String model of black hole microstates

Finn Larsen

Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, Pennsylvania 19104

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The statistical mechanics of black holes arbitrarily far from extremality is modeled by a gas of weakly interacting strings. As an effective low-energy description of black holes the string model provides several highly nontrivial consistency checks and predictions. Speculations on a fundamental origin of the model suggest surprising simplifications in nonperturbative string theory, even in the absence of supersymmetry. [S0556-2821(97)02614-3]

Black holes exhibit thermodynamic properties suggestive of a complicated internal structure. This is somewhat paradoxical within standard general relativity where black holes are absolutely featureless. In the last year or so there has been considerable excitement as it has become clear that string theory can accurately account for the degeneracy implicit in the Bekenstein-Hawking entropy. This development initially concerned the counting of states for extremal [1–3] and near extremal black holes [4]. The effective string model that emerged apparently describes in much detail the dynamics of near extremal black holes [5,6]. This no longer leaves any doubt that string theory describes many features of black holes.

Many questions remain, however. Current models are restricted to very large wave lengths. In the interesting region with possible information loss the appropriate methods are quite different and they still need further development [7]. The focus of this paper is another ill-understood issue, the generalization far off extremality. Here, many new concerns appear, especially because much standard string technology remains untested in a regime that is not even approximately supersymmetric. Some proposals have appeared (including [8]) but there is still much confusion. Moreover, a large region of parameter space has been identified where the D-brane motivated model gives incorrect predictions [9,10]. In this paper we shall argue that, nevertheless, the situation is quite hopeful: the thermodynamics of black holes in string theory has nontrivial features suggesting a surprisingly simple improved effective string model that gives a satisfying description of nonextremal black holes. We shall present arguments motivating the string model and note a number of consistency checks, including highly nontrivial ones. We speculate that the model arises in fundamental string theory as the direct product of two chiral sectors, each a $c=6$ superconformal field theory.

The discussion will assume five-dimensional black holes although the generalization to four dimensions involves no new features. For definiteness we will presume toroidally compactified type-II theory. In this context the most general nonrotating black hole is characterized up to duality by its mass M and three conserved U(1) charges $Q_i : i = 1,2,3$. It is convenient to introduce in intermediate steps the so-called nonextremality parameter μ and also the ''boosts'' $\delta_i : i = 1,2,3$. In the resulting parametric form the physically interesting quantities are [11]

$$M = \frac{1}{2}\mu \sum_i \cosh 2\delta_i, \tag{1}$$

$$Q_i = \frac{1}{2}\mu \sinh 2\delta_i, \tag{2}$$

$$S = 2\pi\mu^{3/2}\prod_i \cosh\delta_i \tag{3}$$

in Planck units $G_N = 1$. When the black hole solution is embedded in string theory the charges arise from specific sources. For example, they may be the winding and momentum charges of a fundamental string as well as solitonic five-brane charge. Or they may be one-brane and five-brane Ramond-Ramond (RR) charges along with Kaluza-Klein momentum. The latter representation is particularly popular because the sources of RR charges are D branes which are accessible to detailed study in string theory [12]. For the present purposes it will not be necessary to make a specific choice between these embeddings (or many others). In fact, nonperturbative string theory in the absence of supersymmetry remains largely unexplored and offers few concrete tools at this point. We will simply rely on the conjectured duality symmetries to ensure that microscopic degeneracies will be identical in all representations. More importantly, duality also gives some guidance in the arguments. Note that the manifestly symmetric appearance of the three charges in the thermodynamic formulas (1)–(3) is a necessary condition for this strategy to be consistent.

Physical charges Q_i occur only in quantized units. The constants of proportionality relating Q_i to integers n_i depend on the precise embedding in string theory. Indeed fundamental charges, RR charges, and solitonic charges depend differently on the microscopic coupling constant, and the geometric moduli of the compactified torus also enter. It is remarkable, however, that, for the black holes embodied in Eqs. (1)–(3), the moduli cancel in the product of charges so

$$\prod_i Q_i = \prod_i n_i. \tag{4}$$

(We have chosen normalizations to avoid a numerical factor in this formula.) Generically physical parameters transform in covariant but nevertheless complicated ways under the

duality symmetry. The right-hand side of Eq. (4) is very special in this respect because here the full duality symmetry group could easily be restored. Indeed, $\Pi_i n_i$ is the manifestation, for our representative class of solutions, of the unique cubic invariant of the U-duality group $E_{6(6)}$ [13]. The cancellation of moduli responsible for this simplification is of great importance because it suggests a direct relation between macroscopic quantities and the underlying microscopic theory. This is particularly clear in the extremal limit $\mu \to 0$, $\delta \to \infty$ with Q_i fixed. Here the entropy becomes

$$S = 2\pi \left(\prod_i Q_i \right)^{1/2} = 2\pi \left(\prod_i n_i \right)^{1/2}. \qquad (5)$$

Note that this is also the degeneracy of a chiral string model with central charge $c = 6$ and an effective level $N = \Pi_i n_i$. For supersymmetric black holes such a model is very well motivated from fundamental string theory.

