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Approximate large N method for lattice chiral models

Stuart Samuel*
Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, Föhringer Ring 6, 80 805 Munich, Germany;
Department of Physics, Columbia University, New York, New York 10027;
and Department of Physics, City College of New York, New York, New York 10031†

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An approximation is used that permits one to explicitly solve the two-point Schwinger-Dyson equations of the U(N) lattice chiral models. The approximate solution correctly predicts a phase transition for dimensions d greater than two. For $d\leqslant 2$, the system is in a single disordered phase with a mass gap. The method reproduces known $N=\infty$ results well for $d=1$. For $d=2$, there is a moderate difference with $N=\infty$ results only in the intermediate coupling constant region.
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I. INTRODUCTION

The generation of a mass gap in two-dimensional spin systems is believed to be analogous to the generation of a nonzero string tension in four-dimensional non-Abelian gauge theories [1,2]. Of particular interest are the $d=2$ matrix chiral models. They are asymptotically free and have properties similar to $d=4$ gauge theories [1–3]. A solution to the $N=\infty$ chiral models has not yet been found. In contrast, it is straightforward to solve the O(N) vector spin models as $N\rightarrow\infty$.

The degrees of freedom of the O(N) models are N-component vectors $v(x)$ assigned to the sites x of a lattice satisfying the constraint $v(x)\cdot v(x)=1$. If it were not for this constraint, the O(N) vector model would be a theory of N noninteracting particles. Thus the effect of the constraint is to introduce interactions. Due to these interparticle forces, an interesting picture of mass generation arises [3]. In terms of bare quanta, the interactions are repulsive and of order $1/N$. The quantum vacuum involves fluctuations in which any one of the N quanta are created and destroyed. From a Euclidean viewpoint, such events correspond to particle loops, and so the vacuum is like a gas of closed loops. Such loops are small and dilute in strong coupling since the mass of a particle is large. As one moves to weaker couplings, the mass decreases and the loops become larger and more plentiful. If interactions could be turned off, the mass would eventually vanish and a phase transition would occur. For dimensions $\leqslant 2$, two large closed loops generically intersect. Because there are repulsive forces at such intersection points, a particular loop feels a pressure from the gas of surrounding loops. For $d\leqslant 2$ and $N\geqslant 3$, this pressure prevents any particular loop from becoming too large and generates an interaction-induced mass which never vanishes. By these means, the vector models avoid a phase transition when $d\leqslant 2$. The models remain in the strong coupling phase in which the particles are massive and the system is disordered: Correlation functions fall off exponentially with distance. In the large N limit, the vector model is exactly solvable: after interactions are incorporated into an effective coupling, particles are free. For $d\leqslant 2$, the effective coupling never attains a value sufficient to produce a phase transition to a massless spontaneously broken phase.

The picture for the matrix chiral models is qualitatively the same [3]. Forces at the intersection of particle loops are repulsive, as can be seen from Fig. 6 of [3]. However, no effective free-theory formulation of the model has been derived, although an exact S matrix has been proposed [4,5]. For SU(N), the spectrum is $M_n=M\sin(n\pi/N)/\sin(\pi/N)$, for $n=1,2,\ldots,N-1$. Interestingly, as $N\rightarrow\infty$, the S matrix becomes the identity matrix to leading order in a $1/N$ expansion. This suggests that perhaps the large N limit of the SU(N) chiral model is a free theory of particles.

II. THE APPROXIMATE LARGE N METHOD

In this work, we obtain an approximate solution for the chiral models. Correlation functions and the mass gap are obtained. Because the method is similar to the one used in the vector models, it is useful to quickly review the vector models in the large N limit. The action $\mathcal{A}$ for the d-dimensional U(N) vector model is

$$\mathcal{A}=\beta N\sum_{x,\Delta} v^*(x)\cdot v(x+\Delta), \tag{1}$$

where $v(x)$ is an N-dimensional complex vector satisfying $v^*(x)\cdot v(x)=1$, β is proportional to the inverse coupling constant, and the sum over Δ is over the $2d$ nearest neighbor sites, i.e., Δ is $\pm e_1,\ \pm e_2,\ \ldots,\ \pm e_d$, where e_i is a unit vector in the ith direction. The U(N) model is equivalent to the O($2N$) vector model.

