PHYSICAL REVIEW D VOLUME 56, NUMBER 2 15 JULY 1997

Unified gauge models and one-loop quantum cosmology

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(Received 20 November 1996)

This paper studies the normalizability criterion for the one-loop wave function of the universe in a de Sitter background, when various unified gauge models are considered. It turns out that the interaction of inflaton and matter fields, jointly with the request of normalizability at one-loop order, picks out nonsupersymmetric versions of unified gauge models. [S0556-2821(97)01714-1]

PACS number(s): 98.80.Hw, 12.10.Dm, 98.80.Cq

The investigations in modern cosmology have been devoted to two main issues. On the one hand, there were the attempts to build a quantum theory of the universe with a corresponding definition and interpretation of its wave function [1,2]. On the other hand, the drawbacks of the cosmological standard model motivated the introduction of inflationary scenarios. These rely on the existence of one or more scalar fields, and a natural framework for the consideration of such fields is provided by the current unified models of fundamental interactions [3]. The unification program started with the proposal and the consequent experimental verification of the electroweak standard model (SM) [$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$], and has been extended to other simple gauge groups, such as SU(5), SO(10) and E_6. All of them in fact, even if with different capability, unlike the electroweak standard model, are able to allocate all matter fields in a few irreducible representations (irreps) of the gauge group, and require a small number of free parameters. However, since these enlarged gauge models predict new physics, a first source of constraints upon them is certainly provided by the experimental bounds on processes like proton decay, neutrino oscillations, etc. [4]. Further restrictions can be obtained from their cosmological applications, as discussed in Ref. [5].

One can say, however, that the majority of investigations, studying the mutual relations between particle physics and cosmology, leave quantum cosmology itself a bit aside, using it only as a tool to provide initial conditions for inflation. Meanwhile, one can get some important restrictions on particle physics models, using general principles of quantum theory such as normalizability of the wave function [6–12] or quantum consistency of the theory [13].

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Our paper, following Refs. [6–12], studies the possible restrictions on unified gauge models resulting from a one-loop analysis of the wave function of the universe and from the request of its normalizability. It is known that the Hartle-Hawking wave function of the universe [1], as well as the tunneling one [2], is not normalizable at tree level [14]. In Ref. [6] it was shown that, by taking into account the one-loop correction to the wave function, jointly with a perturbative analysis of cosmological perturbations at the classical level, one can obtain a normalizable wave function of the universe provided that a restriction on the particle content of the model is fulfilled.

Such a restriction is derived from the formula for the probability distribution for values of the inflaton field [6]:

$$\rho_{\mathrm{HH},T}(\varphi) \cong \frac{1}{H^2(\varphi)} e^{\mp I(\varphi) - \Gamma_{1\mathrm{loop}}(\varphi)}, \tag{1}$$

where HH and T denote the Hartle-Hawking and tunneling wave function, respectively, $H(\varphi)$ is the effective Hubble parameter, and $\Gamma_{1\mathrm{loop}}$ is the one-loop effective action on the compact de Sitter instanton. One can show from Eq. (1) that the normalizability condition of the probability distribution at large values of the inflaton scalar field φ is reduced to the condition [6]

$$Z > -1, \tag{2}$$

where Z is the total anomalous scaling of the theory. This parameter is determined by the total Schwinger-DeWitt coefficient A_2 in the heat-kernel asymptotics [15] and depends on the particle content.

In Ref. [8] the criterion (2) was used to investigate the permissible content of different models. It was noticed that the standard model of particle physics, as well as the minimal SU(5) grand unified theory (GUT) model, does not satisfy the criterion of normalizability, while the standard su-

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persymmetric model, the SU(5) supersymmetry (SUSY) model, and SU(5) supergravity model do satisfy this criterion.

All the analysis in Ref. [8] was carried out in terms of physical degrees of freedom, e.g., three-dimensional transverse photons or three-dimensional transverse-traceless metric perturbations. However, over the last few years, the explicit calculations have shown that a covariant path integral for gauge fields and gravitation yields an anomalous scaling which differs from the one obtained from reduction to physical degrees of freedom. For compact manifolds without a boundary this discrepancy can be appreciated by comparing the results of Ref. [16] and Ref. [17]. For manifolds with a boundary we refer the reader to the work in Refs. [18,19] and references therein.

