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New scenario for the early evolution of the Universe

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We propose that the Universe was created from ''nothing'' with a relatively small number of particles and quickly relaxed to a quasiequilibrium state at the Planck parameters. The classic cosmological solution for this Universe with a Λ term has two branches divided by a gap. The quantum process of tunneling between the cosmological solution branches and kinetics of the second order relativistic phase transition in a supersymmetric SU(5) model on GUT scale are investigated using numerical methods. The Einstein equations are solved together with the equations of relaxation kinetics. Another quantum geometrodynamics process (the bounce from a singularity) and the Wheeler-DeWitt equation are also investigated. The computer experiments show that because of the rapid character of the relaxation processes and the absence in the inflaton potential of peculiarities that are able to delay the system in the overcooled phase, the usual type of inflation regime is not realized. For the formation of the observed number of particles a model of a slowly swelling Universe as the result of the multiple reproduction of cosmological cycles arises naturally. [S0556-2821(97)50110-X]

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The inflation model and its basic modifications [1] are very attractive and explain many cosmological problems. Currently it is the cosmological paradigm [2]. The standard conception of first order cosmological relativistic phase transition (RPT) from a strong overcooled highly symmetric (HS) phase is well developed [3]. An inflaton potential with quite specific properties is necessary to realize this conception (as a rule, this potential must have a wide flat part). It should also be noted that the problem of the formation of the observed number of particles has not been investigated in detail using computer methods. Some ideas about this have been put forward recently [4]. The case when the inflaton potential does not possess specific properties, and a Universe with a Λ term is created from ''nothing,'' and RPT is close to second order is discussed in this article.

We propose the following.

(I) The plasma and vacuum of the Universe, created from ''nothing,'' after relaxation processes taking place near the Planck parameters (see [5]), are in a quasiequilibrium state. In our opinion, the superearly Universe was created from ''nothing'' in an anisotropic state (for example IX type of Bianchi) with some number of particles and with some nonequilibrium state of the vacuum condensate.

(II) The topology of the Universe is closed. Only such a Universe can be created from ''nothing'' (the local properties of this Universe approach to the local properties of a flat Universe if the cosmological scenario solves the problem of flatness/entropy).

(III) After going out of the Universe from a singularity the initial number of particles N_o is large in comparison with unity ($N_0 \sim 10^4 - 10^6$), but it is small in comparison with the number of particles in the observed Universe ($N_{\rm obs} \sim 10^{88}$).

(IV) RPT on the GUT scale ($T \sim 10^{16}$ GeV), which is close to the Planck scale, is not first order RPT; it is second order RPT, for which the generation of a new phase occurs continuously [6].

One possible series of RPT in the early Universe for symmetry breaking is

$$G \Rightarrow [\mathrm{SU}(5)]_{\rm SUSY} \Rightarrow [\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)]_{\rm SUSY}$$
$$\Rightarrow \mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3) \Rightarrow \mathrm{U}(1)\times\mathrm{SU}(3) \Rightarrow \mathrm{U}(1).$$

Here G is the local supersymmetry group joining all physical fields and interactions. The only trace of the first RPT is the initial Λ term (the vacuum energy density) connected with interactions of the local multidimensional supergravity. The remaining RPT are described by modern theories of elementary particles. During RPT, with cooling of the cosmological plasma, a vacuum condensate with a negative energy density is produced. This condensate has the asymptotic state equation $p_{\rm vac} = -\epsilon_{\rm vac} = \mathrm{const}$. Thus RPT series are accompanied by the generation of negative contributions to the cosmological Λ term in the Einstein equations. In accordance with observational data, after all RPT, the final Λ term is practically zero. For simplicity, we have proposed the exact compensation of the Λ term already on the SUSY GUT energy scale. Our paper considers a quantitative model of the RPT $[\mathrm{SU}(5)]_{\rm SUSY} \Rightarrow [\mathrm{U}(1)\times\mathrm{SU}(2)\times\mathrm{SU}(3)]_{\rm SUSY}$ on the scale $\sim 10^{16}$ GeV. Some particles acquire a rest mass, which is proportional to the average value of the Higgs field after spontaneous gauge symmetry breaking. The system considered consists of three subsystems: (1) a gas of massless particles, (2) a gas of massive particles interacting with the vacuum condensate, and (3) vacuum condensate. The reactions of massless and massive particles to the cosmological expansion are different. The changes of these particles' energy spectra due to redshift obey different laws. To obtain evolution equations of the nonequilibrium system we use (a) the order parameter (OP) origin as the C number average of the Higgs field, (b) a method obtaining the relaxation kinetics equations for subsystems of particles analogous to that described in [7], (c) a method for analysis of nonequilibrium relativistic systems analogous to that described in [8], and (d) an estimate of the local particles' creation rate in the variable OP field obtained using a method analogous to that described in [9].