In the general nonextremal case we do not *a priori* know the moduli dependence of the mass Eq. (1) and the entropy Eq. (3). It is possible that the microscopic quantum theory ensures that even among these macroscopic parameters there are moduli-independent combinations, interpretable as quantum numbers. We propose that indeed there is exactly *one* such quantity and it is the entropy. Of course it is the exponential of the entropy that might actually be an integer. The point is, however, that, by definition, quantum numbers do not depend on moduli. It is a consequence of the proposal that, as moduli vary with quantum numbers fixed, the mass changes in a rather complicated way, as implied by Eqs. (1)–(3). The idea that mass depends on moduli but degeneracies do not is familiar from perturbative string theory where degeneracies follow from central charge and level of the conformal field theory, whereas mass depends on both level and (moduli-dependent) physical charges. Our working assumption can be motivated as follows: the semiclassical derivation of the entropy formula shows a remarkable independence of details, such as the low-energy matter content. We interpret this universality as a macroscopic manifestation of the underlying quantum theory. Formally, we note that quantization conditions on the gauge charges derive from topologically nontrivial gauge transformations at infinity, so we expect an analogous condition to follow from general covariance. In fact, the Gibbons-Hawking derivation of the entropy formula from the functional integral appears to realize exactly this [14].

At this point it is instructive to consider briefly the more general case of rotating black holes. In five dimensions the rotation group is $\mathrm{SO}(4) \simeq \mathrm{SU}(2)_R \times \mathrm{SU}(2)_L$, so configurations are characterized by two angular momenta $J_{R,L}$. The entropy is [15]

$$S = 2\pi(\sqrt{N_R} + \sqrt{N_L}), \qquad (6)$$

where

$$N_{R,L} = \frac{1}{4}\mu^3 \left(\prod_i \cosh\delta_i \mp \prod_i \sinh\delta_i \right)^2 - J_{R,L}^2. \qquad (7)$$

In string theory the space of states is the direct product of two terms because of the world sheet decomposition into left- and right-moving modes. Entropy deriving from string degeneracy is therefore expected to be the sum of two terms. It is suggestive that Eq. (6) takes this characteristic form. Concretely we propose that, in analogy with the supersymmetric case, $N_{R,L}$ arise as effective levels of two independent chiral sectors of a single string theory. Assuming central charge $c = 6$ in each sector Eq. (6) expresses a quantitative microscopic interpretation of the entropy in the most general case. It generalizes in a duality invariant fashion the D-brane motivated effective string model (references include [4,16]). A consistency requirement on this proposal is that $N_{R,L}$ are separately quantized with all positive integer values as spectrum. Since the angular momentum is quantized it would certainly take a very complicated conspiracy to make the total entropy moduli independent without N_R and N_L being so separately. Moreover, the angular momenta were normalized in the conventional way taking all integer values, so if indeed the first term in Eq. (7) is quantized with all integers as spectrum, the full $N_{R,L}$ will also be so for all values of the angular momenta. This is an important test of the proposal. It also implies that, accepting the proposal in the absence of angular momenta, generalization to rotating black holes is automatic, with numerical factors matching correctly [17,18]. Vanishing angular momenta can therefore be assumed in the following, without loss of generality. Angular momenta nevertheless play an important auxiliary role in the argument because they indicate how the entropy Eq. (3) should be divided into left- and right-moving contributions, and also because they suggest the precise quantization condition. The first point was noted and repeatedly emphasized by Cvetič, most recently in [19].

