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Primordial black hole formation during the QCD epoch

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We consider the formation of horizon-size primordial black holes (PBH's) from pre-existing density fluctuations during cosmic phase transitions. It is pointed out that the formation of PBH's should be particularly efficient during the QCD epoch due to a substantial reduction of pressure forces during adiabatic collapse, or equivalently, a significant decrease in the effective speed of sound during the color-confinement transition. Our considerations imply that for generic initial density perturbation spectra PBH mass functions are expected to exhibit a pronounced peak on the QCD-horizon mass scale $\sim 1 M_{\odot}$. This mass scale is roughly coincident with the estimated masses for compact objects recently observed in our galactic halo by the MACHO Collaboration. Black holes formed during the QCD epoch may offer an attractive explanation for the origin of halo dark matter evading possibly problematic nucleosynthesis and luminosity bounds on baryonic halo dark matter. [S0556-2821(97)50210-4]

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Ever since the early works by Zeldovich, Novikov [1], and Hawking [2] it is clear that only moderate deviations from a perfectly homogeneous Friedmann universe can lead to copious production of primordial black holes (PBH's) at early epochs. Initially superhorizon-size overdense regions can collapse and convert into PBH's when they enter into the particle horizon. Once a fluctuation passes into the particle horizon its subsequent evolution is essentially a competition between pressure forces and gravity. When the equation of state is hard, $p = \rho/3$ where p is pressure and ρ is energy density, fluctuations with overdensities exceeding a critical value $(\delta \rho / \rho) \gtrsim \delta_c^{\text{RD}} \approx 1/3$ are anticipated to form black holes whereas fluctuations with overdensities less than this critical value are expected to disperse due to pressure forces [3]. Here overdensities are specified at fluctuation horizon crossing in uniform Hubble constant gauge. A typical mass for

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PBH's formed during a radiation dominated era (e.g., $p = \rho/3$) is of the order of the horizon mass. The horizon mass is given by

$$M_H(T) \approx 1 M_{\odot} \left(\frac{T}{100 \text{ MeV}} \right)^{-2} \left(\frac{g_{\text{eff}}}{10.75} \right)^{-1/2} ,$$
 (1)

where *T* is cosmic temperature and g_{eff} are the effective relativistic degrees of freedom contributing to the Hubble expansion. The only numerical, albeit schematic, simulation of PBH formation to date [4] indicates that PBH's form with masses somewhat smaller than the horizon mass. It has been emphasized that black hole production from preexisting adiabatic fluctuations can be very efficient when there is a period during the evolution of the early universe for which the equation of state is soft ($p \approx 0$) [3,5]. In the seventies, the possible overabundant production of PBH's had also been used to rule out a prolonged soft Hagedorn equation of state at $T \sim 100$ MeV [6]. Nevertheless, with the discovery of quarks such speculative "dust like" eras were believed to only occur at temperatures in excess of the electroweak breaking scale ($T \approx 100$ GeV).

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$$p = p_{qg}(T_c) = p_h(T_c), \qquad (5)$$

$$\langle \rho \rangle = f_{qg} \rho_{qg}(T_c) + (1 - f_{qg}) \rho_h(T_c), \qquad (6)$$

$$\langle s \rangle = f_{qg} s_{qg}(T_c) + (1 - f_{qg}) s_h(T_c),$$
 (7)

where f_{qg} is the fraction of space permeated by quark-gluon phase. An adiabatic compression of quark-hadron matter in a state of equilibrium phase coexistence induces a conversion of low-energy density hadron phase into high-energy density quark-gluon phase (f_{qg} rises). During this process the average energy density increases while pressure and temperature remain constant. The pressure response of quark-hadron matter to slow adiabatic expansion, compression, or collapse is therefore negligible. This may be expressed by defining an effective speed of sound for quark-hadron matter in a state of phase mixture, such that in thermodynamic equilibrium

$$v_{S}^{\text{eff}} = \sqrt{\left(\frac{\partial p}{\partial \langle \rho \rangle}\right)_{S}} = 0,$$
 (8)

holds exactly. Equation (8) implies that the universe is Jeans unstable during the quark-hadron transition for scales much smaller than the horizon length. In contrast, the Jeans length during "ordinary" radiation dominated eras ($v_s = 1/\sqrt{3}$) is of the order of the horizon length.

