Long distance contribution to $D \to \pi l^+ l^-$

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We calculate the long distance contribution to $D^{+,0} \to \pi^{+,0} l^+ l^-$ decays by the use of a vector meson dominance model, in which the ϕ meson plays the central role. The branching ratios obtained are 10^{-6} and a few times 10^{-7} for the resonance and nonresonance regions, respectively. The analysis includes a calculation of $D^+ \to \pi^+ \phi$, consistent with the experimental value. [S0556-2821(97)50503-0]

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I. INTRODUCTION AND THE MODEL

New experimental limits for transitions involving change of flavor in charm meson decays were obtained recently at Fermilab [1,2] and at the Cornell Electron Storage Ring using the CLEO II detector [3]. In particular, upper limits in the range $10^{-4}-10^{-5}$ were established for exclusive channels of the type $D^{+,0} \to X^{+,0} l^+ l^-$, where $X^{+,0}$ is a pseudoscalar or vector meson.

The short distance process $c \to u\gamma$, driven by the magnetic penguin diagram, is known to be of little significance for radiative decays of charm mesons as it leads to a branching ratio of $10^{-12}-10^{-11}$ only, despite its enhancement by gluonic corrections [4]. In decays to lepton pairs, the short distance $c \to u l^+ l^-$ transition may be of more relevance, as it contains contributions from both form factors of the electromagnetic penguin diagram, as well as contributions from the Z^0-penguin diagram and from the W-box diagram. Indeed, the rate for the short distance $c \to u l^+ l^-$ transition has been calculated [5–7] to be 1.1×10^{-20} GeV, leading to a branching ratio of 1.8×10^{-8} for the inclusive process. Accordingly, an exclusive process such as $D \to \pi l^+ l^-$, expected to be approximately 10% of the inclusive rate, will have a short distance contribution to the branching ratio of the order of 10^{-9}. On this basis, it has been argued [6] that decays such as $D^{+,0} \to \pi^{+,0} l^+ l^-$, $D_s^+ \to K^+ l^+ l^-$, constitute ''... a large 'discovery' window: seeing this decay occur at a branching ratio above $\sim 10^{-7}$ would be strong evidence for new physics.''

It is therefore of obvious interest to obtain reliable estimates for the long distance contributions to these modes. As one suspects [7] that these contributions are the dominant ones in the standard model, one should have good control of their estimation, in order to perform a meaningful search for new physics in these decays.

In the present paper, we consider the long distance contribution to the helicity-unsuppressed decays $D \to \pi l^+ l^-$, which supposedly are the better suited channel for checking the nature of the flavor-changing neutral transitions. We note that presently the best experimental upper bounds for the branching ratios are as follows: 1.8×10^{-5} for $D^+ \to \pi^+ \mu^+ \mu^-$, 6.6×10^{-5} for $D^+ \to \pi^+ e^+ e^-$ [2], and 1.8×10^{-4} for $D^0 \to \pi^0 \mu^+ \mu^-$ [1], 4.5×10^{-5} for $D^0 \to \pi^0 e^+ e^-$ [3].

Our estimate for the long distance contribution is based on a vector meson dominance mechanism, similar to the approach widely used for calculating the long distance contributions in $b \to s l^+ l^-$ decay [8]. We present the details of our calculation for the $D^+ \to \pi^+ l^+ l^-$ channel, for which the experimental input required by our model is available.

The basic assumption of our model is that the main long distance contribution to $D^+ \to \pi^+ l^+ l^-$ is given by the transition $D^+ \to \pi^+(V) \to \pi^+ l^+ l^-$, where V is a $\overline{qq}$ vector meson state. It is known that [9]

$$\begin{aligned} B(D^+ \to \pi^+ \phi) &= (6.1 \pm 0.6) \times 10^{-3}, \\ B(D^+ \to \pi^+ \rho) &< 1.4 \times 10^{-3}, \\ B(D^+ \to \pi^+ \omega) &< 7 \times 10^{-3}. \end{aligned} \tag{1}$$

Moreover, the branching ratios of ϕ for decays to lepton pairs are [9] $B(\phi \to e^+ e^-) = (3.00 \pm 0.06) \times 10^{-4}$, and $B(\phi \to \mu^+ \mu^-) = (2.48 \pm 0.34) \times 10^{-4}$, while for ρ and ω the similar branching ratios are nearly one order of magnitude lower. Thus, we may restrict our considerations to the role of the ϕ meson only, the contribution of ρ and ω constituting a relatively small correction.

