

Factorization theorems, effective field theory, and nonleptonic heavy meson decays

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(Received 3 January 1997)

The nonleptonic heavy meson decays $B \rightarrow D^{(*)}\pi(\rho)$, $J/\psi K^{(*)}$, and $D \rightarrow K^{(*)}\pi$ are studied based on the three-scale perturbative QCD factorization theorem developed recently. In this formalism the Bauer-Stech-Wirbel parameters a_1 and a_2 are treated as the Wilson coefficients, whose evolution from the W boson mass down to the characteristic scale of the decay processes is determined by effective field theory. The evolution from the characteristic scale to a lower hadronic scale is formulated by the Sudakov resummation. The scale-setting ambiguity, which exists in the conventional approach to nonleptonic heavy meson decays, is removed. Nonfactorizable and nonspectator contributions are taken into account as part of the hard decay subamplitudes. Our formalism is applicable to both bottom and charm decays, and predictions, including those for the ratios R and R_L associated with the $B \rightarrow J/\psi K^{(*)}$ decays, are consistent with experimental data. [S0556-2821(97)01115-6]

PACS number(s): 13.25.Hw, 11.10.Hi, 12.38.Bx, 13.25.Ft

I. INTRODUCTION

The analysis of exclusive nonleptonic heavy meson decays has been a challenging subject because of the involved complicated QCD dynamics. These decays occur through the Hamiltonian

$$H = \frac{G_F}{\sqrt{2}} V_{ij} V_{kl}^* (\bar{q}_i q_k) (\bar{q}_j q_l), \quad (1)$$

with G_F the Fermi coupling constant, V 's the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, q 's the relevant quarks, and $(\bar{q}q) = \bar{q} \gamma_\mu (1 - \gamma_5) q$ the $V-A$ current. Hard gluon corrections cause an operator mixing, and their renormalization-group (RG) summation leads to the effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ij} V_{kl}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \quad (2)$$

where the four-fermion operators $O_{1,2}$ are written as $O_1 = (\bar{q}_i q_k) (\bar{q}_j q_l)$ and $O_2 = (\bar{q}_j q_k) (\bar{q}_i q_l)$. The Wilson coefficients $c_{1,2}$, organizing the large logarithms from the hard gluon corrections to all orders, describe the evolution from the W boson mass M_W to a lower scale μ with the initial conditions $c_1(M_W) = 1$ and $c_2(M_W) = 0$.

The simplest and most widely adopted approach to exclusive nonleptonic heavy meson decays is the Bauer-Stech-Wirbel (BSW) model [1] based on the factorization hypothesis, in which the decay rates are expressed in terms of various hadronic transition form factors. Employing the Fierz transformation, the coefficient of the form factors corresponding to the external W boson emission is $a_1 = c_1 + c_2/N$, and that corresponding to the internal W boson emission is $a_2 = c_2 + c_1/N$, N being the number of colors. The form factors may be related to each other by heavy

quark symmetry, and be modeled by different ansatz. The nonfactorizable contributions which cannot be expressed in terms of hadronic transition form factors, and the nonspectator contributions from the W boson exchange are neglected. In this way the BSW method avoids the complicated QCD dynamics.

Though the BSW model is simple and gives predictions in fair agreement with experimental data, it encounters several difficulties. It has been known that the large N limit of $a_{1,2}$, i.e., the choice $a_1 = c_1(M_c) \approx 1.26$ and $a_2 = c_2(M_c) \approx -0.52$, with M_c the c quark mass, explains the data of charm decays [1]. However, the same large N limit of $a_1 = c_1(M_b) \approx 1.12$ and $a_2 = c_2(M_b) \approx -0.26$, M_b being the b quark mass, does not apply to the bottom case. Even after including the $c_{1,2}/N$ term so that $a_1 = 1.03$ and $a_2 = 0.11$, the BSW predictions are still insufficient to match the data. To overcome this difficulty, parameters χ , denoting the corrections from the nonfactorizable final-state interactions, have been introduced [2]. They lead to the effective coefficients

$$a_1^{\text{eff}} = c_1 + c_2 \left(\frac{1}{N} + \chi_1 \right), \quad a_2^{\text{eff}} = c_2 + c_1 \left(\frac{1}{N} + \chi_2 \right). \quad (3)$$

χ should be negative for charm decays, canceling the color-suppressed term $1/N$, and be positive for bottom decays in order to enhance the predictions. Unfortunately, the mechanism responsible for this sign change has not been understood completely. Furthermore, in such a framework theoretical predictions depend sensitively on the choice of the scale μ for the Wilson coefficients: Setting μ to $2M_b$ or $M_b/2$ gives rise to a more than 20% difference.

Equivalently, one may regard $a_{1,2}$ as free parameters, and determine them by data fitting. However, the behavior of the transition form factors involved in nonleptonic heavy meson decays requires an ansatz as stated above, such that the extraction of $a_{1,2}$ from experimental data becomes model dependent. On the other hand, it was found that the ratio

a_2/a_1 from an individual fit to the CLEO data of $B \rightarrow D^{(*)}\pi(\rho)$ [3] varies significantly [4]. It was also shown that an allowed domain (a_1, a_2) exists for the three classes of decays $\bar{B}^0 \rightarrow D^{(*)+}$, $\bar{B}^0 \rightarrow D^{(*)0}$, and $B^- \rightarrow D^{(*)0}$, only when the experimental errors are expanded to a large extent [5].

Moreover, it has been very difficult to explain the two ratios associated with the $B \rightarrow J/\psi K^{(*)}$ decays [6],

$$R = \frac{\mathcal{B}(B \rightarrow J/\psi K^*)}{\mathcal{B}(B \rightarrow J/\psi K)}, \quad R_L = \frac{\mathcal{B}(B \rightarrow J/\psi K_L^*)}{\mathcal{B}(B \rightarrow J/\psi K^*)}, \quad (4)$$

simultaneously in the BSW framework, where $\mathcal{B}(B \rightarrow J/\psi K)$ is the branching ratio of the decay $B \rightarrow J/\psi K$. It was argued that the inclusion of nonfactorizable contributions is essential for the resolution of this controversy [7]. Such contributions have been analyzed in [7,8] based on the Brodsky-Lepage approach to exclusive processes [9], in which the full Hamiltonian in Eq. (1) was employed. It was found that the nonfactorizable internal W -emission amplitudes are of the same order as the factorizable ones. However, this naive perturbative QCD (PQCD) approach cannot account for the destructive interference between the external and internal W -emission contributions in charm decays. This is obvious from the fact that the coefficient associated with the internal W emissions is $a_2 = 1/N$ in both bottom and charm decays, and thus does not change sign.

Recently, a modified PQCD formalism has been proposed following the series of works [10–16], where the PQCD formalism constructed from the full Hamiltonian H was shown to be applicable to the $B \rightarrow D^{(*)}$ decays [15] in the fast recoil region of final-state hadrons [13]. It was recognized that nonleptonic heavy meson decays involve three scales: the W boson mass M_W , the typical scale t of the decay processes, and the hadronic scale of order Λ_{QCD} . Accordingly, the decay rates are factorized into three convolution factors: the ‘harder’ W -emission function, the hard b quark decay subamplitude, and the nonperturbative meson wave function, which are characterized by M_W , t , and Λ_{QCD} , respectively. Radiative corrections then produce two types of large logarithms $\ln(M_W/t)$ and $\ln(t/\Lambda_{\text{QCD}})$. In this three-scale factorization theorem $\ln(M_W/t)$ are summed to give the evolution from M_W down to t described by the Wilson coefficients $a_{1,2}(t)$, and $\ln(t/\Lambda_{\text{QCD}})$ are summed into a Sudakov factor [17], which describes the evolution from t to the lower hadronic scale. The former has been derived in effective field theory, and the latter has been implemented by the resummation technique [11].

This modified PQCD formalism is μ independent, i.e., RG invariant, and thus the scale-setting ambiguity existing in conventional effective field theory is removed [16]. As the variable t runs to below M_b and M_c , the constructive and destructive interferences involved in bottom and charm decays, respectively, appear naturally. Furthermore, not only the factorizable, but the nonfactorizable and nonspectator contributions are taken into account and evaluated in a systematic way. With the inclusion of the nonfactorizable contributions, we find that $a_{1,2}$ restore their original role of the Wilson coefficients, instead of being treated as the BSW free

parameters. The branching ratios of various decay modes $B \rightarrow D^{(*)}\pi(\rho)$ and $D \rightarrow K^{(*)}\pi$, and the ratios R and R_L associated with the $B \rightarrow J/\psi K^{(*)}$ decays can all be well explained by our formalism.

In Sec. II we derive the three-scale PQCD factorization theorem, concentrating on the separation of the contributions characterized by different scales. The incorporation of the Sudakov resummation is briefly reviewed. In Sec. III the decays $B \rightarrow D^{(*)}\pi(\rho)$ are investigated to demonstrate the importance of the nonfactorizable contributions. We then apply the formalism to the decays $D \rightarrow K^{(*)}\pi$ in Sec. IV, and show that the internal W -emission amplitude can become sufficiently negative in charm decays. In Sec. V we compute the decay rates of $B \rightarrow J/\psi K^{(*)}$ and find that the predictions for R and R_L match the data simultaneously. Section VI is the conclusion, where possible further improvements and applications of our approach are proposed.

II. THREE-SCALE FACTORIZATION THEOREMS

In this section we construct the modified PQCD formalism, that embodies both effective field theory and factorization theorems. The motivation comes from the fact that the Wilson coefficients $c_{1,2}$ of the effective Hamiltonian in Eq. (2) are explicitly μ dependent. Since physical quantities such as the decay rates, which are expressed as the products of $c_{1,2}$ with the matrix elements of the four-fermion operators $O_{1,2}$, do not depend on μ , the latter should contain a μ dependence to cancel that of the former. However, such a cancellation has never been implemented in any previous analysis of nonleptonic heavy meson decays. As stated in the Introduction, the BSW method employs the factorization hypothesis [1], under which the matrix elements of $O_{1,2}$ are factorized into two hadronic matrix elements of the (axial) vector currents $(\bar{q}q)$. Since the current is conserved, the hadronic matrix elements have no anomalous scale dependence, and thus the μ dependence of the Wilson coefficients remains. To remedy this problem, μ should be chosen in such a way that the factorization hypothesis gives dominant contributions. However, the hadronic matrix elements involve both a short-distance scale associated with the heavy quark and a long-distance scale with the mesons. Naively, setting μ to the heavy quark mass will lose large logarithms containing the small scale. It is then quite natural that theoretical predictions are sensitive to the value of μ [18].

We shall show that the cancellation of the μ dependence is explicit in our formalism. We begin with the idea of the conventional PQCD factorization theorem for the $B \rightarrow D$ transition form factors, which describe the amplitude of a b quark decay into a c quark through the current operator $(\bar{c}_L \gamma_\mu b_L)$. Radiative corrections to these form factors are ultraviolet (UV) finite, because the current is not renormalized. However, the corrections give rise to infrared (IR) divergences at the same time, when the loop gluons are soft or collinear to the light partons in the mesons. These IR divergences should be separated from the full radiative corrections and grouped into nonperturbative soft functions.

The separation of IR divergences in one of the higher-order diagrams is demonstrated by Fig. 1(a), where the bubble represents the lowest-order decay subamplitude of the

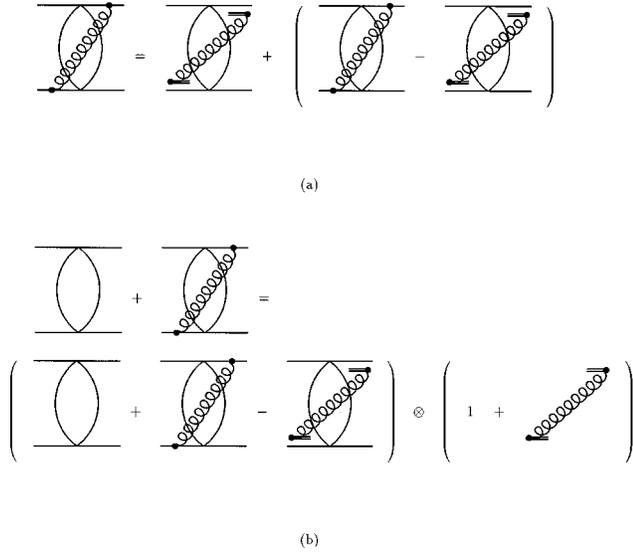


FIG. 1. (a) Separation of the infrared and hard $O(\alpha_s)$ contributions in PQCD. (b) $O(\alpha_s)$ factorization into a soft function and a hard decay subamplitude.