The total system of equations for our theory involves the Einstein equations with the nonequilibrium energy-momentum tensor for a heterogeneous system, which are the nonequilibrium generalization of energy density (ϵ) and pressure (p), corresponding to the equilibrium thermodynamical functional (5),

$$\frac{3}{\kappa}\left(\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\right)=\frac{k\pi^2}{16}T^4+w(J_2+\eta^2 J_1)+\frac{1}{2g^2}\dot{\eta}^2+\frac{1}{8g^2}(\eta^2-m^2)^2\equiv\epsilon,$$

$$\frac{1}{\kappa}\left(2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{1}{a^2}\right)=-\frac{k\pi^2}{48}T^4-\frac{w}{3}J_2+\frac{\eta^2(2J_1+\eta^2 J_0)}{3(4J_2+5\eta^2 J_1+\eta^4 J_0)}wD-\frac{1}{2g^2}\dot{\eta}^2+\frac{1}{8g^2}(\eta^2-m^2)^2\equiv -p, \qquad (1)$$

the evolution equations for the dissipative function D and order parameter η,

$$\dot{D}+\left(4\frac{\dot{a}}{a}+\frac{1}{\tau}\right)D=\frac{k\pi^2T^4\left[(2J_1+\eta^2J_0)\left(\eta^2\frac{\dot{a}}{a}+\eta\dot{\eta}\right)+bT\dot{\eta}^2\right]}{k\pi^2T^4/4+w(4J_2+5\eta^2J_1+\eta^4J_0)}\left[1+\frac{D}{4J_2+5\eta^2J_1+\eta^4J_0}\right], \qquad (2)$$

$$\ddot{\eta}+\left[3\frac{\dot{a}}{a}+bwg^2T\left(1+\frac{D}{4J_2+5\eta^2J_1+\eta^4J_0}\right)\right]\dot{\eta}+\left\{wg^2\left[J_1+\frac{(2J_1+\eta^2J_0)D}{4J_2+5\eta^2J_1+\eta^4J_0}\right]+\frac{1}{2}(\eta^2-m^2)\right\}\eta=0, \qquad (3)$$

and also the entropy equation, which can be transformed to an equation for the temperature of the plasma:

$$\dot{T}+\frac{\dot{a}}{a}T=\frac{wT\left[(2J_1+\eta^2J_o)\left(\eta^2\frac{\dot{a}}{a}+\eta\dot{\eta}\right)+bT\dot{\eta}^2\right]}{(k\pi^2T^4/4)+w(4J_2+5\eta^2J_1+\eta^4J_0)}\left[1+\frac{D}{4J_2+5\eta^2J_1+\eta^4J_0}\right], \qquad (4)$$

where $\kappa\simeq(10^{19}\ \text{GeV})^{-2}$ is the Einstein constant; k is the number of boson degrees of freedom (exactly equal to the number of fermion degrees of freedom), the rest mass of which is equal to zero both in the high symmetric (HS) phase and the low symmetric (LS) phase; w is the analogous number of degrees of freedom of particles with mass η in the LS phase; g is the gauge coupling constant of particles with the vacuum condensate; $m=\text{const}$ is the limiting value of the particle mass when $T\to 0$ in the LS phase; b is a numerical coefficient of order unity; τ is the relaxation time between the subsystems of massless and massive particles (we use units in which $\hbar=c=1$):

$$J_n(T,\eta)=\frac{1}{2\pi^2}\int_0^\infty\frac{p^{2n}}{\omega}\left(\frac{1}{\exp\frac{\omega}{T}-1}+\frac{1}{\exp\frac{\omega}{T}+1}\right)dp,$$

$$n=0,1,2 \quad \omega=(p^2+\eta^2)^{1/2}$$

are characteristic integrals through which the observable magnitudes of our model are expressed.

