More on R-parity- and lepton-family-number-violating couplings from muon(ium) conversion, and τ and π^0 decays

Jihn E. Kim,[1,2,*] Pyungwon Ko,[3,†] and Dae-Gyu Lee[1,‡]
[1]Department of Physics, Seoul National University, Seoul 151-742, Korea
[2]Center for Theoretical Physics, Seoul National University, Seoul 151-742, Korea
[3]Department of Physics, Hong-Ik University, Seoul 121-791, Korea
(Received 23 January 1997)

We present a new class of constraints to the lepton-family-number- and R-parity-violating couplings from muonium conversion $\mu^- + {}^{48}_{22}\mathrm{Ti} \to e^- + {}^{48}_{22}\mathrm{Ti}$, a class of τ decays $\tau \to l +$ (light meson) with $l = \mu$ or e, and J/ψ and π^0 decays into a lepton pair. We find that $\mu^- + {}^{48}_{22}\mathrm{Ti} \to e^- + {}^{48}_{22}\mathrm{Ti}$ provides one of the strongest constraints along with $\Delta m_K, \Delta m_B$, $\mu \to e\gamma$, and the neutrinoless double β decay. The search for these lepton-family-number-violating decays forbidden in the standard model is clearly warranted in various low-energy experiments such as τ-charm factories and PSI, etc. [S0556-2821(97)01613-5]

PACS number(s): 11.30.Fs, 11.30.Er, 13.20.Cz, 13.20.Gd

I. INTRODUCTION

Lepton-family numbers are accidental global symmetries of the standard model (SM), and thus the electron, muon, and τ lepton numbers (denoted by L_e, L_μ, and L_τ, respectively) are separately conserved as well as the total lepton number $L_{\mathrm{tot}} = L_e + L_\mu + L_\tau$. On the contrary, this is no longer true in the minimal supersymmetric standard model (MSSM) [1]. Supersymmetry, gauge invariance, and renormalizability do not forbid the following lepton-number- and/or baryon-number-violating terms in the renormalizable superpotential [2]:

$$W_{\not R_p} = \frac{1}{2}\ \lambda_{ijk}\ L_i L_j E_k^c + \lambda'_{ijk}\ L_i Q_j D_k^c + \frac{1}{2}\ \lambda''_{ijk}\ U_i^c D_j^c D_k^c + \mu_i L_i H_2, \tag{1}$$

where the meaning of L, E^c, Q, D^c, U^c, and H_2 should be self-evident, and the indices i, j, and k refer to families. The $\mathrm{SU}(3)_c$ color and the $\mathrm{SU}(2)_L$ group indices are suppressed for simplicity, and we have $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda''_{ijk} = -\lambda''_{ikj}$. The first two terms and the fourth term in Eq. (2) are lepton-number violating, whereas the third term is baryon-number violating. It has been well known that there is a very tight constraint on $\lambda'\lambda''$ from nonobservation of proton decay [3,4].

The most popular solution to such a stringent bound is to introduce a discrete symmetry called R parity defined as

$$R_p \equiv (-1)^{3B + L_{\mathrm{tot}} + 2S}, \tag{2}$$

where B, L_{tot}, and S are the baryon number, total lepton number, and intrinsic spin of a particle, respectively. Then the ordinary particles appearing in the SM as well as the extra Higgs boson in the MSSM are R-parity even, whereas their superpartners are R-parity odd. Therefore, R-parity conservation implies that the superpartners of ordinary particles be always produced in pairs, and that the lightest supersymmetric particle (LSP) be stable. This property of LSP puts a strong constraint on the possible phenomenology at colliders. Also the LSP plays a potentially important role in cosmology as a (cold) dark matter candidate [5]. This interesting symmetry, R parity, can be introduced without any other symmetry except gauge symmetry and supersymmetry if suitable Higgs representations are chosen [6].

However, the existence of R-parity symmetry itself has not been confirmed. It is clearly worth looking for R-parity-violating processes and deriving the constraints on R-parity-violating couplings. The proton decay originated from R-parity-violating terms can be evaded by assuming a weaker condition than R-parity conservation, either $\lambda' = 0$ or $\lambda'' = 0$. The latter corresponds to the baryon-number conservation. The last term in Eq. (2) can generate neutrino masses [7], and have interesting phenomenological consequences. However, it is irrelevant to the four-fermion processes considered in this paper, and thus will be ignored from now on. In the case of lepton-number conservation $(\lambda = \lambda' = 0)$, constraints on the baryon-number-violating couplings λ''_{ijk} can be obtained from various hadronic processes [8]. In this work, we relax the R-parity conservation assuming the baryon-number conservation $\lambda'' = 0$, and derive new bounds on $\lambda^{(\prime)}\lambda^{(\prime)}$. There are many earlier papers where constraints on $\lambda^{(\prime)}\lambda^{(\prime)}$ (assuming $\lambda'' = 0$) were derived from various low-energy processes [9], including the neutrinoless double β decay [10]. Recently, Choudhury and Roy [11] assumed that $\lambda'' = 0$, and obtained constraints on lepton-number violating-terms $\lambda^{(\prime)}\lambda^{(\prime)}$, considering the neutral meson mixing, the flavor changing decays of K,B mesons, and rare three-body leptonic decays of μ and τ such as $\mu \to 3e$ and $\tau \to 3e, 3\mu, e2\mu$, or $\mu 2e$. They got quite stringent limits on some combinations of these couplings and the masses of superpartners of ordinary matter. Still, some couplings remain either unconstrained (such as $\lambda'_{322}, \lambda'_{323}$) or only weakly constrained (such as $\lambda'_{22k}, \lambda'_{13k}$ except λ'_{133}) from the

*Electronic address: jekim@phyp.snu.ac.kr
†Electronic address: pko@phyb.snu.ac.kr
‡Electronic address: leedaegy@ctp.snu.ac.kr

consideration of Ref. [11]. So far the most stringent limits have come from $\Delta m_K, \Delta m_B, K\to\mu^+\mu^-, \mu\to e\gamma$ [9,11] and the neutrinoless double β decay experiments [10], all of which yield $\lambda^{(\prime)}\lambda^{(\prime)}<10^{-6}-10^{-8}$.