Unfortunately, the reduction to physical degrees of freedom relies on a global foliation by three-dimensional hypersurfaces which is only well defined when the Euler number of the four-dimensional Riemannian manifold vanishes. Moreover, such a reduction does not take explicitly into account gauge and ghost terms in the path integral, and leads to a heat-kernel asymptotics which disagrees with the well-known results of invariance theory [17,20]. For all these reasons, we regard the covariant version of the path integral as more appropriate for one-loop calculations.

In Ref. [9] the investigation of the one-loop wave function was carried out for a nonminimally coupled inflaton field with large negative constant ξ. It was then shown that the behavior of the total anomalous scaling Z is determined by interactions between the inflaton and remaining matter fields.

Here, we study normalizability properties of a wide set of unified gauge models, with or without an interaction with the inflaton field. The models studied are, as shown in Table I, the standard model of particle physics, SU(5), the SO(10) model in the 210-dimensional irreducible representation, E_6, jointly with supersymmetric versions of all these models with or without supergravity. The building blocks of our one-loop analysis are the evaluations of A_2 coefficients for scalar, spinor, gauge, graviton, and gravitino perturbations. All these coefficients (but one) are, by now, well known (e.g., Refs. [20,21]) and are given by

$$A_{2\,\mathrm{scalar}}=\frac{29}{90}-4\xi+12\xi^2-\frac{1}{3}m^2R_0^2+2\xi m^2R_0^2+\frac{1}{12}m^4R_0^4, \tag{3}$$

$$A_{2\,\mathrm{spin\text{-}1/2}}=\frac{11}{180}+\frac{1}{3}m^2R_0^2+\frac{1}{6}m^4R_0^4, \tag{4}$$

$$A_{2\,\mathrm{gauge}}=-\frac{31}{45}+\frac{2}{3}m^2R_0^2+\frac{1}{3}m^4R_0^4, \tag{5}$$

$$A_{2\,\mathrm{gravitino}}=-\frac{589}{180}. \tag{6}$$

It should be stressed that Eq. (3) only holds for scalar fields different from the inflaton. With our notation, m,ξ, and R_0 represent effective mass, (dimensionless) coupling parameter, and four-sphere radius, respectively. Equation (4) holds for a spin-1/2 field with half the number of modes of a Dirac field. Since the results (5) and (6) rely on the Schwinger-DeWitt technique, they incorporate, by construction, the effect of ghost zero modes. However, it has been argued in Ref. [22] that zero modes should be excluded to obtain an infrared finite effective action which is smooth as a function of the de Sitter radius on spherically symmetric backgrounds. On the other hand, the prescription which includes ghost zero modes makes the one-loop results continuous. Strictly, we are considering small perturbations of a de Sitter background already at a classical level (see Refs. [6–12]). There are also deep mathematical reasons for including zero modes, and they result from the spectral theory of elliptic operators [23]. Thus, we use the expressions (5) and (6).

Last, the contribution of gravitons to the total Z should be calculated jointly with the inflaton contribution. What happens is that the second-order differential operator given by the second variation of the action with respect to inflaton and metric is nondiagonal even on shell, by virtue of a nonvanishing vacuum average value of the inflaton [24,25]. The

TABLE I. For various unified gauge models of fundamental interactions, the anomalous scaling factor of the one-loop wave function of the universe is evaluated. The range of values of ξ for which the one-loop normalizability criterion in Eq. (2) is not respected is hence derived.