The lattice Schwinger-Dyson equation for the two-point function is

*Electronic address: samuel@scisun.sci.ccny.cuny.edu
†Permanent address.

$$\delta_{xy}=\langle v^*(x)\cdot v(y)\rangle-\beta\sum_{\Delta}\langle v^*(x)\cdot v(y+\Delta)\rangle+\beta\sum_{\Delta}\langle v^*(x)\cdot v(y)v^*(y+\Delta)\cdot v(y)\rangle. \tag{2}$$

Since the four-point function enters in the last term, the vector model is not a free theory. However, large N factorization allows the four-point function to be expressed in terms of two-point functions via

$$\langle v^*(x)\cdot v(y)v^*(y+\Delta)\cdot v(y)\rangle=\langle v^*(x)\cdot v(y)\rangle\langle v^*(y+\Delta)\cdot v(y)\rangle+\mathrm{O}(1/N). \tag{3}$$

The nearest neighbor expectation, $\langle v^*(y+\Delta)\cdot v(y)\rangle$, is independent of y. A linear equation for $\langle v^*(x)\cdot v(y)\rangle$ is thus obtained when Eq. (3) is substituted in Eq. (2). Consequently, the $N=\infty$ vector model is a free theory governed by an effective coupling

$$\beta_{eff}=\frac{\beta}{1+2d\beta\langle v^*(0)\cdot v(\Delta)\rangle}. \tag{4}$$

For $d\leqslant 2$, no phase transition is encountered as $\beta\to\infty$: The mass gap is a decreasing function of β_{eff} but the effective coupling does not reach the critical value at which the mass gap vanishes. The theory remains in the strong coupling disordered phase. For $d>2$, a phase transition occurs to a weak coupling spin-wave phase in which Goldstone bosons appear due to the breaking of the global U(N) symmetry. This is in agreement with expectations [6,7].

The action $\mathcal{A}$ of the matrix chiral model is

$$\mathcal{A}=\beta N\sum_{x,\Delta}\mathrm{Tr}[U^\dagger(x)U(x+\Delta)], \tag{5}$$

where $U(x)$ is a unitary matrix in a group G. The chiral model is the nonlinear effective low-energy theory of the massless quark model involving N flavors. For this reason when $d=4$, $G=\mathrm{SU}(N)$ is often used with $N=2$ or $N=3$. The matrices $U(x)$ can be regarded as the Goldstone bosons of spontaneous broken chiral $\mathrm{SU}_L(N)\times\mathrm{SU}_R(N)$.

In what follows, we treat the case $G=\mathrm{U}(N)$. The Schwinger-Dyson equation for the two-point function is

$$\delta_{xy}=\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y)]\rangle-\beta\sum_{\Delta}\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y+\Delta)]\rangle+\beta\sum_{\Delta}\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y)U^\dagger(y+\Delta)U(y)]\rangle. \tag{6}$$

Equation (6) is not useful in weak coupling where β becomes large because the last two terms individually become sizable. Since the left-hand side remains constant, a delicate cancellation between the two takes place.

Equation (6) is similar to Eq. (2), except that matrix products arise instead of dot products. Consequently, the four-point function which enters in Eq. (6) cannot be factorized in the large N limit. However, one may consider the vector-model-like replacement

$$\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y)U^\dagger(y+\Delta)U(y)]\rangle\to a\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y+\Delta)]\rangle+b\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y)]\rangle, \tag{7}$$

where a and b are arbitrary functions of β, whose choices are at one's disposal.

It is important to use values of a and b which reproduce the leading orders of perturbation theory, since one is interested in taking the continuum limit for which the coupling g goes to zero. Writing $U(y)=\exp[i\Phi(y)]$ and expanding about the identity matrix, one finds $U(y)U^\dagger(y+\Delta)U(y)=1+2i\Phi(y)-i\Phi(y+\Delta)+\cdots$. Hence, if $U(y)U^\dagger(y+\Delta)U(y)\to aU(y+\Delta)+bU(y)$ then one needs

$$a=-1+O(g^2),\quad b=2+O(g^2). \tag{8}$$

Alternatively, one can perform an operator product expansion. Write

$$U(y)U^\dagger(y+\Delta)U(y)=2U(y)-U(y+\Delta)+[U(y+\Delta)-U(y)]U^\dagger(y+\Delta)\times[U(y+\Delta)-U(y)]. \tag{9}$$

Note that the third term is a higher dimensional operator since it involves two derivatives. One again concludes that a and b have the perturbative expansions of Eq. (8).