In this work, we consider various low-energy processes with lepton-family-number violations (LFNV) which can be induced or affected by the λ and λ' couplings in Eq. (1). In Sec. II, we consider the muonium $(M)\to$ antimuonium $(\overline{M})$ conversion and $\mu^-+{}^{48}_{22}\mathrm{Ti}\to e^-{}^{48}_{22}\mathrm{Ti}$, and find that the latter process gives one of the most stringent limits on $\lambda\lambda^{(\prime)}$. In Sec. III, we consider a class of τ decays with LFNV, $\tau\to l+$ (light meson). Here, $l=e$ or μ, and the ''light meson'' represents a pseudoscalar (P) such as π^0, η, K^0, or a vector meson (V) such as $\rho^0, K^{*0}, \phi, \omega$. In Sec. IV, we derive constraints from J/ψ(or $\pi^0)\to\mu^\pm e^\mp$ and $\pi^0\to e^+e^-$. Then, we briefly summarize our results in Sec. V.

Before closing this section, let us write the R-parity-violating interaction Lagrangian in terms of component fields:

$$\begin{aligned}\mathcal{L}_{\mathrm{int},\not R_p}=&\lambda_{ijk}[\tilde{\nu}_{iL}\overline{e_{kR}}e_{jL}+\tilde{e}_{jL}\overline{e_{kR}}\nu_{iL}+\tilde{e}^*_{kR}\overline{(\nu_{iL})^c}e_{jL}]\\&+\lambda'_{ijk}[(\tilde{\nu}_{iL}\overline{d_{kR}}d_{jL}+\tilde{d}_{jL}\overline{d_{kR}}\nu_{iL}+\tilde{d}^*_{kR}\overline{(\nu_{iL})^c}d_{jL})\\&-V^\dagger_{jp}(\tilde{e}_{iL}\overline{d_{kR}}u_{pL}+\tilde{u}_{pL}\overline{d_{kR}}e_{iL}\\&+\tilde{d}^*_{kR}\overline{(e_{iL})^c}u_{pL})]+\mathrm{H.c.}\end{aligned}\tag{3}$$

We have taken into account the flavor-mixing effects in the up-quark sector in terms of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{jp}. The misalignment between fermion and sfermion fields will be ignored, since it is strongly constrained from the suppression of the flavor-changing neutral current (FCNC) processes. The sparticle fields in Eq. (3) are assumed to be the mass eigenstates.

Integrating out the superparticles such as sneutrinos or u-squarks, we get the effective Lagrangian involving four fermions in the SM. (In this work, we will not be concerned about the four-fermion interactions with neutrinos such as $\pi\to l\nu$.) For example, by integrating out the sneutrino fields, we get the $|\Delta S|=2$ effective Lagrangian

$$\mathcal{L}^{|\Delta S|=2}_{\mathrm{eff}}=-\sum_n\frac{\lambda'_{n21}\lambda'^*_{n12}}{m^2_{\tilde{\nu}_n}}\,\overline{d_R}s_L\,\overline{d_L}s_R,\tag{4}$$

and similarly for the $|\Delta B|=2$ effective Lagrangian. One can also get the effective Lagrangian for $q_i+\overline{q_j}\to e_k+\overline{e_l}$ by integrating out the sneutrino and the squark fields. The resulting effective Lagrangian contributes to the processes $\mu+{}^{48}_{22}\mathrm{Ti}\to e+{}^{48}_{22}\mathrm{Ti}$ and $\tau\to l+P$(or V), where $l=e$ or μ, $P=\pi^0,\eta$, or K, and $V=\rho^0,\omega,K^{*0}$, or ϕ. For $q=d$, we have [1]

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}}(d_i+\overline{d_j}\to e_k+\overline{e_l})=&-\sum_n\frac{1}{m^2_{\tilde{\nu}_{Ln}}}[\lambda'_{nij}\lambda^*_{nkl}\overline{e_{kL}}e_{lR}\overline{d_{jR}}d_{iL}\\&+\lambda_{nlk}\lambda'^*_{nji}\overline{e_{kR}}e_{lL}\overline{d_{jL}}d_{iR}]\\&+\sum_{m,n,p}\frac{V^\dagger_{np}V_{pm}}{2\,m^2_{\tilde{u}_{Lp}}}\\&\times\lambda'_{lnj}\lambda'^*_{kmi}\overline{e_{kL}}\gamma^\mu e_{lL}\overline{d_{jR}}\gamma_\mu d_{iR}.\end{aligned}\tag{5}$$

The first term comes from the sneutrino exchanges, whereas the second comes from u-squark exchanges. We have used the Fierz transformation in order to get the second term. There is another effective Lagrangian for $q_i+\overline{q_j}\to e_k+\overline{e_l}$, with the q's being up-type quarks, which can be obtained from Eq. (3) by integrating out the d-squark fields:

$$\mathcal{L}_{\mathrm{eff}}(u_i+\overline{u_j}\to e_k+\overline{e_l})$$

$$=-\sum_{m,n,p}\lambda'_{lmp}\lambda'^*_{knp}\frac{V^\dagger_{mi}V_{jn}}{m^2_{\tilde{d}_{Rp}}}\,\overline{(e_{lL})^c}u_{iL}\overline{u_{jL}}(e_{kL})^c\tag{6}$$

$$\to-\sum_{m,n,p}\lambda'_{lmp}\lambda'^*_{knp}\frac{V^\dagger_{mi}V_{jn}}{2m^2_{\tilde{d}_{Rp}}}\,\overline{e_{kL}}\gamma^\mu e_{lL}\,\overline{u_{jL}}\gamma_\mu u_{iL},\tag{7}$$

after the Fierz transformation.

