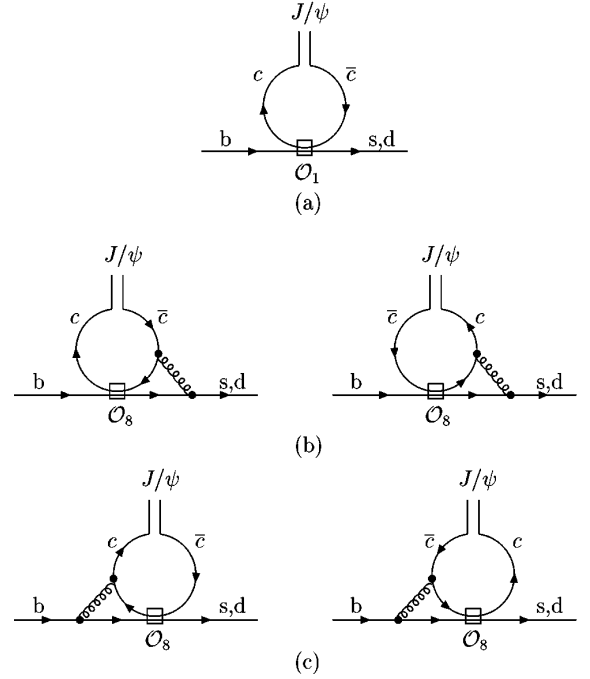

FIG. 1. Feynman graphs contributing to $B \to q J/\psi$ ($q = s, d$), to lowest order in the α_s-L_0/L_2 double expansion.

$$A_{b \to q J/\psi} = -\frac{G_F}{\sqrt{2}}V_{cb}V_{cq}^*\frac{f_{J/\psi}}{m_{J/\psi}}[g_1 m_{J/\psi}^2 \bar{u}_q\gamma_\mu(1-\gamma_5)u_b$$
$$+ g_2 m_b\bar{u}_q i\sigma_{\mu\nu}p_{J/\psi}^\nu(1+\gamma_5)u_b]\varepsilon_{J/\psi}^{*\mu}. \tag{14}$$

The decay rate is then

$$\Gamma(b \to s J/\psi) + \Gamma(b \to d J/\psi)$$
$$= \frac{G_F^2}{16\pi}|V_{cb}|^2\left(\frac{f_{J/\psi}}{m_{J/\psi}}\right)^2 m_b^5(1-r)^2[|g_1|^2 r(1+2r)$$
$$+ |g_2|^2(2+r) - \text{Re}(g_1 g_2^*)6r], \tag{15}$$

where $r \equiv m_{J/\psi}^2/m_b^2$. The J/ψ decay constant $f_{J/\psi}$ is a nonperturbative parameter that describes the hadronization of a color-singlet $c\bar{c}$ pair, with no relative velocity and in a spin $S = 1$ state, into a J/ψ meson. It is defined by

$$\langle 0|\bar{c}\gamma^\mu c|J/\psi\rangle = m_{J/\psi}f_{J/\psi}\varepsilon_{J/\psi}^\mu, \tag{16}$$

and it can be measured from the rate for the decay $J/\psi \to e^+ e^-$.

In the LLA, the decay amplitude is obtained from the tree-level matrix element of the effective weak Hamiltonian in Eq. (1). Only the singlet operator $\mathcal{O}_1$ contributes [as in Fig. 1(a)], and one has

$$g_1 = \tfrac{1}{3}L_0(\mu), \quad g_2 = 0. \tag{17}$$

This leads to a LLA branching ratio (see Fig. 3) that depends strongly on the renormalization scale μ (to the point where it can actually vanish, for $\mu \simeq 2.5$ GeV). As pointed out by Bergström and Ernström [3], this means that higher order contributions to the matrix elements of the operators $\mathcal{O}_{1,8}$, beyond the LLA, need to be considered. More specifically,

the strong μ dependence in the LLA amplitude is due entirely to the Wilson coefficient $L_0(\mu)$. An expansion of this coefficient around some fixed $\mu \sim m_b$ gives

$$L_0(\mu) = L_0(m_b) + \frac{2}{\pi} \, \alpha_s(m_b) L_2(m_b) \ln(\mu^2/m_b^2) + \cdots , \tag{18}$$

and although the μ dependence only appears at order α_s, it is important, since $L_0(m_b)/L_2(m_b) \simeq 0.34$ is of similar size as $\alpha_s(m_b) \simeq 0.21$. From Eq. (18), one can conclude that the terms in the matrix elements of $\mathcal{O}_{1,8}$, which cancel the strong μ dependence that comes from L_0, are terms of order $\alpha_s L_2$. Such higher order terms in α_s appear at the one-loop level; they are absent from the LLA result, where $\mathcal{O}_{1,8}$ only contribute at the tree level.