After the substitution in Eq. (7) is made, one obtains

$$\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y)]\rangle=\frac{\delta_{xy}}{(1+2db\beta)}+\beta_{\text{eff}}\sum_{\Delta}\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(x)U(y+\Delta)]\rangle, \tag{10}$$

where

$$\beta_{\text{eff}}=\frac{\beta(1-a)}{1+2db\beta}. \tag{11}$$

One advantage of the substitution in Eq. (7) is that the effective coupling β_{eff} remains finite as $\beta\to\infty$ so that the corresponding equation for the two-point function does not involve the above-mentioned delicate cancellation between large terms.

One possibility is to choose

$$a=-1,\quad b=2G_\Delta, \tag{12}$$

where the average link G_Δ is

$$G_\Delta\equiv\frac{1}{N}\langle\mathrm{Tr}[U^\dagger(y+\Delta)U(y)]\rangle. \tag{13}$$

The choice in Eq. (12) produces the correct result in Eq. (7) when $y=x$. As a side remark, selecting $a=0$ and $b=G_\Delta$ yields the two-point function of the vector model in the large N limit.

The equation for two-point function $G(x,y) \equiv \langle \mathrm{Tr}[U^\dagger(x)U(y)]\rangle/N$ in Eq. (10) is linear. The solution is

$$G(x,y)=\frac{1}{1+2d\beta b}\int_{-\pi}^{\pi}\frac{d^d p}{(2\pi)^d}\,\frac{\exp[ip\cdot(y-x)]}{1-2\beta_{\mathrm{eff}}\Sigma_{i=1}^{d}\cos(p_i)}. \tag{14}$$

If one requires that $G(x,y)=1$ for $y=x$, then the self-consistency condition

$$1=\frac{1}{1+2d\beta b}\int_{-\pi}^{\pi}\frac{d^d p}{(2\pi)^d}\,\frac{1}{1-2\beta_{\mathrm{eff}}\Sigma_{i=1}^{d}\cos(p_i)} \tag{15}$$

must be satisfied.

A more detailed analysis reveals that the substitution in Eq. (7) with a and b given in Eq. (12) is guaranteed to reproduce the $O(1)$ and $O(g^2)$ terms of correlation functions when a perturbative expansion of the theory is performed. Furthermore, Eqs. (12) and (14) lead to qualitatively the same results as in the vector model case: For $d\leqslant 2$, the chiral model is in the disordered phase and there is a mass gap for all values of β. For $d>2$, a transition occurs at a finite value of β to a phase in which global symmetries are spontaneous broken. This is in agreement with expectations [1–3].

III. THE ONE-DIMENSIONAL CASE

When $d=1$, the chiral model can be solved exactly. Solutions are known for the continuum and lattice cases at finite N and for $N=\infty$ [8,9]. This case allows one to test the approximate large N method. When $d=1$, solving Eqs. (7) and (14) gives

$$G(x,y)=\left(\frac{1-\sqrt{1-4\lambda^2}}{2\lambda}\right)^{|y-x|}, \tag{16}$$

where

$$\lambda=\frac{(1-a)\beta}{1+2\beta b}=\beta_{\mathrm{eff}}. \tag{17}$$

As a consequence of the self-consistency condition in Eq. (15), one also has

$$b=\frac{-1+\sqrt{1+4\beta^2(1-a)^2}}{2\beta}. \tag{18}$$

For large β, the mass gap m is

$$m\Lambda^{-1}=\frac{1}{2(1-a)\beta}-\frac{1}{48\beta^2(1-a)^2}+\cdots, \tag{19}$$

where Λ^{-1} is the lattice spacing, which is often denoted by a.

The continuum limit is obtained by taking $\beta\to\infty$ and $\Lambda^{-1}\to 0$ with $\beta\Lambda=1/(g_c^2N)$ fixed. Here, g_c is the continuum coupling constant. Hence,

$$m=\frac{g_c^2N}{4}. \tag{20}$$

This is the exact $d=1$ result for the U(N) gauge theory.

