Svetitsky-Yaffe conjecture for the plaquette operator

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According to the Svetitsky-Yaffe conjecture, a $(d+1)$-dimensional pure gauge theory undergoing a continuous deconfinement transition is in the same universality class as a d-dimensional statistical model with the order parameter taking values in the center of the gauge group. We show that the plaquette operator of the gauge theory is mapped into the energy operator of the statistical model. For $d=2$, this identification allows us to use conformal field theory techniques to evaluate exactly the correlation functions of the plaquette operator at the critical point. In particular, we can evaluate exactly the plaquette expectation value in the presence of static sources, which gives some new insight into the structure of the color flux tube in mesons and baryons. [S0556-2821(97)06212-7]

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I. INTRODUCTION

Consider a $(d+1)$-dimensional pure gauge theory undergoing a continuous deconfinement transition at the critical temperature T_c. The effective model describing the behavior of Polyakov lines at finite temperature T will be a d-dimensional statistical model with a global symmetry group coinciding with the center of the gauge group. Svetitsky and Yaffe [1] were able to show that this effective model has only short-range interactions. If also the d-dimensional effective model displays a continuous phase transition then it follows from universality arguments that it belongs to the same universality class of the original gauge model.

Therefore all the universal properties of the deconfinement transition can be predicted to coincide with the ones of the dimensionally reduced effective model. These include the values of the critical indices, the finite-size scaling behavior, and the correlation functions at criticality. The conjecture has passed several numerical tests, which became more and more stringent in the last years due to the increased precision reachable with Monte Carlo simulations (see [2], and references therein).

It is clear that the Svetitsky-Yaffe conjecture becomes very predictive for $d=2$, where, using the methods of conformal field theory, the critical behavior can be determined exactly. For example, the critical properties of (2+1)-dimensional [(2+1)D] SU(2) gauge theory at the deconfinement temperature coincide with those of the two-dimensional Ising model. This allows us not only to predict the exact values of the critical indices, but also to write down all the multipoint correlation functions of the Polyakov loop at criticality [3].

What is needed to fully exploit the predictive power of the Svetitsky-Yaffe conjecture is a mapping relating the physical observables of the gauge theory to the operators of the dimensionally reduced model. In $d=2$ the knowledge of this mapping is equivalent to solving the gauge theory at the deconfinement temperature.

The correspondence between the Polyakov line and order parameter of the effective model is the first entry in this mapping and is intrinsically contained in the Svetitsky-Yaffe conjecture. It is natural to ask what operator in the d-dimensional model corresponds to the plaquette operator of the gauge theory: symmetry considerations suggest the energy operator as a natural candidate. In this paper we show that this is actually the case, and we describe some consequences of this identification.

The correctness of the identification plaquette-energy is shown in Sec. II by studying the finite-size behavior of the plaquette operator in (2+1)D Z_2 gauge theory at critical temperature. We show that it coincides with the (highly nontrivial) finite-size behavior of the energy operator in the 2D Ising model at criticality.

In Sec. III we compute correlation functions of the plaquette operator by using conformal field theory techniques. In particular, the expectation value of the plaquette in vacuum modified by static sources can be computed for (2+1)D SU(2) and SU(3) gauge theories at the deconfinement temperature, providing physical insight into the structure of the color flux tube in mesons and baryons.

II. FINITE-SIZE BEHAVIOR OF THE PLAQUETTE EXPECTATION VALUE

Finite-size effects at criticality are typically rather strong, due to scale invariance, and nontrivial. Therefore they are ideally suited to compare theoretical predictions with, for example, results of Monte Carlo simulations. In particular, for two-dimensional statistical systems, the critical behavior, including finite-size effects, is completely understood with the methods of conformal field theory (CFT). We want to exploit this fact to establish the correspondence between the plaquette operator in a $(d+1)$-dimensional lattice gauge theory at the deconfinement transition and the energy operator of the corresponding d-dimensional statistical model.

