Supersymmetric inflation with constraints on superheavy neutrino masses

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We consider a supersymmetric model of inflation in which the primordial density fluctuations are nearly scale invariant (spectral index $n \approx 0.98$) with an amplitude proportional to $(M/M_{\rm Planck})^2$, where $M \sim 10^{16}$ GeV denotes the scale of the gauge symmetry breaking associated with inflation. The 60 or so e foldings take place when all relevant scales are close to M, which helps suppress supergravity corrections. The gravitino and baryogenesis (via leptogenesis) constraints help determine the two heaviest right-handed neutrino masses to be $\approx 2\times10^{13}$ GeV and 6×10^{9} GeV. [S0556-2821(97)06512-0]

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The apparent existence of the supersymmetric grand unification scale of $M_{\rm GUT}(\sim 10^{16}$ GeV), which is hinted at by both theory and an extrapolation of the data from the CERN e^+e^- collider LEP, suggests that the ''small'' ratio $M_{\rm GUT}/M_P \sim 10^{-3}$ (where $M_P = 1.22\times10^{19}$ GeV is the Planck mass) may play an important role in particle physics and cosmology. From the viewpoint of inflationary cosmology, in particular, it seems desirable to have a scenario in which primordial density fluctuations could be related to the above ratio. Moreover, if all scales associated with the relevant inflationary phase are close to $M_{\rm GUT}$, then we are (more or less) assured that the supergravity corrections are adequately suppressed.

A step in this direction was recently taken [1] when it was realized that, within the framework of relatively simple supersymmetric models, a ''hybrid'' inflationary scenario [2] can be implemented with a number of remarkable features. In particular, the primordial density fluctuations are essentially scale invariant (scalar spectral index $n \simeq 0.98$) with magnitude proportional to $(M/M_P)^2$, where M denotes the mass scale of the associated gauge symmetry breaking $(G \to H)$. Cosmic background temperature anisotropy data constrain this scale to be $M \approx (5\text{–}6)\times10^{15}$ GeV which strongly suggests embedding the model in a suitable grand unified scheme (see remarks below). Two other features of this scheme are worth emphasizing: (i) The inflationary phase is ''driven'' by radiative corrections which result from supersymmetry breaking in the very early Universe; (ii) the phase transition from $G \to H$ occurs at the end of the inflationary epoch so that this symmetry breaking should not produce monopoles. In other words, this inflationary scenario will not work for the minimal SU(5) model, even if the scale M had turned out to be precisely the supersymmetric (SUSY) grand unified theory (GUT) scale.

In this report we want to be quite specific about what G and H are, although a detailed discussion on how they are embedded in a supersymmetric grand unified theory such as SO(10) or $SU(3)_c \times SU(3)_L \times SU(3)_R$ ($[SU(3)]^3$ for short) will not be attempted. We will take G to be the subgroup $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ of these two groups, such that the scale M is associated with the breaking of $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$. We will explore, in particular, the ''reheat'' phase that follows inflation. The gravitino constraint on the ''reheat'' temperature T_R leads to important constraints on the masses of the heavy ''right-handed'' neutrinos. In turn, light neutrino masses $m_{\nu_\tau} \sim 4$ eV and $m_{\nu_\mu} \sim 10^{-2.8}$ eV fit nicely into the scheme, and the observed baryon asymmetry of the Universe can be produced via a primordial lepton asymmetry resulting from the decay of right-handed neutrinos. We end with a brief comparison of the resulting cold plus hot dark matter scenario with observations.

We begin with the following globally supersymmetric renormalizable superpotential W [3]:

$$W = \kappa S \overline{\phi}\phi - \mu^2 S \quad (\kappa > 0,\ \mu > 0), \tag{1}$$

where ϕ, $\overline{\phi}$ denote the standard model singlet components of a conjugate pair of $SU(2)_R \times U(1)_{B-L}$ doublet left-handed superfields, and S is a gauge singlet left-handed superfield. An R symmetry, under which $S \to e^{i\alpha}S$, $\overline{\phi}\phi \to \overline{\phi}\phi$, and $W \to e^{i\alpha}W$, can ensure that the rest of the renormalizable terms are either absent or irrelevant. Note that the gauge quantum numbers of ϕ are precisely the same as the ones of the ''matter'' right-handed neutrinos. But they are distinct superfields and, in particular, the latter do not have the conjugate partners. From W, one writes down the potential V as a function of the scalar fields $\phi, \overline{\phi}, S$:

$$V(\phi,\overline{\phi},S) = \kappa^2 |S|^2 [|\phi|^2 + |\overline{\phi}|^2] + |\kappa\phi\overline{\phi} - \mu^2|^2 + D \ \text{terms}. \tag{2}$$