The dissipative function D takes into account the effect of the difference of the particle subsystem energy densities from their equilibrium values corresponding to the quasi-equilibrium temperature T. Equation (3) is for a Higgs field in which two additional effects are included (the creation of particles in a variable Higgs field and the difference of particle states interacting with the Higgs field from equilibrium). We use the self-coordinated field approach, in which only the interaction of quantum particles with the classical Higgs field of the OP is taken into account. Then the Landau free energy is

$$F(T,\eta)=-\frac{k\pi^2}{48}T^4-\frac{w}{3}J_2(T,\eta)+\frac{1}{8g^2}(\eta^2-m^2)^2, \qquad (5)$$

and here the potential entering in this equation and associated with the OP is natural.

In the theory (1)–(4), the properties of the early Universe depend on the total number of particles in a closed space, the critical value of which is $N_{\rm cr}\equiv 7\xi(3)(k+w/2)^{1/4}(12g/\pi\kappa m^2)^{3/2}\simeq 5\times 10^{11}$, where $\xi(3)$ is the zeta function of Riemann. If the initial total number of particles in the HS plasma $N_0<N_{\rm cr}$, then the cosmological solution contains two branches divided by a gap (Fig. 1). Branch I is the Friedmann solution, distorted slightly by Λ term, and branch II is the de Sitter solution, which is distorted slightly by matter. Investigation of the evolution of the plasma temperature regime shows that on branch I, the minimal temperature $T_{\rm I(min)}$ (corresponding to the maximal radius $a_{\rm I(max)}$ is substantially more than the critical temperature $T_c=(4m^2/wg^2)^{1/2}$ defining the boundary of thermodynamical stability of the HS phase. On branch II, the maximal temperature $T_{\rm II(max)}$ (corresponding to the minimal radius $a_{\rm II(min)}$) is substantially less than T_c. From this hierarchy of temperatures, it follows that (1) the Universe evolving on branch I does not undergo RPT in the LS phase (it goes out from an initial singularity and come to final state in the HS phase, (2) on branch II, the Universe cannot be in the HS phase in principle. Branch II is classically prohibited. There are two reasons for this. First, branch II is separated from branch I by a classically uncrossable gap; second, branch II is thermodynamically unstable. Note that if branch II exists formally there is nonzero Λ term. When the plasma and vacuum are in the LS phase and the Λ term is equal to zero,

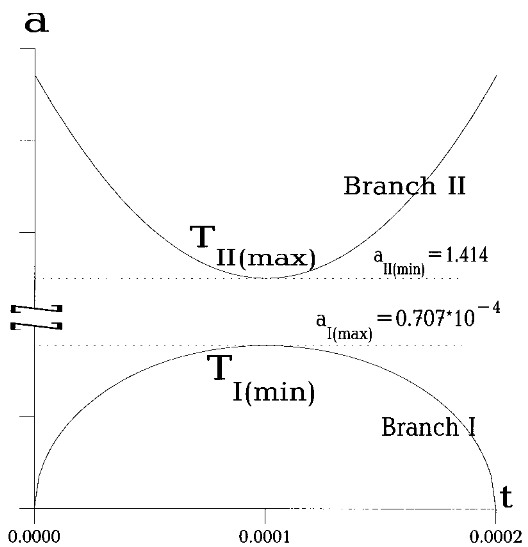

FIG. 1. The cosmological solution for a closed Universe for initial particle number $N_0 \ll N_{cr}$. The units of time and scale factor are $t_0 \simeq 3.5\times 10^{-38}$ sec, $a_0 \simeq 1.5\times 10^{-27}$ cm.

branch II does not appear at all. From the classical point of view branch II does not exist, i.e., in general, it exists only virtually as a classically unrealized version of evolution. Therefore, if a Universe created from ''nothing,'' has a particle number less than the critical number, then this Universe cannot transform into a macroscopic object containing the observed particle number.

However, the situation changes radically in quantum geometrodynamics (QGD) of a closed Universe. In this theory, there is a small, but nonzero, probability of tunneling through the gap dividing branch I and branch II of the classical solution. Let us briefly discuss the mathematical model of this phenomenon.