II. CONSTRAINTS FROM MUON CONVERSION

A. Muonium → antimuonium conversion

Let us first consider the muonium conversion $M(\equiv\mu^+e^-)\to\overline{M}(\equiv\mu^-e^+)$. The four-lepton effective Lagrangian relevant to the muonium conversion $(\Delta L_\mu=-\Delta L_e=-2)$ can be obtained from Eq. (3) by integrating out the sneutrino fields:

$$\mathcal{L}_{\mathrm{eff}}(\mu^+e^-\to\mu^-e^+)=-\frac{\lambda_{321}\lambda^*_{312}}{m^2_{\tilde{\nu}_{L3}}}\,\overline{\mu_R}e_L\overline{\mu_L}e_R.\tag{8}$$

In Eq. (8), we have used the antisymmetry of the couplings $\lambda_{ijk}=-\lambda_{jik}$ in order to simplify the sneutrino contributions. The muonium conversion probability is usually translated into the upper limit on the hypothetical coupling $G_{M\overline{M}}$ defined as

$$\mathcal{L}(M\to\overline{M})=\frac{G_{M\overline{M}}}{\sqrt{2}}\,(\overline{\mu}e)_{V-A}(\overline{\mu}e)_{V+A}+\mathrm{H.c.}\tag{9}$$

Our effective Lagrangian, Eq. (8), is the same as Eq. (9) after the Fierz transformation, with the identification

$$\frac{G_{M\overline{M}}}{\sqrt{2}}=\frac{\lambda_{321}\lambda^*_{312}}{8m^2_{\tilde{\nu}_{L3}}}.\tag{10}$$

Therefore, the conventional limit on $G_{M\overline{M}}$ can be readily translated into the R_p-violating couplings.

[1]We do not agree with D. Choudhury and P. Roy [11], in the detailed form of the effective Lagrangian for $d_i+\overline{d_j}\to e_k+\overline{e_l}$. Compare our Eq. (5) with Eq. (7) of Ref. [11].

The muonium conversion probability depends on the external magnetic field $B_{\rm ext}$ in a nontrivial way. This subject was recently addressed in detail by a few groups [12] and we use their results in the following. From the present upper limit on the transition probability for the external magnetic field $B_{\rm ext}=1.6$ kG,

$$P_{\rm exp}(M\to\overline{M})<2.1\times10^{-9} \qquad (90\%\ \ {\rm C.L.}), \tag{11}$$

one gets the following contraint on $G_{M\overline{M}}<9.6\times10^{-3}G_F$.[2] This in turn implies that

$$|\lambda_{231}\lambda^*_{132}|<6.3\times10^{-3}\left(\frac{m_{\tilde{\nu}_{L3}}}{100\ \ {\rm GeV}}\right)^2. \tag{12}$$

This constraint on R-parity-violating λ couplings is in the same order with other constraints derived from lepton-flavor-violating τ decays such as $\tau\to3l$ or $ll'^+l'^-$ (with $l,l'=\mu$ or e) [11].

B. $\mu^-+{}^{48}_{22}$Ti$\to e^-+{}^{48}_{22}$Ti conversion

In this subsection, let us consider the $\mu^-+{}^{48}_{22}\mathrm{Ti}\to e^-+{}^{48}_{22}\mathrm{Ti}$ induced by the R-parity violating $\lambda'\times\lambda^{(\prime)}$ terms. The relevant effective Lagrangian at the parton level can be written as

$$\mathcal{L}_{\rm eff}=\frac{1}{2}\ \overline{e_L}\gamma_\alpha\mu_L\ \ [A^d_{\mu\mathrm{Ti}}\ \overline{d_R}\gamma_\alpha d_R+A^u_{\mu\mathrm{Ti}}\ \overline{u_L}\gamma_\alpha u_L] + \frac{1}{2}[S^{d,1}_{\mu\mathrm{Ti}}\ \overline{e_L}\mu_R\ \overline{d_R}d_L+S^{d,2}_{\mu\mathrm{Ti}}\ \overline{e_R}\mu_L\ \overline{d_L}d_R], \tag{13}$$

where $A^{u,d}_{\mu\mathrm{Ti}}$ and $S^d_{\mu\mathrm{Ti}}$ can be obtained from Eqs. (5) and (6) as

$$A^d_{\mu\mathrm{Ti}}=+\sum_{m,n,p}\frac{V^\dagger_{np}V_{pm}}{m^2_{\tilde{u}_{Lp}}}\ \lambda'_{2n1}\lambda'^*_{1m1},\to +\sum_n\frac{\lambda'_{2n1}\lambda'^*_{1n1}}{m^2_{\tilde{u}_{Ln}}} \qquad \text{for}\ \ V_{np}=\delta_{np}, \tag{14}$$

$$A^u_{\mu\mathrm{Ti}}=-\sum_{m,n,p}\frac{V^\dagger_{m1}V_{1n}}{m^2_{\tilde{d}_{Rp}}}\ \lambda'_{2mp}\lambda'^*_{1np}, \to -\sum_n\frac{\lambda'_{21n}\lambda'^*_{11n}}{m^2_{\tilde{d}_{Rn}}} \qquad \text{for}\ \ V_{np}=\delta_{np}, \tag{15}$$

$$S^{d,1}_{\mu\mathrm{Ti}}=-\sum_n\frac{2}{m^2_{\tilde{\nu}_{L,n}}}\ \lambda'_{n11}\lambda^*_{n12}, \tag{16}$$

$$S^{d,2}_{\mu\mathrm{Ti}}=-\sum_n\frac{2}{m^2_{\tilde{\nu}_{L,n}}}\ \lambda'^*_{n11}\lambda_{n21}. \tag{17}$$

In many supersymmetric theories with lepton-family-number violation, the $\mu^-\to e^-$ conversion on the ${}^{48}_{22}$Ti nucleus occurs through the electroweak penguin diagram $\mu^-\to e^-+\gamma^*$ (or Z^*) or through the box diagrams $\mu^-+q\to e^-+q$ (with $q=u,d$), where various superparticles run around the loop. In our case with explicit R_p violations, on the contrary, the effective Lagrangian Eq. (13) arises at the tree-level via superparticle exchanges in different channels. Therefore, the usual loop-induced $\mu^-\to e^-$ conversion on the Ti nucleus would be suppressed by $O(\alpha/16\pi^2)$ compared with the tree level contribution from the above effective Lagrangian, and thus will be neglected in this work.

