Perturbing supersymmetric black holes

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(Received 4 October 1996)

An investigation of the perturbations of the Reissner-Nordström black hole in $N=2$ supergravity is presented. We prove in the extremal limit that the black hole responds to the perturbation of each field in the same manner. We conjecture that we can match the modes of the graviton, gravitino, and photon because of supersymmetry transformations. [S0556-2821(97)50108-1]

PACS number(s): 04.70.Dy, 04.25.Nx, 04.30.Nk, 04.65.+e

The extreme Reissner-Nordström black hole is very important particularly in the context of supergravities. First, in supersymmetric theories the mass of the black hole is bounded below by its charge [1]. Then the extreme black hole saturates this bound. This mass bound is surprisingly identical to the bound imposed by the cosmic censorship conjecture. Thus we can naturally avoid naked singularities in supergravity theories [2]. At the same time, the extreme black hole has the symmetry that the nonextreme black holes do not have since the spacetime admits the Killing spinor field, which means the extreme black hole is invariant under the supersymmetric transformations. The state is a close analogy of a Bogomol'ni-Prasad-Sommerfield-(BPS-)saturated state for supersymmetric particles. Second, all orders of quantum corrections should vanish in the extreme state. This feature of the supergravity theories may lead to an insight to the quantum effects around black holes [3].

So far, the extreme black holes in supergravities have been discussed only in the case of static configurations. However, to know more about the role of black holes in the supergravity theories, we need to work on the dynamical aspects of black holes. In the preceding paper [4], we have investigated the quasinormal frequencies of the extreme Reissner-Nordström black hole in the Einstein-Maxwell theory and found an interesting fact that these frequencies were completely identical for both the electromagnetic and the gravitational perturbation. The fact may have something to do with the supersymmetry, since the fields with different spins are related to each other. Thus, we continue our analysis to the O(2) extended supergravity [5] to investigate the perturbations of the Reissner-Nordström black holes to get an insight on the black hole in supergravity theories.

The O(2) extended supergravity field equations reduce to the usual Einstein-Maxwell equations when the gravitino fields vanish. By linearizing the field equations about a solution with vanishing gravitino fields, we obtain a consistent set of equations for the photon, gravitino, and graviton fields on a background solution of the Einstein-Maxwell equations. When we consider the Reissner-Nordström black hole as a background solution, these perturbation equations can all be reduced to the Regge-Wheeler-type equations. For the graviton and photon [6], the radial equations are given by

$$\left[\frac{d^2}{dr_*^2}+\omega^2-V_s(r)\right]Z_s(r)=0, \tag{1}$$

where

$$\frac{dr}{dr_*}=\frac{\Delta}{r^2}, \tag{2}$$

$$V_s=\frac{\Delta}{r^5}\left[Ar-q_s+\frac{4Q^2}{r}\right], \tag{3}$$

$$\Delta=r^2-2Mr+Q^2, \tag{4}$$

$$q_1=3M-\sqrt{9M^2+4Q^2(A-2)}, \tag{5}$$

$$q_2=3M+\sqrt{9M^2+4Q^2(A-2)}, \tag{6}$$

$$A=j_s(j_s+1), \tag{7}$$

$$j_s=l+s \quad (l=0,1,2,\ldots). \tag{8}$$

The cases of $s=1$ and $s=2$ correspond to the electromagnetic and the gravitational perturbation, respectively. Here, M and Q are the mass and charge of the black hole, and j_s is an angular multipole index of the perturbation. For the gravitino [7,8], the same equation as Eq. (1) is obtained. On the other hand, the potential is given by

$$V_{3/2}=G-\frac{dT_1}{dr_*}, \tag{9}$$

$$G=\frac{\Delta}{r^6}(\lambda r^2+2Mr-2Q^2), \tag{10}$$

$$T_1=\frac{1}{F-2Q}\left[\frac{dF}{dr_*}-\lambda\sqrt{\lambda+1}\right], \tag{11}$$