The problem is to construct the quantum analogue of equations (1)–(4), i.e., the Wheeler-DeWitt (WDW) equation for the Universe wave function $\psi = \psi(a,\eta)$. However, the dissipative dynamics of system (1)–(4) is not Hamilton since the formal methods of quantization do not apply to the solution of this problem. The necessity of a new quantum theory that must correlate with the second law of thermodynamics was discussed by Penrose [10]. The absence of such a theory compels us to solve this task in two steps. In the first stage, dissipative processes are not taken into account. We hope the quantum nondissipative geometrodynamics approximately reflects the properties of processes of tunneling through the barrier and bounce from the singularity. In the second stage, dissipative processes are described by the classical method based on Eqs. (1)–(4).

The quantization nondissipative system is the realization of an idea Lapchinsky and Rubakov [11] suggested to describe the presence of matter in a closed Universe using the effective potential in the WDW equation. The fact of the existence of the Hamilton bond requires a separate discussion. This bond is caused by the gauge invariance of the theory relative to time transformation. The arbitrary in time choice is removed by an addition condition put on the gauge variable. This variable is the algebraic form of metric components containing the g_{00} component. For example, if the gauge variable is $g_{00} = \lambda^2$, then the WDW equation does not depend on the choice of the additional condition $F(\lambda, a, \eta) = 0$. However, we can introduce the local conformal time transformation and gauge variable

$$dt = af(a)dt', \quad \lambda = \lambda' af(a), \tag{6}$$

where $f(a)$ is an arbitrary function. In this case, the equation does not depend on the gauge condition $F(\lambda', a, \eta) = 0$, but depends on the generator of the local conformal transformation $f(a)$. Formally, from the mathematical point of view, this dependence is caused by the nonlinear coupling of the gauge variable $\lambda = \lambda' af(a)$ with the square of the generalized impulse $p \sim \dot{a}$. Every change of the order of operators disposition that coincides with the Hermiticity property of the Hamiltonian generates additional terms in the WDW equation. These terms can be interpretated as an additional contribution to the potential of the WDW equation:

$$U_{f(a)} = \frac{\kappa}{24\pi^2}\left[\frac{1}{4f}\frac{d^2f}{da^2} - \frac{3}{16f^2}\left(\frac{df}{da}\right)^2\right]. \tag{7}$$

Thus, there is spontaneous breaking of symmetry relative to the local conformal time transformations (6) in the WDW theory for a closed isotropic Universe. The additional contribution (7) has the meaning of energy of some gravitational vacuum condensate (GVC), the production of which fixes the symmetry breaking in the whole space of the closed Universe. We propose that the GVC must secure the bounce from the singularity, i.e., prolong the time of the Universe's existence. This will take place if

$$f(a) = a^{4(S+1)}, \quad U_{f(a)} \equiv U_{S(a)} = \frac{\kappa}{24\pi^2}\frac{S(S+1)}{a^2}, \tag{8}$$

where $S = \text{const} > 0$ is a parameter of the GVC. The WDW equation in this case is written for the Universe wave function $\psi = \psi_{NS}(a,\eta)$, which depends on the two variables a, η and on the two parameters N and S:

$$-\frac{\kappa}{24\pi^2}\frac{\partial^2\psi_{NS}}{\partial a^2} + \frac{g^2}{4\pi^2 a^2}\frac{\partial^2 \psi_{NS}}{\partial \eta^2} + \left[\frac{\kappa}{24\pi^2}\frac{S(S+1)}{a^2} + \frac{6\pi^2}{\kappa}a^2 - 2\pi^2 a^4 \epsilon_N(a,\eta) - \frac{\pi^2}{4g^2}a^4(\eta^2 - m^2)^2\right]\psi_{NS} = 0, \tag{9}$$

where $\epsilon_N(,a\eta)$ is the energy density of the subsystems of particles, the mathematical form for which coincides with the thermodynamical expression. Equation (9) has a very important property: there is a solution located on the HS vacuum (near $\eta = 0$) for every value of the Universe radius. The main dependence of a wave function quasilocated on the HS vacuum on the radius of the Universe can be factorized by a separate function. This function satisfies an equation that is formally similar to the Schrödinger equation for the stationary states of some conditional ''particle'' in a potential field $U(a)$ with the shape shown in Fig. 2. The asymptotics of the potential $U(a)$ for small a are defined by the GVC energy (8). From Fig. 2 we can see that the GVC provides the quan-