The reason higher order terms in α_s become important is the existence of a second small quantity in the calculation—the ratio L_0/L_2. Then, $\alpha_s L_2/L_0$ is not small compared to unity, and the expansion in α_s fails. Bergström and Ernström advocated instead a double expansion in both α_s and L_0/L_2. Since these are the two small quantities in the calculation, this procedure will correctly identify the dominant terms in the matrix elements of $\mathcal{O}_{1,8}$. Those are also the terms that are needed to cancel the μ dependence that comes from the Wilson coefficients. Here, we will follow this pro-

gram, and calculate all the leading order terms in the double expansion. Inexplicably, this was not done in Ref. [3], where the important terms of order $\alpha_s^2 L_2^2$ in the decay rate were not calculated (and the estimate that was given is incorrect).

At leading order in the α_s-L_0/L_2 expansion for the decay amplitude, there is a tree-level contribution from $\mathcal{O}_1$ [Fig. 1(a)], proportional to C_0, and one-loop contributions from $\mathcal{O}_8$ [Figs. 1(b) and 1(c)], proportional to $\alpha_s C_2$. From the results for the Wilson coefficients in the previous section, and to the order that we are interested in,

$$C_0(\mu) = L_0(\mu) + \alpha_s(\mu) L_2(\mu) \, \frac{1}{3\pi} \left\{ -\kappa + 11 \, \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right.$$
$$\left. + \frac{\alpha_s(M_W) - \alpha_s(\mu)}{\alpha_s(\mu)} \left[(21 + 4n_f) \frac{1}{6\beta_0} - \frac{6\beta_1}{\beta_0^2} \right] \right\}, \tag{19}$$

$$C_2(\mu) = L_2(\mu) \tag{20}$$

[where we have replaced the regularization scheme-dependent parameter $\kappa_\pm$ by $\kappa = 0$ (NDR), 4 (HV) or 5 (DRED)]. After a lengthy calculation, we obtain, for the parameters $g_{1,2}$ in the decay amplitude,

$$g_1 = \frac{1}{3} \, L_0(\mu) + \alpha_s(\mu) L_2(\mu) \, \frac{1}{9\pi} \left\{ \kappa' - \kappa - 6 \ln\left(\frac{\mu^2}{m_b^2}\right) + 11 \, \frac{\alpha_s(M_W)}{\alpha_s(\mu)} + \frac{\alpha_s(M_W) - \alpha_s(\mu)}{\alpha_s(\mu)} \left[(21 + 4n_f) \frac{1}{6\beta_0} - \frac{6\beta_1}{\beta_0^2} \right] \right.$$
$$\left. + \frac{2}{2-r} \left[-16 + 7r + 4r \ln 2 + \frac{2r^2}{1-r} \ln r + \frac{6 - 6r + r^2}{2-r} \ln(1-r) - i\pi \, \frac{6 - 6r + r^2}{2-r} \right] \right\}, \tag{21}$$

$$g_2 = \alpha_s(\mu) L_2(\mu) \, \frac{1}{9\pi} \frac{2r}{2-r} \left[-2 + 4\ln 2 + \frac{2r}{1-r} \ln r + \frac{4 - 3r}{2-r} \ln(1-r) - i\pi \, \frac{4 - 3r}{2-r} \right]. \tag{22}$$

Both g_1 and g_2 are free of infrared divergences. The imaginary parts correspond to the contribution from the on-shell intermediate state with a $c\bar{c}$ color octet, in the one-loop diagrams of Fig. 1(b). They have been previously calculated in Ref. [7]. The ultraviolet divergence in g_1 has been removed using the same $\overline{\text{MS}}$ renormalization scheme as in the calculation of the Wilson coefficients [4]: to leading order in the double expansion, the counterterms that are needed are

$$H_{\text{CT}} = \frac{G_F}{\sqrt{2}} \sum_{q=s,d} V_{cb} V_{cq}^* \left[\frac{2}{\varepsilon} - \gamma_E + \ln(4\pi) \right]$$
$$\times \frac{2}{3} \frac{\alpha_s(\mu)}{\pi} \, C_2(\mu) \mathcal{O}_1 \tag{23}$$

(for $d = 4 - \varepsilon$ dimensions). The regularization of the divergence generates the scheme dependent term $\kappa' = -2$ (NDR), 2 (HV), or 3 (DRED), in the expression for g_1 (see the Appendix for more details on how this term is generated). No-

tice that κ' is such that it cancels the scheme dependence from the Wilson coefficients, parametrized by κ.