In order to evaluate the matrix element of the effective Lagrangian Eq. (13) between the nucleus as well as the initial and final leptons, we assume that the nuclear recoil is negligible, and the nucleus and the initial muon can be treated as nonrelativistic. Under these assumptions, the vector current and the scalar density of the nucleus contribute to the coherent conversion process, basically counting the number of protons and neutrons inside the target nucleus. Then, the conversion rate for the $\mu^-+{}^{48}_{22}\mathrm{Ti}\to e^-+{}^{48}_{22}\mathrm{Ti}$ is given by

$$\Gamma(\mu^-+\mathrm{Ti}\to e^-+\mathrm{Ti})=\frac{\alpha^3}{128\pi^2}\ \frac{Z^4_{\rm eff}}{Z}\times|F(q^2\simeq-m^2_\mu)|^2m^5_\mu\ \ |Q^{\rm eff}_{\mu\mathrm{Ti}}|^2, \tag{18}$$

where

$$|Q^{\rm eff}_{\mu\mathrm{Ti}}|^2=[(Z+2N)(A^d_{\mu\mathrm{Ti}}+S^{d,1}_{\mu\mathrm{Ti}}+S^{d,2}_{\mu\mathrm{Ti}})+A^u_{\mu\mathrm{Ti}}(2Z+N)]^2 +[(Z+2N)(A^d_{\mu\mathrm{Ti}}+S^{d,1}_{\mu\mathrm{Ti}}-S^{d,2}_{\mu\mathrm{Ti}})+A^u_{\mu\mathrm{Ti}}(2Z+N)]^2. \tag{19}$$

For ${}^{48}_{22}$Ti, one has $Z=22$, $N=26$, $Z_{\rm eff}=17.6$, and $F(q^2\simeq-m^2_\mu)\simeq0.54$ [15].

The experimental limit for the search for $\mu^-+{}^{48}_{22}\mathrm{Ti}\to e^-+{}^{48}_{22}\mathrm{Ti}$ is commonly given in terms of the above conversion rate divided by the muon capture rate in ${}^{48}_{22}$Ti, $\Gamma(\mu\ \ \text{capture in}\ \ {}^{48}_{22}\mathrm{Ti})=(2.590\pm0.012)\times10^6/\text{sec}$ [16]:

$$\frac{\Gamma(\mu+\mathrm{Ti}\to e+\mathrm{Ti})}{\Gamma(\mu\ \ \text{capture in}\ \ {}^{48}_{22}\mathrm{Ti})}<4.3\times10^{-12}. \tag{20}$$

This puts a strong contraint on $|Q_{\mu\mathrm{Ti}}|^2$,

$$|Q^{\rm eff}_{\mu\mathrm{Ti}}|<1.2\times10^{-9}\ \ \mathrm{GeV}^{-2}, \tag{21}$$

which can be translated into

$$\left|(A^d_{\mu\mathrm{Ti}}+S^{d,1}_{\mu\mathrm{Ti}}\pm S^{d,2}_{\mu\mathrm{Ti}})+\frac{70}{74}\ A^u_{\mu\mathrm{Ti}}\right|<1.6\times10^{-7}, \tag{22}$$

for $m_{\tilde{u}_L}=m_{\tilde{d}_R}=m_{\tilde{\nu}_L}=100$ GeV. This is a new strong constraint which was not considered before to our knowledge.

[2]This type of interaction also arises in theories with dilepton-gauge bosons $(Y^\pm,Y^{\pm\pm})$ [13], such as the $\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_X$ (3-3-1) model considered by Frampton and Ng [14]. This limit on the coupling $G_{M\overline{M}}$ is translated into a lower bound on the mass of the dilepton-gauge boson $M^2_{Y^{\pm\pm}}>(690\ \ \mathrm{GeV})^2$.

This is as good as those obtained from Δm_K, Δm_B [11], or the neutrinoless double β decay experiments [10]. It also constrains different combinations of R_p-violating couplings.

III. CONSTRAINTS FROM τ DECAYS

Now, we consider lepton-family-number-violating (LFNV) τ decays into a meson and a lepton $\tau \to l+P$ (or V), where $l=e$ or μ, $P=\pi^0,\eta$, or K^0, and $V=\rho^0,\omega,K^*$, or ϕ. The relevant effective Lagrangian has been already constructed in the previous subsection, Eqs. (5) and (7). The matrix element for $\langle l,P(\text{or } V)|\mathcal{L}_{\text{eff}}|\tau\rangle$ can be evaluated using PCAC (partial conservation of axial-vector current) conditions:

$$\langle \pi^0(p)|\overline{u}\gamma_\mu\gamma_5 u(0)|0\rangle = i f_\pi p_\mu = -\langle \pi^0(p)|\overline{d}\gamma_\mu\gamma_5 d(0)|0\rangle,$$

$$\langle \eta(p)|\overline{u}\gamma_\mu\gamma_5 u(0)|0\rangle = \frac{i f_\pi}{\sqrt{3}}\, p_\mu = \langle \eta(p)|\overline{u}\gamma_\mu\gamma_5 u(0)|0\rangle, \tag{23}$$

$$\langle \eta(p)|\overline{s}\gamma_\mu\gamma_5 s(0)|0\rangle = -\frac{2 i f_\pi}{\sqrt{3}}\, p_\mu,$$

$$\langle K(p)|\overline{d}\gamma_\mu\gamma_5 s(0)|0\rangle = i\sqrt{2} f_K p_\mu,$$

and, using CVC (conserved vector current) conditions,

$$\langle \rho^0(p,\epsilon)|\overline{u}\gamma_\mu u(0)|0\rangle = m_\rho f_\rho \epsilon_\mu{}^* = -\langle \rho^0(p,\epsilon)|\overline{d}\gamma_\mu d(0)|0\rangle,$$

$$\langle \omega^0(p,\epsilon)|\overline{u}\gamma_\mu u(0)|0\rangle = m_\omega f_\omega \epsilon_\mu{}^* = \langle \omega^0(p,\epsilon)|\overline{d}\gamma_\mu d(0)|0\rangle, \tag{24}$$

$$\langle \phi(p,\epsilon)|\overline{s}\gamma_\mu s(0)|0\rangle = m_\phi f_\phi \epsilon_\mu{}^*,$$

$$\langle K^*(p,\epsilon)|\overline{d}\gamma_\mu s(0)|0\rangle = m_{K^*} f_{K^*} \epsilon_\mu{}^*.$$

The pseudoscalar meson decay constants $f_\pi = 93$ MeV and $f_K = 113$ MeV are extracted from the leptonic decay of each pseudoscalar meson, whereas the vector meson decay constants $f_\rho = 153$ MeV, $f_\omega = 138$ MeV, $f_\phi = 237$ MeV, and $f_{K^*} = 224$ MeV can be obtained from $\rho^0(\text{or } \omega,\phi) \to e^+e^-$ and $\tau \to K^* + \nu_\tau$.

