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New class of quark mass matrix and calculability of the flavor mixing matrix

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We discuss a new general class of mass matrix *Ansatz* that respects the fermion mass hierarchy and *calculability* of the flavor mixing matrix. This is a generalization and justification of the various specific forms of the mass matrix by successive breaking of the maximal permutation symmetry. By confronting the experimental data, a large class of the mass matrices are shown to survive, while certain specific cases are phenomenologically ruled out. Also the CP violation turns out to be maximal, when the phase of the (1,2) element of the mass matrix is $\pi/2$. [S0556-2821(97)03815-0]

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With the discovery of the top quark [1], the three family structure of the fermion sector has been completely determined. Nevertheless, the flavor mixing and fermion masses and their hierarchical patterns remain to be one of the basic problems in particle physics.

Within the standard model, all masses and flavor mixing angle are free parameters and no relations among them are provided. Perhaps, a new theory could predict all masses and flavor mixing parameters in terms of some new, few fundamental parameters, but we lack such theory at the moment and are unable to derive the masses and the flavor mixing parameters from the first principles. One can at the best take a phenomenological standpoint in that one assumes a special form for the mass matrices and hopes to be able to derive phenomenologically viable relations for the flavor mixing parameters in terms of the quark masses.

As an attempt to derive relationship between the quark masses and flavor mixing hierarchies, mass matrix *Ansatz* based on flavor democracy with a suitable breaking so as to allow mixing between the quarks of near kinship was suggested about two decades ago [2]. This, in fact, reflects the *calculability* [2,3] that all flavor mixing parameters depend solely on, and are determined by, the quark masses. In general, the *calculability* condition does not determine the CP violation phase, for which either additional *Ansatz* or input is needed to determine. Of several *Ansätze* proposed, the canonical mass matrices of the Fritzsch-type [2,4] have been generally assumed to predict the entire Kobayashi-Maskawa (KM) matrix [5] or the Wolfenstein mixing matrix [6]. Though the *Ansatz* of the Fritzsch texture [2,3] is attractive because of its maximal *calculability*, it predicts a top quark mass to be no larger than 100 GeV and thus is ruled out [4].

Alternatively, one may introduce a modification to the Fritzsch texture of mass matrix by allowing a nonvanishing (2,2) elements in the ''hierarchical'' mass eigenstates. Such scheme was proposed sometime ago by Kaus and Meshkov [7] based on a postulate of the ''BCS mechanism'' for the quarks and assuming that the heaviest third generation quark mass is to be identified by the nonzero eigenvalue of the ''democratic mass matrix.'' More recently, Fritzsch and co-workers [8] have suggested the same type of mass matrix by assuming that the ''democratic'' maximal permutation symmetry may be broken in a simple and analogous manner as the mass mixing pattern of the η-η' system. As a result, the mass matrices contain only three zero elements at (1,1), (1,3), and (3,1) positions in the hierarchical mass eigenstates. Nevertheless, this does not necessarily imply lack of *calculability* because the additional nonvanishing (2,2) element may be related to the (2,3) or (3,2) elements.

With one such form for the mass matrices, Fritzsch and co-workers [8] described the KM matrix in terms of the quark mass ratios to the lowest order approximation and claimed that they are in good agreement with the experimental values. However, this is not true at least for V_{cb} element because one gets $|V_{cb}|\simeq(1/\sqrt{2})(m_s/m_b-m_c/m_t)$ so that $m_t(\mu=1\ \mathrm{GeV})$ can be at most 113 GeV from the experimental range $|V_{cb}|=0.036$–0.046 [9]. Several other authors [10,11] have also discussed specific forms of this type of mass matrices.

We present in this paper a generalization of this class of mass matrices in such a way that it can maintain the *calculability* property and consistency with experiments, while accommodating a CP violation phase. We will show that this can be achieved by breaking the democratic flavor symmetry $S(3)_L\times S(3)_R$ successively down to $S(2)_L\times S(2)_R$ and to $S(1)_L\times S(1)_R$, so that the (2,2) element can be related to (2,3) element appropriately in the hierarchical mass eigenstates.