IV. THE TWO-DIMENSIONAL CASE

When $d=2$, the approximate large N method leads to a mass gap which is exponentially small as $\beta\to\infty$:

$$m\Lambda^{-1}\approx\exp[-2\pi\beta(1-a)]. \tag{21}$$

For $a=-1$, $m\approx\Lambda\exp[-4\pi\beta]$. It is remarkable that an exponentially suppressed mass gap is obtained, given that the substitution in Eq. (7) is guaranteed only to reproduce perturbative results to order g^2. However, the mass gap in Eq. (21) for $a\to -1$ is not as suppressed as the result obtained from perturbative renormalization group analysis, which gives $m\approx\Lambda\exp[-8\pi\beta]$ [10].

One possibility to overcome this discrepancy is to permit the coefficients a and b in Eq. (7) to depend on the distance between x and y. For weak coupling, $a\sim -1$ at short distances but at large distances $a\sim -3$. However, one would have to find some theoretical justification for this dependence of a on $|y-x|$.

V. THE MATCHING OF BOTH STRONG AND WEAK COUPLING

Although the values of a and b in Eq. (12) produce good results for weak couplings, they fail to do so in the strong coupling region: For example, the mass gap behaves as $-\ln(2\beta)+O(\beta)$ instead of $-\ln(\beta)+O(\beta)$. To obtain results which are accurate in both strong and weak coupling, we have found that

$$a=-(G_\Delta)^2,\quad b=G_\Delta+(G_\Delta)^3 \tag{22}$$

works reasonably well. These values of a and b satisfy the weak coupling limits in Eq. (8) since $G_\Delta=1-g^2N/(4d)+O(g^4)$. In addition, one can show that they reproduce correlation functions correctly to $O(g^2)$ in a perturbative expansion. It turns out that Eq. (22) produces the correct $N=\infty$ strong coupling expansions for the mass gap and G_Δ to order β^3. Since Eqs. (14) and (22) give good results in both the strong and weak coupling limits, there is a reasonable chance that results are good throughout the entire coupling constant region. The idea of weak-strong coupling interpolation recently was used successfully in an approximate analytic computation of the 0^{++} glueball mass in the SU(3) gauge theory [11].

To examine how well Eq. (22) works for intermediate couplings, one can use the solvable $d=1$ case as a guide. When $N=\infty$, $G_\Delta=\beta$ for $\beta\leqslant 0.5$ and $G_\Delta=1-1/(4\beta)$ for $\beta\geqslant 0.5$ [8,9], and the two-point function is given by

$$G(x,y)=(G_\Delta)^{|y-x|}. \tag{23}$$

Using the approximate large N method with Eqs. (18) and (22), one obtains the following equation for G_Δ:

$$(G_\Delta)^4+\frac{G_\Delta}{\beta}=1. \tag{24}$$

For the approximate large N method, the two-point correlation function is again given by Eq. (23) but with G_Δ determined from Eq. (24). Hence the difference between the exact and approximate results is governed by G_Δ. Figure 1 dis-

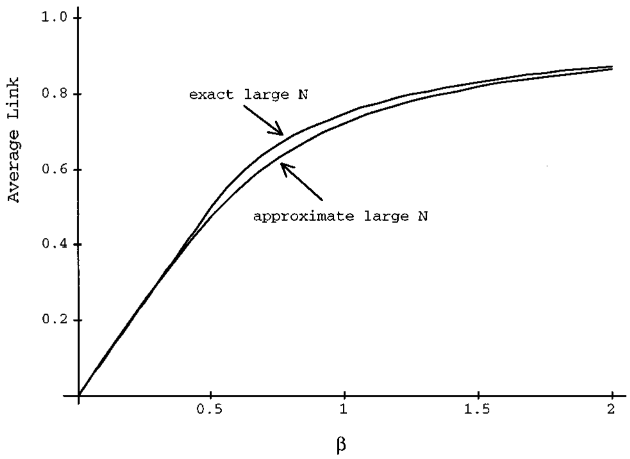

FIG. 1. The average link G_Δ vs β for the $N=\infty$ $d=1$ case.

plays G_Δ. The approximate large N method yields values of G_Δ which are slightly less than the exact $N=\infty$ value. The difference reaches a maximum of 6.7% near $\beta=0.6$. Very good agreement is, of course, obtained for β small or large.