Consider for example the 2D Ising model on a rectangle of sides L_1, L_2, and periodic boundary conditions in both

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directions, i.e., on a torus. The shape and size dependence at criticality of the expectation value of the internal energy are given by [4–6]

$$\langle \epsilon \rangle = \frac{\pi \sqrt{\mathrm{Im}\tau} |\eta(\tau)|^2}{\sqrt{A} Z_{1/2}(\tau)}, \tag{1}$$

where $A \equiv L_1 L_2$ and $\tau \equiv i L_1 / L_2$ are, respectively, the area and the modular parameter of the torus, and $Z_{1/2}$ is the Ising partition function at the critical point:

$$Z_{1/2} = \frac{1}{2} \sum_{\nu=2}^{4} \left| \frac{\theta_\nu(0,\tau)}{\eta(\tau)} \right|. \tag{2}$$

Here θ_ν are the Jacobi theta functions and η is the Dedekind function (for notations and conventions see Ref. [6]).

Comparing Eq. (1) with the finite-size behavior of the plaquette operator in a 3D lattice gauge theory (LGT) such that the center of the gauge group is Z_2 provides a stringent test of our identification. The simplest choice is the 3D Z_2 gauge model, for which it is possible to achieve very high precision in the Monte Carlo evaluation of physical quantities, and accurate estimates of the deconfinement temperature are available.

Therefore we considered the (2+1)D Z_2 gauge model on lattices of size $L_1 \times L_2 \times L_t$, where $L_t \ll L_1, L_2$ with periodic boundary conditions on all directions, and we studied it at the critical coupling $\beta_c(L_t)$, which is known to high accuracy for several values of L_t [2]. By performing Monte Carlo simulations at different values of L_1, L_2, we can compare the finite-size behavior of the plaquette expectation value with Eq. (1).

More precisely, we will show that the plaquette operator is a mixture of the identity and energy operators of the 2D CFT: on the one hand, both these operators transform as singlets under Z_2, and therefore can contribute to the plaquette operator; on the other hand, we know that the plaquette expectation value does not vanish in infinite volume, unlike the energy operator of the 2D Ising model [see Eq. (1)]. Therefore, a nonvanishing contribution of the identity operator must be expected in the plaquette expectation value. Hence our conjecture is

$$\langle \Box \rangle = c_1 \langle 1 \rangle + c_\epsilon \langle \epsilon \rangle, \tag{3}$$

where the expectation value in the left-hand side (LHS) is taken in the LGT, while the ones in the RHS refer to the CFT. The prediction for the finite-size behavior of the plaquette expectation value is, therefore,

$$\langle \Box \rangle_{L_1 L_2} = c_1 + c_\epsilon \frac{F(\tau)}{\sqrt{L_1 L_2}} + O(1/L_1 L_2), \tag{4}$$

where F is a function of the modular parameter $\tau \equiv i L_1 / L_2$ only:

$$F(\tau) = \frac{\pi \sqrt{\mathrm{Im}\tau} |\eta(\tau)|^2}{Z_{1/2}(\tau)}. \tag{5}$$

We performed our Monte Carlo simulations at $L_t = 6$ with critical coupling

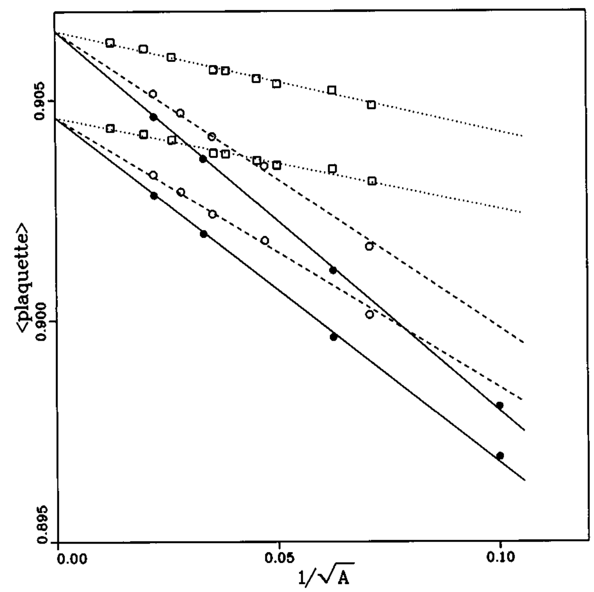

FIG. 1. Size and shape dependence of the plaquette expectation value. Black dots correspond to square lattices ($\mathrm{Im}\tau = 1$). White dots and squares correspond to rectangular lattices with $\mathrm{Im}\tau = 2$ and $\mathrm{Im}\tau = 4$, respectively. Both timelike and spacelike plaquettes are shown, the latter having lower expectation values. The lines correspond to the best fit to Eq. (4).

