Choptuik scaling and quantum effects in 2D dilaton gravity

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We study numerically the collapse of massless scalar fields in two-dimensional dilaton gravity, both classically and semiclassically. At the classical level, we find that the black hole mass scales at threshold like $M_{\mathrm{BH}} \propto |p-p^*|^{\gamma}$, where $\gamma \simeq 0.53$. At the semiclassical level, we find that in general M_{BH} approaches a nonzero constant as $p \to p^*$. Thus, one-loop effects suggest a mass gap not present classically at the onset of black hole formation. [S0556-2821(97)50508-X]

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The discovery of universality and scaling at the onset of black hole formation [1] may have important implications in understanding the structure of solution-space in gravitational theory. Choptuik studied numerically the collapse of a spherically symmetric self-gravitating real scalar field in four-dimensional (4D) Einstein gravity and considered one-parameter families of initial data, $\mathcal{S}[p]$, where p is a parameter specifying the strength of the gravitational self-interaction of the scalar field. He found that there exists a critical value, $p=p^*$, such that for $p<p^*$, the ''subcritical case,'' no black hole is formed and the solution is regular, while for $p>p^*$, the ''supercritical case,'' a black hole is formed. Furthermore, by careful analysis of the solutions near criticality, $p=p^*$, he found that as p approaches p^* from above, the mass of the created black hole approaches zero, and when a black hole just forms, its mass scales as $M_{\mathrm{BH}} \propto |p-p^*|^{\gamma}$, where the critical exponent $\gamma \simeq 0.37$. The critical solutions also exhibit discrete self-similarity [1]. Similar behavior was found in other models of nonlinear gravity [2].

In the semiclassical scenario, i.e., for a quantum field propagating on a classical dynamical background metric, the created black hole of mass M radiates in 4D with the Hawking temperature $T_{\mathrm{H}} \propto M^{-1}$ [3]. As $M \to 0$ near criticality, T_{H} becomes large and quantum effects are clearly important. To gain some understanding of how quantum effects may alter the classical scenario near criticality, we investigate a tractable two-dimensional (2D) model that exhibits classical scaling as well as significant quantum effects near criticality.

The model we study is 2D dilaton gravity coupled to a massless scalar field. We first consider the classical theory and then include quantum effects. We find that classically there is universal scaling of the black hole mass near criticality, $M_{\mathrm{BH}} \propto |p-p^*|^{\gamma}$, where $\gamma \simeq 0.53 \pm 0.01$ is independent of initial data. In addition, we find that the ground state of the classical theory gives a lower bound on the energy of spacetimes resulting from dynamical processes in which a black hole is formed. This is related to a radiation energy deficit Δ_{rad} that we describe later. At the semiclassical level, although the Hawking temperature in our 2D model is independent of M, we find that quantum effects alter significantly the critical behavior. Most interestingly, when p is sufficiently close to p^*, the classical scaling behavior breaks down and M_{BH} approaches a nonzero constant value that depends on the initial data. Thus our one-loop calculations indicate the possiblity for the existence of a mass gap. This is a new result that may be of fundamental interest. Also, since the breakdown of the classical scaling law occurs in the domain of validity of the semiclassical approximation, we prove the absence of Choptuik scaling in the full quantum theory.

The classical theory is described by the 2D Callan-Giddings-Harvey-Strominger (CGHS) action [4]

$$S_{\mathrm{CGHS}} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\Bigg\{ e^{-2\phi}[\mathcal{R}+4(\nabla\phi)^2+4\lambda^2] - \frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\Bigg\}, \tag{1}$$

where ϕ is the dilaton field, $\mathcal{R}$ is the 2D Ricci scalar formed from the metric tensor $g_{\mu\nu}$, λ is a positive constant, and the f_i are N massless scalar matter fields conformally coupled to the 2D geometry. We work in the conformal gauge, $g_{--}=g_{++}=0$ and $g_{+-}=-(1/2)\exp(2\rho)$, (i.e., $ds^2=-\exp[2\rho(x^+,x^-)]dx^+dx^-$), where (x^+,x^-) are the ''Kruskal'' null coordinates in which $\phi(x^+,x^-)$ $\phi(x^+,x^-) = \rho(x^+,x^-)$ [5].