Another kind of possible long distance contributions comes from the W-annihilation (or W-exchange) diagram, which is found for example to be large in $B \to \rho\gamma$ [10], and needs therefore to be discussed further. In the present case of $D \to \pi e^+ e^-$, the mechanism for the W-annihilation (or W-exchange) diagram is through $D \to \pi\gamma^*$ and $\gamma^* \to e^+ e^-$, where the virtual photon comes from one of the four quark lines. Because naive (constituent) quark model does not work for the light pseudo Goldstone meson π, we need to use a hadronic model to estimate these contributions. At the hadronic level, the process $D \to \pi\gamma^*$ corresponds to the electromagnetic transition $D \to D\gamma^*$ followed by the weak transition $D \to \pi$, or $D \to \pi$ followed by $\pi \to \pi\gamma^*$. These electromagnetic transitions are described by the electromagnetic formfactors of the D and the π mesons, which can be taken in the vector meson dominance (VMD) model as dominated by vector mesons as ψ, ρ, and ω. The contribution from ψ is smaller than those from the other two since this ψ is highly offshell. The remaining contributions from ρ and ω in the W-annihilation (or W-exchange) mechanism correspond to the full processes $D \to \pi\rho$ and $D \to \pi\omega$. These effects have been included in our VMD model and found to be small.

In the second section we present our approach to the calculation of $D^+ \to \pi^+ \phi$, and in the third section we calculate its contribution to the $D \to \pi l^+ l^-$ modes.

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II. $D^+\to\pi^+\phi$ TRANSITION

We begin with a treatment of the process $D^+\to\pi^+\phi$, which is the main contribution to $D^+\to\pi^+l^+l^-$ in our approach. The effective Hamiltonian for this transition is

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}V_{us}^*V_{cs}a_2\overline{s}\gamma^\mu(1-\gamma_5)s\overline{u}\gamma_\mu(1-\gamma_5)c, \tag{2}$$

where a_2 is QCD coefficient which is taken as $|a_2|=0.55\pm0.1$ from the overall fit to nonleptonic D decays [11]. We shall assume the usefulness of the factorization hypothesis [11]; thus,

$$\langle\pi^+\phi|\mathcal{H}_{\rm eff}|D^+\rangle=\frac{G_F}{\sqrt{2}}V_{us}^*V_{cs}a_2\langle\phi|\overline{s}\gamma^\mu(1-\gamma_5)s|0\rangle \times\langle\pi^+|\overline{u}\gamma_\mu(1-\gamma_5)c|D^+\rangle, \tag{3}$$

and we define g_ϕ by

$$\langle\phi|\overline{s}\gamma^\mu s|0\rangle=ig_\phi\epsilon_\phi^{*\mu}. \tag{4}$$

g_ϕ will be determined from the observed lepton pair decay $\phi\to e^+e^-$.

The hadronic matrix element is parametrized by two independent form factors f_+ and f_- as

$$\langle\pi^+|\overline{u}\gamma_\mu(1-\gamma_5)c|D^+\rangle=f_+(p_D+p_\pi)_\mu+f_-(p_D-p_\pi)_\mu. \tag{5}$$

The f_- form factor does not contribute in the decay to a lepton pair; f_+ is related by isospin symmetry to the form factor of the semileptonic decay $D^+\to\pi^0l^+\overline{\nu_l}$. Accordingly, we turn to the latter for learning the f_+ form factor.

Using the heavy meson chiral perturbation theory, Wise [12] has calculated the f_+ form factor near the zero recoil point $q_m^2=(p_D-p_\pi)_m=(m_{D^+}-m_{\pi^0})^2$ in $D^+\to\pi^0l^+\overline{\nu_l}$ to be

$$f_+(q_m^2)=-\frac{f_D}{2\sqrt{2}f_\pi}\left(1-g\frac{p_D\cdot p_\pi-m_D^2}{p_D\cdot p_\pi+m_D\Delta}\right) \sim-\frac{f_D}{2\sqrt{2}f_\pi}\left(1+g\frac{m_D-m_\pi}{m_\pi+\Delta}\right),$$
$$\Delta\equiv m_{D^*}-m_D. \tag{6}$$

The analysis of various existing experiments gives the following bounds on the parameters in Eq. (6) [9,13]:

$$f_D\leqslant0.31\ \text{GeV},\qquad |g|\leqslant0.63. \tag{7}$$

On the other hand, two measurements [14] of the form factor $f_+(q^2)$ in the semileptonic decay have been analyzed under the assumption of a monopole behavior for it,

$$f_+(q^2)=f_+(q_m^2)\frac{1-q_m^2/m_{D^*}^2}{1-q^2/m_{D^*}^2}, \tag{8}$$

and the data may be summarized as [9]

$$f_+(0)=(1.0\pm0.3)\times f_+^K(0)=(1.0\pm0.3)\times(0.75\pm0.03), \tag{9}$$

which shows the consistency of Eqs. (6)–(8), within the accuracy obtained so far. A new recent measurement by E687 gave $|f_+^\pi(0)/f_+^K(0)|=1.03\pm0.16\pm0.02$ [15], which is consistent with the value cited in Eq. (9).