The D terms vanish along the D-flat direction $\phi = \overline{\phi}^*$ which contains the supersymmetric minimum

$$\langle S\rangle=0,$$
$$\langle|\phi|\rangle=\langle|\overline{\phi}|\rangle=\mu/\sqrt{\kappa}\equiv M. \tag{3}$$

Using an appropriate R transformation, S can be brought to the real axis, i.e., $S=\sigma/\sqrt{2}$, where σ is a normalized real scalar field.

The important point now is that in the early Universe the scalar fields are displaced from the above minimum. In particular, for $S>S_c=M$, the potential V is minimized by $\phi=\overline{\phi}=0$. The energy density is dominated by μ^4 which, therefore, leads to an exponentially expanding inflationary phase (hybrid inflation). As emphasized in [1], there are important radiative corrections under these conditions [4]. At one loop, and for S sufficiently larger than S_c, the inflationary potential is given by

$$V_{\rm eff}(S)=\mu^4\left\{1+\frac{\kappa^2}{16\pi^2}\left[\ln\left(\frac{\kappa^2S^2}{\Lambda^2}\right)+\frac{3}{2}-\frac{S_c^4}{12S^4}+\cdots\right]\right\}. \tag{4}$$

Using Eq. (4), one finds [5] that the cosmic microwave quadrupole anisotropy amplitude, $(\Delta T/T)_Q\approx 8\pi(N_Q/45)^{1/2}(x_Q/y_Q)(M/M_P)^2$, and the primordial density fluctuation spectral index $n\simeq 0.98$. Here $N_Q\approx$ 50–60 denotes the relevant number of e foldings experienced by the Universe between the time the quadrupole scale exited the horizon and the end of inflation, $y_Q=x_Q[1-7/(12x_Q^2)+\cdots]$ with $x_Q=S_Q/M$, and S_Q is the value of the scalar field S when the scale which evolved to the present horizon size crossed outside the de Sitter horizon during inflation. Also from Eq. (4), one finds $\kappa\approx(8\pi^{3/2}/\sqrt{N_Q})y_Q(M/M_P)$.

The inflationary phase ends as S approaches S_c from above. Write $S=xS_c$, where $x=1$ corresponds to the phase transition from $G\rightarrow H$ which, it turns out, more or less coincides with the end of the inflationary phase [this is checked by noting the amplitude of the quantities $\epsilon=(M_P^2/16\pi)(V'/V)^2$ and $\eta=(M_P^2/8\pi)(V''/V)$, where the prime refers to derivatives with respect to the field σ]. Indeed, the 50–60 e foldings needed for the inflationary scenario can be realized even with $x\approx 2$. An important consequence of this is that with $S\sim 10^{16}$ GeV, the supergravity corrections are negligible [6].

In order to estimate the ''reheat'' temperature we take account of the fact that the inflaton consists of the two complex scalar fields S and $\theta=(\delta\phi+\delta\overline{\phi})/\sqrt{2}$, where $\delta\phi=\phi-M$, $\delta\overline{\phi}=\overline{\phi}-M$, with mass $m_{infl}=\sqrt{2}\kappa M$. We mainly concentrate on the decay of θ. Its relevant coupling to ''matter'' is provided by the nonrenormalizable superpotential coupling (in symbolic form):

$$\frac{1}{2}\left(\frac{M_{\nu^c}}{M^2}\right)\overline{\phi\phi}\nu^c\nu^c, \tag{5}$$

where M_{ν^c} denotes the Majorana mass of the relevant right-handed neutrino ν^c. Without loss of generality we assume that the Majorana mass matrix of the right-handed neutrinos has been brought to diagonal form with positive entries. Clearly, θ decays predominantly into the heaviest right-handed neutrino permitted by phase space. (The field S can rapidly decay into Higgsinos through the renormalizable superpotential term $\xi S h^{(1)}h^{(2)}$ allowed by the gauge symmetry, where $h^{(1)},h^{(2)}$ denote the electroweak Higgs doublets which couple to the up- and down-type quarks, respectively, and ξ is a suitable coupling constant. Note that after supersymmetry breaking, $\langle S\rangle\sim M_S$, where $M_S\sim$ TeV denotes the magnitude of the breaking.)