Let us consider $\tau(k,s) \to e_k(k',s') + V(p,\epsilon)$. From the effective Lagrangians Eqs. (5)–(7), one gets the corresponding amplitude as

$$\mathcal{M}(\tau \to e_k + V) = \frac{1}{8}\, A_V f_V m_V \epsilon^*_\mu\, \overline{e_k}\gamma^\mu(1-\gamma_5)\tau, \tag{25}$$

where

$$A_V = A_{V=(u_j\overline{u_i})} + A_{V=(d_j\overline{d_i})}, \tag{26}$$

$$A_{V=(u_j\overline{u_i})} = -\sum_{m,n,p} \frac{V^\dagger_{mi} V_{jn}}{m^2_{\tilde{d}_{Rp}}}\, \lambda'_{3mp}\lambda'^*_{knp} \tag{27}$$

$$\to -\sum_p \frac{\lambda'_{3ip}\lambda'^*_{kjp}}{m^2_{\tilde{d}_{Rp}}} \qquad \text{for } K_{np} = \delta_{np}, \tag{28}$$

$$A_{V=(d_j\overline{d_i})} = \sum_{m,n,p} \frac{V^\dagger_{np} V_{pm}}{m^2_{\tilde{u}_{Lp}}}\, \lambda'_{3nj}\lambda'^*_{kmi} \tag{29}$$

$$\to \sum_p \frac{\lambda'_{3pi}\lambda'^*_{kpj}}{m^2_{\tilde{u}_{Lp}}} \qquad \text{for } K_{np} = \delta_{np}. \tag{30}$$

TABLE I. Constraints from $\tau \to l+V$ with $l=e$ or μ, and $V=\rho^0$, K^* or ϕ. In the table we use the notation $u_p \equiv (100\ \text{GeV}/m_{\tilde{u}_{Lp}})^2$ and $d_p \equiv (100\ \text{GeV}/m_{\tilde{d}_{Rp}})^2$. Data are taken from the recent results reported by the CLEO Collaborations, Ref. [17]. Sum over $m,n,p=1,2,3$ is to be understood.

Final state	B_{expt}	Combinations constrained	Constraint
$e\rho^0$	$<4.2\times10^{-6}$	$V^\dagger_{np}V_{pm}\lambda'_{3n1}\lambda'^*_{1m1}u_p$	$<3.5\times10^{-3}$
		$V^\dagger_{n1}V_{1m}\lambda'_{3np}\lambda'^*_{1mp}d_p$	$<3.5\times10^{-3}$
eK^{*0}	$<6.3\times10^{-6}$	$V^\dagger_{np}V_{pm}\lambda'_{3n1}\lambda'^*_{1m2}u_p$	$<3.0\times10^{-3}$
$\mu\rho^0$	$<5.7\times10^{-6}$	$V^\dagger_{np}V_{pm}\lambda'_{3n1}\lambda'^*_{2m1}u_p$	$<4.2\times10^{-3}$
		$V^\dagger_{n1}V_{1m}\lambda'_{3np}\lambda'^*_{2mp}d_p$	$<4.2\times10^{-3}$
μK^{*0}	$<9.4\times10^{-6}$	$V^\dagger_{np}V_{pm}\lambda'_{3n1}\lambda'^*_{2m2}u_p$	$<3.8\times10^{-3}$

The decay rate for the $\tau \to e_k + V$ is given by

$$\Gamma(\tau \to e_k + V) = \frac{1}{128\pi}\,|A_V|^2 f_V^2\,[2k\cdot pk'\cdot p + m_V^2 k\cdot k']\,\frac{|\vec{p}|}{m_\tau^2}. \tag{31}$$

The limit on the A_V is given in Table I. Note that these limits in Table I are comparable to those from $\tau \to 3e, e\mu^+\mu^-, 3\mu$, and so on. However, these two classes of tau decays constrain different combinations of λ and λ' from $\tau \to 3e, e\mu^+\mu^-$, or 3μ. Therefore, it is worthwhile to consider $\tau \to e_k + V$, in addition to $\tau \to e_k + \gamma$ and $\tau \to ll'^+l'^-$, as an independent probe of lepton-family-number violation beyond SM. These decays are also easier to study experimentally compared with another decays $\tau \to e_k + P$ to be considered below, since one can tag the dilepton emerging from the decay of a vector meson V (except for K^{*0} which decays mainly into $K\pi$).

Next, consider $\tau(k,s) \to e_k(k',s') + P(p)$. There are two contributions: one from the axial vector current of quarks, and the other from the pseudoscalar density of quarks. Using the equations of motion for the lepton spinors and $p=k-k'$, one can transform the former to the latter:

$$p^\mu\,\overline{l}(k',s')\gamma_\mu(1-\gamma_5)\tau(k,s) \to \overline{l}(-m_l(1-\gamma_5) + m_\tau(1+\gamma_5))\tau \simeq m_\tau\,\overline{l}(1+\gamma_5)\tau, \tag{32}$$

ignoring the final lepton mass. Therefore, the corresponding amplitude derived from the effective Lagrangians, (5) and (6), can be written as

TABLE II. Constraints from $\tau \to l + P$ with $l = e$ or μ, and P being a light pseudoscalar meson. In the table we use the notations, $n_n \equiv (100\ \text{GeV}/m_{\tilde{\nu}_{Ln}})^2$, $u_p \equiv (100\ \text{GeV}/m_{\tilde{u}_{Lp}})^2$, and $d_p \equiv (100\ \text{GeV}/m_{\tilde{d}_{Rp}})^2$. Data are taken from Ref. [[15]]. Sum over m,n,p is to be understood.