Using now, for example, the reasonable set of values $f_D=0.20$ GeV and $g=0.6$, as well as Eq. (8), we find $\Gamma(D^+\to\pi^+\phi)=(3.7\pm1.4)\times10^{-15}$ GeV, which is in remarkable agreement with the experimental average [9] $^{\rm exp}\Gamma(D^+\to\pi^+\phi)=(3.8\pm0.4)\times10^{-15}$ GeV. This approach leads to $f_+(0)=0.92$ which agrees with the results of Refs. [14,15].

In the calculation of the leptonic decay $D^+\to\pi^+l^+l^-$, we shall normalize to the observed $D^+\to\pi^+\phi$ rate. Nevertheless, it is appropriate to emphasize at this point that the theoretical treatment described here accounts well for this mode; this provides the needed confidence in its use as the major contribution to the $D\to\pi l^+l^-$ decays.

III. LONG DISTANCE CONTRIBUTION TO $D^+\to\pi^+l^+l^-$

The effective coupling between ϕ and l^+l^- via a photon propagator is

$$g_{\phi l\overline{l}}=[e\overline{u_{l^-}}\gamma^\alpha v_{l^+}]\frac{g_{\nu\alpha}}{(p_++p_-)^2}\left[\frac{1}{3}eg_\phi\epsilon_\phi^\nu\right] =\frac{e^2}{3}\overline{u_{l^-}}\gamma^\nu v_{l^+}\frac{g_\phi}{(p_++p_-)^2}\epsilon_\phi^\nu. \tag{10}$$

For the ϕ meson decaying on shell, we replace $1/(p_++p_-)^2\to1/m_\phi^2$

We shall assume that $g_\phi(q^2)$ defined in Eq. (4) does not vary appreciably with q^2 in the region of interest for our calculation [16], which is taken as $m_\eta\leqslant\sqrt{m_{+-}^2}\leqslant(m_D-m_\pi)$, and we use for it the value determined from $\phi\to e^+e^-$ decay

$$g_\phi=(492\ \text{MeV})^2. \tag{11}$$

The amplitude for the decay is then

$$\begin{aligned}\mathcal{A}(D^+\to\pi^+l^+l^-)&=\frac{G_F}{\sqrt{2}}a_2V_{us}^*V_{cs}\langle\pi^+|\overline{u}\gamma_\mu(1-\gamma_5)c|D^+\rangle \times g_\phi\frac{g_{\mu\nu}-(p_++p_-)_\mu(p_++p_-)_\nu/m_\phi^2}{(p_++p_-)^2-m_\phi^2+i\Gamma_\phi m_\phi} \times\frac{e^2}{3}\overline{u_{l^-}}\gamma^\nu v_{l^+}\frac{g_\phi}{(p_++p_-)^2}\\ &=\frac{G_F}{\sqrt{2}}a_2V_{us}^*V_{cs}g_\phi^2\frac{2e^2}{3}\frac{1}{m_{+-}^2}\frac{1}{m_{+-}^2-m_\phi^2+i\Gamma_\phi m_\phi} \times f_+(m_{+-}^2)\overline{u_{l^-}}\not{p}_\pi v_{l^+},\end{aligned} \tag{12}$$

where we used for the ϕ propagator a Breit-Wigner form to account for the behavior throughout the region of decay.

Since at the η mass the sizable $D^+\to\pi^+(\eta)\to\pi^+\mu^+\mu^-$ opens $[B(D^+\to\pi^+\eta)=(7.5\pm2.5)\times10^{-3}]$, we shall impose a lower cut on the m_{+-} spectrum of the muon pairs above the η mass. We have also checked that changing this cut up to 700 MeV does not affect practically our results. We shall make for convenience the same cut in the e^+e^- channel. Then, the spectra for e^+e^- and $\mu^+\mu^-$ are essentially identical in the chosen region for m_{+-}. By restricting our considerations to this region, we also avoid a possible ambiguity concerning th q^2 dependence in the region close to $q^2=0$.

The contribution of ϕ to the long distance $D^+\to\pi^+l^+l^-$ decay in the resonance region is given by

$$\Gamma_R=\int_{(m_\phi-\Gamma_\phi/2)^2}^{(m_\phi+\Gamma_\phi/2)^2} dm_{+-}^2\,\frac{d\Gamma(D^+\to\pi^+l^+l^-)}{dm_{+-}^2}, \qquad (13)$$

and we find it to contribute to the branching ratio

$$^{(\Gamma_R)}B(D^+\to\pi^+l^+l^-)=0.82\times10^{-6}. \qquad (14)$$

If we extend the limits in Eq. (13) to $m_\phi\pm\Gamma_\phi$, this branching ratio becomes 1.22×10^{-6}.