Gauge group	Version	Z	Forbidden ξ range
	Non-SUSY	$36\xi^2-12\xi-\frac{543}{20}$	$-0.701\leqslant\xi\leqslant1.035$
SM	SUSY	$1164\xi^2-388\xi+\frac{389}{180}$	$0.008\leqslant\xi\leqslant0.325$
	SUGRA	$1164\xi^2-388\xi+\frac{163}{30}$	$0.017\leqslant\xi\leqslant0.316$
	Non-SUSY	$396\xi^2-132\xi-\frac{103}{4}$	$-0.134\leqslant\xi\leqslant0.467$
SU(5)	SUSY	$1884\xi^2-628\xi+\frac{1919}{180}$	$0.020\leqslant\xi\leqslant0.314$
	SUGRA	$1884\xi^2-628\xi+\frac{209}{15}$	$0.026\leqslant\xi\leqslant0.308$
	Non-SUSY	$5772\xi^2-1924\xi+\frac{4678}{45}$	$0.069\leqslant\xi\leqslant0.265$
SO(10)	SUSY	$12444\xi^2-4148\xi+\frac{11321}{45}$	$0.080\leqslant\xi\leqslant0.253$
	SUGRA	$12444\xi^2-4148\xi+\frac{5097}{20}$	$0.082\leqslant\xi\leqslant0.252$
	Non-SUSY	$10932\xi^2-3644\xi+\frac{39197}{180}$	$0.078\leqslant\xi\leqslant0.255$
E_6	SUSY	$12876\xi^2-4292\xi+\frac{42719}{180}$	$0.070\leqslant\xi\leqslant0.263$
	SUGRA	$12876\xi^2-4292\xi+\frac{1203}{5}$	$0.072\leqslant\xi\leqslant0.262$

resulting A_2 coefficient turns out to be independent of the value of ξ and equal to [12]

$$A_{2\,\mathrm{graviton+inflaton}} = -\frac{171}{10}. \tag{7}$$

In Table I, we report the total Z for some relevant examples of GUT theories, whenever one neglects the mass terms. This ansatz is correct, if the interaction between inflaton and the other particles is not considered. In this case, in fact, the term $m^2R_0^2 \sim \varphi^{-2}$ is very small due to the large value of φ. The analysis starts with the electroweak standard model, which contains, in its non-SUSY version, 45 Weyl spinors (we neglect for simplicity right-handed neutrinos and their antiparticles), 24 gauge bosons, and one doublet of complex Higgs fields. The particle content changes for the SUSY version of this model in its minimal form (MSSM) [26]. In this case, in fact, to the 45 Weyl leptons and quarks one has to add 4 Higgsinos and 12 gauginos, whereas the scalar sector consists now of 90 sleptons and squarks plus 8 real scalar fields. A similar analysis is performed for the SU(5) GUT model [27], which in its non-SUSY version, apart from the 24 gauge bosons, needs scalars belonging to $\mathbf{24}\oplus\mathbf{5}\oplus\overline{\mathbf{5}}$ irreps to accomplish the spontaneous symmetry-breaking pattern. The matter content of the SUSY extension of the model [28] is obtained by doubling the number of Higgs irreps used and by adding superpartners to any degrees of freedom. As far as SO(10) gauge theories are concerned, we have considered the particular model containing $\mathbf{210}\oplus(\mathbf{126}\oplus\overline{\mathbf{126}})\oplus\mathbf{10}\oplus\mathbf{10}$ irreps of Higgs fields, which is still compatible with the present experimental limit on the proton lifetime and neutrino phenomenology [4]. Furthermore, we have also considered the SUSY extension of SO(10), which, to be consistent also with cosmological constraints, needs complex Higgs fields belonging to $\mathbf{1}\oplus\mathbf{10}\oplus\mathbf{10}'\oplus\mathbf{45}\oplus\mathbf{45}'\oplus\mathbf{54}\oplus\mathbf{54}'\oplus\mathbf{126}\oplus\mathbf{126}'$ irreps [29].

Last, we have also considered E_6 GUT theories, for which fermions are allocated in three **27** fundamental irreps, and scalars belong to two $(\mathbf{78}\oplus\mathbf{27}\oplus\mathbf{351})$ [30]. For the SUSY extension of this model, we have just added the superpartner degrees of freedom. Concerning the SUGRA versions of all the above models, they have been obtained from the supersymmetric ones, just by adding the gravitino contribution [i.e., subtracting the A_2 coefficient in Eq. (6), because of the fermionic statistics]. Indeed, we have considered particular versions of SO(10) and E_6 gauge models, but we expect that the qualitative features of the results (see below) should remain unaffected.

In Table I, we have assumed that one of the Higgs fields plays the role of the inflaton. The forbidden range denotes the range of values of ξ for which the normalizability criterion (2) is not satisfied. Interestingly, conformal coupling (i.e., $\xi=1/6$) is ruled out by all 12 models listed in Table I. Moreover, for the standard and SU(5) models, minimal coupling (i.e., $\xi=0$) is also ruled out. At this stage, supersymmetric models are hence favored, as well as nonsupersymmetric models with a large number of scalar fields.