Final state	B_{expt}	Combinations constrained	Constraint
$e\pi^0$	$<1.4\times10^{-4}$	$\lambda_{n31}\lambda'^{*}_{n11}n_n, \lambda^{*}_{n13}\lambda'_{n11}n_n$	$<6.4\times10^{-2}$
		$V^{\dagger}_{np}V_{pm}\lambda'_{3n1}\lambda'^{*}_{1m1}u_p, V^{\dagger}_{m1}V_{n1}\lambda'_{3mp}\lambda'^{*}_{1np}d_p$	$<6.6\times10^{-2}$
$\mu\pi^0$	$<4.4\times10^{-5}$	$\lambda_{n32}\lambda'^{*}_{n11}n_n, \lambda^{*}_{n23}\lambda'_{n11}n_n$	$<3.6\times10^{-2}$
		$V^{\dagger}_{np}V_{pm}\lambda'_{3n1}\lambda'^{*}_{2m1}u_p, V^{\dagger}_{m1}V_{n1}\lambda'_{3mp}\lambda'^{*}_{2np}d_p$	$<3.7\times10^{-2}$
eK^0	$<1.3\times10^{-3}$	$\lambda_{n31}\lambda'^{*}_{n12}n_n, \lambda^{*}_{n13}\lambda'_{n21}n_n$	$<8.5\times10^{-2}$
		$V^{\dagger}_{np}V_{pm}\lambda'_{3n1}\lambda'^{*}_{1m2}u_p$	$<4.0\times10^{-1}$
μK^0	$<1.0\times10^{-3}$	$\lambda_{n32}\lambda'^{*}_{n12}n_n, \lambda^{*}_{n23}\lambda'_{n21}n_n$	$<7.6\times10^{-2}$
		$V^{\dagger}_{np}V_{pm}\lambda'_{3n1}\lambda'^{*}_{2m2}u_p$	$<3.6\times10^{-1}$
$e\eta$	$<6.3\times10^{-5}$	$\lambda_{n31}\lambda'^{*}_{n11}n_n, \lambda^{*}_{n13}\lambda'_{n11}n_n$	$<4.5\times10^{-3}$
		$\lambda_{n31}\lambda'^{*}_{n22}n_n, \lambda^{*}_{n13}\lambda'_{n22}n_n$	$<4.5\times10^{-2}$
		$V^{\dagger}_{np}V_{pm}(\lambda'_{3n1}\lambda'^{*}_{1m1}-2\lambda'_{3n2}\lambda'^{*}_{1m2})u_p$	$<7.8\times10^{-2}$
		$V^{\dagger}_{m1}V_{1n}\lambda^{\dagger}_{3mp}\lambda'^{*}_{1np}d_p$	$<7.8\times10^{-2}$
$\mu\eta$	$<7.3\times10^{-5}$	$\lambda_{n32}\lambda'^{*}_{n11}n_n, \lambda^{*}_{n23}\lambda'_{n11}n_n$	$<4.8\times10^{-3}$
		$\lambda_{n32}\lambda'^{*}_{n22}n_n, \lambda^{*}_{n23}\lambda'_{n22}n_n$	$<4.8\times10^{-2}$
		$V^{\dagger}_{np}V_{pm}(\lambda'_{3n1}\lambda'^{*}_{2m1}-2\lambda'_{3n2}\lambda'^{*}_{2m2})u_p$	$<8.2\times10^{-2}$
		$V^{\dagger}_{m1}V_{1n}\lambda^{\dagger}_{3mp}\lambda'^{*}_{2np}d_p$	$<8.2\times10^{-2}$

$$\mathcal{M}(\tau \to e_k + P) = \overline{e_k}(A^P_L P_L + A^P_R P_R)\tau, \tag{33}$$

which leads to the decay rate

$$\Gamma(\tau \to e_k + P) = \frac{m_\tau}{64\pi}\,[|A^P_L + A^P_R|^2 + |A^P_L - A^P_R|^2], \tag{34}$$

where $P(=\pi^0, \eta, K)$ denotes the final pseudoscalar meson. We have ignored the final lepton mass compared to the τ mass. The relevant $A^P_{L,R}$'s for $P = \pi^0, \eta, K^0$ are given by the expressions

$$A^{\pi^0}_L = \sum_n \frac{\lambda'^{*}_{n11}\lambda_{n3k}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{f_\pi m^2_\pi}{2m_d}, \tag{35}$$

$$A^{\pi^0}_R = -\sum_n \frac{\lambda'_{n11}\lambda^{*}_{nk3}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{f_\pi m^2_\pi}{2m_d} - \sum_{m,n,p} \frac{V^{\dagger}_{np}V_{pm}}{4m^2_{\tilde{u}_{Lp}}}\, m_\tau f_\pi \lambda'_{3n1}\lambda'^{*}_{km1} + \sum_{m,n,p} \frac{V^{\dagger}_{m1}V_{1n}}{4m^2_{\tilde{d}_{Rp}}}\, m_\tau f_\pi\ \lambda'_{3mp}\lambda'^{*}_{knp}, \tag{36}$$

$$A^{\eta}_L = -\sum_n \frac{\lambda'^{*}_{n11}\lambda_{n3k}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{f_\pi m^2_\eta}{\sqrt{3}\times 2m_d} + \sum_n \frac{\lambda'^{*}_{n22}\lambda_{n3k}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{2f_\pi m^2_\eta}{\sqrt{3}\times 2m_s}, \tag{37}$$

$$A^{\eta}_R = +\sum_n \frac{\lambda'_{n11}\lambda^{*}_{nk3}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{f_\pi m^2_\eta}{\sqrt{3}\times 2m_d} - \sum_n \frac{\lambda'_{n22}\lambda^{*}_{nk3}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{2f_\pi m^2_\eta}{\sqrt{3}\times 2m_s} + \sum_{m,n,p} \frac{V^{\dagger}_{np}V_{pm}}{4m^2_{\tilde{u}_{Lp}}}\,(\lambda'_{3n1}\lambda'^{*}_{km1} - 2\lambda'_{3n2}\lambda'^{*}_{km2})\,\frac{f_\pi m_\tau}{\sqrt{3}} + \sum_{m,n,p} \frac{V^{\dagger}_{m1}V_{1n}}{4m^2_{\tilde{d}_{Rp}}}\,\lambda'_{3mp}\lambda'^{*}_{knp}\,\frac{f_\pi m_\tau}{\sqrt{3}}, \tag{38}$$

$$A^{K^0}_L = -\sum_n \frac{\lambda_{n3k}\lambda'^{*}_{n12}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{\sqrt{2}f_K m^2_K}{(m_d + m_s)}, \tag{39}$$

$$A^{K^0}_R = \sum_n \frac{\lambda^{*}_{nk3}\lambda'_{n21}}{2m^2_{\tilde{\nu}_{Ln}}}\,\frac{\sqrt{2}f_K m^2_K}{(m_d + m_s)} + \sum_{m,n,p} \frac{V^{\dagger}_{np}V_{pm}}{4m^2_{\tilde{u}_{Lp}}}\,\lambda'_{3n1}\lambda'^{*}_{km2}\ (\sqrt{2}f_K m_K). \tag{40}$$