The lattice $N=\infty$ $d=2$ chiral model has not been exactly solved, but Monte Carlo data are available for finite values of N and there exist analytic weak and strong coupling results [12–14]. In Fig. 2, we compare the approximate large N method for the values of a and b in Eq. (22) versus weak and strong coupling results. For the average link G_Δ, displayed in Fig. 2(a), the approximate method agrees well with the 11th-order $N=\infty$ strong coupling series of [3] for $\beta<0.25$. It also agrees the weak coupling $N=\infty$ series, $1-1/(8\beta)-1/(256\beta^2)+\cdots$ for large β, but in the intermediate coupling region it is below both curves and below the U(2) Monte Carlo data [15]. Since the data for U(2) do not differ greatly from the data for SU(10) of [14], the dots in Fig. 2(a) are probably close to the $N=\infty$ results. By allowing a and b to be a polynomial in G_Δ of sufficient high order, the approximate large N method can be adjusted to produce more terms in the strong coupling series. When this is done, agreement for the average link is obtained to less than 5% throughout the entire coupling constant region.

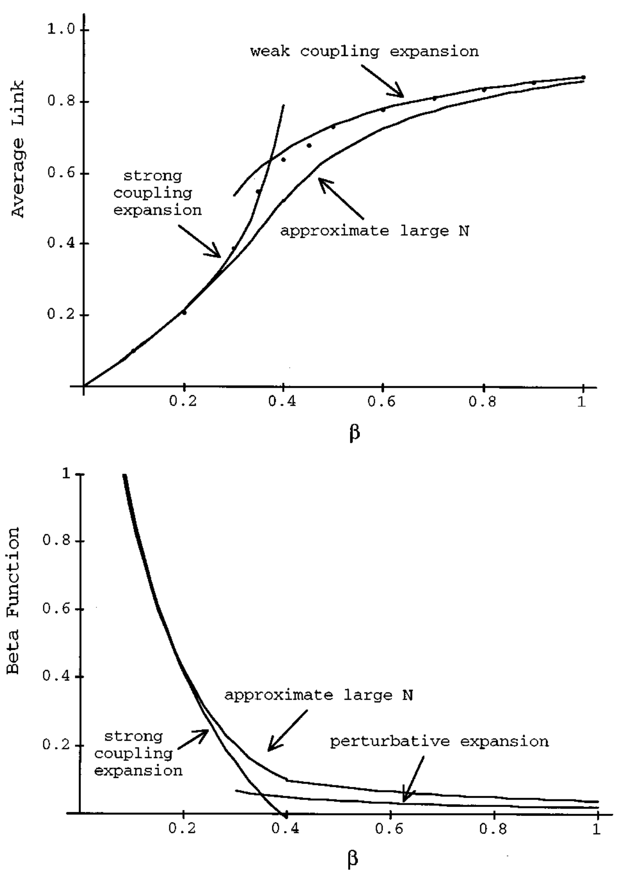

FIG. 2. (a) The average link G_Δ vs β for the $N=\infty$ $d=2$ case. The dots are $N=2$ Monte Carlo data. (b) The β function $-\beta(g)/g$ vs the inverse coupling β. The $N=\infty$ perturbative term comes from renormalization group analysis.

From the mass gap, $m\Lambda^{-1}\equiv f(g)$, one can obtain a lattice β function $\beta(g)$ (not to be confused with the inverse coupling) via $-\beta(g)/g=f/[g(\partial f/\partial g)]$. In Fig. 2(a), the approximate method is compared to the strong coupling result [3] and to perturbation theory for which $-\beta(g)/g=g^2N/(16\pi)+\ldots$. Excellent agreement is obtained for $\beta<0.25$. In weak coupling, the approximate large N method gives results for $-\beta(g)/g$ which are twice as large because the mass gap is exponentially suppressed only by half as much as the renormalization group result.

VI. CONCLUSION

In this work, we have used a vector-model-like method to linearize the lattice Schwinger-Dyson two-point function in the U(N) chiral model. For $d\leqslant 2$, the approximate method produces no phase transition. This is in agreement with expectations for the finite N case: The system is believed to be in the disordered phase and to have a mass gap for all couplings. For $N=\infty$, a phase transition occurs [8,9,16], even though the system is disordered on both sides of the transition. For $d\leqslant 2$, our approximate method does not see this large N transition, probably because the transition is rather mild and because it occurs in the intermediate coupling constant region, where the method is least accurate. For $d>2$, a phase transition occurs for all N and is expected to be first order [7,17]. The approximate method correctly sees the transition and correctly predicts the nature of the phases: symmetry breaking with massless Goldstone bosons at weaking coupling, while a disordered phase with a mass gap in strong coupling. Thus the method gives what-is-believed-to-be the correct qualitative phase diagram in all dimensions for the finite N case. By adjusting the method to agree with strong coupling expansions, the approximate method gives reasonably good results throughout the entire coupling constant regime for the average link and mass gap.