The general vacuum solution of the classical theory (1) is $\phi=-(1/2)\ln(-\lambda^2x^+x^-+C)$, where λC is its Arnowitt-Deser-Misner (ADM) mass [6]. For $C>0$ the vacuum solutions describe static 2D black holes. The $C=0$ solution is the linear dilaton vacuum (LDV), which is the ground state of the theory. Solutions with $C<0$ have timelike naked singularities in the strong coupling region where $\exp(2\phi)\to\infty$. The weak coupling, asymptotically flat region will be taken to be on the right-hand side (RHS) of the spacetime [5]. In this work we take $C<0$, but we avoid the region of strong coupling and the singularity by considering the solutions only in the weak coupling region $\exp(2\phi)\leqslant\exp(2\phi_c)$ for a given constant ϕ_c, and by imposing reflecting boundary conditions on the timelike boundary curve $\phi(x^+,x^-)=\phi_c$. A previous study of such a system at the onset of black hole formation [7] was based on boundary conditions that mix classical and

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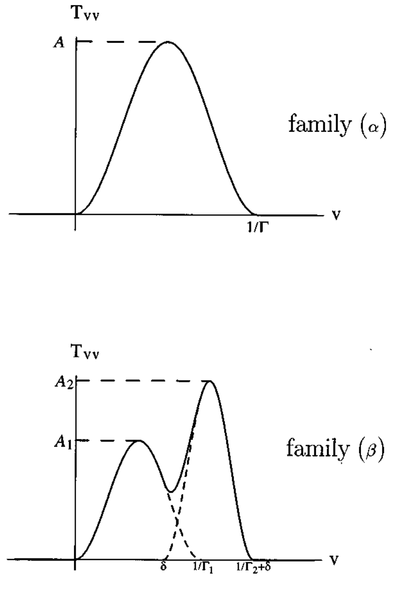

FIG. 1. Two families of initial data described by the stress tensor T_{vv}.

one-loop contributions and do not have a standard classical limit. The reflecting boundary conditions [8,9] that we impose here have a standard classical limit. Let the boundary be described by the curve $x^{+}=x_{B}^{+}(x^{-})$. Then the reflecting boundary condition is

$$T_{--}(x^{-})=\left(\frac{\partial x_{B}^{+}}{\partial x^{-}}\right)^{2}T_{++}[x_{B}^{+}(x^{-})], \tag{2}$$

where $T_{\pm\pm}=(1/2)\Sigma_{i}(\partial_{\pm}f_{i})^{2}$ are the components, in the (x^{+},x^{-}) coordinates, of the stress tensor of the massless scalar fields. The general solution for the conformally coupled matter fields is $f_{i}(x^{+},x^{-})=f_{i}^{+}(x^{+})+f_{i}^{-}(x^{-})$. The initial data at right asymptotic past null infinity, $\Im_{R}^{-}$, are therefore given by $f_{i}^{+}(x^{+})$ or $T_{++}(x^{+})$. Working in the large N limit [5] we are able to choose the boundary curve to be at $\exp(-2\phi_{c})\to 0^{+}$ and get a second order nonlinear ordinary differential equation (ODE) for the classical boundary curve [5]

$$[x^{-}+P_{+}(x_{B}^{+})/\lambda^{2}]\frac{d^{2}x_{B}^{+}}{dx^{-2}}+2\frac{dx_{B}^{+}}{dx^{-}}+2\lambda^{-2}T_{++}(x_{B}^{+})\left(\frac{dx_{B}^{+}}{dx^{-}}\right)^{2}=0, \tag{3}$$