B meson. This diagram is reexpressed into two terms: The first term, with proper eikonal approximation for quark propagators, picks up the IR structure of the full diagram. The second term, containing an IR subtraction, is finite. The first term, being IR sensitive, is absorbed into a meson wave function $\phi(b, \mu)$, if the radiative correction is two-particle reducible, or into a soft function $U(b, \mu)$, if the radiative correction is two-particle irreducible as shown in Fig. 1(a). Here b is the conjugate variable of the transverse momentum k_T carried by a valence quark of the meson, and thus can be regarded as the transverse extent of the meson. It will become clear later that the scale $1/b$ serves as an IR cutoff of the associated loop integral. The function U corresponds, in some sense, to the nonfactorizable final-state interactions in the literature of nonleptonic heavy meson decays [2]. The second term, being IR finite, is absorbed into the hard decay subamplitude $H(t, \mu)$ as a higher-order correction, where t is its typical scale.

The above factorization procedure is graphically described by Fig. 1(b), where the diagrams in the first parentheses contribute to H , and those in the second parentheses to ϕ or U . Below we shall neglect U , because of the pair cancellation between the diagram in Fig. 1(a) and the diagram with the right end of the gluon attaching the lower quark line. The combination of these two diagrams leads to an integrand proportional to a factor $1 - e^{i l_T \cdot b}$, l_T being the transverse loop momentum. It is then apparent that U is unimportant, if the main contributions to the form factors came from the small b region. It will be shown that the Sudakov factor mentioned in the Introduction exhibits a strong suppression at large b , and thus justifies the neglect of U .

Note that U has no IR pole due to the factor $1 - e^{i l_T \cdot b}$, which vanishes as $l_T \rightarrow 0$, though each individual diagram contributing to U has, while the meson wave function, to which such a pair cancellation does not happen, possesses an IR pole. The disappearance of the IR pole of U has been employed to deduce the universality of hadron wave functions in the literature. However, when moving to the large

b region, U contains large logarithms $\ln(1/b)$, characterizing its soft nature. If Sudakov suppression is not strong enough, U should be included, and the large $\ln(1/b)$ need to be summed for a more accurate analysis. On the other hand, U involves complicated color flows. Hence, we leave its discussion to a separate work [19].

Though the full diagrams are UV finite, the IR factorization introduces UV divergences into ϕ and H , which have opposite signs. This observation hints a RG treatment of the factorization formula derived above. Let γ_ϕ be the anomalous dimension of ϕ . Then the anomalous dimension of H must be $-\gamma_\phi$. We have the RG equations

$$\mu \frac{d}{d\mu} \phi = -\gamma_\phi = -\mu \frac{d}{d\mu} H, \quad (5)$$

whose solutions are given by

$$\phi(b, \mu) = \phi(b, 1/b) \exp \left[- \int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) \right], \quad (6)$$

$$H(t, \mu) = H(t, t) \exp \left[- \int_{\mu}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) \right]. \quad (7)$$

Equation (6) describes the evolution of ϕ from the IR cutoff $1/b$ to an arbitrary scale μ , and Eq. (7) describes the evolution of H from μ to the typical scale t . The physics characterized by momenta smaller than $1/b$ is absorbed into the initial condition $\phi(b, 1/b)$, which is of nonperturbative origin. After the RG treatment, the large logarithms $\ln(t/\mu)$ in H are grouped into the exponent, and thus the initial condition $H(t, t)$ can be computed by perturbation theory. Combining Eqs. (6) and (7), the factorization formula becomes free of the μ dependence as indicated by

$$H(t, \mu) \phi(b, \mu) = H(t, t) \phi(b, 1/b) \times \exp \left[- \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) \right]. \quad (8)$$

The effective Hamiltonian H_{eff} in Eq. (2) can be constructed in a similar way. Consider the nonleptonic b quark decays through a W boson emission up to $O(\alpha_s)$. We reexpress the full diagram, which does not possess UV divergences because of the current conservation and the presence of the W boson propagator, into two terms as shown in Fig. 2(a). The first term, obtained by shrinking the W boson line into a point, corresponds to the local four-fermion operators $O_{1,2}$ appearing in H_{eff} , and is absorbed into the hard decay subamplitude $H(t, \mu)$. This subamplitude is characterized by momenta smaller than the W boson mass M_W , that is, by the typical scale t of the heavy meson decays, since gluons in H do not “see” the W boson. The second term, characterized by momenta of order M_W due to the subtraction term, is absorbed into a “harder” function $H_r(M_W, \mu)$ (not an amplitude). Note that the factorization in H is not complete yet, because it still contains IR divergences, i.e., the contributions characterized by the hadronic scale.

We then obtain the $O(\alpha_s)$ factorization formula shown in Fig. 2(b), where the diagrams in the first parentheses contrib-

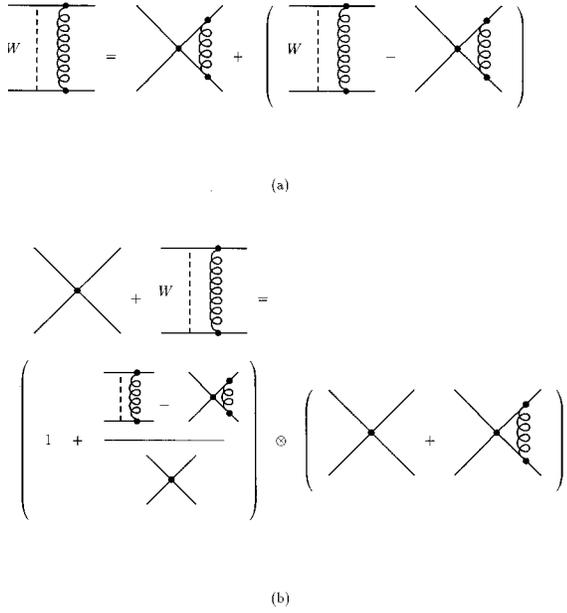


FIG. 2. (a) Separation of the hard and harder $O(\alpha_s)$ contributions in effective field theory. (b) $O(\alpha_s)$ factorization into a ‘harder’ function and a hard decay subamplitude.

ute to H_r , and those in the second parentheses to H . This formula should be interpreted as a matrix relation because of the mixing between the operators O_1 and O_2 , or equivalently, the four-fermion vertex should be regarded as the linearly combined operators $O_1 \pm O_2$, which evolve independently. The four-fermion vertex in the denominator means that H_r does not carry Dirac and color matrix structures. Similarly, UV divergences are introduced in the above factorization procedure, when the W boson line is shrunk, and thus both H and H_r need renormalization. The RG improved factorization formula is written as

$$H_r(M_W, \mu)H(t, \mu) = H_r(M_W, M_W)H(t, t) \exp \left[\int_t^{M_W} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_{H_r}(\alpha_s(\bar{\mu})) \right], \quad (9)$$

with γ_{H_r} the anomalous dimension of H_r . It is easy to identify the exponential in Eq. (9) as the Wilson coefficient, implying that the scale μ in the Wilson coefficient should be set to the hard scale t . The function $H_r(M_W, M_W)$ can now be safely taken as its lowest-order expression $H_r^{(0)} = 1$, since the large logarithms $\ln(M_W/\mu)$ have been organized into the exponent. Note that the appropriate active flavor number should be substituted into $\alpha_s(\bar{\mu})$, when $\bar{\mu}$ evolves from M_W down to t . The continuity conditions for the transition of $\alpha_s(\bar{\mu})$ between regions with different active flavor numbers [20] are understood.

We are now ready to construct the three-scale factorization theorem by combining Eqs. (8) and (9). Start with the nonleptonic heavy meson decay amplitude up to $O(\alpha_s)$ without integrating out the W boson. The IR sensitive functions are first factorized according to Fig. 3(a), such that the diagrams in the first parentheses are characterized by momenta larger than the IR cutoff. Employing Fig. 2(b) to separate H_r , we arrive at the factorization formula described by Fig.

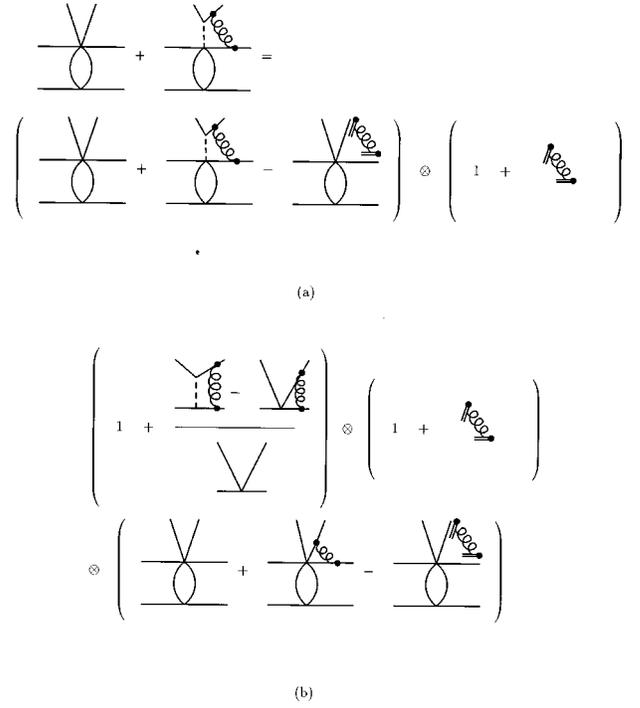


FIG. 3. (a) $O(\alpha_s)$ factorization of a soft function from a full decay amplitude. (b) $O(\alpha_s)$ three-scale factorization formula for a decay amplitude.

3(b). The diagrams in the last parentheses are identified as the hard decay subamplitude H . It is obvious that its anomalous dimension is given by $\gamma_H = -\gamma_\phi - \gamma_{H_r}$. Applying the RG analysis to each convolution factor, we derive

$$H_r(M_W, \mu)H(t, \mu)\phi(b, \mu) = c(t)H(t, t)\phi(b, 1/b) \times \exp \left[- \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) \right], \quad (10)$$

where the Wilson coefficient $c(t)$ represents the exponential in Eq. (9). The cancellation of the μ dependences among the three convolution factors is explicit. The two-stage evolutions from $1/b$ to t and from t to M_W have been established. We emphasize that the Wilson coefficient appears as a convolution factor of the three-scale factorization formula, which is, however, a constant coefficient (once its argument μ is set to a value) in the conventional approach of effective field theory.

In the leading logarithmic approximation $c_{1,2}$ are given, in terms of the combination $c_\pm(\mu) = c_1(\mu) \pm c_2(\mu)$, by

$$c_\pm(\mu) = \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{-6\gamma_\pm/(33-2n_f)}, \quad (11)$$

with the constants $2\gamma_+ = -\gamma_- = -2$, and n_f the number of active quark flavors. Below we shall employ the more complicated two-loop expressions of $c_{1,2}$ presented in the Appendix A, that include next-to-leading logarithms [20].

A remark is in order. Though Eq. (10) is μ independent, another ambiguity arises from the nonperturbative nature of the meson wave function $\phi(b, 1/b)$, which describes physics

below the IR cutoff $1/b$. This cutoff is arbitrary, and the others differing from $1/b$ by a factor of order unity, for example, $2/b$, are equally good. For a different cutoff, ϕ should change accordingly [for example, to $\phi(b/2, 2/b)$] such that predictions remain the same. However, in practice, the exact behavior of ϕ with b is unknown, and is neglected usually as in the following sections. It turns out that our predictions depend on the cutoff slightly.

At last, we explain how to incorporate the Sudakov factor into the above factorization formula. The RG solution in Eq. (6) sums only the single logarithms contained in the meson wave function ϕ . In fact, there exist also double logarithms coming from the overlap of collinear and soft divergences. Hence, an extra large-scale P^+ (or P^-), the dominant light-cone component of the meson momentum in the Breit frame, should be added into ϕ as an argument. The scale P^+ is associated with the collinear divergences, while the small-scale $1/b$ is associated with the soft divergences as stated at the beginning of this section. Before reaching Eq. (5), one performs the resummation for these double logarithms, and obtains

$$\phi(P^+, b, \mu) = \phi(b, \mu) \exp[-s(P^+, b)]. \quad (12)$$

e^{-s} is the Sudakov factor, which exhibits a strong suppression in the large b region. The single-scale wave function $\phi(b, \mu)$, discussed above, is then identified as the initial condition of the resummation for the two-scale wave function $\phi(P^+, b, \mu)$. For the detailed derivation of Eq. (12), refer to [11, 13].