As is well known, the 3×3 ''democratic mass matrix''

$$\frac{c}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{1}$$

exhibits the maximal $S(3)_L\times S(3)_R$ permutation symmetry. This can be achieved by breaking of the chiral symmetry $U(3)_L\times U(3)_R$ to $S(3)_L\times S(3)_R$, where U(3) is the symmetry group connecting the three generations [7,12]. One may say that the scale of this chiral symmetry breaking is the electroweak symmetry-breaking scale at which the third generation quarks get heavy masses. Indeed, one can see this by making unitary transformation of Eq. (1) with the help of

$$U=\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{2}$$

This matrix is in fact reminiscent of the matrix for the mass squared of the neutral pseudoscalar mesons in QCD in the chiral limit. In order to account for the hierarchical pattern of the second and first generation quark masses, one has to break the $S(3)_L\times S(3)_R$ symmetry successively in two stages to $S(2)_L\times S(2)_R$ and $S(1)_L\times S(1)_R$. This can be achieved by adding the following two matrices to the ''democratic matrix'' (1):

$$\begin{pmatrix} 0 & 0 & a \\ 0 & 0 & a \\ a & a & b \end{pmatrix}, \quad d\begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix}, \tag{3}$$

where the parameters (a,b) and d are responsible for the breakdown of $S(3)_L\times S(3)_R$ and $S(2)_L\times S(2)_R$ symmetries, respectively. It is also reasonable to anticipate that this two-stage breaking happens to be at around 1 GeV scale in view of the proximity of the second and first generation quark masses compared to the third generation quarks.

Note that the two scales in proximity are related to the generation of the second and first generation quark masses and the evolution from the electroweak scale to 1 GeV scale cannot alter the '' democratic '' pattern of the mass matrix because of the symmetry $S(3)_L\times S(3)_R$. Thus the resulting mass matrix can be regarded as the one at 1 GeV scale.

In principle, the most general form of $S(3)_L\times S(3)_R\rightarrow S(2)_L\times S(2)_R$ breaking can allow different parameters at the (1,3), (2,3), (3,1), and (3,2) elements. But to maintain the *calculability* property, the form of Eq. (3) containing only two parameters (a,b) is necessary, which is general enough to cover all different specific forms proposed by others [7,8,10,11] as a special case. Then in the hierarchical basis after the unitary transformation with Eq. (2), the resulting mass matrix M_H becomes

$$M_H=\begin{pmatrix} 0 & A & 0 \\ A & D & B \\ 0 & B & C \end{pmatrix}, \tag{4}$$

where $A=\sqrt{3}d$, $D=-\frac{2}{3}(2a-b)$, $B=-(\sqrt{2}/3)(a+b)$, and $C=\frac{1}{3}(4a+b)+c$.

Note that in order to get a Hermitian mass matrix instead of Eq. (4), one can use the following two matrices:

$$\begin{pmatrix} p & p & a+q \\ p & p & a+q \\ a+q^* & a+q^* & b-2p \end{pmatrix}, \tag{5}$$

$$d\begin{pmatrix} \cos\sigma & -i\sin\sigma & -e^{-i\sigma} \\ i\sin\sigma & -\cos\sigma & e^{-i\sigma} \\ -e^{i\sigma} & e^{i\sigma} & 0 \end{pmatrix},$$

where $p=\frac{4}{9}(a+b)\sin^2\delta/2$ and $q=p(1+i\frac{3}{2}e^{i\delta/2}/\sin\delta/2)$, in such a way that the matrices (3) are replaced by those of Eq. (5). Then, after the unitary transformation with Eq. (2), the (1,2) and (2,3) elements become $Ae^{-i\sigma}$ and $Be^{-i\delta}$, respectively. However, since only one phase factor is sufficient to describe the CP violation in the standard model containing three family generations of quarks, we may introduce only one phase factor in the Hermitian matrix M_H such that only (1,2) and (2,1) elements are complex and conjugate to each other. In this way, a Hermitian mass matrix of the type (4), with a complex element at (1,2) and its conjugate at (2,1), can be obtained from a general permutation symmetry-breaking chain: i.e., $S(3)_L\times S(3)_R\rightarrow S(2)_L\times S(2)_R \rightarrow S(1)_L\times S(1)_R$.