In numerical analyses, we use the current quark masses

$$m_u = 5\ \text{MeV}, \qquad m_d = 10\ \text{MeV}, \qquad m_s = 200\ \text{MeV}. \tag{41}$$

Comparing with the experimental upper limits on these SM-forbidden decays, we get the constraints shown in Table I. For the superparticle masses of 100 GeV, the constraints are all order of $10^{-2} - 10^{-3}$, which are in the similar range as the constraints obtained from the $\tau \to e_k + V$. (See Table II.)

IV. CONSTRAINTS FROM J/ψ AND π^0 DECAYS

Finally, let us consider $J/\psi \to e_i \overline{e_j}$ with $i \neq j$, and similar decays for Υ and π^0. Since the J/ψ and Υ mainly decay via strong and electromagnetic interactions, these particles would give weaker constraints on LFNV couplings compared to the weak transitions or decays we have considered before. However, in these decays, the relevant LFNV couplings from the effective Lagrangian (7) differ from those in the others, and are simpler than those in the $\tau \to l + P$. Normalizing the decay rate for the $J/\psi \to e_i \overline{e_j}$ (with $i \neq j$) to the SM process $J/\psi \to e^+ e^-$, we get (summing over two charged modes)

$$\frac{\Gamma(J/\psi \to e_i \overline{e_j} + \overline{e_i} e_j)}{\Gamma(J/\psi \to e^+ e^-)} = \frac{9}{64} \frac{m_\psi^4}{(4\pi\alpha)^2} |A_{J/\psi}^{(ij)}|^2, \tag{42}$$

with

$$A_{J/\psi}^{(ij)} = \sum_{m,n,p} \frac{V_{m2}^\dagger V_{2n}}{m_{\tilde{d}_{Rp}}^2} \lambda_{imp}'^* \lambda_{jnp}'. \tag{43}$$

We have neglected the final lepton masses. For the upsilon decays into $e_i \overline{e_j}$, one can replace m_ψ by m_Υ, and multiply the above ratio by a factor of 4.[3]

Unfortunately, there is no published upper limit on J/ψ[or $\Upsilon(1S)$]$\to e\mu, \mu\tau$, or $e\tau$. For example, the upper limit on the ratio

$$\frac{\Gamma(J/\psi \to e^\pm \mu^\mp)}{\Gamma(J/\psi \to e^+ e^-)} < 10^{-4} \tag{44}$$

would imply $|A_{J/\psi}^{(12)}| < 7.2$ for $m_{\tilde{d}_R} = 100$ GeV. As one might expect, this limit is not that stringent, since J/ψ (and Υ) decays mainly through strong and electromagnetic annihilations, and not through weak annihilation. However, one may still try to search for the LFNV J/ψ decays at τ-charm factories. Note that $\lambda_{22p}' \lambda_{12p}'$ has never been constrained before.

Similarly, the effective Lagrangians, (5) and (7) contribute to the decays $\pi^0 \to e^+ e^-$ and $\eta \to l^+ l^-$ as well as the LFNV decay $\pi^0 \to e^\pm \mu^\mp$. In these decays, the (pseudo)scalar $\times$ (pseudo)scalar couplings in Eq. (5) give the largest contributions because they are enhanced by a factor of m_π^2/m_d compared with m_π or m_μ, if the couplings and the masses of the superparticles are in the same order of magnitude. So we ignore the contributions from $(V-A)$ quark currents in Eqs. (5)–(7). In this approximation, the amplitude for $\pi^0 \to e_i \overline{e_j}$ becomes

$$\mathcal{M}(\pi^0 \to e_i \overline{e_j}) = A_{P,L} \, \overline{e_{j,L}} e_{i,R} + A_{P,R} \, \overline{e_{j,R}} e_{i,L}, \tag{45}$$

with

$$A_{P,L} = \frac{f_\pi m_\pi^2}{8 m_d} \sum_n \frac{\lambda_{n11}' \lambda_{nij}^*}{m_{\tilde{\nu}_{Ln}}^2}, \tag{46}$$

$$A_{P,R} = -\frac{f_\pi m_\pi^2}{8 m_d} \sum_n \frac{\lambda_{n11}'^* \lambda_{nji}}{m_{\tilde{\nu}_{Ln}}^2}. \tag{47}$$

The resulting decay rate is (after summing over the charge conjugate state)

$$\Gamma(\pi^0 \to \mu^\pm e^\mp) = \frac{m_\pi}{16\pi} [|A_{P,L} + A_{P,R}|^2 + |A_{P,L} - A_{P,R}|^2] \times \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2. \tag{48}$$

For the LFNV decays $\pi^0 \to e^\pm \mu^\mp$, there is a tight upper bound on the branching ratio 1.72×10^{-8}. This implies that (for $m_{\nu_{\tilde{L}}n} = 100$ GeV)

$$\left| \sum_n (\lambda_{n11}' \lambda_{n12}^* \pm \lambda_{n11}'^* \lambda_{n21}) \right| < 0.14. \tag{49}$$