In summary, the large logarithms $\ln(M_W/t)$ are grouped into the Wilson coefficients $c_{1,2}$, and $\ln(tb)$ are organized by the resummation technique and by the RG method. Combining Eqs. (10) and (12), we derive the final expression of the three-scale factorization formula

$$\begin{aligned} & H_r(M_W, \mu) H(t, \mu) \phi(x, P^+, b, \mu) \\ &= c(t) H(t, t) \phi(x, b, 1/b) \\ & \times \exp\left[-s(P^+, b) - \int_{1/b}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu}))\right], \quad (13) \end{aligned}$$

where the momentum fraction x associated with a valence quark of the meson has been inserted. Note that Eq. (13) is in fact the integrand appearing in the factorization formula for the exclusive nonleptonic heavy meson decays, and all the variables x , t , and b will be integrated over. That is, the Wilson coefficient c , the hard decay subamplitude H , and the meson wave function ϕ are in a convolution relation, instead of the multiplication relation as shown in the effective Hamiltonian equation (2). Hence, our formalism is a non-trivial extension of effective field theory.

III. THE $B \rightarrow D^{(*)} \pi(\rho)$ DECAYS

In the conventional BSW approach the branching ratios of the exclusive nonleptonic heavy meson decays are parameterized only by the factorizable contributions from the external and internal W emissions as stated in the Introduction. The associated nonfactorizable contributions, which cannot be expressed in terms of hadronic form factors, are ignored.

The nonspectator contributions from the W -exchange diagrams, which may be factorizable or nonfactorizable, are not included either. However, the naive PQCD analysis based on the full Hamiltonian in Eq. (1) has shown that the nonfactorizable contributions are comparable to the factorizable ones for the internal W emissions and for the W exchanges [8].

In this section we shall investigate the importance of the nonfactorizable and nonspectator contributions to exclusive nonleptonic heavy meson decays employing the more sophisticated three-scale PQCD factorization theorem developed in the previous section [16]. We evaluate the branching ratios of the $B \rightarrow D^{(*)} \pi(\rho)$ decays, taking into account the factorizable, nonfactorizable, and nonspectator contributions, and letting the Wilson coefficients $c_{1,2}$ evolve according to effective field theory. In this framework the external W emissions also give nonfactorizable contributions. The relevant effective Hamiltonian is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \quad (14)$$

with the four-fermion operators $O_1 = (\bar{d}u)(\bar{c}b)$ and $O_2 = (\bar{c}u)(\bar{d}b)$. The full Hamiltonian H is then a special case with the choice $c_1 = 1$ and $c_2 = 0$.

We first study the $B \rightarrow D^{(*)} \pi$ decays. The analysis of the $B \rightarrow D^{(*)} \rho$ decays is similar. The factorizable external W -emission amplitudes define the $B \rightarrow D^{(*)}$ transition form factors ξ through the hadronic matrix elements:

$$\begin{aligned} \langle D(P_2) | V^\mu | B(P_1) \rangle &= \sqrt{M_B M_D} [\xi_+(\eta)(v_1 + v_2)^\mu \\ & \quad + \xi_-(\eta)(v_1 - v_2)^\mu], \\ \langle D^*(P_2) | V^\mu | B(P_1) \rangle &= i \sqrt{M_B M_{D^*}} \xi_V(\eta) \epsilon^{\mu\nu\alpha\beta} \epsilon_\nu^* v_{2\alpha} v_{1\beta}, \\ \langle D^*(P_2) | A^\mu | B(P_1) \rangle &= \sqrt{M_B M_{D^*}} [\xi_{A_1}(\eta)(\eta + 1) \epsilon^{*\mu} \\ & \quad - \xi_{A_2}(\eta) \epsilon^* \cdot v_1 v_1^\mu \\ & \quad - \xi_{A_3}(\eta) \epsilon^* \cdot v_1 v_2^\mu]. \quad (15) \end{aligned}$$

P_1 (P_2), M_B ($M_{D^{(*)}}$), and v_1 (v_2) are the momentum, the mass, and the velocity of the B ($D^{(*)}$) meson, satisfying the relation $P_1 = M_B v_1$ ($P_2 = M_{D^{(*)}} v_2$). ϵ^* is the polarization vector of the D^* meson. The velocity transfer $v_1 \cdot v_2$ in two-body nonleptonic decays takes the maximal value $\eta = (1 + r^2)/(2r)$ with $r = M_{D^{(*)}}/M_B$. In the rest frame of the B meson P_1 and P_2 are expressed as $P_1 = (M_B/\sqrt{2})(1, 1, \mathbf{0}_T)$ and $P_2 = (M_B/\sqrt{2})(1, r^2, \mathbf{0}_T)$ [13]. For the analysis below, we define k_1 (k_2) the momentum of the light valence quark in the B ($D^{(*)}$) meson. k_1 may have a minus component k_1^- , giving the momentum fraction $x_1 = k_1^-/P_1^-$, and small transverse components \mathbf{k}_{1T} . k_2 may have a large plus component k_2^+ , giving $x_2 = k_2^+/P_2^+$, and small \mathbf{k}_{2T} . The pion then carries the momentum $P_3 = P_1 - P_2$, whose nonvanishing component is only P_3^- . One of its valence quark carries the fractional momentum $x_3 P_3^-$, and small transverse momenta \mathbf{k}_{3T} . In the infinite mass limit of M_B and $M_{D^{(*)}}$ the form factors ξ_i with $i = +, -, V, A_1, A_2$, and A_3 obey the relations

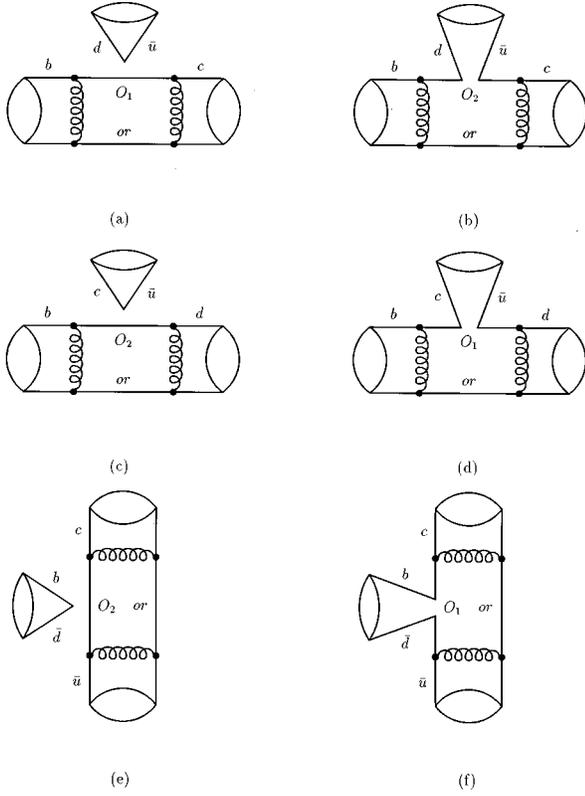


FIG. 4. Factorizable external W emissions from (a) the operator O_1 and from (b) O_2 , factorizable internal W emissions from (c) O_2 and from (d) O_1 , and factorizable W exchanges from (e) O_2 and from (f) O_1 .

$$\xi_+ = \xi_V = \xi_{A_1} = \xi_{A_3} = \xi, \quad \xi_- = \xi_{A_2} = 0. \quad (16)$$

ξ is the so-called Isgur-Wise function [21], which is normalized to unity at zero recoil $\eta \rightarrow 1$ by heavy quark symmetry.

Equation (15) is the standard definition of the form factors ξ_i . In our approach, however, the factorization formulas for ξ_i will contain the Wilson coefficients as the extra convolution factors. ξ_i include the contributions from the hadronic matrix element of O_1 shown in Fig. 4(a) and from the color-suppressed matrix element of O_2 in Fig. 4(b). Therefore, their factorization formulas involve the Wilson coefficient $a_1 = c_1 + c_2/N$. We define the form factors $\xi_{\text{int}}^{(*)}$ for the internal W -emission diagrams, which include the factorizable contributions from the matrix elements of O_2 in Fig. 4(c), and from the color-suppressed matrix element of O_1 in Fig. 4(d). These form factors then contain the Wilson coefficient $a_2 = c_2 + c_1/N$. Similarly, we define the form factors $\xi_{\text{exc}}^{(*)}$ for the W -exchange diagrams, which include the factorizable contributions from the matrix elements of O_2 in Fig. 4(e), and from the color-suppressed matrix element of O_1 in Fig. 4(f). Hence, these form factors also contain the Wilson coefficient a_2 .

For the nonfactorizable contributions to the $B \rightarrow D^{(*)}\pi$ decays, the possible diagrams are exhibited in Fig. 5, which correspond to those in Fig. 4. Figures 5(a), 5(c), and 5(e) do not contribute at $O(\alpha_s)$ simply because a trace of odd number of color matrices vanishes. Hence, all the nonfactorizable contributions come from Figs. 5(b), 5(d), and 5(f), denoted by the amplitudes $\mathcal{M}_b^{(*)}$, $\mathcal{M}_d^{(*)}$, and $\mathcal{M}_f^{(*)}$, respectively,

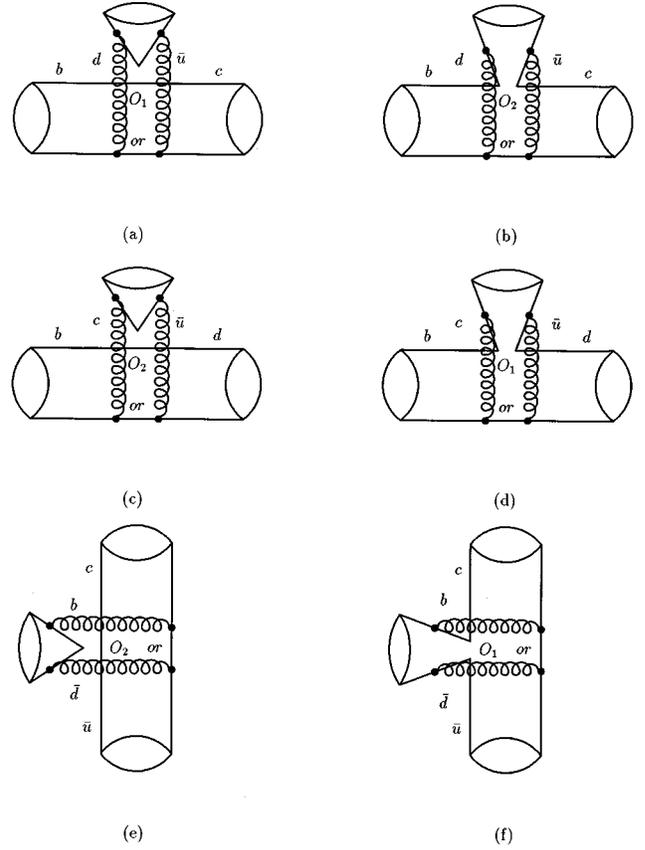


FIG. 5. Nonfactorizable external W emissions from (a) the operator O_1 and from (b) O_2 , nonfactorizable internal W emissions from (c) O_2 and from (d) O_1 , and nonfactorizable W exchanges from (e) O_2 and from (f) O_1 .

and are thus color suppressed. Their expressions are more complicated, and cannot be written in terms of hadronic form factors. The amplitudes $\mathcal{M}_b^{(*)}$ for the nonfactorizable external W emissions depend on the Wilson coefficient c_2/N . They have the same expressions for the charged and neutral B meson decays, because replacing the spectator \bar{u} quark in the B^- meson by the \bar{d} quark does not change the Feynman rules. The amplitudes $\mathcal{M}_d^{(*)}$ for the nonfactorizable internal W emissions contain the Wilson coefficient c_1/N . $\mathcal{M}_f^{(*)}$ for the nonfactorizable W exchanges involve the Wilson coefficient c_1/N .