where $P_{+}(x^{+})=\int_{0}^{x^{+}}T_{++}(\overline{x^{+}})d\overline{x^{+}}$. We solve the ODE (3) numerically for different initial data, $T_{++}(x^{+})$, with compact support, $x_{1}^{+}<x^{+}<x_{2}^{+}$. To the past of x_{1}^{+}, i.e., for $x^{+}<x_{1}^{+}$, we have a vacuum solution $\phi=-(1/2)\ln(-\lambda^{2}x^{+}x^{-}+C)$, with C being a negative constant. In this region we have an analytical solution for the boundary curve: $x_{B}^{+}(x^{-})=C/x^{-}$, where $x^{-}<C/x_{1}^{+}$. For $x^{-}>C/x_{1}^{+}$, we integrate Eq. (3) numerically to find the boundary curve and the corresponding solutions for the metric and dilaton field.

We study in detail the two families of initial data shown in Fig. 1. Family (α) is a two-parameter family specified by the amplitude, A, and the inverse width, Γ, of the profile $T_{vv}(v)$, where $v=\lambda^{-1}\ln(\lambda x^{+})$ and $u=\lambda^{-1}\ln[-\lambda x^{-}-P_{+}(\infty)/\lambda]$ are the manifestly asymptotically flat null coordinates on $\Im_{R}^{\pm}$. Family (β) is a five-parameter family described by A_{1}, A_{2}, Γ_{1}, Γ_{2}, and δ. All the parameters are scaled by appropriate powers of λ to make them dimensionless. In these two cases (and in other cases that we studied) we find that fixing all but one arbitrarily chosen parameter, say p, yields a regular evolution with no black hole formation *if and only if* $p<p^{*}$. The critical value, p^{*}, depends on the values of the other fixed parameters. As in the 4D solution-space, the only ''intermediate solution'' separating black hole solutions from regular ones is the critical solution with $p=p^{*}$.

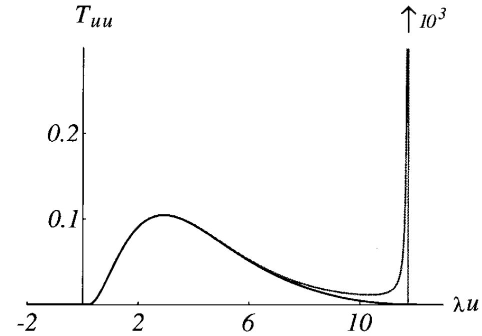

FIG. 2. The stress tensor T_{uu}, in units of λ^{2}, describing the outgoing radiation. The upper curve corresponds to a subcritical solution just below criticality and the lower curve corresponds to a supercritical solution just above criticality.

Next we show that there is a universal scaling of the black hole mass near criticality in the classical theory. In order to find the mass of the created black hole we calculate the outgoing radiation reaching right asymptotic future null infinity, $\Im_{R}^{+}$, after being reflected from the boundary. This radiation is described by the stress tensor, $T_{uu}(u)$. Then we find that the Bondi mass at late times $(u\to\infty)$ on $\Im_{R}^{+}$ is $M_{\rm Bondi}^{(\infty)}\equiv M_{\rm Bondi}(u\to\infty)=M_{\rm ADM}-\int_{-\infty}^{\infty}T_{uu}(u)du$. Here $M_{\rm ADM}$ is the ADM mass [10] of the spacetime, defined such that the LDV ground state has zero ADM mass. In Fig. 2 we show the numerical results for the reflected outgoing radiation, $T_{uu}(u)$, for two nearby sets of initial data. The two solutions correspond to family (α) with the same width, $\Gamma=0.2$, but with different amplitudes, A. The upper curve corresponds to a subcritical solution near criticality, $A=A_{\rm sub}=A^{*}-\epsilon_{1}=1.546$. The lower curve corresponds to a supercritical case near criticality, $A=A_{\rm super}=A^{*}+\epsilon_{2}=1.547$, where A^{*} is the critical value of the amplitude parameter A.