Following standard procedures (we will soon comment on the issue of parametric resonance), and assuming the minimal supersymmetric standard model (MSSM) spectrum, the 'reheat' temperature T_R is given by

$$T_R\approx\tfrac{1}{7}(\Gamma_\theta M_P)^{1/2}, \tag{6}$$

where $\Gamma_\theta\approx(1/16\pi)(\sqrt{2}M_{\nu^c}/M)^2\sqrt{2}\kappa M$ is the decay rate of θ. Substituting κ as a function of N_Q, y_Q, and M, we find

$$T_R\approx\frac{1}{12}\left(\frac{56}{N_Q}\right)^{1/4}\sqrt{y_Q}M_{\nu^c}. \tag{7}$$

Several comments are in order.

(i) For x_Q on the order of unity the ''reheat'' temperature is essentially determined by the mass of the heaviest right-handed neutrino the inflaton can decay into.

(ii) The well-known gravitino problem requires that T_R lie below 10^8–10^{10} GeV, unless a source of late stage entropy production is available. Given the uncertainties, we will interpret the gravitino constraint as the requirement that $T_R\lesssim 10^9$ GeV.

(ii) In deriving Eq. (7) we have ignored the phenomenon of parametric resonance. This is justified because the oscillation amplitude is of order M (not M_P), such that the induced scalar mass ($\sim M_{\nu^c}$) is smaller than the inflaton mass $\sqrt{2}\kappa M$. Note that here M_{ν^c} denotes the mass of the heaviest right-handed neutrino supermultiplet the inflaton can decay into.

To proceed further we will need some details from the seesaw mechanism for the generation of light neutrino masses. For simplicity, we will ignore the first family of quarks and leptons. The Majorana mass matrix of the right-handed neutrinos can then be brought (by an appropriate unitary transformation on the right-handed neutrinos) to the diagonal form with real positive entries

$$\mathcal{M}=\begin{pmatrix}M_1 & 0\\ 0 & M_2\end{pmatrix}\quad(M_1,M_2>0). \tag{8}$$

An appropriate unitary rotation can then be further performed on the left-handed neutrinos so that the (approximate) seesaw light neutrino mass matrix $m_D\mathcal{M}^{-1}\widetilde{m}_D$, m_D being the neutrino Dirac matrix, takes the diagonal form

$$m_D\frac{1}{\mathcal{M}}\widetilde{m}_D=\begin{pmatrix}m_1 & 0\\ 0 & m_2\end{pmatrix} \tag{9}$$

(m_1,m_2 are, in general, complex) [7]. In this basis of right- and left-handed neutrinos, the elements of

$$m_D=\begin{pmatrix}a & b\\ c & d\end{pmatrix}, \tag{10}$$

are not all independent. They can be expressed in terms of only three complex parameters a, d, and η, where $\eta=-[M_1/M_2]^{1/2}(b/a)=[M_2/M_1]^{1/2}(c/d)$.

We will now assume that m_D coincides asymptotically (at the SUSY GUT scale $M_{\rm GUT}\simeq 2\times 10^{16}$ GeV) with the up-type quark mass matrix as is the case in many GUT models. Restricting ourselves, from now on, to the case where $|\eta|\sim 1$ and $M_1/M_2\gg 1$, we have $|a|\gg|b|$ and $|c|\gg|d|$. Without much loss of generality we can further take $|c|\ll|a|$ so that a is the dominant element in m_D. In fact, one can numerically show that the primordial lepton asymmetry of the Universe (see below) is maximized in this region of the parameter space. Under these assumptions the asymptotic top and charm quark masses are $|m_t|\approx|a|$ and $|m_c|\approx|d||1+\eta^2|$. Since $|m_2|\ll|m_1|$, we can make the following identification of the light neutrino mass eigenstates

$$m_{\nu_\tau}=|m_1|=\frac{|a|^2}{M_1}|1+\eta^2|, \quad m_{\nu_\mu}=|m_2|=\frac{|d|^2}{M_2}|1+\eta^2|. \tag{11}$$