The decay rates of $B \rightarrow D^{(*)}\pi$ have the expression

$$\Gamma_i = \frac{1}{128\pi} G_F^2 |V_{cb}|^2 |V_{ud}|^2 M_B^3 \frac{(1-r^2)^3}{r} |\mathcal{M}_i|^2, \quad (17)$$

where $i=1, 2, 3$, and 4 denote the modes $B^- \rightarrow D^0 \pi^-$, $\bar{B}^0 \rightarrow D^+ \pi^-$, $B^- \rightarrow D^{*0} \pi^-$, and $\bar{B}^0 \rightarrow D^{*+} \pi^-$, respectively. With the above form factors and the nonfactorizable amplitudes, the decay amplitudes \mathcal{M}_i are written as

$$\mathcal{M}_1 = f_\pi [(1+r)\xi_+ - (1-r)\xi_-] + f_D \xi_{\text{int}} + \mathcal{M}_b + \mathcal{M}_d, \quad (18)$$

$$\mathcal{M}_2 = f_\pi [(1+r)\xi_+ - (1-r)\xi_-] + f_B \xi_{\text{exc}} + \mathcal{M}_b + \mathcal{M}_f, \quad (19)$$

$$\begin{aligned} \mathcal{M}_3 &= \frac{1+r}{2r} f_\pi [(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2} + \xi_{A_3})] + f_{D^*} \xi_{\text{int}}^* \\ &+ \mathcal{M}_b^* + \mathcal{M}_d^*, \end{aligned} \quad (20)$$

$$\begin{aligned} \mathcal{M}_4 &= \frac{1+r}{2r} f_\pi [(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2} + \xi_{A_3})] + f_B \xi_{\text{exc}}^* \\ &+ \mathcal{M}_b^* + \mathcal{M}_f^*, \end{aligned} \quad (21)$$

where f_B , $f_{D^{(*)}}$, and f_π are the B meson, $D^{(*)}$ meson, and pion decay constants. We have made explicit that the charged B meson decays $B^- \rightarrow D^{(*)0} \pi^-$ contain the external and internal W -emission contributions, and the neutral B meson decays $\bar{B}^0 \rightarrow D^{(*)+} \pi^-$ contain the external W -emission and W -exchange contributions.

In the considered maximal recoil region with $P_2^+ \gg M_{D^{(*)}}/\sqrt{2} \gg P_2^-$, we regard the $D^{(*)}$ meson as a light meson for simplicity [13]. Double logarithms contained in the B meson, $D^{(*)}$ meson, and pion wave functions ϕ_B , $\phi_{D^{(*)}}$, and ϕ_π , respectively, are organized into the corresponding Sudakov factors using the resummation technique [13,17,22]:

$$\phi_B(x_1, P_1^-, b_1, \mu) = \phi_B(x_1, b_1, 1/b_1) \exp[-S_B(\mu)],$$

$$\phi_{D^{(*)}}(x_2, P_2^+, b_2, \mu) = \phi_{D^{(*)}}(x_2, b_2, 1/b_2) \exp[-S_{D^{(*)}}(\mu)],$$

$$\phi_\pi(x_3, P_3^-, b_3, \mu) = \phi_\pi(x_3, b_3, 1/b_3) \exp[-S_\pi(\mu)], \quad (22)$$

with the exponents

$$\begin{aligned} S_B(\mu) &= s(x_1 P_1^-, b_1) + 2 \int_{1/b_1}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \\ S_{D^{(*)}}(\mu) &= s(x_2 P_2^+, b_2) + s((1-x_2) P_2^+, b_2) \\ &+ 2 \int_{1/b_2}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})), \\ S_\pi(\mu) &= s(x_3 P_3^-, b_3) + s((1-x_3) P_3^-, b_3) \\ &+ 2 \int_{1/b_3}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma(\alpha_s(\bar{\mu})). \end{aligned} \quad (23)$$

The quark anomalous dimension $\gamma = -\alpha_s/\pi$ is related to $\gamma_\phi = 2\gamma$, introduced in Sec. II. The spatial extents b_i of the mesons are the Fourier conjugate variables of k_{iT} . We shall

neglect the intrinsic b dependence of the wave functions, denoted by the argument b , and the $O(\alpha_s(1/b))$ corrections, denoted by the argument $1/b$. That is, we assume $\phi(x, b, 1/b) = \phi(x)$. The wave functions $\phi_i(x)$, $i=B, D, D^*$, and π , satisfy the normalization

$$\int_0^1 \phi_i(x) dx = \frac{f_i}{2\sqrt{6}}. \quad (24)$$

The exponent s is written as [23]

$$s(Q, b) = \int_{1/b}^Q \frac{d\mu}{\mu} \left[\ln\left(\frac{Q}{\mu}\right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right], \quad (25)$$

where the anomalous dimensions A to two loops and B to one loop are given by

$$\begin{aligned} A &= C_F \frac{\alpha_s}{\pi} + \left[\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln\left(\frac{e^{\gamma_E}}{2}\right) \right] \left(\frac{\alpha_s}{\pi} \right)^2, \\ B &= \frac{2}{3} \frac{\alpha_s}{\pi} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right), \end{aligned} \quad (26)$$

with $C_F = 4/3$ the color factor and γ_E the Euler constant. The two-loop expression of the running coupling constant,

$$\frac{\alpha_s(\mu)}{\pi} = \frac{4}{\beta_0 \ln(\mu^2/\Lambda^2)} - \frac{16\beta_1}{\beta_0^3} \frac{\ln \ln(\mu^2/\Lambda^2)}{\ln^2(\mu^2/\Lambda^2)}, \quad (27)$$

will be substituted into Eq. (25), with the coefficients

$$\beta_0 = \frac{33-2n_f}{3}, \quad \beta_1 = \frac{153-19n_f}{6}, \quad (28)$$

and the QCD scale $\Lambda \equiv \Lambda_{\text{QCD}}$.

Combined with the evolution of the hard decay subamplitudes H , the variables μ in Eq. (23) are replaced by the hard scales t as shown in Eq. (13), leading to the RG-invariant Sudakov exponents $S_B(t)$, $S_{D^{(*)}}(t)$, and $S_\pi(t)$. Since large logarithms have been organized, we compute H to lowest order by sandwiching Figs. 4 and 5 with the matrix structures $(\mathcal{P}_1 + M_B) \gamma_5 / \sqrt{2N}$ from the initial B meson, with $\gamma_5 (\mathcal{P}_2 + M_D) / \sqrt{2N}$, $\not{\epsilon} (\mathcal{P}_2 + M_{D^*}) / \sqrt{2N}$, and $\gamma_5 \mathcal{P}_3 / \sqrt{2N}$ from the final D meson, D^* meson, and pion, respectively.

The expressions of all the form factors and nonfactorizable amplitudes for the $B \rightarrow D^{(*)} \pi$ decays are listed below. The form factors ξ_i , $i=+, A_1$, and A_3 , and ξ_j , $j=-$ and A_2 , derived from Figs. 4(a) and 4(b), are given by

$$\begin{aligned} \xi_i = & 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^{(*)}}(x_2) a_1(t) \alpha_s(t) \\ & \times [(1 + \zeta_i x_2 r) h(x_1, x_2, b_1, b_2, m) + (r + \zeta'_i x_1) h(x_2, x_1, b_2, b_1)] \exp[-S_B(t) - S_{D^{(*)}}(t)], \end{aligned} \quad (29)$$

$$\begin{aligned} \xi_j = & 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^{(*)}}(x_2) a_1(t) \alpha_s(t) [\zeta_j x_2 r h(x_1, x_2, b_1, b_2) + \zeta'_j x_1 h(x_2, x_1, b_2, b_1)] \\ & \times \exp[-S_B(t) - S_{D^{(*)}}(t)], \end{aligned} \quad (30)$$

with the constants [15]

$$\begin{aligned} \zeta_+ = \zeta'_+ = & \frac{1}{2} \left[\eta - \frac{3}{2} + \sqrt{\frac{\eta-1}{\eta+1}} \left(\eta - \frac{1}{2} \right) \right], & \zeta_{A_2} = 0, \quad \zeta'_{A_2} = -1 - \frac{\eta}{\sqrt{\eta^2-1}}, \\ \zeta_- = -\zeta'_- = & -\frac{1}{2} \left[\eta - \frac{1}{2} + \sqrt{\frac{\eta+1}{\eta-1}} \left(\eta - \frac{3}{2} \right) \right], & \zeta_{A_3} = -\frac{1}{2} - \frac{\eta-2}{2\sqrt{\eta^2-1}}, \quad \zeta'_{A_3} = \frac{1}{2\sqrt{\eta^2-1}}. \end{aligned} \quad (31)$$

Here η takes the maximal velocity transfer $\eta = (1 + r^2)/(2r)$ as stated before. For the behavior of the above form factors at other values of velocity transfer, refer to [15].

The form factors $\xi_{\text{int}}^{(*)}$ from Figs. 4(c) and 4(d), and $\xi_{\text{exc}}^{(*)}$ from Figs. 4(e) and 4(f) are written as

$$\begin{aligned} \xi_{\text{int}}^{(*)} = & 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \phi_\pi(x_3) a_2(t_{\text{int}}) \alpha_s(t_{\text{int}}) \\ & \times \{ [1 + x_3(1 - r^2)] h_{\text{int}}(x_1, x_3, b_1, b_3, m_{\text{int}}) + \zeta_{\text{int}}^{(*)} x_1 r^2 h_{\text{int}}(x_3, x_1, b_3, b_1, m_{\text{int}}) \} \exp[-S_B(t_{\text{int}}) - S_\pi(t_{\text{int}})], \end{aligned} \quad (32)$$

$$\begin{aligned} \xi_{\text{exc}}^{(*)} = & 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_{D^{(*)}}(x_2) \phi_\pi(x_3) a_2(t_{\text{exc}}) \alpha_s(t_{\text{exc}}) \\ & \times \{ [x_3(1 - r^2) - \zeta_{\text{exc}}^{(*)} r^2] h_{\text{exc}}(x_2, x_3, b_2, b_3, m_{\text{exc}}) + x_2 h_{\text{exc}}(x_3, x_2, b_3, b_2, m_{\text{exc}}) \} \exp[-S_{D^{(*)}}(t_{\text{exc}}) - S_\pi(t_{\text{exc}})], \end{aligned} \quad (33)$$

with the constants $\zeta_{\text{int}} = \zeta_{\text{exc}} = -\zeta_{\text{int}}^* = -\zeta_{\text{exc}}^* = 1$. In the derivation of $\xi_{\text{int}}^{(*)}$ we have assumed that k_1 has a plus component $k_1^+ = x_1 P_1^+$. It is obvious that $\xi_{\text{int}}^{(*)}$ and $\xi_{\text{exc}}^{(*)}$ are exactly the $B \rightarrow \pi$ and $D^{(*)} \rightarrow \pi$ transition form factors, respectively, evaluated at maximal recoil.

In Eqs. (29), (30), (32), and (33) the functions h 's, obtained from the Fourier transform of the lowest-order H , are given by

$$h(x_1, x_2, b_1, b_2, m) = K_0(\sqrt{x_1 x_2 m b_1}) [\theta(b_1 - b_2) K_0(\sqrt{x_2 m b_1}) I_0(\sqrt{x_2 m b_2}) + \theta(b_2 - b_1) K_0(\sqrt{x_2 m b_2}) I_0(\sqrt{x_2 m b_1})], \quad (34)$$

$$h_{\text{int}}(x_1, x_3, b_1, b_3, m_{\text{int}}) = h(x_1, x_3, b_1, b_3, m_{\text{int}}), \quad (35)$$

$$\begin{aligned} h_{\text{exc}}(x_2, x_3, b_2, b_3, m_{\text{exc}}) = & \frac{\pi^2}{4} H_0^{(1)}(\sqrt{x_2 x_3 m_{\text{exc}} b_2}) [\theta(b_2 - b_3) H_0^{(1)}(\sqrt{x_3 m_{\text{exc}} b_2}) J_0(\sqrt{x_3 m_{\text{exc}} b_3}) + \theta(b_3 - b_2) H_0^{(1)} \\ & \times (\sqrt{x_3 m_{\text{exc}} b_3}) J_0(\sqrt{x_3 m_{\text{exc}} b_2})], \end{aligned} \quad (36)$$

with $m = M_B^2$ and $m_{\text{int}} = m_{\text{exc}} = (1 - r^2)M_B^2$. We observe that the W -exchange contributions are complex due to the exchange of timelike hard gluons. The hard scales t are chosen as

$$t = \max(\sqrt{x_1 m}, \sqrt{x_2 m}, 1/b_1, 1/b_2), \quad (37)$$

$$t_{\text{int}} = \max(\sqrt{x_1 m_{\text{int}}}, \sqrt{x_3 m_{\text{int}}}, 1/b_1, 1/b_3), \quad (38)$$

$$t_{\text{exc}} = \max(\sqrt{x_2 m_{\text{exc}}}, \sqrt{x_3 m_{\text{exc}}}, 1/b_2, 1/b_3), \quad (39)$$

where we consider only the energies $\sqrt{x_i m}$ and $\sqrt{x_j m}$ from

the longitudinal momenta of the internal quarks in the diagrams of Fig. 4, because the gluon energies $\sqrt{x_i x_j m}$ are always smaller. The scales $1/b_i$ are associated with the transverse momenta. Then Sudakov suppression guarantees that the main contributions come from the large t region, where the running coupling constant $\alpha_s(t)$ is small, and perturbation theory is reliable.