Although the initial data are very similar in the two cases, the late-time outgoing radiation is quite different. For the subcritical solution near criticality, the boundary curve becomes almost null and the late-time outgoing radiation is highly blue-shifted [5]. In the supercritical case on the other hand, this blue-shifted radiation is absent, and we find that part of the incoming radiation does not reach future asymptotic null infinity. Since T_{uu} is quite different in the two cases, the Bondi mass defined through the integral $\int T_{uu}(u)du$ is obviously different. For all the subcritical solutions the Bondi mass at late times, $M_{\rm Bondi}^{(\infty)}$, is equal to the negative ADM mass of the initial spacetime, λC. However, for the supercritical solutions $M_{\rm Bondi}^{(\infty)}$ is always positive. We define the ''radiation energy deficit,'' $\Delta_{\rm rad}$, to be $\Delta_{\rm rad}\equiv \lim_{\epsilon\to 0}\{M_{\rm Bondi}^{(\infty)}[p=p^*+\epsilon]-M_{\rm Bondi}^{(\infty)}[p=p^*-\epsilon]\}$. The fact that $\Delta_{\rm rad}$ turns out to be nonzero is a striking manifestation of the nonlinearity of our problem.

We find that $\Delta_{\rm rad}$ does not depend on the profile of the infalling matter, but only on the initial vacuum geometry. Explicitly,

$$\Delta_{\rm rad}=-\lambda C=\lambda|C|, \tag{4}$$

where C is the constant specifying the initial vacuum geometry. The existence of this nonzero $\Delta_{\rm rad}$ can be interpreted as implying that in the classical theory the LDV is a ''stable ground state.'' In particular, we have started with a negative-mass geometry and find that the process of throwing in matter to form a black hole results in spacetimes having non-negative mass relative to the LDV. In agreement with cosmic censorship, this suggests that systems in this classical theory will not evolve to states of energy lower than the LDV, which have naked singularities. Once the infalling matter configuration is dense enough, $p>p^*$, part of the incoming energy is ''swallowed'' by the strong curvature region to compensate for the negative mass of the initial spacetime and make the future spacetime a solution with positive energy compared to the LDV. For our initial geometry with negative ADM mass λC, the minimum energy that must be swallowed in order to get a positive-energy spacetime, is precisely $\Delta_{\rm rad}$. The resulting positive-energy spacetime is a 2D black hole. We find that in this classical theory the mass of the created black hole, $M_{\rm BH}\equiv M_{\rm Bondi}^{(\infty)}[p>p^*]$, approaches zero as p approaches p^* from above. Moreover, the black hole mass scales as

$$M_{\rm BH}\propto|p-p^*|^{\gamma} \tag{5}$$

at threshold, where $\gamma=0.53\pm0.01$. The value of γ does not appear to depend on the profile of the infalling matter or the initial geometry. In Fig. 3(a) we plot $\ln(M_{\rm BH}/\lambda)+a_i$ versus $\ln|p_i-p_i^*|$ for different parameters p_i. The constants a_i are chosen such that the three curves intersect at a given point. The slopes of all the lines are the same within our numerical accuracy.

Next we consider quantum effects. We quantize the scalar fields on a classical dynamical background geometry consisting of the metric and the dilaton field. The semiclassical effective action that we study is [11]

$$S_{\rm eff}=S_{\rm CGHS}-\frac{\kappa}{8\pi}\int d^2x\sqrt{-g(x)}\int d^2x'\sqrt{-g(x')}\times\mathcal{R}(x)G(x,x')\mathcal{R}(x')+\frac{\kappa}{2\pi}\times\int d^2x\sqrt{-g}[(\nabla\phi)^2-\phi\mathcal{R}], \tag{6}$$