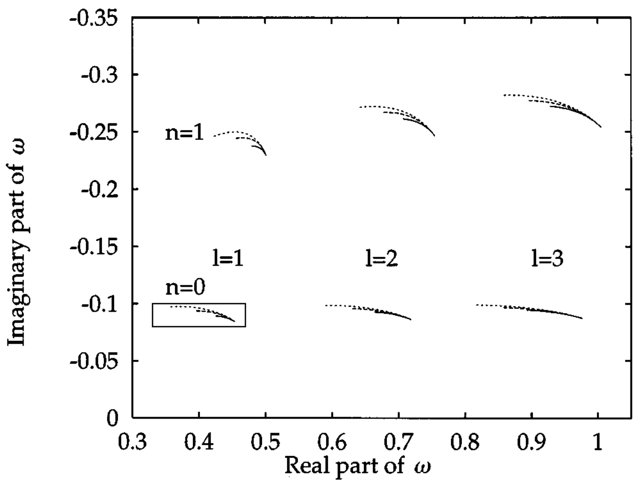

FIG. 1. Solid lines, short dashed lines, and long dashed lines are the trajectories of the first order WKB quasinormal frequencies of the photon, gravitino, and graviton, respectively. Each left endpoint of lines corresponds to the frequency of a charged black hole of $Q=0.8$, and each right end point corresponds to the frequency in the limit of maximal charge. A trajectory of each quasinormal frequency meets at the right end with the two corresponding trajectories. This means that supersymmetry is recovered in the extremal limit.

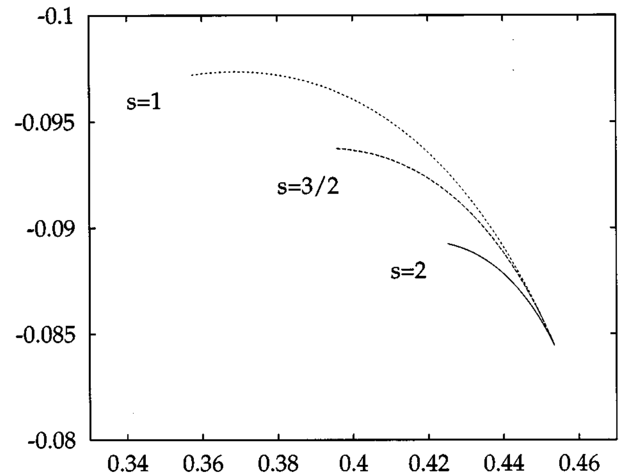

FIG. 2. The bounded box in the above figure is zoomed in. The quasinormal frequencies are equal only in the extremal limit. This is consistent with the fact that the black hole preserves supersymmetry only in the extremal limit.

$$F=\frac{r^6}{\Delta^{3/2}}G, \tag{12}$$

$$\lambda=(j_{3/2}-\tfrac{1}{2})(j_{3/2}+\tfrac{3}{2}), \tag{13}$$

$$j_{3/2}=l+\tfrac{3}{2} \quad (l=0,1,2,\dots). \tag{14}$$

In the extremal limit under the normalization of $M=Q=1$, the potentials for the three fields surprisingly reduce to similar forms:

$$V_1=+(j_1+1)\frac{df}{dr_*}-4f^3+(j_1+1)^2f^2, \tag{15}$$

$$V_{3/2}=+\left(j_{3/2}+\frac{1}{2}\right)\frac{df}{dr_*}-4f^3+\left(j_{3/2}+\frac{1}{2}\right)^2f^2, \tag{16}$$

$$V_2=-j_2\frac{df}{dr_*}-4f^3+j_2^2f^2, \tag{17}$$

$$j_s=l+s \quad (l=0,1,2,\dots), \tag{18}$$

where f is defined by

$$f=\frac{r-1}{r^2}, \tag{19}$$

and

$$r_*=r-1+\ln(r-1)^2-\frac{1}{r-1}. \tag{20}$$

These three potentials are related in the following way:

$$V_1(r_*,j_1=j)=V_{3/2}(r_*,j_{3/2}=j+\tfrac{1}{2})=V_2(-r_*,j_2=j+1), \tag{21}$$

where j is a positive integer. The first equality is obvious in Eqs. (15) and (16); the potential of the Rarita-Schwinger field is identical to that of the electromagnetic field if we shift a multipole index by 1/2. The second equality is proved by using the readily verifiable relation

$$f(r_*)=f(-r_*). \tag{22}$$

Therefore V_2 can be obtained by reflecting V_1 or $V_{3/2}$ about $r_*=0$. It is noteworthy that this transformation corresponds to the exchange of the horizon and infinity. Equation (21) means that a scattering problem for each perturbed field with a corresponding multiple index results in the same transmission and reflection amplitudes. Obviously, the relation $V_1(r_*,j_1=j)=V_2(-r_*,j_2=j+1)$ proves our numerical results in the previous paper [4].