One may wonder if during a cosmic QCD phase transition, thermodynamic equilibrium, in particular, constant pressure and temperature are maintained. Consider, for example, a region of quark-hadron matter at T_c . Upon compression or collapse such a region could, in principle, superheat, which would yield a pressure response, such that $v_s^{\text{eff}} \neq 0$. Superheated quark-hadron matter may cool by either the growth of existing quark-gluon phase or the nucleation of new quark-gluon bubbles. One may estimate the amount of superheating $\eta = (T - T_c)/T_c$ at which heating due to adiabatic collapse is balanced by cooling due to the nucleation of critically sized quark-gluon bubbles [13]

$$\frac{\langle \rho \rangle}{t_H} \simeq 30 \frac{\sigma^3 T_c^4}{L^2 \eta^3} \exp\left(-\frac{16\pi}{3} \frac{\sigma^3}{L^2 \eta^2 T_c}\right),\tag{9}$$

where t_H , L, and σ are Hubble time, latent heat of the transition, and surface free energy of the phase boundary, respectively, and we have assumed $\eta \ll 1$. The right hand side of Eq. 9 gives the cooling rate from nucleation of new phase with the exponent the change in free energy due to the spontaneous appearance of a critically sized bubble of quarkgluon phase divided by the temperature [13,14], whereas the left hand side of Eq. (9) is simply the rate of change in energy density during collapse in the absence of cooling. This latter rate is approximately given by the energy density of the mildly nonlinear fluctuation $\sim \langle \rho \rangle$ over the gravitational collapse time scale $\sim t_H$. Quite independent of the prefactors, cooling is efficient when the exponent in Eq. (9) is approximately 10–20, which yields $\eta \simeq \sigma^{3/2}/LT_c^{1/2}$. Typical parameters of the phase transition have been estimated to be approximately $\sigma \approx 0.02 - 0.1T_c^3$ [15–18], $L \approx 2 - 15T_c^4$ [19–22] with $T_c \approx 100$ MeV. This implies that quark-hadron matter can not sustain superheating by more than $\eta \approx 10^{-3} - 10^{-5}$ during adiabatic collapse.

There are also various schemes for the spontaneous, phase-transition dynamics related, generation of density perturbations on subhorizon scales during cosmological first order transitions and the concomitant production of PBH's. In the context of the QCD confinement transition, Crawford and Schramm [7] have argued that the long-range color force could lead to the generation of subhorizon density fluctuations which may turn into planetary sized black holes even though the details of this mechanism remain to be explored. Hall and Hsu [8] proposed the formation of PBH's during a first order OCD transition from imploding, supercooled quark-gluon plasma bubbles. Katalinić and Olinto [9] point out that the possible tendency of extreme focusing of inwardly traveling sound waves in a quark-gluon plasma bubble, reminiscent of the process of sonoluminescence, can lead to density fluctuations at the center of the bubble sufficiently large for the production of PBH's. Note that these schemes predict PBH masses which are far below the QCDhorizon mass. In the following we will show that the universe effectively has a soft equation of state during a firstorder QCD transition which, depending on cosmic initial conditions, may lead to the abundant formation of PBH's on the QCD horizon mass scale.

A first-order color-deconfinement QCD phase transition is characterized by the coexistence of high-energy density quark-gluon phase (quarks and gluons with strong mutual many-body interactions) with low-energy density hadron phase (mostly color-singlet pions for a gas at vanishing chemical potential) at a coexistence temperature of the order of $T_c \sim 100 \text{ MeV} [10,11,13]$. Within the simplistic bag model pressure p, energy density ρ , and entropy density s of the hadronic (h) and quark-gluon (qg) phases may be written as [12,13]

$$p_h(T) = \frac{1}{3}g_h \frac{\pi^2}{30}T^4, \quad p_{qg}(T) = \frac{1}{3}g_{qg} \frac{\pi^2}{30}T^4 - B,$$
 (2)

$$\rho_h(T) = g_h \frac{\pi^2}{30} T^4, \quad \rho_{qg}(T) = g_{qg} \frac{\pi^2}{30} T^4 + B, \quad (3)$$

$$s_h(T) = \frac{4}{3}g_h \frac{\pi^2}{30}T^3, \quad s_{qg}(T) = \frac{4}{3}g_{qg} \frac{\pi^2}{30}T^3.$$
 (4)