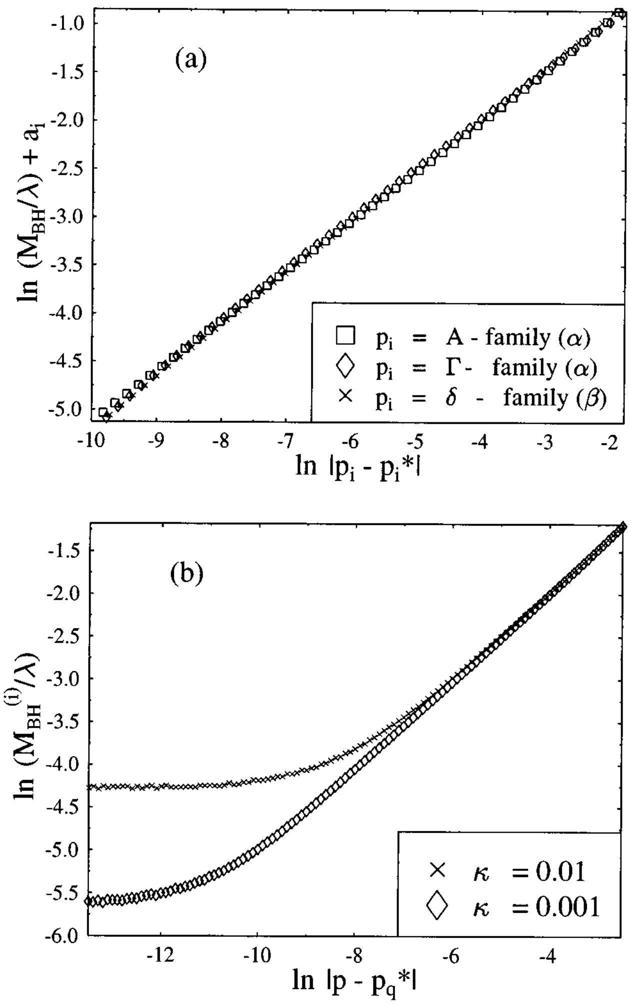

FIG. 3. $\ln(M_{\rm BH}/\lambda)$ vs $\ln|p-p^*|$ in the classical case, (a), and the quantum case, (b).

where $\kappa=N\hbar/12$ and $G(x,x')$ is an appropriate Green function for the massless scalar fields. The second term on the RHS of Eq. (6) is the Polyakov-Liouville effective action derived from the trace anomaly of the 2D massless scalar fields [12], and the last term on the RHS of Eq. (6) is a local counter-term that we add to our effective theory in order to make it exactly solvable [11]. The vacuum solutions of our semiclassical theory are $\phi=-(1/2)\ln[-\lambda^2x^+x^--(\kappa/4)\ln(-\lambda^2x^+x^-)+C]$, where C is a constant. The ground state is the solution with $C=C_0\equiv(\kappa/4)[\ln(\kappa/4)-1]$. As in the classical case, we impose reflecting boundary conditions on a timelike boundary curve and study the solutions only in the weak coupling region. The reflecting boundary condition is a modification of Eq. (2) where $T_{\pm\pm}$ are now components of the total stress tensor, including the classical and one-loop contributions. There is also an extra term due to possible particle creation from the boundary, which is effectively a moving mirror [13]. One can eliminate the moving mirror term by taking the large N limit together with $\exp(-2\phi_c)\to0^+$ [5]. In contrast to Eq. (3), the resulting nonlinear ODE for the semiclassical boundary curve, $x^+=x_B^+(x^-)$, is [5]

$$\left[x^- + \frac{P_+(x_B^+)}{\lambda^2} + \frac{\kappa}{4\lambda^2 x_B^+}\right]\frac{d^2x_B^+}{dx^{-2}} + 2\frac{dx_B^+}{dx^-} + \left[2\lambda^{-2}T_{++}(x_B^+) - \frac{\kappa}{4\lambda^2 x_B^{+2}}\right]\left(\frac{dx_B^+}{dx^-}\right)^2 = 0. \quad (7)$$