Next, we consider the cases of the nonextreme black holes. It is necessary to see how the supersymmetry (SUSY) breaking of the black hole influences on these three modes. Since the potentials are complicated in the nonextreme cases, it is difficult to analytically find a symmetry even if it exits. Instead, we will numerically calculate the quasinormal frequencies, which are the resonant poles of the problem. The coincidence of these quasinormal modes is then a necessary condition for all fields to have the same transmission amplitude. The quasinormal modes of black holes have so far been calculated through several methods [9–14]. We use the WKB method [11] to calculate the quasinormal frequencies. The WKB frequencies are related to the potential in the following:

$$\omega_n^2=V_0-i\left(n+\frac{1}{2}\right)\sqrt{-2V_0''}, \tag{23}$$

where the subscript 0 denotes the value at the top of the potential and the subscript n is 0 or a positive integer called the mode number. Thus we can evaluate how these potentials are close to each other. Using the above formula, the first two quasinormal modes of the electromagnetic, gravitino's, and gravitational perturbations for $l=0,1,2$ are calculated for nearly extreme black holes. As easily seen in Figs. 1 and 2, three modes approach continuously as the charge of the black hole increases and then meet in the extremal limit. This is consistent with the fact that only the extreme black hole preserves the supersymmetry. Quasinormal frequencies of the electromagnetic field move fastest in the complex ω plane when we change the charge of the black hole, because the electromagnetic field is most sensitive to the change of the charge and quasinormal frequencies of the gravitino field is approximately the average value of the electromagnetic and the gravitational frequencies.

We have found the relation (21) for the effective potentials of the perturbed fields and proved the coincidence of the quasinormal mode frequencies which we had found in the previous paper [4]. This coincidence found among the graviton, gravitino, and photon in the extremal limit can be considered a consequence of the supersymmetry. It is well known that the extreme Reissner-Nordström black hole admits a Killing spinor field [1]. It means that the background solution is invariant under the supersymmetry transformations with respect to the Killing spinor field. This implies that all the perturbed fields can be related to each other using the supersymmetry transformations, which conserves the S matrix. Our results show that the gravitino is matched with the photon by adding 1/2 to the multipole index. Since the multipole index j_s here describes the total angular momentum, this shift is explained by the supersymmetric transformation that increases the spin by 1/2 and hence increases the total angular momentum by 1/2.

Here we reach a new insight on the black hole in supergravities. It is expected that for any supergravity theory, the perturbed fields such as a dilaton studied in [15] around the BPS saturated black hole should have a similar behavior; a scattering problem around the black hole is the same from one field to another. This is because all the fields can be matched with each other using supersymmetry transformations and the character of the black hole does not change whatever field we consider around it. Recently the extreme black holes have been intensively studied in the context of the strings and D-branes and these studies have led to the first microscopic description of the black hole entropy [16]. It is interesting to obtain explicitly the supersymmetric transformations and relate them to Eq. (21), because our scenario can be clearly understood and will shed new light on this subject. This is under investigation.

We would like to thank Professor G. W. Gibbons for pointing out the possible relation between our previous results and his joint work with Professor C. M. Hull. We would like to thank Professor A. Hosoya for his continuous encouragement and Dr. B. Meister for reading the manuscript. The research was supported in part by the Scientific Research Fund of the Ministry of Education.

[1] G. Gibbons and C. Hull, Phys. Lett. **109B**, 190 (1982).
[2] R. Kallosh, A. Linde, T. Ortín, and A. Peet, Phys. Rev. D **46**, 5278 (1992).
[3] R. Kallosh, Phys. Lett. B **282**, 80 (1992).
[4] H. Onozawa, T. Mishima, T. Okamura, and H. Ishihara, Phys. Rev. D **53**, 7033 (1996).
[5] S. Ferrara and P. Nieuwenhuizen, Phys. Rev. Lett. **37**, 1669 (1976).
[6] S. Chandrasekhar, *The Mathematical Theory of Black Holes* (Clarendon, Oxford, 1983).
[7] P. Aichburg and R. Güven, Phys. Rev. D **24**, 2066 (1981).
[8] G. Torres del Castillo and G. Silva-Ortigoza, Phys. Rev. D **46**, 5395 (1992).
[9] S. Chandrasekhar and S. Detweiler, Proc. R. Soc. London **A344**, 441 (1975).
[10] E. Leaver, Proc. R. Soc. London **A402**, 285 (1985).
[11] B. Schutz and C. Will, Astrophys. J. **291**, L33 (1985).
[12] S. Iyer and C. Will, Phys. Rev. D **35**, 3621 (1987).
[13] H. Nollert and B. Schmidt, Phys. Rev. D **45**, 2617 (1992).
[14] N. Andersson and S. Linnæus, Phys. Rev. D **46**, 4179 (1992).
[15] C. Holzhey and F. Wilczek, Nucl. Phys. **B380**, 447 (1992).
[16] A. Strominger and C. Vafa, Phys. Lett. B **379**, 99 (1992). (See G. Horowitz, Report No. gr-gc/9604051 (unpublished), for a review and other references.)