In the formulation of physical models, however, one has to move gradually from the original, simplified case, towards a more involved problem which is physically more realistic. In our investigation, this means having to deal with the interactions between the inflaton and remaining fields, since such interactions are responsible for the reheating in the early universe [5]. This is a stage as important as the inflationary phase. Indeed, as shown in Refs. [9,12], for a scalar field with mass m_χ and constant ξ_χ of nonminimal interaction [which differs from ξ in Eq. (3)], one finds, on a de Sitter background,

$$\zeta_\chi(0) = \frac{29}{90} - 4\xi_\chi + 12\xi_\chi^2 - \frac{1}{3}\frac{m_\chi^2}{H^2} + \frac{1}{12}\frac{m_\chi^4}{H^4}, \tag{8}$$

where $m_\chi^2 = \lambda_\chi \varphi_0^2/2$. Moreover, for a spin-1 gauge field with mass m_A and a massive Dirac field with mass m_ψ, one finds [9,12]

$$\zeta_A(0) = 48\xi^2 \frac{g_A^2}{\lambda^2}\left[1 + \frac{1+2\delta}{4\pi}\frac{m_p^2}{|\xi|\varphi_0^2} + O(1/|\xi|)\right], \tag{9}$$

$$\zeta_\psi(0) = -48\xi^2 \frac{f_\psi^2}{\lambda^2}\left[1 + \frac{1+2\delta}{4\pi}\frac{m_p^2}{|\xi|\varphi_0^2} + O(1/|\xi|)\right], \tag{10}$$

where the coupling constants g_A and f_ψ are related to the masses by the formulas $m_A^2 = g_A^2\varphi_0^2$, $m_\psi^2 = f_\psi^2\varphi_0^2$, and the parameter δ is defined by [9,12]

$$\delta \equiv -\frac{8\pi|\xi|m^2}{\lambda m_p^2}, \tag{11}$$

λ being the parameter of self-interaction for the inflaton. Thus, if one considers supersymmetry, jointly with a Wess-Zumino scalar multiplet interacting with the inflaton, the terms of order $m^4R_0^4$ in Eqs. (3) and (4) cancel each other exactly after combining contributions proportional to [12]

$$\sum_\chi \lambda_\chi^2 + 16\sum_A g_A^4 - 16\sum_\psi f_\psi^4.$$

By contrast, terms of order $m^2R_0^2$ have opposite signs, since they are proportional to

$$-8\sum_\chi \lambda_\chi + 32\sum_A g_A^2 - 32\sum_\psi f_\psi^2.$$

At this stage, one has to bear in mind that, by virtue of cosmological perturbations, one can prove that $m^2R_0^2$ is of order 10^4 [31]. The effect of all these properties is hence a negative value of Z which cannot be greater than -1 [cf. Eq. (2)]. Thus, inflaton interactions reverse completely the conclusions that, otherwise, would be drawn from Table I. In particular, our analysis proves that the ''pseudosupersymmetric'' combination of coupling constants considered in Refs. [9,10,12] does not improve the situation with respect to the criterion in Eq. (2).

One can thus conclude that, in the attempt to understand whether or not supersymmetric theories are better than nonsupersymmetric theories for constructing inflationary universe models, the one-loop normalizability criterion for the wave function of the universe picks out nonsupersymmetric versions of unified gauge models. The open problem now remains of proving that such a conclusion is not affected by higher-order effects in the perturbative evaluation of the wave function of the universe. These effects cannot be treated, to our knowledge, with ζ-function methods, and represent a fascinating problem both in quantum gravity and in the theory of the early universe.

A.K. is grateful to A. O. Barvinsky for useful discussions. A.K. is indebted to the Istituto Nazionale di Fisica Nucleare for financial support and to the Dipartimento di Scienze Fisiche of the University of Naples for hospitality. A.K. was partially supported by RFBR via Grant No. 96-02-16220, by INTAS via Grant No. 93-3364-Ext., and by the Russian research project ''Cosmomicrophysics.''

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