We can then get the useful relations

$$M_2\approx\frac{m_c^2m_t^2}{m_{\nu_\mu}m_{\nu_\tau}}\frac{1}{M_1}, \qquad |1+\eta^2|\approx\frac{m_{\nu_\tau}}{m_t^2}M_1. \tag{12}$$

We are now ready to draw some important conclusions concerning neutrino masses that are more or less model independent. Assuming that the inflaton predominantly decays to the heaviest right-handed neutrino [i.e., $M_{\nu^c}=M_1$ in Eq. (7)] and employing condition (ii), we obtain $M_1\lesssim 9.3\times 10^9$ GeV for $N_Q\approx 56$ and $x_Q\approx 2$. Equation (11) then implies an unacceptably large m_{ν_τ} for $|\eta|\sim 1$. Thus, we are led to our first important conclusion: the inflaton should decay to the second heaviest right-handed neutrino and, consequently, $M_{\nu^c}=M_2$ in Eq. (7). Combining this equation with Eq. (12) we obtain

$$T_R\approx\frac{1}{12}\left(\frac{56}{N_Q}\right)^{1/4}\frac{m_c^2m_t^2}{m_{\nu_\mu}m_{\nu_\tau}}\frac{y_Q^{1/2}}{M_1}\approx 9.2\times 10^{21}\ \frac{y_Q^{1/2}}{M_1}\ \text{GeV}. \tag{13}$$

Here we put $N_Q=56$ which is easily justifiable by standard methods at the end of the calculation after having fixed the values of all relevant parameters. Also, we took $m_t=110$ GeV, $m_c=0.24$ GeV, which are consistent with the assumption that below $M_{\rm GUT}$ the theory reduces to the minimal supersymmetric standard model (MSSM) with large $\tan\beta$ [8]. Moreover, we took $m_{\nu_\mu}\approx 10^{-2.8}$ eV which lies at the center of the region consistent with the resolution of the neutrino solar puzzle via the small angle Mikheyev-Smirnov-Wolfenstein (MSW) mechanism. The value $m_{\nu_\tau}\approx 4$ eV is consistent with the light τ neutrino playing an essential role in the formation of large scale structure in the Universe.

The value of M_1 is restricted by the fact that the inflaton should not decay to the corresponding right-handed ''τ'' neutrino

$$M_1\geqslant\frac{m_{infl}}{2}=\frac{\kappa M}{\sqrt{2}}\approx\left(\frac{45\pi}{2}\right)^{1/2}\frac{y_Q^2}{N_Qx_Q}M_P\left(\frac{\Delta T}{T}\right)_Q$$
$$\approx y_Q^2x_Q^{-1}1.2\times 10^{13}\ \text{GeV}. \tag{14}$$

It is interesting to note that since the right-handed neutrinos acquire their masses from superpotential terms $\lambda\overline{\phi}\overline{\phi}\nu^c\nu^c/M_c$, where $M_c=M_P/\sqrt{8\pi}\approx 2.4\times 10^{18}$ GeV and $\lambda\lesssim 1$, $M_1=2\lambda M^2/M_c\lesssim(y_Q/x_Q)2.9\times 10^{13}$ GeV [for $N_Q=56$, $(\Delta T/T)_Q=6.6\times 10^{-6}$]. Thus, from Eq. (14), $y_Q\lesssim 2.4$ which implies $x_Q\lesssim 2.6$, and restricts the relevant part of inflation at values of $S\sim 10^{16}$ GeV.

To maximize the primordial lepton asymmetry (see below) we choose the bound in Eq. (14) to be saturated. Equation (13) then gives

$$T_R\approx x_Qy_Q^{-3/2}7.6\times 10^8\ \left(\frac{\Delta T/T}{6.6\times 10^{-6}}\right)^{-1}\left(\frac{N_Q}{56}\right)^{3/4}$$
$$\times\left(\frac{m_c}{0.24\ \text{GeV}}\frac{m_t}{110\ \text{GeV}}\right)^2\left(\frac{m_{\nu_\mu}}{10^{-2.8}\ \text{eV}}\frac{m_{\nu_\tau}}{4\ \text{eV}}\right)^{-1}\text{GeV}, \tag{15}$$

which satisfies condition (ii) for all allowed values of y_Q. Equation (12) implies

$$M_2\approx x_Qy_Q^{-2}9\times 10^9\ \text{GeV},$$
$$|1+\eta^2|\approx 4\ \ y_Q^2x_Q^{-1}. \tag{16}$$

This implies that the errors in the asymptotic formulas for the top and charm quark masses are $<1\%$.