We consider initial data with no quantum radiation on $\mathfrak{I}_R^-$ and classical incoming radiation corresponding to the profiles shown in Fig. 1. We find that as in the classical case, the solutions are regular and no black hole is formed if and only if $p<p_q^*$ (where p_q^* is the critical value of the parameter p in the semiclassical case). For $p>p_q^*$, the black hole that is formed evaporates by emitting Hawking radiation [11]. To look for scaling analogous to Eq. (5), we calculate the *initial* mass of the created black hole. It is this mass that reduces in the limit $N\hbar\to 0$ to the mass of the classical black hole that appears in Eq. (5). The initial black hole mass, $M_{\rm BH}^{(i)}$, is the Bondi mass at $u=u_0$, where u_0 is the minimum of the apparent horizon $u=u_{\rm ah}(v)$. The apparent horizon is the solution of the equation $\partial_+\phi=0$. Explicitly, $M_{\rm BH}^{(i)}=M_{\rm ADM}-\int_{-\infty}^{u_0}T_{uu}du$, where $T_{uu}(u)$ is the total stress tensor of the outgoing radiation and $M_{\rm ADM}$ is the ADM mass of the spacetime, defined such that the semiclassical ground state, i.e., the static vacuum solution with $\overline{C}=\overline{C}_0$, has zero ADM mass.

We examine quantum effects by considering cases with different values of $\kappa=N\hbar/12$. In Fig. 3(b) we plot $\ln(M_{\rm BH}^{(i)}/\lambda)$ versus $\ln|p-p_q^*|$ for two different values of κ. We do so in the case of the family (α) of initial data shown in Fig. 1, where the free parameter p is the amplitude A, and the parameter $\Gamma=0.2$ is fixed. For large values of $M_{\rm BH}^{(i)}$ the initial mass of the black hole behaves like that of the classical black hole [see Eq. (5)]. However, as $M_{\rm BH}^{(i)}$ decreases, deviations from the classical behavior appear. Unlike the classical case, as p approaches p_q^* the initial mass, $M_{\rm BH}^{(i)}$ of the created black hole *does not* generally approach zero, but rather approaches a constant, $M_{\rm gap}$. We find that this mass gap, $M_{\rm gap}$, depends not only on the value of κ but also on the initial data. For the cases shown in Fig. 3(b) we have $M_{\rm gap}/\lambda\simeq 0.014$ for $\kappa=10^{-2}$, and $M_{\rm gap}/\lambda\simeq 0.0037$ for $\kappa=10^{-3}$. The radiation energy deficit in the quantum case is $\Delta_{\rm rad}^q=\lambda(\overline{C}_0-\overline{C})+M_{\rm gap}$.

The deviations from the classical scaling law (5) become significant for values of $M_{\rm BH}^{(i)}$ that are of order $\kappa\lambda$. When $M_{\rm BH}^{(i)}$ takes values that are of order or less than $\kappa\lambda$, the semiclassical approximation remains valid provided N is sufficiently large [5,14]. However for any fixed value of N, no matter how large, the semiclassical approximation breaks down when p is sufficiently close to p_q^* [5], i.e., as one moves to the far left well beyond the region shown in Fig. 3(b). Thus, determining the behavior of the curve as $\ln|p-p_q^*|\to-\infty$ requires full quantization of the theory. Although we do not prove the presence of a mass gap in the full quantum theory, our one-loop calculations strongly suggest that such a mass gap might persist even when higher order corrections are incorporated. Moreover, since breakdown of the classical scaling law occurs in the domain of validity of the semiclassical approximation, such a scaling will be absent even in the full quantum theory. We are currently studying the phase structure of the semiclassical theory near threshold which appears to be richer than that in the classical case.

In this work we investigate numerically the collapse of massless scalar fields in 2D dilaton gravity. We find that classically the black hole mass at threshold obeys a power-law, $M_{\rm BH}\propto|p-p^*|^\gamma$, where $\gamma=0.53\pm0.01$. However, quantum effects destroy the classical scaling and strongly suggest the formation of a mass gap that depends on the initial data.

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