As for the dependence of $g_{1,2}$ on the renormalization scale μ, there is also an exact cancellation between the μ dependence of the Wilson coefficients and that which originates in the one-loop matrix elements of the $\mathcal{O}_{1,8}$ operators. The latter is shown explicitly in the expressions for $g_{1,2}$. The former can be obtained from the expansion of $L_0(\mu)$ around $\mu \sim m_b$ in Eq. (18), and the analogous results for $\alpha_s(\mu)$ and $L_2(\mu)$:

$$L_2(\mu) = L_2(m_b) + \cdots , \quad \alpha_s(\mu) = \alpha_s(m_b) + \cdots . \tag{24}$$

As in Eq. (18), the terms not shown are of higher order in the double expansion in α_s-L_0/L_2. In our final result (see Fig. 3), we have chosen to keep the full expressions for $\alpha_s(\mu)$ and $L_{0,2}(\mu)$, rather than use the above expansions. This leads to a residual μ dependence in the branching ratio, since higher order terms in Eqs. (18) and (24) have not been matched by the corresponding higher order contributions to
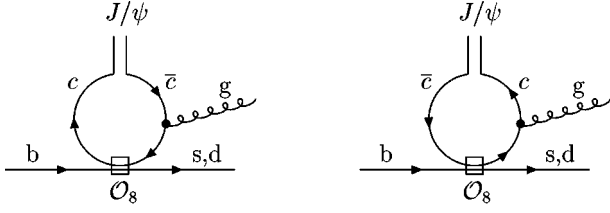
FIG. 2. Feynman graphs contributing to $B \to q J/\psi g$ $(q = s, d)$, to lowest order in the α_s-L_0/L_2 double expansion.

the matrix elements of $\mathcal{O}_{1,8}$. The fact that the residual μ dependence is small suggests that such higher order terms in the matrix elements are negligible; keeping only the leading order in the double expansion, as we do in here, is a good approximation.

In order to obtain the branching ratio for $B \to J/\psi +$ anything, one must consider, in addition to $b \to q J/\psi$, the contribution to the inclusive decay from other possible parton processes. The dominant process is $b \to q J/\psi g$, as it contributes to the rate at order $\alpha_s L_2^2$ in our double expansion, through the diagrams in Fig. 2. After integrating over the three-body phase space, the result for the decay rate is

$$\Gamma(b \to s J/\psi g) + \Gamma(b \to d J/\psi g)$$

$$= \frac{G_F^2}{162\pi^2} |V_{cb}|^2 \alpha_s L_2^2 \left(\frac{f_{J/\psi}}{m_{J/\psi}}\right)^2 m_b^5 \left[\frac{1}{6} \ r(1-r) \right.$$

$$\times (1 + 37r - 8r^2) - (1 - 6r) r \ln r \bigg], \qquad (25)$$

with $r \equiv m_{J/\psi}^2/m_b^2$. Notice that this contribution to the $B \to J/\psi +$ anything decay rate is of lower order in the double expansion than the contribution from $b \to q J/\psi$. However, we have checked that the three-body phase space suppresses the rate, so that both contributions are quantitatively of similar size. This allows us to neglect corrections to $b \to q J/\psi g$ and other parton processes such as $b \to q J/\psi g g$ or $b \to q J/\psi g q' \bar{q}'$, which appear in the $B \to J/\psi +$ anything rate at order $\alpha_s^2 L_2^2$ or $\alpha_s L_0 L_2$. They are of the same order as our calculation for the $b \to q J/\psi$ rate, but quantitatively smaller because of the phase space suppression.