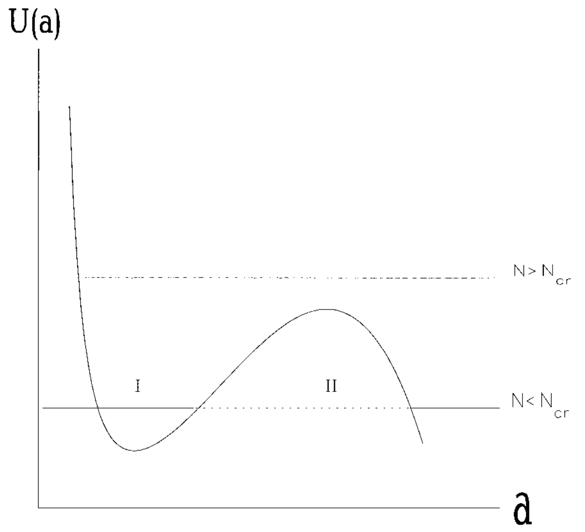

FIG. 2. The dependence of the Universe wave function factorized by the separate function $U(a)$ on scale factor a.

tum bounce from the singularity. According to the quantum bounce hypothesis, a Universe with a small number of particles oscillates quasiclassically in region I of the potential $U(a)$. Sector II of potential $U(a)$ corresponds to branch II of the classical cosmological solution (compare Fig. 1 and Fig. 2). The probability of tunneling through the barrier dividing branches I and II is exponentially small when the number of particles is small. It increases monotonously with an increase in the number of oscillation cycles in region I. If the number of oscillation cycles is large, then the cause-effect connections have been set up among all its space-time points and there is no horizon problem. We propose that after tunneling, the cause-effect connections among different points of the Universe are preserved.

Thus, our Universe was created from ''nothing'' with a small number of particles, and performed exponentially long oscillations in region I of the effective potential $U(a)$. Here

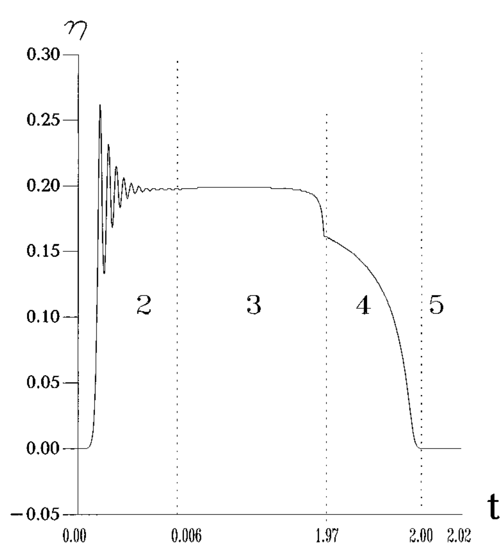

FIG. 3. The change of order parameter (η) during the evolution of the Universe. The units of time and scale factor are the same as in Fig. 1.

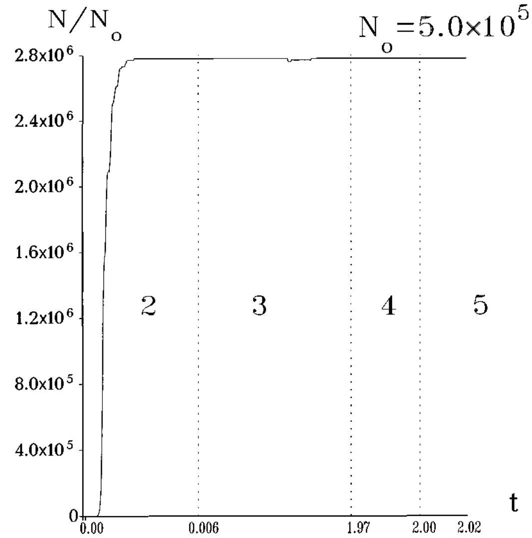

FIG. 4. The change of relative particle number (N/N_0) during the evolution of the Universe. The units of time and scale factor are the same as in Fig. 1.

the Universe existed in the HS phase. After a large number of oscillations the Universe underwent a tunneling transition in the HS thermodynamically unstable phase. If after tunneling the Universe appears in the direct vicinity at the barrier, i.e., on the left boundary of region II, then its size increases by the factor

$$\frac{a_{\mathrm{II(min)}}}{a_{\mathrm{I(max)}}}=2\left(\frac{N_{\mathrm{cr}}}{N_0}\right)^{2/3}. \qquad (10)$$

According to Eq. (10) when the initial particle number $N_0=5\times10^5$, the radius increases by a factor 2×10^4 as the result of quantum tunneling. This phenomenon can be considered as an analogue of classical inflation. After tunneling the Universe will be in a strongly nonequilibrium state. The relaxation of the nonequilibrium plasma and vacuum to the new equilibrium state corresponding to the stable LS phase occurs in the relaxation kinetics regime, and is accompanied by a sharp entropy increase. This process is described by Eqs. (1)–(4).