Here $g_h \approx 17.25$ and $g_{qg} \approx 51.25$ are the statistical weights of relativistic particles for the individual phases, including the contributions of pions and leptons for the hadronic phase and quarks, gluons, and leptons for the quark-gluon phase. The bag constant *B* acts effectively as a vacuum energy density for the quark-gluon phase and accounts for the strong mutual interactions of quarks and gluons. For a first-order transition at coexistence temperature T_c , the conditions of thermodynamic equilibrium are the equality of pressure $p = -(\partial E/\partial V)_S$ and temperature $T = (\partial E/\partial S)_V$ between hadronic phase and quark-gluon phase.

One may consider a region of quark-hadron matter sufficiently large to include material in both phases. The pressure, average energy density $\langle \rho \rangle$, and average entropy density $\langle s \rangle$ of such a region of quark/hadron matter are

These considerations illustrate that the pressure response of quark-hadron matter to adiabatic compression, or equivalently the effective speed of sound, is dependent on the amplitude and time scale of the compression process. For example, small-amplitude, $(\delta T/T) \leq \eta$, sound waves in pure hadronic phase, or quark-gluon phase, at T_c will propagate with the ordinary speed of sound $v_s = (\partial p / \partial \rho)_s^{1/2} \sim 1/\sqrt{3}$. However, the pressure response of "mixed" quark-gluon and hadron phase to significant adiabatic collapse, $(\delta \langle \rho \rangle / \langle \rho \rangle) \sim 1$, will be reduced substantially by a factor $\sim \eta$ when compared to the pressure response of "ordinary" relativistic matter to collapse. Note that the anomaly in the speed of sound during a first-order QCD transition, which was independently discovered by Schmid et al. [24], may have interesting implications for the growth of subhorizon size initial density perturbations [24].

The universe has effectively a soft equation of state during the QCD transition when considering the pressure response to gravitational collapse of density fluctuations. It is interesting to note that the evolution of average energy density $\langle \rho \rangle$ as a function of scale factor *R* during a first-order QCD transition is given by

$$\langle \rho \rangle(R) = \left(\frac{R_0}{R}\right)^3 \left[\rho_{qg} + \frac{1}{3}\rho_h\right] - \frac{1}{3}\rho_h \quad , \tag{10}$$

which yields for the time evolution of *R* approximately $R(t) \sim t^{2/3}$, akin to a dustlike era. Nevertheless, the duration of this dustlike phase is brief. By using the bag model we may derive for the ratio of scale factors at the beginning of the transition, R_0 , to the value at completion of the transition, R_1 , from conservation of entropy $(R_1/R_0) = (g_{qg}/g_h)^{1/3} \approx 1.44$, which implies that the duration of the transition is of order of the Hubble time at the QCD epoch. The duration of the transition may be even substantially shorter than this estimate if the latent heat is smaller than the bag model value, $L \approx 15T_c^4$, as indicated by most lattice QCD simulations [19–22].

Moderate amplitude, overdense fluctuations entering into the horizon during the QCD transition will experience an almost complete reduction in dispersing pressure forces for approximately the duration of the transition. Such fluctuations will have a fraction of a Hubble time longer to collapse unhindered by pressure forces than their counterparts passing into the horizon during "ordinary" radiation dominated eras. Therefore the demand on fluctuation amplitude δ_c^{QCD} for successful formation of a black hole during the QCD transition should be lessened when compared to the analogous quantity δ_c^{RD} :

$$\delta_c^{\text{QCD}} < \delta_c^{\text{RD}} \approx \frac{1}{3} \quad . \tag{11}$$

For almost scale-invariant initial adiabatic perturbations of the Harrison-Zeldovich type, which have approximately equal amplitudes at horizon crossing on all scales, PBH's would form more abundantly during the QCD epoch than during radiation dominated epochs leading to a peak in the PBH mass function at $M_{\rm BH} \sim 1 M_{\odot}$. Note that a peak in PBH mass functions on a given mass scale may also result from nontrivial initial fluctuation spectra which, however, require fine-tuning of initial conditions [23,25].