At a glance, the matrix M_H contains four independent parameters even in the case of real parameters so that the *calculability* is lost. However, one can make additional *Ansatz* to relate a to b, so that $a=kb$ in general, with the same ratio parameter k for both the up- and down-quark sectors, so as to maintain the *calculability*. On the other hand, one may think that other choices than $a=kb$ for both up and down quarks might be interesting, but the choice of $a=kb$ meets clearly the elegance of simplicity. Then, the (2,2) element is related to (2,3) element by $w\equiv B/D =(k+1)/\sqrt{2}(2k-1)$ in the hierarchical mass eigenstate and various specific mass matrices proposed by others can be identified as a special case of different ratios, i.e., $w=5/3$ $(k=0.9)$ for Ref. [7], $w=-(1/\sqrt{2})$ $(k=0)$ for Fritzsch and co-workers [8], $w=\pm 2\sqrt{2}$ $(k=\frac{5}{7}$ or $\frac{1}{3})$ for Ref. [10], and $w=\sqrt{2}(k=1)$ for Ref. [11]. The case of $k=\frac{1}{2}$ reduces to the old Fritzsch-type with $D=0$ which is ruled out by experiments as we said before. We are, therefore, interested in the general case but $k\neq\frac{1}{2}$ in this paper.

The next step is then to constrain k for the general class of mass matrix by confronting the experiments for consistency. Obviously, a careful analysis with exact flavor mixing elements predicted from the new *Ansatz* is desired to confront the experiments. The mass matrix M_H of the type (4) can be brought to a diagonal form by appropriate rotation of the fermion fields in the hierarchical eigenstates via a biunitary transformation,

$$U_L^{(u)}M_H^{(u)}U_R^{(u)\dagger}=\mathrm{diag}[m_u,m_c,m_t],$$

$$U_L^{(d)}M_H^{(d)}U_R^{(d)\dagger}=\mathrm{diag}[m_d,m_s,m_b],$$

and the quark fields in the physical mass eigenstates are related to the hierarchical mass eigenstates by

$$q_{L(R)}^{(u)}=U_{L(R)}^{(u)}u_{L(R)}^0,$$

$$q_{L(R)}^{(d)}=U_{L(R)}^{(d)}d_{L(R)}^0,$$

where $(q^{(u)},q^{(d)})$ and (u^0,d^0) denote the physical mass eigenstates and the hierarchical mass eigenstates for the up- and down-quark sectors, respectively. We note that a phase factor is attached to the (1,2) and (2,1) elements as $Ae^{-i\sigma}$ and $Ae^{i\sigma}$ in M_H, where A will be assumed to be positive without loss of generality. Then both $U_L M_H U_L^\dagger$ and $U_R M_H U_R^\dagger$ are diagonal so that $U_L U_R^\dagger \equiv K$ is again diagonal. In our *Ansatz*, it turns out in general that, because of the empirical mass hierarchy $m_1 \ll m_2 \ll m_3$, $K=\mathrm{diag}[1,-1,1]$ irrespective of the sign of D and $K=\mathrm{diag}[-1,1,1]$ only for positive D. This point was not clearly understood in previous works [8,10,11,13]. Fritzsch and co-workers [8] chose the relative signs of the $\mathrm{S}(3)_L \times \mathrm{S}(3)_R$-breaking terms different, so that the sign of m_c is opposite to that of m_s, while keeping m_u and m_d to be negative as mass eigenvalues, which is clearly inconsistent with the empirical quark mass hierarchy. Other authors [13] assumed the same form of the mass matrix without basing on a symmetry consideration and thus treating the up- and down-quark sectors unevenly.

The parameters A, B, C, and D can be expressed in terms of the quark masses. As emphasized earlier, in this paper, we will deal with the same pattern for both the up- and down-quark mass matrices so that the calculability of the flavor mixing matrix from the quark masses is retained. In view of the hierarchical pattern of the quark masses, it is natural to expect that $A < |D| \ll C$, and then the case of $K=\mathrm{diag}[1,-1,1]$ for positive D can be excluded if the same ratio parameter w is required for both up- and down-quark sectors. Otherwise, the masses of the second family could be unacceptably large.

The Case $K=\mathrm{diag}[-1,1,1]$. Because a Hermitian matrix can be expressed as a unitary transformation of a real symmetric matrix, one can write $M_H^{(u,d)} = P^{(u,d)} M_r^{(u,d)} \widetilde{P}^{(u.d)}$, where $P^{(u,d)} = \mathrm{diag}[\exp(-i\sigma^{(u,d)}),1,1]$, and the real matrix $M_r^{(u,d)}$ can be diagonalized by a real orthogonal matrix $R^{(u,d)}$ so that $R^{(u,d)} M_r^{(u,d)} \widetilde{R}^{(u,d)} = \mathrm{diag}[-m_{(u,d)}, m_{(c,s)}, m_{(t,b)}]$. Then, $U_L^{(u)} = \widetilde{R}^{(u)} P^{(u)\dagger}$ and $U_L^{(d)} = \widetilde{R}^{(d)} P^{(d)\dagger}$. The flavor mixing matrix is given by $V = U_L^{(u)} U_L^{(d)\dagger} = \widetilde{R}^{(u)} P^{(u)\dagger} P^{(d)} R^{(d)} = \widetilde{R}^{(u)} P R^{(d)}$ where $P=\mathrm{diag}[e^{i\sigma},1,1]$ with $\sigma = \sigma^{(u)} - \sigma^{(d)}$.