The classical state corresponding to the minimum curve a_{II} must appear with the greatest probability. The evolution of a Universe with the initial state $a(t_0)=a_{\mathrm{II(min)}}$ and with

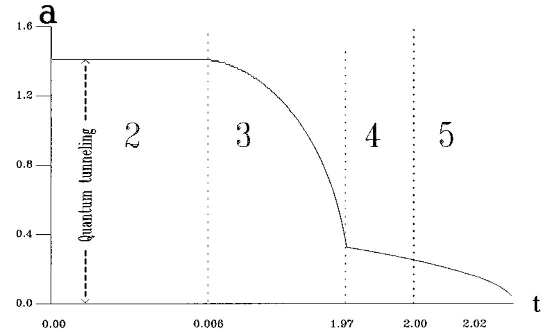

FIG. 5. The change of scale factor *(a)* during the evolution of the Universe. The units of time and scale factor are the same as in Fig. 1.

$N_0 \simeq 5 \times 10^5$ is shown in Figs. 3–5. The RPT begins in fact immediately after the tunneling and is accompanied by nonlinear OP oscillations with frequency $\sim T_c$ (Fig. 3) and particles creation (Fig. 4). After damping of these oscillations and the end of the RPT, the particle number in the Universe is increased by a factor of 2.6×10^6 (for $N_0 = 5 \times 10^5$). Our numerical experiments show that the total particle number in the Universe after relaxation approximately coincides with N_{cr} for various values $N_0 \sim 10^2 - 10^9$. Thus the particle number in the macroscopic Universe after tunneling and RPT is expressed through the fundamental constants

$$N_{cr} \sim \kappa^{-3/2} m^{-3} = \left(\frac{10^{19} \ \text{GeV}}{m} \right)^3 . \tag{11}$$

The classical evolution of the closed Universe shown in Figs. 3–5 ends in a singularity. However, the calculation of QGD effects transfers the entry to the singularity to the quantum bounce at the Planck parameters. The number of particles is conserved during this bounce since the next classical evolution cycle starts for $N > N_{cr}$. Numerical studies of this Universe evolution cycle were also performed. The main result is the conclusion that in a Universe with $N > N_{cr}$ the second order RPT happens quasiadiabatically. The relative increase of particle number for the total evolution cycle is smaller than one percent. Thus we obtain a model of a slowly swelling Universe. In this context we note Ref. [12].

Further, more detailed investigation of this scenario of the evolution of the Universe requires the calculation of RPT on other energetic scales. During its evolution the Universe must overcome some potential barriers similar to the barrier shown in Fig. 2. According to Eq. (11) after the last RPT on a scale ~ 100 MeV, we obtain the macroscopic Universe with particles number $N \sim 10^{60}$. The problem is that $N \sim 10^{88}$ in the observed Universe. We can formulate some hypotheses explaining this. (1) The observed particle number has been created after multiple cosmological cycles containing all series of RPT. During each cycle, the particle number is increased because of dissipative processes accompanying the RPT. (2) After tunneling through some barriers in some evolutionary cycles the Universe can appear at a point of the trajectory that is rather far from the left boundary of region II (see Fig. 2). From the solution of the WDW equations it follows that the probability of this process decreases exponentially with movement away from the barrier. Our numerical experiments show that the number of particles created during a RPT that is delayed in time, increases exponentially with time delay. (3) Effects of strong nonlinear interaction of different vacuum condensates in more complex series of RPT lead to time delays of these RPT (for example, the series $G \Rightarrow E_6 \Rightarrow O(10) \Rightarrow SU(5), \ldots,$ can be considered). (4) The fourth possibility is connected with the hypothetical possibility of a dynamic chaos regime in the region of nonequilibrium RPT.

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