Now we turn to the region outside the resonance, which is the main object of the present investigation. We denote this contribution as $\Gamma_{\rm NR}$, and we calculate it from the amplitude of Eq. (12) for the region $m_\eta\leqslant m_{+-}\leqslant(m_\phi-\Delta)$ and $(m_\phi+\Delta)\leqslant m_{+-}\leqslant(m_D-m_\pi)$. The branching ratios for the long distance contribution outside the resonance region, for several values of Δ, are given in Table I.

We remind the reader now that the short distance contribution [5] to these decays is about two orders of magnitude smaller than the values of Tables I. Hence, the typical interference distribution in m_{+-} expected in $b\to sl^+l^-$ [4] should not appear in $D\to\pi l^+l^-$, the spectrum of the lepton pair in our case being given by the matrix element of Eq. (12) and phase space only.

TABLE I. The long distance contribution $\Gamma_{\rm NR}$ to the $D^+\to\pi^+\mu^+\mu^-$ or $D^+\to\pi^+e^+e^-$ decay rates, outside the ϕ-resonance (beyond $m_\phi\pm\Delta$).

Δ (MeV)	$\Gamma_{\rm NR}$ (10^{-19} GeV)	Branching ratio
5	3.66	5.9×10^{-7}
10	1.92	3.1×10^{-7}
20	0.95	1.5×10^{-7}
40	0.45	0.73×10^{-7}

In the present paper, we concentrated on the $D^+\to\pi^+l^+l^-$ modes. For the parallel decays $D^0\to\pi^0l^+l^-$, we expect in our model a branching ratio smaller by approximately 5, due to a factor of 2 from the $\Delta I=\frac{1}{2}$ weak $D\to\pi$ transition and a factor of 2.5 from the D^+-D^0 lifetime difference.

We remark that a previous long-distance calculation of $D\to\pi l^+l^-$ (first Ref. of [7]) has considered it as evolving from the $D\to D^*\pi$ process, with the virtual D^* decaying to a lepton pair. A rate of 10^{-8} has been found for this contribution; this is consistent with our result as this specific diagram is obviously only part of the form factor we considered here for the transition (5).

We conclude by stressing that our results, presented in Eq. (13) and in Table I, invalidate the earlier expectations [6] that branching ratios above $\sim10^{-7}$ would constitute a signal for physics beyond the standard model.

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[1] E653 Collaboration, K. Kodama *et al.*, Phys. Lett. **B345**, 85 (1995).
[2] E791 Collaboration, E. M. Aitala *et al.*, Phys. Rev. Lett. **76**, 364 (1996).
[3] CLEO Collaboration, A. Freyberger *et al.*, Phys. Rev. Lett. **76**, 3065 (1996).
[4] G. Burdman, E. Golowich, J. L. Hewett, and S. Pakvasa, Phys. Rev. D **52**, 6383 (1995).
[5] Wei-Shu Hou, R. S. Willey, and A. Soni, Phys. Rev. Lett. **58**, 1608 (1987); **60**, 2337(E) (1988).
[6] A. J. Schwartz, Mod. Phys. Lett. **A8**, 967 (1993).
[7] K. S. Babu, X. G. He, X.-Q. Li, and S. Pakvasa, Phys. Lett. B **205**, 540 (1988); J. L. Hewett, presented at LAFEX International School on High Energy Physics (LISHEP 95), Report No. SLAC-PUB-95-6821 (unpublished).
[8] C. S. Lim, T. Morozumi, and A. I. Sanda, Phys. Lett. B **218**, 343 (1989); N. G. Deshpande, J. Trampetic, and K. Panose, Phys. Rev. D **39**, 1461 (1989); P. J. O'Donnell and H. K. K. Tung, *ibid.* **43**, 2067 (1991); P. J. O'Donnell, M. Sutherland, and H. K. K. Tung, *ibid.* **46**, 4091 (1992).
[9] Particle Data Group, M. Barnett, *et al.*, Phys. Rev. D **54**, 1 (1996).
[10] A. Khodjamirian, G. Stoll, and D. Wyler, Phys. Lett. B **358**, 129 (1995); A. Ali and V. M. Braun, *ibid.*, **359**, 223 (1995).
[11] M. Baur, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987).
[12] M. B. Wise, Phys. Rev. D **45**, 2188 (1992).
[13] ACCMOR Collaboration, S. Barlag *et al.*, Phys. Lett. B **278**, 480 (1992).
[14] MARK-III Collaboration, J. Adler *et al.*, Phys. Rev. Lett. **62**, 1821 (1989); CLEO Collaboration, F. Butler *et al.*, Phys. Rev. D **52**, 2656 (1995).
[15] E687 Collaboration, P. L. Frabetti *et al.*, Phys. Lett. B **382**, 312 (1996).
[16] G. Eilam, A. Ioannissian, R. R. Mendel, and P. Singer, Phys. Rev. D **53**, 3629 (1996).