For first order transitions which are much longer than the QCD transition, the computation of an analogous critical overdensity δ_c may be well approximated by using a spherical top-hat model for the evolution of the fluctuation [26], since there are no pressure forces between fluctuation and environment. For such transitions δ_c may be substantially below δ_c^{RD} for nonrotating fluctuations. However, in the case of a first-order QCD transition we only expect mild (order unity) decrease in δ_c^{QCD} when compared to δ_c^{RD} since the epoch of phase coexistence is short. For successful PBH formation during the QCD era fluctuation and environment may well exist in different regimes during the pre-PBH formation evolution, for example, the overdense fluctuation may be in a quark-gluon/hadron mixture in phase coexistence at T_c whereas the environment may be purely in hadronic phase at $T \leq T_c$. In this case the reduction in pressure gradients is only partial. Nevertheless, since pressure gradients are always reduced a decrease in δ_c^{QCD} compared to δ_c^{RD} seems inevitable. A reliable estimate of δ_c^{QCD} would have to be obtained with the help of a numerical general-relativistic hydrodynamics code, using an appropriate metric which asymtotically approaches the Robertson-Walker metric in regions far away from a fluctuation, and under the inclusion of detailed modeling of the OCD equation of state, in regimes at, but also below and above, the transition point.

We have so far restricted our attention to the QCD transition and furthermore implicitly assumed that colorconfinement in the early universe proceeds via a first order transition. Lattice gauge simulations are still not conclusive as to the order of the QCD transition, mainly due to finite resolution effects and the difficulties associated with simulating bare quark masses [20,27,28]. We wish to stress that a possible reduction of the effective speed of sound may be a generic feature of cosmic phase transitions and may not necessarily be tied to the character of a transition. In fact, a reduction in the speed of sound of order 10-20% for a few Hubble times does occur during the cosmic e^+e^- annihilation [29] (the nature of the e^+e^- annihilation is very different from that of a first-order QCD color-confinement transition). This effect of pressure reduction during phases of e^+e^- creation and annihilation is also well known in stellar evolution calculations, commonly referred to as the pair instability, and relates to the conversion of relativistic energy density (photons) to e^+e^- rest mass energy density. We note that there is the possibility of an enhancement in PBH formation on the e^+e^- annihilation horizon mass scale of approximately, $M \sim 10^5 M_{\odot}$.

PBH's formed during the QCD epoch (or any other early epoch) may contribute a significant fraction Ω_{BH} to the closure density today if only a tiny fraction ϵ of the radiation energy density at the QCD epoch is converted into black hole mass density:

$$\Omega_{\rm BH} = 5.8 \times 10^7 \, \epsilon(T) \left(\frac{T}{100 \text{ MeV}} \right) \left(\frac{g_{\rm eff}}{10.75} \right) h^{-2} \quad . \tag{12}$$

Here *h* is the Hubble constant in units of 100 km s⁻¹ Mpc⁻¹. This is because radiation energy density redshifts as $1/R^4$, whereas black hole mass density redshifts as $1/R^3$ dur-

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ing the subsequent expansion of the universe. Production of PBH's during the QCD epoch would also lead to the spontaneous generation of isocurvature perturbations on superhorizon scales, even though this isocurvature component is not expected to play a role in the formation of large-scale structure unless $M_{\rm BH} \gtrsim 10^5 M_{\odot}$ [30,31]. It is, however, interesting to note that PBH's form on the peaks of the underlying adiabatic perturbations and PBH number density is therefore strongly correlated with the adiabatic density fluctuations.

The efficiency $\epsilon(T_c)$ for PBH formation during the QCD transition can be obtained from the statistics of the initial density perturbations and from δ_c^{QCD} . It is given by the fraction of QCD horizon volumes which are overdense by more than δ_c^{QCD} :

$$\boldsymbol{\epsilon}(T_c) = \int_{\delta_c^{\text{QCD}}}^{\infty} f(\delta, T_c) d\delta.$$
 (13)