The observed baryon asymmetry of the Universe can be generated by first producing a primordial lepton asymmetry via the out-of-equilibrium decay of the right-handed neutrinos, which emerge as decay products of the inflaton field at ''reheating'' [9]. It is important though to ensure that the lepton asymmetry is not erased by lepton number-violating 2-2 scatterings at all temperatures between T_R and 100 GeV [10]. In our case this requirement is automatically satisfied since at temperatures above 10^7 GeV the lepton asymmetry is protected [11] by supersymmetry, whereas at temperatures between 10^7 and 100 GeV, as one can easily show, these 2-2 scatterings are well out of equilibrium. The out-of-equilibrium condition for the decay of the right-handed neutrinos is also satisfied since $M_2\gg T_R$ for all relevant values of x_Q. The primordial lepton asymmetry is estimated to be [9]

$$\frac{n_L}{s}\approx\frac{9}{8\pi}\frac{T_R}{m_{infl}}\frac{M_2}{M_1}\frac{\text{Im}(m_D^\dagger m_D/|\langle h^{(1)}\rangle|^2)_{21}^2}{(m_D^\dagger m_D/|\langle h^{(1)}\rangle|^2)_{22}}. \tag{17}$$

Equation (10) combined with the fact that $|c||d|\ll|a||b|$ then gives

$$\frac{n_L}{s}\lesssim\frac{9}{8\pi}\frac{T_R}{m_{infl}}\frac{M_2}{M_1}\frac{m_t^2}{|\langle h^{(1)}\rangle|^2}, \tag{18}$$

which, using Eqs. (12)–(16) and the fact that $|\langle h^{(1)}\rangle|\approx 174$ GeV for large $\tan\beta$, becomes

$$\frac{n_L}{s} \lesssim x_Q^4 y_Q^{-15/2} 3.4\times 10^{-9} \left(\frac{\Delta T/T}{6.6\times 10^{-6}}\right)^{-4} \left(\frac{N_Q}{56}\right)^{15/4} \times \left(\frac{m_c}{0.24\ \mathrm{GeV}}\right)^{4} \left(\frac{m_t}{110\ \mathrm{GeV}}\right)^{6} \left(\frac{m_{\nu_\mu}}{10^{-2.8}\ \mathrm{eV}}\ \frac{m_{\nu_\tau}}{4\ \mathrm{eV}}\right)^{-2}. \tag{19}$$

For $x_Q \approx 2$ ($y_Q \approx 1.7$), this gives $n_L/s \lesssim 10^{-9}$ which is large enough to account for the observed baryon asymmetry. Also $M \approx 5.47\times 10^{15}$ GeV, $T_R \approx 6.8\times 10^{8}$ GeV, $M_1 \approx 1.75\times 10^{13}$ GeV, $M_2 \approx 6.2\times 10^{9}$ GeV, and $m_{infl} \approx 3.5\times 10^{13}$ GeV for the same value of x_Q.

In supersymmetric models the lightest supersymmetric particle (LSP) is expected to be stable and is a leading cold dark matter candidate. If we couple this with a τ neutrino of mass $\sim$ 2–6 eV we are led to the well-tested cold plus hot dark matter (CHDM) model [12] of large scale structure formation, with a spectral index of $n = 0.98$. This model [12] provides a consistent picture for the formation of large scale structure in the Universe, and was used to correctly predict [13] the primordial cosmic background radiation fluctuation amplitude seen by the Cosmic Background Explorer satellite [14].

To summarize, among the key features of the inflationary models we have discussed one could list the role played by radiative corrections in the early Universe, the realization of inflation at scales well below M_P so that the gravitational corrections can be adequately suppressed, and the constraints on the two heaviest right-handed neutrino masses. The resulting cold plus hot dark matter combination which results is an added bonus. One of the remaining challenges is to embed the scheme described here within a fully unified framework.

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