For the nonfactorizable amplitudes, the factorization formulas involve the kinematic variables of all the three mesons, and the Sudakov exponent is given by $S = S_B + S_{D^{(*)}} + S_\pi$. The integration over b_3 can be performed trivially, leading to $b_3 = b_1$ or $b_3 = b_2$. Their expressions are

$$\begin{aligned} \mathcal{M}_b^{(*)} &= 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^{(*)}}(x_2) \phi_\pi(x_3) \\ &\times \left\{ \alpha_s(t_b^{(1)}) \frac{c_2(t_b^{(1)})}{N} \exp[-S(t_b^{(1)})|_{b_3=b_2}] [(1-r^2)(1-x_3) - x_1 + \zeta_b^{(*)}(r-r^2)(x_1-x_2)] h_b^{(1)}(x_i, b_i) - \alpha_s(t_b^{(2)}) \frac{c_2(t_b^{(2)})}{N} \right. \\ &\times \left. \exp[-S(t_b^{(2)})|_{b_3=b_2}] [(2-r)x_1 - (1-r)x_2 - (1-r^2)x_3] h_b^{(2)}(x_i, b_i) \right\}, \quad (40) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_d^{(*)} &= 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^{(*)}}(x_2) \phi_\pi(x_3) \zeta_d^{(*)} \\ &\times \left\{ \alpha_s(t_d^{(1)}) \frac{c_1(t_d^{(1)})}{N} \exp[-S(t_d^{(1)})|_{b_3=b_1}] [x_1 - x_2 - x_3(1-r^2)] h_d^{(1)}(x_i, b_i) + \alpha_s(t_d^{(2)}) \frac{c_1(t_d^{(2)})}{N} \right. \\ &\times \left. \exp[-S(t_d^{(2)})|_{b_3=b_1}] [(x_1+x_2)(1+\zeta_d^{(*)}r^2) - 1] h_d^{(2)}(x_i, b_i) \right\}, \quad (41) \end{aligned}$$

$$\begin{aligned} \mathcal{M}_f^{(*)} &= 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^{(*)}}(x_2) \phi_\pi(x_3) \\ &\times \left\{ \alpha_s(t_f^{(1)}) \frac{c_1(t_f^{(1)})}{N} \exp[-S(t_f^{(1)})|_{b_3=b_2}] [x_1(1+\zeta_f^{(*)}r^2) - \zeta_f^{(*)}x_2r^2 - x_3(1-r^2)] h_f^{(1)}(x_i, b_i) - \alpha_s(t_f^{(2)}) \frac{c_1(t_f^{(2)})}{N} \right. \\ &\times \left. \exp[-S(t_f^{(2)})|_{b_3=b_2}] [(x_1+x_2)(1+\zeta_f^{(*)}r^2) - \zeta_f^{(*)}r^2] h_f^{(2)}(x_i, b_i) \right\}, \quad (42) \end{aligned}$$

with the definition $[dx] \equiv dx_1 dx_2 dx_3$. The constants are $\zeta_{b,d,f} = -\zeta_{b,d,f}^* = 1$.

The functions $h^{(j)}$, $j=1$ and 2 , appearing in Eqs. (40)–(42), are written as

$$h_b^{(j)} = [\theta(b_1 - b_2) K_0(BM_B b_1) I_0(BM_B b_2) + \theta(b_2 - b_1) K_0(BM_B b_2) I_0(BM_B b_1)] \begin{pmatrix} K_0(B_j M_B b_2) & \text{for } B_j \geq 0 \\ \frac{i\pi}{2} H_0^{(1)}(|B_j| M_B b_2) & \text{for } B_j \leq 0 \end{pmatrix}, \quad (43)$$

$$h_d^{(j)} = [\theta(b_1 - b_2) K_0(DM_B b_1) I_0(DM_B b_2) + \theta(b_2 - b_1) K_0(DM_B b_2) I_0(DM_B b_1)] \begin{pmatrix} K_0(D_j M_B b_2) & \text{for } D_j \geq 0 \\ \frac{i\pi}{2} H_0^{(1)}(|D_j| M_B b_2) & \text{for } D_j \leq 0 \end{pmatrix}, \quad (44)$$

$$h_f^{(j)} = i \frac{\pi}{2} \left[\theta(b_1 - b_2) H_0^{(1)}(FM_B b_1) J_0(FM_B b_2) + \theta(b_2 - b_1) H_0^{(1)}(FM_B b_2) J_0(FM_B b_1) \right] \begin{pmatrix} K_0(F_j M_B b_1) & \text{for } F_j \geq 0 \\ \frac{i\pi}{2} H_0^{(1)}(|F_j| M_B b_1) & \text{for } F_j \leq 0 \end{pmatrix}, \quad (45)$$

with the variables

$$\begin{aligned} B^2 &= x_1 x_2 (1 - r^2), \\ B_1^2 &= (x_1 - x_2) x_3 (1 - r^2) + x_1 x_2 (1 + r^2), \\ B_2^2 &= x_1 x_2 (1 + r^2) - (x_1 - x_2) (1 - x_3) (1 - r^2), \\ D^2 = F^2 &= x_1 x_3 (1 - r^2), \\ D_1^2 = F_1^2 &= (x_1 - x_2) x_3 (1 - r^2), \\ D_2^2 &= (x_1 + x_2) r^2 - (1 - x_1 - x_2) x_3 (1 - r^2), \\ F_2^2 &= (x_1 + x_2) + (1 - x_1 - x_2) x_3 (1 - r^2). \end{aligned} \quad (46)$$

The scales $t^{(j)}$ are chosen as

$$\begin{aligned} t_b^{(j)} &= \max(BM_B, |B_j| M_B, 1/b_1, 1/b_2), \\ t_d^{(j)} &= \max(DM_B, |D_j| M_B, 1/b_1, 1/b_2), \\ t_f^{(j)} &= \max(FM_B, |F_j| M_B, 1/b_1, 1/b_2). \end{aligned} \quad (47)$$

Here we include also the gluon energies BM_B , DM_B , and FM_B except for the energies $|B_j| M_B$, $|D_j| M_B$, and $|F_j| M_B$ of the internal quarks, because the former may not be smaller than the latter.

The corresponding form factors and the nonfactorizable amplitudes for the $B \rightarrow D^{(*)} \rho$ decays can be computed in a similar way, and their expressions are presented in the Appendix B. The only differences are the matrix structures of the ρ meson in the calculation of the hard decay subamplitudes, and the extra transverse-mode contributions from the ρ_T meson, except for the longitudinal-mode contributions from the ρ_L meson. The matrix structures associated with the

final ρ_L and ρ_T mesons are $\not{P}_3 / \sqrt{2N}$ and $\not{\epsilon} \not{P}_3 / \sqrt{2N}$ with $\epsilon \cdot P_3 = 0$, respectively. We also assume a vanishing ρ meson mass.

In the evaluation of the various form factors and amplitudes, we adopt $G_F = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$, the decay constants $f_B = 200 \text{ MeV}$, $f_D = f_{D^*} = 220 \text{ MeV}$, $f_\pi = 132 \text{ MeV}$ [3], and $f_\rho = 200 \text{ MeV}$, the CKM matrix elements $|V_{cb}| = 0.043$ [13,15] and $|V_{ud}| = 0.974$, the masses $M_B = 5.28 \text{ GeV}$, $M_D = 1.87 \text{ GeV}$, and $M_{D^*} = 2.01 \text{ GeV}$ [25], and the \bar{B}^0 (B^-) meson lifetime $\tau_{B^0} = 1.53$ ($\tau_{B^-} = 1.68$) ps [26]. As to the wave functions, we employ the model

$$\phi_{B,D^{(*)}}(x) = \frac{N_{B,D^{(*)}}}{16\pi^2} \frac{x(1-x)^2}{M_{B,D^{(*)}}^2 + C_{B,D^{(*)}}(1-x)}, \quad (48)$$

for the B and $D^{(*)}$ mesons, and the Chernyak-Zhitnitsky models from QCD sum rules [24],

$$\phi_\pi(x) = \frac{5\sqrt{6}}{2} f_\pi x(1-x)(1-2x)^2, \quad (49)$$

$$\phi_\rho^L(x) = \frac{5\sqrt{6}}{2} f_\rho x(1-x)[0.25(1-2x)^2 + 0.15], \quad (50)$$

$$\phi_\rho^T(x) = \frac{5\sqrt{6}}{2} f_\rho x^2(1-x)^2, \quad (51)$$

for the pion and the ρ meson, respectively.

The normalization constant N_B and the shape parameter C_B are determined by two constraints from the relativistic constituent quark model [10]. They are given by $N_B = 604.332 \text{ GeV}^3$ and $C_B = -27.5 \text{ GeV}^2$, which correspond to $f_B = 200 \text{ MeV}$ listed above. The shape parameters C_D and C_D^* are adjusted such that our predictions for the branching ratios of the various modes of $B \rightarrow D^{(*)} \pi$ fall into

TABLE I. Predictions for the branching ratios of the $B \rightarrow D^{(*)} \pi(\rho)$ decays from the PQCD formalism based on H (I), on H_{eff} (II), and from the BSW model with the parameters $a_1 = 1.15$ and $a_2 = 0.26$ (BSWI) [3] and with $a_1 = 1.012$ and $a_2/a_1 = 0.224$ (BSWII) [4]. The CLEO data [3] are also shown.

Modes	I (%)	II (%)	BSWI (%)	BSWII (%)	Data (%)
$B^- \rightarrow D^0 \pi^-$	0.52	0.50	0.57	0.51	0.534 ± 0.025
$\bar{B}^0 \rightarrow D^+ \pi^-$	0.33	0.33	0.35	0.28	0.308 ± 0.026
$B^- \rightarrow D^{*0} \pi^-$	0.49	0.48	0.56	0.56	0.497 ± 0.044
$\bar{B}^0 \rightarrow D^{*+} \pi^-$	0.32	0.32	0.34	0.27	0.304 ± 0.024
$B^- \rightarrow D^0 \rho^-$	1.34	1.21	1.07	1.11	1.022 ± 0.067
$\bar{B}^0 \rightarrow D^+ \rho^-$	0.63	0.68	0.82	0.69	0.861 ± 0.078
$B^- \rightarrow D^{*0} \rho^-$	1.34	1.62	1.27	1.48	1.444 ± 0.134
$\bar{B}^0 \rightarrow D^{*+} \rho^-$	0.58	0.83	0.93	0.83	0.844 ± 0.071

TABLE II. Contributions to the $B \rightarrow D^{(*)}\pi$ decays from the factorizable external W emissions and internal W emissions (or W exchanges), and from the nonfactorizable amplitudes $\mathcal{M}_{b,d,f}^{(*)}$ in Eqs. (18)–(21). The unit is 10^{-3} GeV.

Amplitudes	External W (factorizable)	Internal W (factorizable)	$M_b^{(*)}$	$M_d^{(*)}$
\mathcal{M}_1	95.1	2.5	$-5.3 + 14.8i$	$18.5 - 10.4i$
\mathcal{M}_3	86.5	2.6	$6.6 - 20.6i$	$17.0 - 10.8i$
Amplitudes	External W (factorizable)	W exchange (factorizable)	$M_b^{(*)}$	$M_f^{(*)}$
\mathcal{M}_2	95.1	$-0.6 + 0.4i$	$-5.3 + 14.8i$	$2.2 - 3.1i$
\mathcal{M}_4	86.5	$-0.6 + 1.3i$	$6.6 - 20.6i$	$3.0 - 3.1i$

the errors of the experimental data [3] shown in Table I. We determine $C_D = -3.372$ GeV² and $C_{D^*} = -3.772$ GeV², and the corresponding normalization constants $N_D = 92.85$ GeV³ and $N_{D^*} = 119.51$ GeV³ from $f_D = f_{D^*} = 220$ GeV. Results along with those from the naive PQCD formalism based on the full Hamiltonian H (i.e., with $c_1 = 1$ and $c_2 = 0$), are exhibited in Table I. We find that the predictions for B meson decays from these two approaches are close to each other. For comparison, we quote the BSW results from [3] (BSWI), and from [4] (BSWII), in which a modified pole ansatz is employed to make the extraction of the ratio a_2/a_1 less mode dependent.