IV. RESULTS

Using the expressions for the decay rates, in Eqs. (15) and (25), together with the results of Eqs. (21) and (22), we obtain our prediction for the $B \to J/\psi +$ anything branching ratio,

$$B(B \to J/\psi + \text{anything}) = \tau_B \sum_{q=s,d} [\Gamma(b \to q J/\psi)$$

$$+ \Gamma(b \to q J/\psi g)]. \qquad (26)$$

In order to eliminate the factor of m_b^5 that appears in the decay rates, and so minimize the uncertainty in our result, we divide the right-hand side of Eq. (26) by the expression for the inclusive semileptonic B decay rate [9],
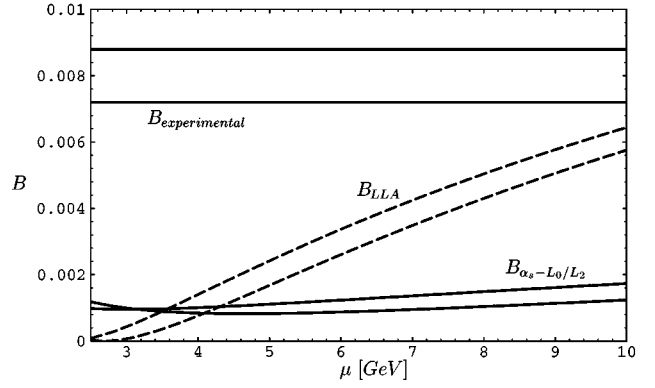

FIG. 3. The $B \to J/\psi +$ anything branching ratio as a function of the renormalization scale μ. The theoretical curves, in the leading logarithm approximation (LLA) and in the α_s-L_0/L_2 double expansion, correspond to $f_{J/\psi} = 515$ MeV, $m_b = 4.8$ GeV and $\Lambda^{(5)}_{\overline{\text{MS}}} = 209^{+39}_{-33}$ MeV.

$$B(B \to X_c e^- \bar{\nu}_e) = \tau_B \frac{G_F^2}{192\pi^3} |V_{cb}|^2 m_b^5 f\left(\frac{m_c}{m_b}\right) h\left(\frac{m_c}{m_b}, m_b\right), \qquad (27)$$

and multiply it by the experimental result [5]

$$B(B \to X_c e^- \bar{\nu}_e) = (10.4 \pm 0.4)\%. \qquad (28)$$

In Eq. (27),

$$f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z \qquad (29)$$

is a phase space factor and

$$h(z, \mu) = 1 - \frac{2\alpha_s(\mu)}{3\pi} \tilde{h}(z), \qquad (30)$$

with [9]

$$\tilde{h}(z) \simeq (\pi^2 - \tfrac{31}{4})(1-z)^2 + \tfrac{3}{2}, \qquad (31)$$

is a QCD correction factor. In our quantitative analysis, we take for the b-quark pole mass $m_b = 4.8 \pm 0.15$ GeV and $m_b - m_c = 3.40$ GeV, as in [10]. As for the J/ψ decay constant, it can be determined from $\Gamma(J/\psi \to e^+ e^-) = (5.26 \pm 0.37)$ keV [5]. Within the color-singlet approximation (now applied to the J/ψ decay) and including perturbative QCD corrections,

$$\Gamma(J/\psi \to e^+ e^-) = \frac{4\pi}{3} Q_c^2 \alpha(m_{J/\psi})^2 \left(\frac{f_{J/\psi}}{m_{J/\psi}}\right)^2 m_{J/\psi}$$

$$\times \left[1 - \frac{16\alpha_s(m_{J/\psi})}{3\pi} \right], \qquad (32)$$

with $\alpha(m_{J/\psi}) = 1/133$ and $\alpha_s(m_{J/\psi}) = 0.245$, we find $f_{J/\psi} \simeq 515$ MeV. Given the importance of the QCD corrections of order α_s, it is quite possible that higher order corrections, not included in Eq. (32), will be significant. Because of this potentially large theoretical uncertainty, we choose to factor out the dependence on $f_{J/\psi}$ in our final result.

In Fig. 3, we show the predicted $B \to J/\psi +$ anything

branching ratio as a function of the renormalization scale μ, for μ ranging between $m_b/2$ and $2m_b$. The error band corresponds to the uncertainty in $\Lambda_{\overline{\mathrm{MS}}}^{(5)}$. Adding to this the uncertainty in m_b and that due to the residual μ dependence, we find

$$B(B \to J/\psi + \text{anything}) = \left(\frac{f_{J/\psi}}{515 \text{ MeV}} \right)^2 \times 0.9^{+1.1}_{-0.3} \times 10^{-3}. \tag{33}$$

(We have not included the error due to the use of the parton process to describe the inclusive decay of the B meson [11]. That error is small when compared to the sensitivity of our result to the exact value of $\Lambda_{\overline{\mathrm{MS}}}^{(5)}$ and m_b.) The central value in Eq. (33) corresponds to $\mu = m_b = 4.8$ GeV and $\Lambda_{\overline{\mathrm{MS}}}^{(5)} = 209$ MeV.