From the characteristic equation for the M_r, the mass matrix M_r can be written by

$$M_r = \begin{pmatrix} 0 & \sqrt{\dfrac{m_1 m_2}{1-\dfrac{\epsilon}{m_3}}} & 0 \\ \sqrt{\dfrac{m_1 m_2}{1-\dfrac{\epsilon}{m_3}}} & m_2 - m_1 + \epsilon & w(m_2 - m_1 + \epsilon) \\ 0 & w(m_2 - m_1 + \epsilon) & m_3 - \epsilon \end{pmatrix}, \tag{6}$$

in which the small parameter ϵ is related to w, i.e., $w \simeq \pm(\sqrt{\epsilon m_3}/m_2)(1 + m_1/m_2 - m_2/2m_3)$, whose range is to be determined from the experiments. Note the sign of B is undetermined from the characteristic equation but the KM matrix elements are independent of the sign of B. Then, we can obtain analytic expressions for the flavor mixing matrix V in the leading approximation such as

$$|V_{us}| \simeq |\sqrt{m_d/m_s}\exp(i\sigma) - \sqrt{m_u/m_c}|, \tag{7}$$

$$|V_{cb}| \simeq |w(m_s/m_b - m_c/m_t)|, \tag{8}$$

$$|V_{ub}|/|V_{cb}| \simeq \sqrt{m_u/m_c}, \quad |V_{td}|/|V_{ts}| \simeq \sqrt{m_d/m_s}. \tag{9}$$

Notice that $|V_{cb}|$ depends on the quark mass ratios and w. In fact, the w dependence appears in the four elements V_{ub}, V_{cb}, V_{ts}, and V_{td} only. Since the second term of $|V_{cb}|$ is negligible compared to the first term, it is easy to examine the range of w for which $|V_{cb}|$ is compatible with experiments. We use the light quark masses [14], $m_u = 5.1 \pm 0.9$ MeV, $m_d = 9.3 \pm 1.4$ MeV, and $m_s = 175 \pm 25$ MeV, and the heavy quark masses [15], $m_c = 1.35 \pm 0.05$ GeV and $m_b = 5.3 \pm 0.1$ GeV, all of which correspond to the masses at a modified minimal subtraction ($\overline{\mathrm{MS}}$) renormalization point of 1 GeV. The top quark mass m_t of the recent measurement $m_t = 175 \pm 6$ GeV corresponds to the running mass $m_t(\mu = 1$ GeV$) \simeq 280$–450 GeV for $\Lambda_{\overline{\mathrm{MS}}} = 150$–$200$ MeV [16].

Using the value $|V_{cb}| = 0.036$–0.046 from experiments [9], Eq. (8) leads to $1.01 \lesssim |w| \lesssim 2.02$ so that $0.82 \lesssim k \lesssim 1.31$ if $w > 0$ and $0.11 \lesssim k \lesssim 0.28$ if $w < 0$ in the leading approximation, which is close to the exact result $0.97 \lesssim |w| \lesssim 1.87$ so that $0.85 \lesssim k \lesssim 1.36$ if $w > 0$ and $0.10 \lesssim k \lesssim 0.26$ if $w < 0$. Note that $\epsilon = O(m_1)$ for the allowed range of k and w.

Next, we examine if this range of w preserves the consistency with experiments for other KM elements. Since several KM elements depend on the phase factor σ, we have to determine the allowed range of the phase factor first. We see from Eq. (7) that $|V_{us}|$ depends on the phase factor σ, while independent of w. Using the experimental value $|V_{us}| \simeq 0.219$–0.224 [9], the allowed range of σ turns out to be $26°$–$111°$. In particular, the maximal weak CP phase conjecture $\sigma = \pi/2$ suggested previously by Ref. [17] follows when $m_s \simeq 0.206$ GeV from Eq. (7) and when $m_s \simeq 0.194$ from exact calculation for the central values of the parameters m_u, m_d, m_c, and $|V_{us}|$. The exact numerical result gives $39° \lesssim \sigma \lesssim 117°$. In addition, we find that all other KM elements are in good agreement with experiments for the above ranges of w and σ.

