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Constraints on the τ neutrino mass and mixing from precise measurements of τ decay rates

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We have derived constraints on the τ neutrino mass and fourth generation mixing from an analysis of the partial widths of τ lepton decays, in particular, $\tau^-\to e^-\overline{\nu_e}\nu_\tau$, $\tau^-\to\mu^-\overline{\nu_\mu}\nu_\tau$, $\tau\to\pi^-\nu_\tau$, and $\tau\to K^-\nu_\tau$. We present predictions for the τ decay widths, allowing for a nonzero τ neutrino mass m_{ν_τ} and for mixing with a neutrino of mass $m_{\nu_L}>M_Z/2$, which is parametrized using a Cabibbo-like mixing angle θ_L. By comparison of these theoretical predictions with the experimental measurements, we obtain the following bounds at the 90% confidence level: $m_{\nu_\tau}<42$ MeV and $\sin^2\theta_L<0.014$. [S0556-2821(97)50101-9]

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I. INTRODUCTION

Massive neutrinos feature in many extensions to the standard model [1]. They have been suggested as a source of dark matter [2] and, if they oscillate in the Sun, as an explanation for the deficit in the observed solar neutrino flux [3, 4]. The best experimental upper limit on the τ neutrino mass is $m_{\nu_\tau}<24$ MeV at the 95% confidence level [5], which was obtained using many-body hadronic decays of the τ.

In this paper we describe a complementary method for constraining the τ neutrino mass m_{ν_τ} from precise measurements of τ partial widths for the following decays:[1] $\tau^-\to e^-\overline{\nu_e}\nu_\tau$, $\tau^-\to\mu^-\overline{\nu_\mu}\nu_\tau$, $\tau^-\to\pi^-\nu_\tau$, and $\tau^-\to K^-\nu_\tau$. The dependence of the purely leptonic decay rates on m_{ν_τ} has been considered by others [6–8], whereas, to the best of our knowledge, the hadronic decays have not previously been analyzed for this purpose. The τ partial decay widths are also sensitive to mixing in the leptonic sector with a fourth generation weak isospin lepton doublet (ν_L, L^-) [6,9–13], where the neutrino has a mass $M>M_Z/2$.

We present calculations of the τ partial widths for these channels which allow for the effects of nonzero τ neutrino mass and mixing. Then, using data from e^+e^- experiments, we derive constraints on the τ neutrino mass and mixing.

This approach is complementary to traditional analyses of the kinematic end point of multihadron τ decays, which yield tighter constraints on m_{ν_τ} but which are insensitive to fourth generation mixing. Moreover, the channels we consider are statistically independent and theoretically very well understood.

II. PARTIAL WIDTHS FOR τ DECAYS

In the standard model describing the electroweak interaction, the partial width Γ_ℓ for the decay $\tau^-\to\ell^-\overline{\nu_\ell}\nu_\tau(X_{\rm EM})$, with $\ell^-=e^-$, μ^- and $X_{\rm EM}=\gamma$, $\gamma\gamma$, $e^+e^-,\ldots$, is given by

$$\Gamma_\ell\equiv\frac{\mathcal{B}_\ell}{\tau_\tau}=\frac{G_F^2 m_\tau^5}{192\pi^3}R_\ell F_\ell, \tag{1}$$

where G_F is the Fermi constant, m_τ and τ_τ are the τ mass and lifetime. The radiative–correction function R_ℓ has been calculated [14–17], with $\alpha(m_\tau)\simeq 1/133.3$ [17], to be

[1] Henceforth we denote the branching ratios for these processes as $\mathcal{B}_e$, $\mathcal{B}_\mu$, $\mathcal{B}_\pi$, $\mathcal{B}_K$, respectively; $\mathcal{B}_\ell$ denotes either $\mathcal{B}_e$ or $\mathcal{B}_\mu$ while $\mathcal{B}_h$ denotes either $\mathcal{B}_\pi$ or $\mathcal{B}_K$.

$$R_\ell = \left[1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right]\left[1 + \frac{3}{5}\frac{m_\tau^2}{m_W^2} + \cdots\right] \simeq 0.9960. \tag{2}$$

If the neutrino masses are zero for all generations, then the phase-space factor F_ℓ^0 is given by the well-known expression

$$F_\ell^0(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4 \simeq \begin{cases} 1 & (\ell = e), \\ 0.9726 & (\ell = \mu), \end{cases} \tag{3}$$

where $x = m_\ell^2/m_\tau^2$. A value of $G_F = (1.16639 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2}$ [18] is obtained from the measurement of the muon lifetime using Eq. (1), with $\ell^- = e^-$ and substituting $\tau \to \mu$. G_F implicitly includes the residual effects of radiative corrections not explicitly included in Eq. (2).

The partial widths for the decays $\tau^- \to h^- \nu_\tau(\gamma)$, with $h^- = \pi^-/K^-$, are given by

$$\Gamma_h \equiv \frac{\mathcal{B}_h}{\tau_\tau} = \frac{G_F^2 m_\tau^3}{16\pi} f_h^2 R_h |V_{\alpha\beta}|^2 F_h, \tag{4}$$

where f_h are the hadronic form factors, f_π and f_K, and $V_{\alpha\beta}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, V_{ud} and V_{us}, for π^- and K^-, respectively. From an analysis of $\pi^- \to \mu^- \overline{\nu_\mu}$ and $K^- \to \mu^- \overline{\nu_\mu}$ decays, one obtains $f_\pi |V_{ud}| = (127.4 \pm 0.1)$ MeV and $f_K |V_{us}| = (35.18 \pm 0.05)$ MeV [19, and references therein]. The radiative correction factor R_h is given by [17]

$$R_h = 1 + \frac{2\alpha}{\pi} \ln\left(\frac{m_Z}{m_\tau}\right) + \cdots \simeq 1.02. \tag{5}$$

The ellipsis represents terms, estimated to be $\sim \pm 0.01$ [19], which are neither explicitly treated nor implicitly absorbed into G_F, $f_\pi |V_{ud}|$, or $f_K |V_{us}|$. For massless neutrinos the phase-space function F_h^0 is given by

$$F_h^0(x) = (1-x)^2 \simeq \begin{cases} 0.9877 & (h = \pi), \\ 0.8516 & (h = K), \end{cases} \tag{6}$$

where m_h is the hadron mass and $x = m_h^2/m_\tau^2$.