Certainly, Eqs. (48)–(51) are not the only choices of the meson wave functions. For example, the oscillator wave function for the B meson,

$$\phi'_B(x) = \frac{N'_B}{8\pi^2} \sqrt{x(1-x)} \exp\left(-\frac{M_B^2}{2\omega^2} x^2\right), \quad (52)$$

with $\omega = 0.4$ GeV, has been proposed in [27], and the asymptotic pion wave function

$$\phi'_\pi(x) = \frac{\sqrt{6}}{2} f_\pi x(1-x) \quad (53)$$

is widely adopted in the literature. The predictions from these models have been evaluated, and presented in Fig. 7 and Table I of [12]. ϕ'_B peaks at $x \rightarrow 0$ much more sharply than ϕ_B , their second moments which carry the information of the function profiles differing by five times. However, the results of the form factors ξ_+ and ξ_- from ϕ'_B are only 2.5 times larger than those from ϕ_B . We also find that the relative decay rate of each mode is insensitive to the models of the wave functions. This is satisfactory enough, since the variation of the absolute rates can always be compensated by other overall parameters.

Different kinds of contributions to the decay amplitudes \mathcal{M}_i in Eqs. (18)–(21) associated with the decays $B \rightarrow D^{(*)}\pi$ are presented in Table II. It is obvious that the nonfactorizable internal W -emission amplitudes $\mathcal{M}_d^{(*)}$ play an essential role for the explanation of the branching ratios of the $B \rightarrow D^{(*)}\pi$ decays: In the charged B meson decays $\mathcal{M}_d^{(*)}$ is about 20% of the factorizable external W -emission contributions, while in the neutral B meson decays only the factorizable external W emissions dominate, and all other

kinds of contributions are small. Hence, the branching ratios of the former are predicted to be $(1.2)^2 \times (\tau_{B^-} / \tau_{B^0}) \approx 1.6$ times of the latter, which is well consistent with the data. This conclusion differs from the previous one drawn in [8], which is based on the naive PQCD formalism: The factorizable internal W emissions give 20% of the factorizable external W -emission contributions, and are responsible for the ratio of the charged B to neutral B decay rates. Though the values in column I and in column II of Table I are close, the relative weights of the various contributions change. The W -exchange contributions are always negligible, which are only about 5% of the external W -emission amplitudes. If the conventional factorization hypothesis for nonleptonic B meson decays is correct, only the diagrams in Fig. 4 are considered. However, our analysis has indicated that Figs. 4(c) and 4(d) give small contributions. This is the reason the naive choice $\mu = M_b$ for the arguments of the Wilson coefficients in the BSW model cannot match the data.

The results for the branching ratios of the $B \rightarrow D^{(*)}\rho$ decays listed in Table I are also satisfactory. Note that after fixing the $D^{(*)}$ meson wave function from the data of $B \rightarrow D^{(*)}\pi$, there is no free parameter left in the analysis of the $B \rightarrow D^{(*)}\rho$ decays. Hence, the consistency of our predictions with the data is nontrivial. The cutoff dependence of the modified PQCD formalism can be tested simply by substituting $2/b$ for $1/b$ in the factorization formulas. The predictions change by about 5%. In the conventional approach of effective field theory the substitution of M_b by $2M_b$ for the arguments of the Wilson coefficients $c_{1,2}$ results in a more than 20% difference. Hence, the scale-setting ambiguity is indeed moderated in our approach.

IV. THE $D \rightarrow K^{(*)}\pi$ DECAYS

We have stated that the naive choice of the BSW parameters $a_1 = c_1(M_c) + c_2(M_c)/N$ and $a_2 = c_2(M_c) + c_1(M_c)/N$ cannot explain the data of charm decays. To do it, the large N ansatz $a_1 = c_1(M_c) \approx 1.26$ and $a_2 = c_2(M_c) \approx -0.51$ must be assumed [1]. However, the same ansatz $a_2 = c_2(M_b) \approx 0.11$ does not work for bottom decays, because the best fit to the experimental data gives $a_2 \approx 0.22$ [4]. We argue that the large N ansatz is the consequence of the factorization hypothesis employed in the BSW model. If the nonfactorizable contributions along with the evolution of the Wilson coefficients are taken into account, such an ansatz is

not necessary. In this section we apply the three-scale factorization theorem to the decays $D \rightarrow K^{(*)}\pi$, and explore in details how the contributions from Figs. 4 and 5 vary, when they are evaluated at different energy scales. In our approach the factorizable and nonfactorizable contributions change with the characteristic scales t of the decay processes in different ways, such that their combination can explain both the bottom and charm data. That is, our work provides a unified viewpoint to the exclusive nonleptonic heavy meson decays.

Before proceeding with the calculation of the decay rates, we emphasize that the applicability of PQCD to D meson decays with $M_D = 1.87$ GeV is marginal. This is hinted by the conclusion in [28], which showed that the PQCD analysis of the proton form factor with the Sudakov effects included is reliable for the energy scale above 2–3 GeV. Therefore, we concentrate only on the mechanism of the destructive interference involved in charm decays. For this purpose, it is enough to consider the ratios of the charged D meson decay rates to the neutral D meson decay rates,

$$R_1 = \frac{\mathcal{B}(D^- \rightarrow K^0 \pi^-)}{\mathcal{B}(\bar{D}^0 \rightarrow K^+ \pi^-)}, \quad R_2 = \frac{\mathcal{B}(D^- \rightarrow K^{*0} \pi^-)}{\mathcal{B}(\bar{D}^0 \rightarrow K^{*+} \pi^-)}, \quad (54)$$

instead of the individual branching ratios.

The $D \rightarrow K^{(*)}\pi$ decays occur through a similar effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \quad (55)$$

where the four-fermion operators are $O_1 = (\bar{d}u)(\bar{s}c)$ and $O_2 = (\bar{s}u)(\bar{d}c)$. The analysis of the nonleptonic B meson decays in the previous section can be copied to the D meson decays directly. The expressions of the decay rates Γ_i are similar to Eq. (17), but with the subscripts $i = 1, 2, 3$, and 4 denoting the modes $D^- \rightarrow K^0 \pi^-$, $\bar{D}^0 \rightarrow K^+ \pi^-$, $D^- \rightarrow K^{*0} \pi^-$, and $\bar{D}^0 \rightarrow K^{*+} \pi^-$, respectively. At the same time, the CKM matrix element $|V_{cs}| = 1.0$ is substituted for $|V_{cb}|$, and the meson masses M_D and $M_{K^{(*)}}$ for M_B and $M_{D^{(*)}}$, respectively. In all the form factors and nonfactorizable amplitudes the kinematic variables of the B ($D^{(*)}$) meson are replaced by those of the D ($K^{(*)}$) meson. The \bar{D}^0 (D^-) meson lifetime is $\tau_{\bar{D}^0} = 0.415$ ($\tau_{D^-} = 1.05$) ps [29]. The D meson wave function has been determined in the study of the

TABLE III. Predictions for the ratios R_1 and R_2 associated with the $D \rightarrow K^{(*)}\pi$ decays from the PQCD formalism based on H (I) and on H_{eff} (II). The experimental data [29] are also shown.

Modes	I	II	Data
R_1	4.96	0.74	0.72
R_2	5.00	0.35	0.38

B meson decays. For the kaon, we have the masses $M_K = 0.497$ GeV and $M_{K^*} = 0.892$ GeV, the decay constants $f_K = 160$ and $f_{K^*} = 220$ MeV, and the model wave functions derived from QCD sum rules [24],

$$\phi_K(x) = \frac{\sqrt{6}}{2} f_K x(1-x)[3.0(1-2x)^2 + 0.4], \quad (56)$$

$$\phi_{K^*}^L(x) = \frac{\sqrt{6}}{2} f_{K^*} x(1-x)[0.5(1-2x)^2 + 0.9], \quad (57)$$

$$\phi_{K^*}^T(x) = \sqrt{6} f_{K^*} x(1-x)[0.7 - (1-2x)^2], \quad (58)$$

for the K , K_L^* , and K_T^* mesons, respectively. Note that we take $\phi_{K^*}^L$ as the K^* meson wave functions throughout the analysis of the D meson decays for simplicity. Then all the factorization formulas in Sec. III can be adopted directly without further modification. Compared to the pion wave function ϕ_π in Eq. (49), ϕ_K 's do not possess dips at the middle of the momentum fraction x . $\phi_{K^*}^T$ has a single hump at $x = 1/2$, differing from the behavior of ϕ_K and $\phi_{K^*}^L$.

Because of the smaller D meson mass, the transverse degrees of freedom are more important in the definitions of the hard scales t . Hence, we choose the maximum of the scales $1/b_i$ for the arguments t of the Wilson coefficients. In this case Sudakov suppression is weaker, and thus insufficient to diminish the contributions from the region with t close to Λ_{QCD} , where $c_{1,2}$ diverge. To have meaningful predictions, a lower bound $t_c = (1 + \epsilon)\Lambda_{\text{QCD}}$ must be introduced for the variables t in the numerical analysis, where ϵ is a small number. We then have $t = \max(1/b_i, t_c)$. The results of R_1 and R_2 for $\epsilon \approx 0.0002$ are exhibited in Table III, which are well consistent with the data. Note that t_c can be regarded as

TABLE IV. Contributions to the $D \rightarrow K^{(*)}\pi$ decays from the factorizable external W emissions and internal W emissions (or W exchanges), and from the nonfactorizable amplitudes $\mathcal{M}_{b,d,f}^{(*)}$. The unit is 10^{-3} GeV.

Amplitudes	External W (factorizable)	Internal W (factorizable)	$M_b^{(*)}$	$M_d^{(*)}$
\mathcal{M}_1	368.0	-192.4	-19.7 + 17.1 <i>i</i>	18.5 - 24.4 <i>i</i>
\mathcal{M}_3	752.0	-576.7	44.9 - 10.3 <i>i</i>	119.2 - 39.8 <i>i</i>
Amplitudes	External W (factorizable)	W exchange (factorizable)	$M_b^{(*)}$	$M_f^{(*)}$
\mathcal{M}_2	368.0	3.6 - 2.1 <i>i</i>	-19.7 + 17.1 <i>i</i>	-10.7 + 24.7 <i>i</i>
\mathcal{M}_4	752.0	-2.5 - 37.8 <i>i</i>	44.9 - 10.3 <i>i</i>	51.6 - 5.6 <i>i</i>

one and the only one free parameter in the analysis of the D meson decays. Therefore, the simultaneous fit to R_1 and R_2 is not trivial.

The contributions from different diagrams are listed in Table IV. The W -exchange contributions to the neutral D meson decays are negligible as in the neutral B meson decays. It is easy to observe that with t running to below M_c , the factorizable internal W -emission contributions to the charged D meson decays become very negative due to the evolution of a_2 , and overcome the positive nonfactorizable internal W -emission amplitudes $\mathcal{M}_d^{(*)}$. Note that the factorizable internal W -emission contributions are positive and small in the B meson decays. The naive PQCD formalism based on the full Hamiltonian H without considering the evolution of the Wilson coefficients [7,8] cannot account for this sign change, since the corresponding coefficient a_2 is always equal to $1/N$. It then predicts that the charged D meson decay rates are larger than the neutral ones (the values of R_1 and R_2 in column I of Table III are greater than unity) as in the B meson case.

V. THE $B \rightarrow J/\psi K^{(*)}$ DECAYS

As mentioned in the Introduction, it has been very difficult to explain the ratios R and R_L associated with the $B \rightarrow J/\psi K^{(*)}$ decays simultaneously, which were defined in Eq. (4), in the BSW framework based on the factorization hypothesis. We have observed in Secs. III and IV that the nonfactorizable contributions play an essential role in the decays $B \rightarrow D^{(*)} \pi(\rho)$ and $D \rightarrow K^{(*)} \pi$. Therefore, it is expected that the nonfactorizable effects are also important in the decays $B \rightarrow J/\psi K^{(*)}$. In fact, it has been suspected that the discrepancy between model-dependent BSW predictions and the experimental data [6] is attributed to the breakdown of the factorization hypothesis [30].