The Case $K=\mathrm{diag}[1,-1,1]$. For a negative D, the real symmetric matrix $M_r^{(u,d)}$ can be diagonalized as $R^{(u,d)} M_r^{(u,d)} \widetilde{R}^{(u,d)} = \mathrm{diag}[m_{(u,d)}, -m_{(c,s)}, m_{(t,b)}]$, thus reversing the signs of m_1 and m_2 in Eq. (6). As we noted, a positive D in this case is excluded for the reasons of naturalness due to the quark mass hierarchy and *calculability*. Following the similar analysis as in the previous case, we get $1.14 \lesssim |w| \lesssim 2.76$ so that $0.72 \lesssim k \lesssim 1.17$ if $w > 0$ and $0.14 \lesssim k \lesssim 0.33$ if $w < 0$, and the same range of σ as in the previous case in the exact numerical calculation, while we find the same result of w and σ as in the previous case in the leading approximation. Consequently, the *Ansatz* adopted by Fritzsch and co-workers [8], corresponding to $k=0$, is not consistent with experimental data of V_{cb} and the *Ansatz* adopted by Ref. [10], corresponding to $w^2 = 8$, is slightly beyond the upper bound of the allowed w.

Now, we note that the predicted ratio $|V_{ub}|/|V_{cb}|$ ($\lesssim 0.07$) tends to be on the low side of (but consistent with) the present experimental range, $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$ [9] or 0.08 ± 0.016 [18].

In terms of the three inner angles of the unitarity triangle [19], $\alpha = \arg(-V_{ub}^* V_{ud}/V_{tb}^* V_{td})$, $\beta = \arg(-V_{tb}^* V_{td}/V_{cb}^* V_{cd})$, and $\gamma = \arg(-V_{cb}^* V_{cd}/V_{ub}^* V_{ud})$, we obtain $\alpha \simeq \sigma \simeq 26°–111°$, $\beta \simeq 6°–13°$, and $\gamma \simeq 180° - \alpha - \beta \simeq 148° - 56°$ in the leading order approximation. These angles are independent of k. From the Jarlskog determinant [20], $\mathrm{Det}C \simeq [(k+1)^2/(2k-1)^2]\sqrt{m_u m_d m_s m_c} m_t^2 m_s^2 \sin\sigma$, we see that the CP violation becomes maximal for $\sigma = 90°$ which is an allowed value from our results.

Finally, the Wolfenstein parameters [6] can be determined from $|V_{us}| \simeq \lambda$, $|V_{cb}| \simeq \lambda^2 A$, $V_{ub} \simeq \lambda^3 A(\rho - i\eta)$, and $V_{td} \simeq \lambda^3 A(1-\rho-i\eta)$ in terms of the quark masses. Since $\lambda \approx \sqrt{(m_d/m_s)}(1-\sqrt{(m_s m_u/m_d m_c)}\cos\sigma) \simeq |V_{12}|$ from Eq. (7), we obtain $\sigma \simeq 80.17°$ for all central values of quark masses and $\lambda = 0.22$. From the element V_{cb}, we get $A \simeq 0.74–0.95$ and since $|V_{ub}|/|V_{cb}| = \lambda\sqrt{\rho^2+\eta^2}$ ranges 0.06–0.10 from the semileptonic B decays [9], we get $\rho^2+\eta^2 \simeq 0.074–0.207$ for the central values of quark masses, while from Eq. (9), $\rho^2+\eta^2 = 0.0781$.

In conclusion, we suggested a general class of Hermitian mass matrices that can be obtained from successive breaking chain $U(3)_L \times U(3)_R \to S(3)_L \times S(3)_R \to S(2)_L \times S(2)_R \to S(1)_L \times S(1)_R$ so as to reflect the quark mass hierarchy and to maintain the *calculability* of the flavor mixing matrix and its consistency with experiments. There are four regions of k, the ratio parameter of the two elements of the $S(2)_L \times S(2)_R$ symmetric matrix, for which the generalized mass matrix *Ansatz* is compatible with experiments. In particular, the CP violation turns out to be maximal when the phase of the mass matrix is $\pi/2$.

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