Here $f(\delta,T)$ is the probability distribution to find a horizon volume overdense by $\delta = (\delta \rho / \rho)$, normalized such that $\int_{-\infty}^{\infty} f d \delta = 1$. Currently favored mechanisms for the generation of primordial density perturbations involve quantum fluctuations of scalar fields which drive an extended inflationary period of expansion in the very early universe ($T \ge 1$ TeV). Many of such models predict a Gaussian probability distribution

$$f = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma(M)} \exp\left(-\frac{1}{2} \frac{\delta^2}{\sigma^2(M)}\right).$$
(14)

with approximate variance

$$\sigma(M) \approx 5 \times 10^{-6} \left(\frac{M}{5 \times 10^{23} M_{\odot} h^{-1}} \right)^{(1-n)/6},$$
 (15)

where n is a spectral index [32]. Simple inflationary scenarios predict equal perturbation amplitudes for radiation and pressureless matter (CDM). The reader should be advised that the mass scale M in Eq. 15 denotes the horizon mass in CDM only. For the QCD epoch, and assuming a closed uni- $M_{\rm CDM}^{H} \approx 3$ this mass verse, scale is $\times 10^{-8} M_{\odot} (T/100 \text{ MeV})^{-3} h^2$. The simplest inflationary models predict scale-invariance, n = 1 [33]. There are, however, inflationary scenarios which predict the generation of blue spectra [34] (n > 1). Using Eqs. (13)–(15) and assuming δ_c^{QCD} in the range $\delta_c^{\text{QCD}} = 0.05 - 0.2$, one can infer that for spectral index in the range n = 1.67 - 1.79 cosmologically significant PBH formation during the QCD epoch $(\Omega_{PBH} \sim 1)$ may occur. This estimate assesses the sensitivity of the required spectral index for significant QCD black hole formation on the undetermined δ_c^{QCD} . We note that in Gaussian models a δ_c^{QCD} only somewhat smaller than δ_c^{RD} results in PBH formation essentially only on the QCD scale due to the steep decrease of f with δ .

A spectral index as large as n = 1.7 is incompatible with observed cosmic microwave background radiation (CMBR) anisotropies [35] and spectral distortions [36], which imply $n \gtrsim 1.5-1.6$. Moreover, the required spectral index for signifi-

cant QCD black hole formation may even exceed the above estimate when non-Gaussian, skew-negative features, resulting for most inflationary scenarios producing blue spectra, are taken into account [37]. This constraint may be circumvented in two ways. First, it may be that the effective spectral index increases with decreasing mass scale. Cosmologically significant PBH formation at the OCD scale may then still be compatible with the CMBR limits, since those limits are derived on scales much larger than the QCD horizon scale. However, in this case the primordial density perturbation spectrum has to be such that constraints derived from PBH formation on mass scales smaller than the QCD-horizon mass are not violated [38]. These limits may be fairly stringent, in particular, when the reheating temperature after an inflationary epoch is high $(T_{RH} \ge 1)$ TeV) and/or the inflationary epoch is followed by a prolonged reheating period. As a second possibility it may be that the initial density perturbations are scale-invariant (n=1), but non-Gaussian [39,40] and conspire to have a skew-positive tail, such that a fraction of approximately $\sim 10^{-8}$ horizon volumes are overdense by more than δ_c^{QCD} .

Our findings are particularly interesting in light of the recent observations of the MACHO collaboration [41] that a significant fraction of the dark matter in the galactic halo may be composed of massive compact objects. Even though the statistics of gravitational microlensing events is still poor, there is accumulating evidence for a sharp cutoff on the distribution of masses of compact halo objects, such that essentially all compact halo objects have $M \ge 0.1 M_{\odot}$ for a standard halo model. The identification of these objects as red or white dwarfs may be problematic. Searches for a dwarf population in our halo with the wide field camera on the Hubble Space Telescope yield fairly stringent limits on the fraction of mass contributed by dwarfs to the halo [42,43]. In the case of an abundant halo white dwarf population, one may also have to address the problem of overproducing metals.

In summary, we have argued that PBH formation from pre-existing adiabatic fluctuations may be particularly efficient during cosmic phase transitions and periods of particle annihilation due to a softening of the equation of state. In the case of the QCD color-confinement transition we have found that PBH's may form abundantly on the mass scale $M_{\rm BH} \sim 1 M_{\odot}$, which is surprisingly close to the inferred masses of compact objects recently discovered in our galactic halo by the MACHO Collaboration [41]. A peak in the mass function of compact halo dark matter may, in the future, be observationally verified by gravitational microlensing experiments. PBH's formed during the QCD epoch provide a natural explanation for such a peak, and furthermore evade possibly problematic bounds on baryonic compact halo dark matter.

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