$$\beta_c(L_t = 6) = 0.746\,035. \tag{6}$$

We measured the plaquette expectation value for lattices of area $100 \leq A \leq 6400$ and asymmetry ratio $\mathrm{Im}\tau = 1,2,4$. The spacelike and timelike plaquettes have different expectation values, therefore must be fitted separately with Eq. (4). The Monte Carlo simulations were actually performed in the 3D Ising (spin) model, which is exactly equivalent through duality to the Z_2 gauge model. This choice allowed us to use a nonlocal cluster simulation algorithm.

The agreement is very good, giving $\chi^2_{\mathrm{red}} = 0.7$ for spacelike plaquettes and $\chi^2_{\mathrm{red}} = 0.9$ for timelike plaquettes.[1] These data are plotted in Fig. 1.

III. CORRELATION FUNCTIONS OF THE PLAQUETTE OPERATOR

In this section we will exploit the new entry we added to the Svetitsky-Yaffe mapping to compute correlation functions of the plaquette operator at the deconfinement temperature. This will provide some new insight into the structure of color flux tubes in mesons and baryons.

Consider for example (2+1)D SU(2) LGT at deconfinement temperature. To study the flux tube structure in a "static meson" we can consider the plaquette expectation value in the vacuum modified by the presence of two static sources, i.e., the correlation function of the plaquette operator with two Polyakov loops:

[1] It must be noted that the expectation values of spacelike and timelike plaquettes for a given lattice are not statistically uncorrelated, since they were extracted from the same sample of configurations.

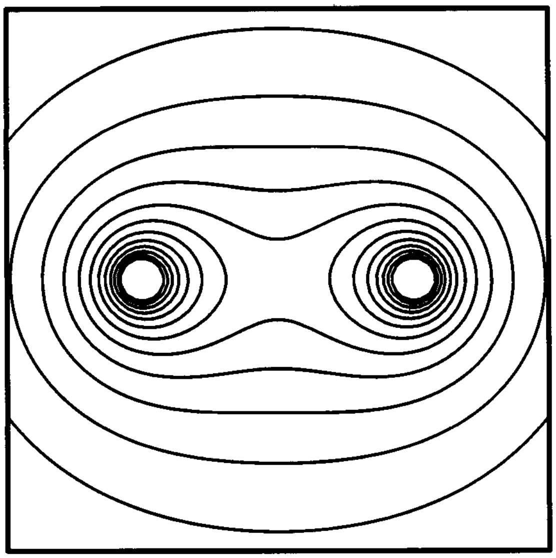

FIG. 2. Structure of the flux tube in a ''static meson'' at the deconfinement temperature, Eq. (9).

$$G(x,x_1,x_2)=\langle\Box(x)P(x_1)P(x_2)\rangle-\langle\Box\rangle\langle P(x_1)P(x_2)\rangle, \tag{7}$$

where x,x_1,x_2 are points in the 2D space. This will be given by the correlation of the energy operator with 2 spin operators in the 2D critical Ising model:

$$G(x,x_1,x_2)\propto\langle\epsilon(x)\sigma(x_1)\sigma(x_2)\rangle_{\text{Ising}}. \tag{8}$$

The RHS is easily computed in CFT and we find

$$G(x,x_1,x_2)\propto\frac{|x_1-x_2|^{3/8}}{(|x-x_1||x-x_2|)^{1/2}}. \tag{9}$$

We have plotted this function in Fig. 2.