It is of interest to adapt the method to lattice gauge theories. The Schwinger-Dyson equations lead to relations among Wilson loops [18–20]. A non-self-intersecting loop is related to loops modified at a link by the addition of plaquettes in the $2d$ directions minus loops modified by the addition of $2d$ "twisted plaquettes." The structure of the equation is similar to Eq. (6) in the sense that the inverse coupling β multiplies the two terms and there is a sizable cancellation between the two terms in weak coupling. The analog approximate large N method involves replacing each twisted-plaquette term by an untwisted loop and an original loop multiplied, respectively, by coefficients a and b. We have three comments on this approximate approach to large N for lattice gauge theories. (i) Although Wilson loops are related to Wilson loops after the substitution of the twisted-plaquette terms is made, there appears to be no way to find an analytic expression for Wilson loops when $d\neq 2$. This problem is probably related to the lack of an analytic description of free lattice string theory. (ii) When $d>2$, the method appears to depend on the link on which the Schwinger-Dyson equations are derived. This implies that certain consistency issues must be addressed. (iii) When $d=2$, the approximation can be consistently applied to the lattice gauge theory. For a U(N) gauge group, the results are identical to those of the U(N) chiral model. The value of a non-self-intersecting Wilson loop WL of area A is

$$WL=\left(\frac{1-\sqrt{1-4\lambda^2}}{2\lambda}\right)^A, \tag{25}$$

where λ is given in Eq. (17). The choice $a=-(\langle \mathrm{Tr}U_P\rangle/N)^2$ and $b=(\langle \mathrm{Tr}U_P\rangle/N)+(\langle \mathrm{Tr}U_P\rangle/N)^3$, where $\langle \mathrm{Tr}U_P\rangle/N$ is the average plaquette [these values of a and b are the gauge-theory analog of Eq. (22)], leads to good results. The self-consistency condition becomes $(\langle \mathrm{Tr}U_P\rangle/N)^4+\langle \mathrm{Tr}U_P\rangle/(N\beta)=1$. The value of the Wilson loop then becomes

$$(\langle \mathrm{Tr}U_P\rangle/N)^A. \tag{26}$$

In the exact large N limit, the Wilson loop value is given by Eq. (26) with $\langle \mathrm{Tr}U_P\rangle/N=\beta$ for $\beta\leqslant 0.5$ and $\langle \mathrm{Tr}U_P\rangle/N=1-1/(4\beta)$ for $\beta\geqslant 0.5$. Thus the discrepancy between the approximate and exact large N results is the same as in the $d=1$ chiral model: The error is at most 6.7% near $\beta=0.6$ and there is good agreement in the strong and weak coupling regimes. When a continuum limit is taken, the exact continuum solution is obtained.

The picture of mass-gap generation for spin models in $d\leqslant 2$ has an analogy for gauge models in $d\leqslant 4$. In the lattice gauge theory, the strong coupling expansion involves sums over closed surfaces. From a Euclidean viewpoint, such surfaces can be thought of as the propagation of strings. When two surfaces overlap, there are repulsive forces. In fact, the surface-surface interactions involve potentials which are exactly the same as in the chiral-model case [3]. The quantum vacuum involves fluctuations in which strings appear, expand, contract, and self-annihilate. This creates a medium in which any individual string must propagate. When $d\leqslant 4$, two large surfaces generically intersect, and since forces are repulsive, the medium creates a pressure which inhibits strings from becoming large. If this mechanism is sufficient robust as β increases and g^2 becomes smaller, then the string tension will remain nonzero in the continuum limit. The same mechanism can inhibit the spanning surface of a Wilson-loop computation from becoming large. Hence one arrives at a possible physical picture for confinement and mass-gap generation in $d\leqslant 4$ gauge theories. The idea fails for $d>4$ because two two-dimensional surfaces do not generally intersect.

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