For the lepton number conserving decay $\pi^0 \to e^+ e^-$, the branching ratio is known to be

$$B(\pi^0 \to e^+ e^-) = (7.5 \pm 2.0) \times 10^{-8}, \tag{50}$$

which is dominated by the so-called unitarity bound coming from $\pi^0 \to \gamma\gamma \to e^+ e^-$. This unitarity bound is calculable, and known to be [18]

$$\frac{\Gamma_{\text{unit}}(\pi^0 \to e^+ e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{\alpha^2}{2\beta_e} \frac{m_e^2}{m_\pi^2} \left[\ln\left(\frac{1+\beta_e}{1-\beta_e}\right) \right]^2 = 4.75 \times 10^{-8}, \tag{51}$$

with $\beta_e = \sqrt{1 - 4m_e^2/m_{\pi^0}^2}$. Extracting this unitary bound from the experimental branching ratio and assuming no large contributions from the dispersive part of the two-photon contributions ($\mathrm{Re}A_{\gamma\gamma}$) or large cancellation between $\mathrm{Re}A_{\gamma\gamma}$ and the R_p-violating contributions, we can put the (90 % C.L.) limit on the contribution from the R_p-violating interactions in Eq. (5):

$$\left| \sum_n (\lambda_{n11}' \lambda_{n11}^* \pm \lambda_{n11}'^* \lambda_{n11}) \right| < 0.15 \tag{52}$$

for $m_{\tilde{\nu}_L} = 100$ GeV.

V. CONCLUSIONS

In conclusion, we considered several different LFNV processes: (i) the muonium conversion, (ii) $\mu^- + {}^{48}_{22}\mathrm{Ti} \to e^- + {}^{48}_{22}\mathrm{Ti}$, (iii) τ decays into a lepton and a meson, $\tau \to l + P$(or V), and (iv) $J/\psi(\Upsilon, \pi^0) \to e_i \overline{e_k}$. From these processes, we got constraints on the R-parity-violating couplings and the superparticle masses. Some of these constraints are new and/or stronger than constraints from other popular processes such as $\tau \to \mu(e) + \gamma$, $\tau \to l l'^+ l'^-$, etc. We got one of the strongest constraints, Eq. (22), from $\mu^- \to e^-$ conversion on the ${}^{48}_{22}$Ti nucleus. This originates from the fact that the R-parity-violating terms can give tree-level contributions to the processes considered in this work. In many supersymmetric models with R-parity conservation, on the other hand, LFNV processes usually arise from one-

[3]Note that $Q_b = -|e|/3 = -Q_c/2$.

loop Feynman diagrams, so that the most important one is often the electromagnetic penguin ($\mu^- \to e^- \gamma$) contribution to $\mu^- + {}^{48}_{22}\mathrm{Ti} \to e^- + {}^{48}_{22}\mathrm{Ti}$. Therefore, dedicated searches for these decays at PSI, τ-charm factories, and other facilities are clearly warranted, and are very important, because they will provide us with hints of new physics beyond the SM via lepton-flavor violations from any origin, including those from supersymmetric models with or without R-parity conservations.

ACKNOWLEDGMENTS

P.K. is grateful to Dr. E. J. Chun for discussions on R-parity violations. This work was supported in part by KOSEF through CTP at Seoul National University, by the Basic Science Research Program, Ministry of Education Project No. BSRI–96–2418, Korea-Nagoya Exchange Program, and by the Distinguished Scholar Exchange Program of Korea Research Foundation.

[1] H. P. Nilles, Phys. Rep. **110**, 1 (1984); H. E. Haber and G. L. Kane, *ibid.* **117**, 75 (1985).

[2] S. Weinberg, Phys. Rev. D **26**, 287 (1982); N. Sakai and T. Yanagida, Nucl. Phys. **B197**, 533 (1982).

[3] I. Hinchliffe and T. Kaeding, Phys. Rev. D **47**, 279 (1993). They obtained $|\lambda' \lambda''| \leqslant 10^{-24}$ for squark masses about 1 TeV; A. Y. Smirnov and F. Vissani, Phys. Lett. B **380**, 317 (1996).

[4] Most works assume that the LSP mass is around ~ 100 GeV. For other scenarios with a very light gravitino (G) or axino ($\tilde{a}$), see K. Choi, E. J. Chun, and J. S. Lee, Phys. Rev. D **55**, 392 (1997). These authors obtain $\lambda''_{112} \sim 10^{-15}$, independent of λ' from $p \to K^+ G$ (or $K^+ \tilde{a}$).

[5] For a recent review and references, see G. Jungman, M. Kaminokowski, and K. Griest, Phys. Rep. **267**, 195 (1996).

[6] D.-G. Lee and R. N. Mohapatra, Phys. Rev. D **51**, 1353 (1995).

[7] L. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).

[8] See, for example, J. L. Goity and M. Sher, Phys. Lett. B **346**, 69 (1995).

[9] V. Barger, G. F. Giudice, and T. Han, Phys. Rev. D **40**, 2987 (1989); F. Vissani, Report No. hep-ph/9602395, 1996 (unpublished); G. Bhattacharyya, in *Supersymmetry '96 Theoretical Perspectives and Experimental Outlook*, Proceedings of the Conference, College Park, Maryland, edited by R. Mohapatra and A. Rasin [Nucl. Phys. B (Proc. Suppl.) **52A**, 83 (1997)], hep-ph/9608415, and references therein; Ji-Ho Jang, J. K. Kim, and J. S. Lee, Phys. Rev. D **55**, 7296 (1997).

[10] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **75**, 2276 (1995).

[11] D. Choudhury and P. Roy, Phys. Lett. B **378**, 153 (1996).

[12] W. S. Hou, in *Mu$^+$MU$^-$ Colliders*, Proceedings of the 3rd Symposium, San Francisco, California, 1995, edited by D. Cline [Nucl. Phys. B (Proc. Suppl.) **51A**, 40 (1996)], hep-ph/9605204, and references therein.

[13] H. Fujii, S. Nakamura, and K. Sasaki, Phys. Lett. B **299**, 342 (1993); P. Ko, Nuovo Cimento A **107**, 809 (1994).

[14] P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); P. H. Frampton and D. Ng, Phys. Rev. D **45**, 4240 (1992).

[15] H. C. Chiang, E. Oset, T. S. Kosmas, A. Faessler, and J. D. Vergados, Nucl. Phys. **A559**, 526 (1993), and references therein.

[16] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[17] CLEO Collaborations, Phys. Rev. Lett. **73**, 1890 (1994).

[18] P. Ko, Phys. Rev. D **45**, 174 (1992), and references therein.