It was argued from general relativity that, apart from charges, there should be only one quantum number visible in the macroscopic theory, namely, the entropy. On the other hand, the string model introduces two quantum numbers, the levels of the left- and right-moving states. The reason that there is no conflict is the constraint

$$N_L - N_R = \mu^3 \prod_i \cosh\delta_i \sinh\delta_i = \prod_i Q_i = \prod_i n_i \qquad (8)$$

that follows from, in turn, Eqs. (7), (2), and (4). This not only avoids an apparent contradiction but it also relates quantities, purely within the microscopic theory, in a fashion reminiscent of the matching condition in perturbative string theory. It is indeed natural to suspect that in the full nonperturbative theory some condition arises from world sheet reparametrization invariance. Duality is a powerful restriction on its possible form and Eq. (8) is the simplest expression consistent with duality. As an independent consistency check note that both sides of Eq. (8) have all integers as spectrum. These nontrivial and desired results lend some support to the underlying assumptions.

In the preceding it was argued that Eq. (3) should be represented as the sum of two terms $S_{R,L} = 2\pi\sqrt{N_{R,L}}$ that are separately quantized. The difference of the two terms is $S_- = 2\pi\mu^{3/2}\Pi_i\sinh\delta_i$. It is amusing to note that this is $\frac{1}{4}$ the area of the inner horizon. Hence, whereas the Bekenstein-Hawking formula suggests that somehow the *outer* horizon is quantized in Planck units the present proposal implies that

also the *inner* horizon is quantized. This is a tantalizing prospect, although the precise geometric significance escapes us at this point.

In the string model there are separate left- and right-moving gases of weakly interacting strings. The terminology of left- and right-moving modes is independent of whether there is a net Kaluza-Klein momentum: all charges are treated democratically. Independent inverse temperatures of left- and right-moving modes follow from the thermodynamic relations

$$\beta_{R,L} = \left(\frac{\partial S_{R,L}}{\partial M}\right)_{Q_i} = \pi\mu^{1/2}\left(\prod_i \cosh\delta_i \pm \prod_i \sinh\delta_i\right) \quad (9)$$

with the physical (Hawking) temperature given by $\beta = \beta_R + \beta_L$. Assuming weak coupling between left and right sectors the emission spectrum is expected to be proportional to the product of the left and right occupation numbers and, specifically, their characteristic statistical occupation factors. For this to be possible while, at the same time, the black hole exhibits the correct overall thermality predicted by Hawking, certain gray body factors must take a very special form. The absorption cross section of a black hole is calculable in classical field theory and the gray body factor is easily extracted. Remarkably, Maldacena and Strominger found a regime where it has exactly the right form for this interpretation to be consistent [5]. For our purposes it is the precise value of the temperatures that is of interest as they lead to a test of our ideas. The temperatures are only known in the near extremal limit $\delta_i \gg 1$, where [9,10]

$$\beta_{R,L} \simeq \pi\mu^{1/2}\prod_i \sinh\delta_i\left[\left(1+\sum_i \frac{1}{\sinh^2}\right)^{1/2} \pm 1\right]. \quad (10)$$

In its region of validity Eq. (10) agrees with the prediction Eq. (9). This is particularly satisfying because the D-brane model only accounts for the temperatures when, in addition to $\delta_i \gg 1$, one boost parameter is much smaller than the other two, say, $\delta_3 \ll \delta_{1,2}$ [9,10]. Moreover, the model here provides a precise prediction for the general case: when $\delta_i \sim 1$ the expression under the square root should be augmented with terms that makes it a perfect square $\prod_i(1+1/\sinh^2\delta_i) = \prod_i \coth^2\delta_i$.

The black hole also interacts with its surroundings through absorption and emission of charged particles. We consider a particle of the species referred to by index 3. Statistical properties of charged particles are more complicated because they depend on both the energy ω and the charge q_3 of the particle. The combined effect is captured by statistical distributions with characteristic exponent $E = \beta(\omega - q_3\Phi_3)$. Potentials follow from Eqs. (1)–(3) and are given by

$$\Phi_i = -\frac{1}{\beta}\left(\frac{\partial S}{\partial Q_i}\right)_{Q_{j\neq i},M} = \tanh\delta_i. \quad (11)$$

In the string model the background charge Q_3 couples to both left and right sectors, so the effective left- and right-moving distribution functions have exponents that exhibit

intricate structure. Using the left and right entropies, respectively, and proceeding as in Eq. (11) we find

$$E_{R,L} = \pi\mu^{1/2}\left[\omega_+\prod_i \cosh\delta_i \pm \omega_-\prod_i \sinh\delta_i\right]. \quad (12)$$

where $\omega_+ = (\omega - q_3\tanh\delta_3)$ and $\omega_- = (\omega - q_3\coth\delta_3)$. Analogous exponents were inferred from the absorption cross section in the near extremal limit [10]. They are

$$E_{R,L} \simeq \pi\mu^{1/2}\prod_i \sinh\delta_i$$
$$\times \left\{\left[\omega_-^2\left(1+\frac{1}{\sinh^2\delta_1}+\frac{1}{\sinh^2\delta_2}\right)\right.\right.$$
$$\left.\left.+\frac{\omega^2-q_3^2}{\sinh^2\delta_3}\right]^{1/2} \pm \omega_-\right\}. \quad (13)$$