In this section we apply the three-scale factorization theorem to the $B \rightarrow J/\psi K^{(*)}$ decays, and show that our predictions for the branching ratios of the various decay modes and for R and R_L are in good agreement with the data. Similarly, the decays $B \rightarrow J/\psi K^{(*)}$ occur through the effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* [c_1(\mu) O_1 + c_2(\mu) O_2], \quad (59)$$

with the four-fermion operators $O_1 = (\bar{s}c)(\bar{c}b)$ and $O_2 = (\bar{c}c)(\bar{s}b)$. The relevant decay rates have the expression

$$\Gamma_i = \frac{1}{128\pi} G_F^2 |V_{cb}|^2 |V_{cs}|^2 m_B^3 \frac{(1-r^2)^3}{r} |\mathcal{M}_i|^2, \quad (60)$$

with $r = M_{J/\psi}/M_B$, $M_{J/\psi} = 3.096$ GeV being the J/ψ meson mass. The subscript $i=1$ denotes the modes $B^- \rightarrow J/\psi K^-$ and $\bar{B}^0 \rightarrow J/\psi K^0$, which possess the same factorization formulas, and $i=2$ denotes $B^- \rightarrow J/\psi K^{*-}$ and $\bar{B}^0 \rightarrow J/\psi K^{*0}$. Since the decay amplitudes \mathcal{M}_i contain only the internal W -emission contributions from Figs. 4(c) and 4(d) as the factorizable part, and from Fig. 5(d) as the nonfactorizable part, their expressions are given by

$$\mathcal{M}_1 = f_{J/\psi} \xi_{\text{int}}^{\xi(J/\psi)} + \mathcal{M}_d^{(J/\psi)}, \quad (61)$$

$$\mathcal{M}_2^L = f_{J/\psi} \xi_{\text{int}}^{\xi(J/\psi)L} + \mathcal{M}_d^{(J/\psi)L}, \quad (62)$$

$$\mathcal{M}_2^T = f_{J/\psi} \xi_{\text{int}}^{\xi(J/\psi)T} + \mathcal{M}_d^{(J/\psi)T}, \quad (63)$$

where the superscripts L and T denote the longitudinal and transverse modes, $B \rightarrow J/\psi K_L^*$ and $B \rightarrow J/\psi K_T^*$, respectively, and $f_{J/\psi} = 390$ MeV is the J/ψ meson decay constant [7]. Equation (61) is similar to Eq. (18) and Eqs. (62) and (63) to Eq. (20), but with the external W -emission contributions dropped. The type of the mode $\bar{B}^0 \rightarrow J/\psi K^0$ corresponds to that of $\bar{B}^0 \rightarrow D^0 \pi^0$, which was not considered in Sec. III. Note that the $\bar{B}^0 \rightarrow D^0 \pi^0$ decay involves not only the internal W -emission but also the W -exchange diagrams.

Employing the matrix structure $\not{\epsilon}(\not{P}_2 + M_{J/\psi})/\sqrt{2N}$ for the final J/ψ meson, and the vanishing kaon masses $M_K = M_{K^*} = 0$ for simplicity, the factorization formulas of the form factors and of the nonfactorizable amplitudes are derived straightforwardly. They are written as

$$\begin{aligned} \xi_{\text{int}}^{\xi(J/\psi)} = -\xi_{\text{int}}^{\xi(J/\psi)L} &= 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \phi_{K^{(*)}}^L(x_3) a_2(t_{\text{int}}) \alpha_s(t_{\text{int}}) \\ &\times \{ [1 + x_3(1-r^2)] h_{\text{int}}(x_1, x_3, b_1, b_3, m_{\text{int}}) - x_1 r^2 h_{\text{int}}(x_3, x_1, b_3, b_1, m_{\text{int}}) \} \exp[-S_B(t_{\text{int}}) - S_K(t_{\text{int}})], \end{aligned} \quad (64)$$

$$\begin{aligned} \xi_{\text{int}}^{\xi(J/\psi)T} &= 32\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \phi_{K^{(*)}}^T(x_3) a_2(t_{\text{int}}) \alpha_s(t_{\text{int}}) r h_{\text{int}}(x_1, x_3, b_1, b_3, m_{\text{int}}) \\ &\times \exp[-S_B(t_{\text{int}}) - S_K(t_{\text{int}})], \end{aligned} \quad (65)$$

$$\begin{aligned} \mathcal{M}_d^{(J/\psi)} = -\mathcal{M}_d^{(J/\psi)L} &= 16\pi \sqrt{2N} C_F \sqrt{r} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{J/\psi}^L(x_2) \phi_{K^{(*)}}^L(x_3) \\ &\times \left\{ \alpha_s(t_d^{(1)}) \frac{c_1(t_d^{(1)})}{N} \exp[-S(t_d^{(1)})]_{b_3=b_1} [2-r^2 - 2(x_1+x_2)(1-r^2)] h_d^{(1)}(x_i, b_i) + \alpha_s(t_d^{(2)}) \frac{c_1(t_d^{(2)})}{N} \right. \\ &\times \left. \exp[-S(t_d^{(2)})]_{b_3=b_1} [4x_1 - r^2 - 2x_2(1+r^2) - 2x_3(1-r^2)] h_d^{(2)}(x_i, b_i) \right\}, \end{aligned} \quad (66)$$

TABLE V. Predictions for the branching ratios of the $B \rightarrow J/\psi K^{(*)}$ decays from the PQCD formalism based on H (I) and on H_{eff} (II), and from the BSW model with the parameters $a_1=1.012$ and $a_2/a_1=0.224$ (BSWII) [4]. The CLEO and CDF data [31] are also shown.

Modes	I (%)	II (%)	BSWII (%)	Data (%)
$B^- \rightarrow J/\psi K^-$	0.11	0.11		0.102 ± 0.014
$\bar{B}^0 \rightarrow J/\psi K^0$	0.10	0.10		0.115 ± 0.023
$B^- \rightarrow J/\psi K^{*-}$	0.14	0.15		0.158 ± 0.047
$\bar{B}^0 \rightarrow J/\psi K^{*0}$	0.13	0.14		0.136 ± 0.027
R	1.32	1.47	1.84	$1.36 \pm 0.17 \pm 0.04$ $1.32 \pm 0.23 \pm 0.16$ (CDF)
R_L	0.62	0.56	0.56	$0.52 \pm 0.07 \pm 0.04$ $0.65 \pm 0.10 \pm 0.04$ (CDF)

$$\begin{aligned} \mathcal{M}_d^{(J/\psi)T} = & 32\pi\sqrt{2N}C_F\sqrt{r}M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{J/\psi}^T(x_2) \phi_{K^{(*)}}^T(x_3) \left\{ \alpha_s(t_d^{(1)}) \frac{c_1(t_d^{(1)})}{N} \exp[-S(t_d^{(1)})]_{b_3=b_1} \right. \\ & \left. \times 2r(1-x_1-x_2)h_d^{(1)}(x_i, b_i) - \alpha_s(t_d^{(2)}) \frac{c_1(t_d^{(2)})}{N} \exp[-S(t_d^{(2)})]_{b_3=b_1} \right\} 2r(1-x_1+x_2)h_d^{(2)}(x_i, b_i). \end{aligned} \quad (67)$$

The total Sudakov exponent for the nonfactorizable amplitudes is given by $S = S_B + S_{J/\psi} + S_K$, where $S_{J/\psi}$ (S_K) has the same expression as $S_{D^{(*)}}$ (S_π) in Eq. (23). The hard functions h_{int} and $h_d^{(j)}$, $j=1$ and 2 , are the same as those appearing in Eqs. (35) and (44), but with the arguments

$$m_{\text{int}} = M_B^2 - M_{J/\psi}^2, \quad (68)$$

$$D^2 = x_1 x_3 (1 - r^2), \quad (69)$$

$$\begin{aligned} D_1^2 = & (1-x_2)x_1(1+r^2) - (3-2x_2-x_2^2)\frac{r^2}{4} \\ & + (x_1+x_2-1)x_3(1-r^2), \end{aligned} \quad (70)$$

$$D_2^2 = x_1 x_2 (1 + r^2) + (x_1 - x_2) x_3 (1 - r^2) + (x_2 - \frac{1}{4}) r^2. \quad (71)$$

The hard scales t are also similar to those in the analysis of the $B \rightarrow D^{(*)}\pi$ decays but with the insertion of the above arguments.

To evaluate the form factors and the nonfactorizable amplitudes, we need the information of the J/ψ meson wave function. Unfortunately, there are not yet convincing models for them. However, it should be most possible that the two charm quarks in the J/ψ meson carry equal fractional momenta. Hence, we assume, for convenience, that the wave function $\phi_{J/\psi}^T$ for the $(J/\psi)_T$ meson with transverse polarization possesses the same form as $\phi_\rho^T \propto x^2(1-x)^2$ for the ρ_T meson in Eq. (51), because ϕ_ρ^T has a maximum at $x=1/2$. We further assume that $\phi_{J/\psi}^L$ for the $(J/\psi)_L$ meson with longitudinal polarization is proportional to $x^n(1-x)^n$ with n a free parameter, which will be determined by the data of the decay $B \rightarrow J/\psi K$. That is, we propose the model wave functions

$$\phi_{J/\psi}^L(x) = N_{J/\psi} x^n (1-x)^n, \quad (72)$$

$$\phi_{J/\psi}^T(x) = \frac{5\sqrt{6}}{2} f_{J/\psi} x^2 (1-x)^2. \quad (73)$$

The constant $N_{J/\psi}$ is related to the normalization condition

$$\int_0^1 dx \phi_{J/\psi}^L(x) = \frac{f_{J/\psi}}{2\sqrt{6}}. \quad (74)$$

We stress that the particular form of the J/ψ meson wave functions is not important. We have tried other models, such as $x(1-x)\exp[-(1-2x)^2]$, and found that it works equally well. The kaon wave functions have been shown in Eqs. (56)–(58).

The experimental data of the branching ratios of the $B \rightarrow J/\psi K^{(*)}$ decays, and of R and R_L [31] are exhibited in Table V. We determine the parameter $n=1.25$ from the branching ratio $\mathcal{B}(B \rightarrow J/\psi K)$, and then the normalization $N_{J/\psi} = 0.858 \text{ GeV}^3$ from Eq. (74). After fixing the J/ψ meson wave functions, we evaluate the branching ratios of the decays $B \rightarrow J/\psi K_L^*$ and $B \rightarrow J/\psi K_T^*$. Results and the corresponding factorizable and nonfactorizable contributions are presented in Tables V and VI, respectively. Obviously, both

TABLE VI. Contributions to the $B \rightarrow J/\psi K^{(*)}$ decays from the factorizable internal W emissions and from the nonfactorizable amplitudes $\mathcal{M}_d^{(J/\psi)}$. The unit is 10^{-3} GeV .

Amplitudes	Internal W (factorizable)	$M_d^{(J/\psi)}$
\mathcal{M}_1	165.7	$-39.3 + 6.45i$
\mathcal{M}_2^L	-158	$42.8 - 0.3i$
\mathcal{M}_2^T	47.9	$45.6 + 40.3i$

the branching ratios of the various decay modes, and R and R_L are explained successfully. If the nonfactorizable amplitudes $\mathcal{M}_d^{(*)}$ are excluded, we shall have $R=1.48$ and $R_L=0.92$, which are too large. It implies that the nonfactorizable contributions are indeed essential for the decays $B \rightarrow J/\psi K^{(*)}$.

VI. DISCUSSION

In this paper we have developed a modified PQCD formalism for the study of the exclusive nonleptonic heavy meson decays, which embodies effective field theory and factorization theorems. It involves three scales: the W boson mass M_W , the characteristic energy t of the decay processes, and the transverse extent b of the mesons. The evolution of the Wilson coefficients from M_W to t and of the Sudakov factor from t to $1/b$ are established to make the factorization formulas explicitly μ independent. The factorizable, nonfactorizable, and nonspectator contributions from the external W emissions, the internal W emissions, and the W exchanges are all taken into account, and have been evaluated reliably. We emphasize again that the Wilson coefficient appears as a convolution factor of the factorization formulas in our analysis, instead of a constant coefficient as in the conventional approach of effective field theory.

Basically, the two main controversies in the exclusive nonleptonic heavy meson decays, i.e., the extraction of $a_{1,2}$ from bottom and charm decays, and the simultaneous explanation of R and R_L , have been resolved by our formalism: The nonfactorizable internal W -emission contributions $\mathcal{M}_d^{(*)}$ alone, which are 20% of the factorizable one, account for the data of the $B \rightarrow D^{(*)} \pi(\rho)$ decays. The factorizable internal W -emission contributions, becoming negative enough to overcome $\mathcal{M}_d^{(*)}$ in the D meson case, successfully explain the data of the $D \rightarrow K^{(*)} \pi$ decays. That is, the evolution of the Wilson coefficients can lead to the necessary constructive and destructive interferences involved in bottom and charm decays. It is the nonfactorizable contributions that make trivial to account for the ratios R and R_L associated with the decays $B \rightarrow J/\psi K^{(*)}$.