More interesting is the case of (2+1)D SU(3) LGT, where we can consider a ''static baryon'' by modifying the vacuum with three static sources and compute

$$G(x,x_1,x_2,x_3)=\langle\Box(x)P(x_1)P(x_2)P(x_3)\rangle -\langle\Box\rangle\langle P(x_1)P(x_2)P(x_3)\rangle. \tag{10}$$

Our identification gives

$$G(x,x_1,x_2,x_3)\propto\langle\epsilon(x)\sigma(x_1)\sigma(x_2)\sigma(x_3)\rangle_{\text{3-state Potts}}, \tag{11}$$

where the correlation function on the RHS must be computed in the $c=4/5$ CFT describing the three-state Potts model at criticality. This is done using the methods introduced in Ref. [7] (see also [6]) and gives

$$G(x,x_1,x_2,x_3)\propto\frac{(|x_1-x_2||x_1-x_3||x_2-x_3|)^{1/15}}{(|x-x_1||x-x_2||x-x_3|)^{4/15}}\times|y(1-y)|^{7/15}\left[|f_1(y)|^2+\frac{9}{4}\frac{\Gamma^3(3/5)\Gamma(1/5)}{\Gamma^3(2/5)\Gamma(4/5)}|f_2(y)|^2\right], \tag{12}$$

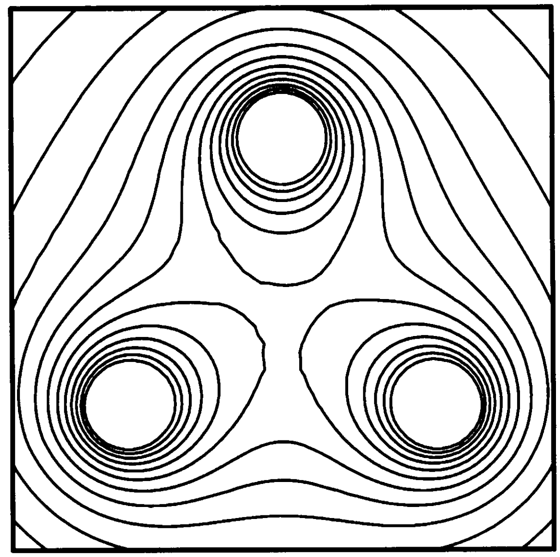

FIG. 3. The structure of the flux tube in a ''static baryon,'' Eq. (12).

where, introducing a complex coordinate z in 2D space, y is the conformally invariant cross ratio

$$y=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)} \tag{13}$$

and f_1 and f_2 are hypergeometric functions:

$$f_1(y)=F(4/5,7/5;8/5;y), \tag{14}$$

$$f_2(y)=y^{-3/5}F(1/5,4/5;2/5;y). \tag{15}$$

$G(x,x_1,x_2,x_3)$ is plotted in Fig. 3, for the case in which the three static sources form an equilateral triangle. Notice that this calculation brings strong support to the ''Y'' structure of the flux tube in baryons (see, e.g., [8] and references therein), as opposed to the ''Δ'' structure [9].

IV. CONCLUSIONS

In this paper we have added a new entry to the Svetitsky-Yaffe mapping between $(d+1)$-dimensional LGT's at deconfinement temperature and d-dimensional critical statistical models, namely, we have shown that the plaquette operator of the LGT is mapped into the energy operator of the statistical model. For $d=2$, this identification allows in principle the exact evaluation of all correlations of the plaquette operator at the deconfinement point, providing a useful tool for the study of the color flux tube in mesons and baryons.

For $d>2$, the critical behavior of statistical models is not completely understood, therefore our conjecture does not hold all the predictive power we have shown in the $d=2$ case. However, many useful predictions can still be made. Consider for example SU(2) in $d+1$ dimensions: if d is such that the deconfinement transition of the gauge model is of second order, as it is in the realistic case $d=3$, we can predict the following finite-size scaling behavior of the plaquette operator at the deconfinement point (L is the spatial size of the lattice):

$$\langle \Box \rangle_L = \langle \Box \rangle_\infty + cL^{1/\nu_d - d} \tag{16}$$

where ν_d is the correlation length critical index of the d-dimensional Ising model.

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