It is straightforward to check that only terms of order at most $\sinh^{-4}\delta_i$ for large δ_i need to be added under the square root in order that the expression becomes a perfect square, the square root can be taken, and Eq. (12) follows. Hence Eq. (12) agrees with Eq. (13) in the regime where Eq. (13) is valid. This detailed agreement between complicated functions is a highly nontrivial check on the string model. The unusual situation where the approximate expression is much more complicated than the proposed exact one suggests some unrecognized underlying simplicity.

The mass is not an independent parameter, but rather a definite function of the charges and levels given implicitly by Eqs. (1)–(3). In a complete microscopic theory this relation should be derived from arguments internal to the theory. It seems plausible that this should be possible, using dualities and Lorentz invariance. Incidentally, it should be noted that the arguments in the preceding paragraphs relied on no ''mass renormalizations'' or ''redshifts'' and indeed none are allowed by the agreement with gray body factors. Therefore it can be argued in the converse direction that the string model parametrize the low-energy interaction of the nonextremal black holes.

Up to this point the string model has been presented as an effective theory that embodies the statistical mechanics of nonextremal black holes. It should correctly capture the low-energy physics of black holes in string theory. However, the precise relation to more conventional ideas in string theory is certainly not clear. To elucidate the question consider a bound state of n_1 $D1$ branes and n_5 $D5$ branes, carrying momentum k. Strominger and Vafa [3] proposed that, when wrapped on $K3\times S_1$, the exact degeneracy of this supersymmetric state is described by level k of a superconformal σ model with target space

$$C = (K3)^{\otimes(n_1n_5+1)}/\Sigma(n_1n_5+1), \quad (14)$$

where the quotient by the symmetric group is implemented by orbifolding the product manifold. Remarkably, the central charge depends on the quantum numbers of the configuration at hand. The formula is not manifestly duality invariant, as the three charges do not appear symmetrically. An alternative approach, due to Dijkgraaf, Verlinde, and Verlinde [13,20], was motivated by ideas about second-quantized

string theory and yields manifestly duality invariant expressions. It was subsequently shown that the proposed degeneracies are in fact identical [21] and specifically the Strominger-Vafa formula is consistent with duality. The rather involved mathematics describing the space of Bogomol'nyi-Prasad-Sommerfield (BPS) states in various cases is currently being explored [22]. For our purposes we note that, for large charges, the degeneracies derive from a superconformal field theory with a more familiar, state-independent, central charge $c = 6$, but an unusual relation between the level and spacetime charges, namely, $N = \Pi_i n_i$. As explained earlier the corresponding string degeneracy accounts for the entropy of extremal black holes. The point here is the following: statistical mechanics of nonextremal black holes suggests that the generalization to the nonsupersymmetric context simply involves two such structures with the full space of states realized as the direct product. Each sector describe BPS excitations of some brane configuration wrapped on compact manifolds, not necessarily the same on the two sides. Moreover, each sector is supersymmetric and duality invariant (or covariant, according to the precise context) and the two sectors are related by a matching condition Eq. (8) (suitably generalized) that respects these features. As usual the condition on the 0 modes that relates this microscopic physics to a specific spacetime interpretation introduces moduli. Generically duality transformations do not leave moduli invariant, so the duality symmetry is spontaneously broken. In fact it is broken to a compact subgroup and this is quite welcome, as explicitly realized noncompact symmetry is unstable. The novelty in the present proposal is that the introduction of moduli also spontaneously breaks supersymmetry. As the analogous breaking of duality is no cause of concern, it is reasonable to expect that also the breaking of supersymmetry is sufficiently mild that string theory remains under full control.

It should be cautioned that the scenario in the previous paragraph has only been realized in the regime where quantum numbers are large and black hole physics is a guide. Clearly much needs to be done to make the proposal precise in the context of the full algebra of BPS states and to establish the consistency of the resulting vacua in the interacting theory. However, if this endeavor should be successful it could be quite rewarding: it would provide a precise tool for nonperturbative string theory in the absence of supersymmetry.

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