It is clear that the prediction for the $B \to J/\psi + $ anything decay rate, in the color-singlet approximation for J/ψ production and decay, falls well short of the experimental value $B_{\mathrm{expt}} = (0.80 \pm 0.08)\%$. Although one might have suspected that this was so, already, from the LLA prediction, that result could not be trusted because of the strong dependence on the renormalization scale (see Fig. 3). With a prediction that is much more stable in μ, it can now be concluded with certainty that new, nonperturbative, contributions to the description of the J/ψ bound state are indeed important [1]. A calculation of the $B \to J/\psi + $ anything decay rate which attempts to include such contributions is given in Ref. [2]. Our result can be used [12] to improve the color-singlet part of the $B \to J/\psi + $ anything decay rate that appears in there.

ACKNOWLEDGMENTS

APPENDIX

The origin of the scheme dependent term κ', in the expression for g_1 [see Eq. (21)], deserves a brief comment. We summarize the more detailed discussion of Ref. [8] (notice that our result for κ' can be recovered from the more general case presented there).

The κ' term in g_1 originates from the regularization of the divergence in the unrenormalized one-loop $b \to qJ/\psi$ amplitude. That divergence is of the form

$$A_{\mathrm{div}} = -\frac{G_F}{\sqrt{2}} V_{cb} V_{cq}^* C_2 \alpha_s \frac{1}{36\pi} \left[\frac{2}{\varepsilon} - \gamma_E + \ln(4\pi) \right]$$
$$\times \langle qJ/\psi | \Omega | b \rangle, \tag{A1}$$

with

$$\Omega = \bar{q}[\gamma_\mu(1-\gamma_5)\gamma_\alpha\gamma_\nu - \gamma_\nu\gamma_\alpha\gamma_\mu(1-\gamma_5)]b\bar{c}[\gamma^\nu\gamma^\alpha\gamma^\mu$$
$$\times (1-\gamma_5) - \gamma^\mu(1-\gamma_5)\gamma^\alpha\gamma^\nu]c. \tag{A2}$$

In $d = 4$ dimensions, $\mathcal{O}_{1,8}$ form a complete basis under QCD corrections, and Ω is proportional to $\mathcal{O}_1$: $\Omega = -24\mathcal{O}_1$. In $d = 4 - \varepsilon$ dimensions, however, the $\mathcal{O}_{1,8}$ basis must be extended to include evanescent operators E, and

$$\Omega = a_1(\varepsilon)\mathcal{O}_1 + E. \tag{A3}$$

The evanescent operators E do not exist in four dimensions, and they do not contribute to the physical amplitude. On the other hand, the terms of order ε in $a_1(\varepsilon)$ will contribute to the finite part of the $b \to qJ/\psi$ amplitude, when inserted in Eq. (A2). In order to determine these terms, we must completely fix the regularization scheme by giving the form of the evanescent operators E (and the same form for these operators must be used in the calculation of the Wilson coefficients). One way to do this is to define the evanescent operators by the condition [8]

$$E_{\alpha\beta,\gamma\delta}\Gamma_{\beta\gamma}\Gamma_{\delta\alpha} = 0, \tag{A4}$$

where the two pairs of fermion fields were removed from E, and each one was replaced by $\Gamma \equiv \gamma_\mu(1+\gamma_5)$. Applying this operation to both sides of Eq. (A3), we obtain

$$a_1(\varepsilon) = \Omega_{\alpha\beta,\gamma\delta}\Gamma_{\beta\gamma}\Gamma_{\delta\alpha}(\mathcal{O}_{1\,\alpha\beta,\gamma\delta}\Gamma_{\beta\gamma}\Gamma_{\delta\alpha})^{-1}$$
$$= -12[2 - \kappa'\varepsilon + \mathcal{O}(\varepsilon^2)]. \tag{A5}$$

This is the origin of the scheme-dependent κ' term in the expression for g_1 of Eq. (21).

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