Note that the free parameters contained in our formalism are less than the decay modes considered. Hence, the match of the theoretical predictions with the experimental data is nontrivial, and indicates that we may have explored the correct mechanism responsible for the nonleptonic heavy meson decays. It is worthwhile to compute other decay modes, whose consistency with the data will further justify our approach. Inclusive nonleptonic heavy meson decays are another important subject to which our formalism can be applied. The nonfactorizable soft corrections U will give the fine-tuning of our predictions. These topics will be investigated in separate works.

ACKNOWLEDGMENTS

We thank C. H. Chang, H. Y. Cheng, and B. Tseng for useful discussions. This work was supported by the National Science Council of Republic of China under the Grant No. NSC-86-2112-M-194-007.

APPENDIX A: TWO-LOOP EXPRESSIONS OF $C_{1,2}(\mu)$

In this appendix we present the expressions of the Wilson coefficients $c_{1,2}(\mu)$ to two loops, which are given in terms of $c_{\pm} = c_1 \pm c_2$ by [20]

$$c_{\pm}(\mu) = \left[1 + \frac{\alpha_s(\mu)}{4\pi} J_{\pm} \right] \left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{d_{\pm}} \times \left[1 + \frac{\alpha_s(M_W)}{4\pi} (B_{\pm} - J_{\pm}) \right], \quad (\text{A1})$$

with the constants

$$J_{\pm} = \frac{d_{\pm}}{\beta_0} \beta_1 - \frac{\gamma_{\pm}^{(1)}}{2\beta_0}, \quad d_{\pm} = \frac{\gamma_{\pm}^{(0)}}{2\beta_0},$$

$$\gamma_{\pm}^{(0)} = \pm 12 \frac{N_{\mp} - 1}{2N},$$

$$\gamma_{\pm}^{(1)} = \frac{N_{\mp} - 1}{2N} \left[-21 \pm \frac{57}{N} \mp \frac{19}{3} N \pm \frac{4}{3} n_f - 2\beta_0 k_{\pm} \right],$$

$$B_{\pm} = \frac{N_{\mp} - 1}{2N} [\pm 11 + k_{\pm}]. \quad (\text{A2})$$

The scheme-dependent parameters k_{\pm} are

$$k_{\pm} = 0 \quad \text{NDR},$$

$$= \mp 4 \quad \text{'t Hooft-Veltman}. \quad (\text{A3})$$

The constants β_0 and β_1 have been defined in Eq. (28). In this paper we adopt the naive dimensional reduction (NDR) scheme. However, we have tested the sensitivity of our predictions to these two schemes, and found that the scheme dependence can be absorbed into the meson wave functions. Namely, the wave functions vary with the scheme such that the predictions almost remain the same.

When the scale μ in $\alpha_s(\mu)$ evolves from above M_b to below M_b , the flavor number n_f changes from 5 to 4. A similar change from $n_f=4$ to 3 occurs as μ evolves from above M_c to below M_c . To make α_s continuous at these thresholds, Λ_{QCD} must change accordingly. However, again, the dependence on Λ_{QCD} can also be absorbed into the wave functions, such that our predictions are insensitive to whether or not the continuity conditions of α_s are implemented. Hence, for simplicity and within the errors of the data, we assign the value $\Lambda_{\text{QCD}}=0.2$ GeV, and $n_f=4$ for bottom decays and $n_f=3$ for charm decays in the numerical analysis.

APPENDIX B: FACTORIZATION OF THE $B \rightarrow D^{(*)} \rho$ DECAYS

The factorization formulas for the $B \rightarrow D^{(*)} \rho$ decays can be derived straightforwardly. The only differences from the $B \rightarrow D^{(*)} \pi$ case are the matrix structures of the ρ meson in the calculation of the hard decay subamplitudes H , and the extra contributions from the transverse modes involving the ρ_T meson, as stated in Sec. III.

The decay rates are given by Eq. (17) with $i=1, 2, 3$, and 4, representing the modes $B^- \rightarrow D^0 \rho^-$, $\bar{B}^0 \rightarrow D^+ \rho^-$, $B^- \rightarrow D^{*0} \rho^-$, and $\bar{B}^0 \rightarrow D^{*+} \rho^-$, respectively. The decay amplitudes \mathcal{M}_i are written as

$$\mathcal{M}_1 = f_\rho [(1+r)\xi_+ - (1-r)\xi_-] + f_D \xi_{\text{int}} - \mathcal{M}_b + \mathcal{M}_d, \quad (\text{B1})$$

$$\mathcal{M}_2 = f_\rho [(1+r)\xi_+ - (1-r)\xi_-] + f_B \xi_{\text{exc}} - \mathcal{M}_b + \mathcal{M}_f, \quad (\text{B2})$$

$$\mathcal{M}_3^L = \frac{1+r}{2r} f_\rho [(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2} + \xi_{A_3})] + f_{D^*} \xi_{\text{int}}^* + \mathcal{M}_b^* - \mathcal{M}_d^*, \quad (\text{B3})$$

$$\mathcal{M}_3^T = f_{D^*} \xi_{\text{int}}^{T*} + \mathcal{M}_d^{T*}, \quad (\text{B4})$$

$$\mathcal{M}_4^L = \frac{1+r}{2r} f_\rho [(1+r)\xi_{A_1} - (1-r)(r\xi_{A_2} + \xi_{A_3})] + f_B \xi_{\text{exc}}^* + \mathcal{M}_b^* - \mathcal{M}_f^*, \quad (\text{B5})$$

$$\mathcal{M}_4^T = f_B \xi_{\text{exc}}^{T*} + \mathcal{M}_f^{T*}, \quad (\text{B6})$$

where the superscript L (T) denotes the longitudinal (transverse) mode $B \rightarrow D^* \rho^{L(T)}$. We have used the same notation as that for the $B \rightarrow D^{(*)} \pi$ decays without confusion.

The form factors ξ_i , $i=+, -, A_1, A_2$, and A_3 , related only to the $B \rightarrow D^{(*)}$ transitions, are the same as those associated with the $B \rightarrow D^{(*)} \pi$ decays. The form factors $\xi_{\text{int}}^{(*)}$ and $\xi_{\text{exc}}^{(*)}$ and the nonfactorizable amplitudes $\mathcal{M}_{b,d,f}^{(*)}$ are similar to those in the $B \rightarrow D^{(*)} \pi$ decays, but with the pion wave function $\phi_\pi(x_3)$ replaced by the ρ_L meson wave function $\phi_\rho^L(x_3)$ given in Eq. (50). The Sudakov exponents S_ρ for the ρ meson and S_π for the pion are the same. Below we give only the form factors ξ_{int}^{T*} and ξ_{exc}^{T*} and the nonfactorizable amplitudes $\mathcal{M}_{d,f}^{T*}$ involved in the transverse modes $B \rightarrow D^* \rho_T$:

$$\xi_{\text{int}}^{T*} = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1) \phi_\rho^T(x_3) a_2(t_{\text{int}}) \alpha_s(t_{\text{int}}) r h_{\text{int}}(x_1, x_3, b_1, b_3, m_{\text{int}}) \times \exp[-S_B(t_{\text{int}}) - S_\rho(t_{\text{int}})], \quad (\text{B7})$$

$$\xi_{\text{exc}}^{T*} = 16\pi C_F \sqrt{r} M_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \phi_{D^*}(x_2) \phi_\rho^T(x_3) a_2(t_{\text{exc}}) \alpha_s(t_{\text{exc}}) r^2 h_{\text{exc}}(x_2, x_3, b_2, b_3, m_{\text{exc}}) \times \exp[-S_{D^*}(t_{\text{exc}}) - S_\rho(t_{\text{exc}})], \quad (\text{B8})$$

$$\mathcal{M}_d^{T*} = 32\pi \sqrt{2N} C_F \sqrt{r} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^*}(x_2) \phi_\rho^T(x_3) \left\{ \alpha_s(t_d^{(1)}) \frac{c_1(t_d^{(1)})}{N} \exp[-S(t_d^{(1)})]_{b_3=b_1} \right. \\ \left. \times r[1-x_1-x_2] h_d^{(1)}(x_i, b_i) + \alpha_s(t_d^{(2)}) \frac{c_1(t_d^{(2)})}{N} \exp[-S(t_d^{(2)})]_{b_3=b_1} r(x_1-x_2) h_d^{(2)}(x_i, b_i) \right\}, \quad (\text{B9})$$

$$\mathcal{M}_f^{T*} = 32\pi \sqrt{2N} C_F \sqrt{r} M_B^2 \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1) \phi_{D^*}(x_2) \phi_\rho^T(x_3) \left\{ \alpha_s(t_f^{(1)}) \frac{c_1(t_f^{(1)})}{N} \exp[-S(t_f^{(1)})]_{b_3=b_2} \right. \\ \left. \times r^2[1-x_1-x_2] h_f^{(1)}(x_i, b_i) + \alpha_s(t_f^{(2)}) \frac{c_1(t_f^{(2)})}{N} \exp[-S(t_f^{(2)})]_{b_3=b_2} r^2(x_1-x_2) h_f^{(2)}(x_i, b_i) \right\}, \quad (\text{B10})$$

with the Sudakov exponent $S = S_B + S_{D^*} + S_\rho$. The functions h_{int} , $h_d^{(j)}$, and $h_f^{(j)}$, $j=1$ and 2 , and the hard scales t have been defined in Sec. III.

- [1] M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **29**, 637 (1985); **34**, 103 (1987).
[2] H. Y. Cheng, Phys. Lett. B **335**, 428 (1994).
[3] J. L. Rodriguez, Report No. hep-ph/9604011 (unpublished); CLEO Collaboration, M. S. Alam *et al.*, Phys. Rev. D **50**, 43 (1994).

- [4] H. Y. Cheng and B. Tseng, Phys. Rev. D **51**, 6259 (1995).
[5] M. Gourdin, A. N. Kamal, Y. Y. Keum, and X. Y. Pham, Phys. Lett. B **333**, 507 (1994).
[6] M. Gourdin, A. N. Kamal, and X. Y. Pham, Phys. Rev. Lett. **73**, 3355 (1994).
[7] C. E. Carlson and J. Milana, Phys. Rev. D **49**, 5908 (1994).

- [8] C. Y. Wu, T. W. Yeh, and H-n. Li, Phys. Rev. D **55**, 237 (1997).
- [9] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); A. Szczepaniak, E. M. Henley, and S. J. Brodsky, Phys. Lett. B **243**, 287 (1990).
- [10] R. Akhouri, G. Sterman, and Y. P. Yao, Phys. Rev. D **50**, 358 (1994).
- [11] H-n. Li and H. L. Yu, Phys. Rev. Lett. **74**, 4388 (1995); Phys. Lett. B **353**, 301 (1995).
- [12] H-n. Li and H. L. Yu, Phys. Rev. D **53**, 2480 (1996).
- [13] H-n. Li, Phys. Rev. D **52**, 3958 (1995).
- [14] H-n. Li, Phys. Lett. B **348**, 597 (1995).
- [15] C. Y. Wu, T. W. Yeh, and H-n. Li, Phys. Rev. D **53**, 4982 (1996).
- [16] C. H. Chang and H-n. Li, Phys. Rev. D **55**, 5577 (1997).
- [17] J. C. Collins and D. E. Soper, Nucl. Phys. **B193**, 381 (1981).
- [18] M. Luke, M. J. Savage, and M. B. Wise, Phys. Lett. B **343**, 329 (1995); **345**, 301 (1995).
- [19] B. Tseng and H-n. Li (in preparation).
- [20] G. Buchalla, A. Buras, and M. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
- [21] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990).
- [22] H-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
- [23] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).
- [24] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).
- [25] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).
- [26] CLEO Collaboration, B. Barish *et al.*, Phys. Rev. D **51**, 1014 (1995).
- [27] M. Bauer and M. Wirbel, Z. Phys. C **42**, 671 (1989).
- [28] H-n. Li, Phys. Rev. D **48**, 4243 (1993).
- [29] Particle Data Group, E. J. Weinberg *et al.*, Phys. Rev. D **54**, 1 (1996).
- [30] R. Aleksan *et al.*, Phys. Rev. D **51**, 6235 (1995); P. Gosdzinsky, Phys. Lett. B **351**, 546 (1995).
- [31] T. Miao, talk given at the annual meeting of APS, DPF96 (unpublished).