We now consider the effects of neutrino masses and mixing on the τ decay rates. The expression for Γ_ℓ, allowing for nonzero neutrino masses and mixing between n lepton generations, is given by

$$\Gamma_\ell = \frac{G_F^2 m_\tau^5}{192\pi^3} R_\ell \sum_{i=1}^{n} \sum_{j=1}^{n} |U_{\tau i}|^2 |U_{\ell j}|^2 F_\ell, \tag{7}$$

where U_{ai} is the lepton mixing matrix [18, p. 276] and F_ℓ, the phase-space factor, depends on the neutrino and charged lepton masses.

The electron and muon neutrinos couple dominantly to ν_1 and ν_2 and have small masses [18]. We therefore neglect the masses of ν_1 and ν_2 and their mixing with ν_3. We do, however, consider the possible existence of a fourth generation neutrino, ν_L, of mass $m_{\nu_L} > M_Z/2$ which mixes with ν_τ (data from the CERN e^+e^- collider LEP have constrained the number of neutrinos of mass $m_\nu < M_Z/2$ to be

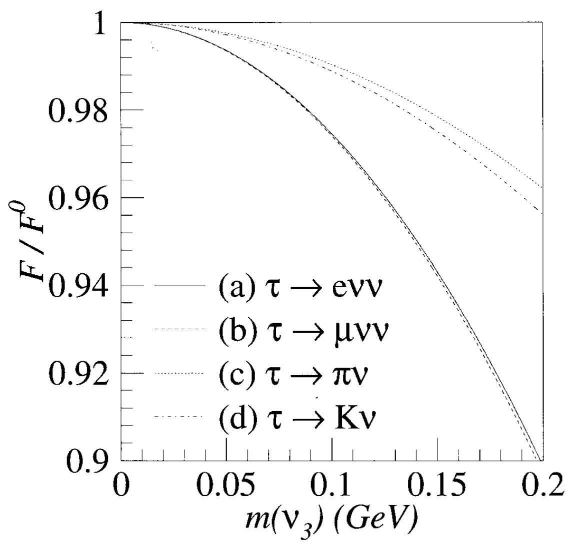

FIG. 1. Relative phase-space suppression factors, as a function of τ neutrino mass for (a) $\tau^- \to e^- \overline{\nu_e} \nu_\tau$, (b) $\tau^- \to \mu^- \overline{\nu_\mu} \nu_\tau$, (c) $\tau^- \to \pi^- \nu_\tau$, and (d) $\tau^- \to K^- \nu_\tau$.

$N_\nu = 2.991 \pm 0.016$ [18]). Since such a neutrino is kinematically forbidden in τ decays, the corresponding phase-space factor is zero. Γ_ℓ is therefore suppressed by a factor which depends on the strength of the mixing of the third and fourth generations, but not on the mass m_{ν_L}. From the above, Eq. (7) reduces to

$$\Gamma_\ell = \frac{G_F^2 m_\tau^5}{192\pi^3} R_\ell (1 - \sin^2\theta_L) F_\ell, \tag{8}$$

where the factor of $(1 - \sin^2\theta_L) \equiv |U_{\tau 3}|^2$ allows for the Cabibbo-like suppression due to fourth generation mixing, and the phase-space factor is given by

$$F_\ell(x,y) = F_\ell^0(x) - 8y(1-x)^3 + \cdots, \tag{9}$$

where $y = m_{\nu_3}^2/m_\tau^2$, F_ℓ^0 is given by Eq. (3) and the ellipsis represents negligible terms of higher order in m_{ν_3}. Figures 1(a) and 1(b) show the variation of F_ℓ/F_ℓ^0 with m_{ν_3} for $\tau^- \to e^- \overline{\nu_e} \nu_\tau$ and $\tau^- \to \mu^- \overline{\nu_\mu} \nu_\tau$ decays, respectively.

The expression for Γ_h, allowing for nonzero neutrino masses and mixing in a similar fashion, is given by

$$\begin{aligned} \Gamma_h &= \frac{G_F^2 m_\tau^3}{16\pi} f_h^2 R_h |V_{\alpha\beta}|^2 \sum_{i=1}^{n} |U_{\tau i}|^2 F_h \\ &= \frac{G_F^2 m_\tau^3}{16\pi} f_h^2 R_h |V_{\alpha\beta}|^2 (1 - \sin^2\theta_L) F_h \end{aligned} \tag{10}$$

Where the phase-space function has been calculated to be

$$F_h(x,y) = F_h^0(x)\left[1 - y\left(\frac{2+x-y}{1-x}\right)\right]\sqrt{1 - y\left[\frac{2+2x-y}{(1-x)^2}\right]}, \tag{11}$$

with $y = m_{\nu_3}^2/m_\nu^2$. Figures 1(c) and 1(d) show the variation of F_h/F_h^0 with m_{ν_3} for $\tau \to \pi^- \nu_\tau$ and $\tau \to K^- \nu_\tau$ decays, respectively. Note the lower sensitivity to m_{ν_3} of the (two-

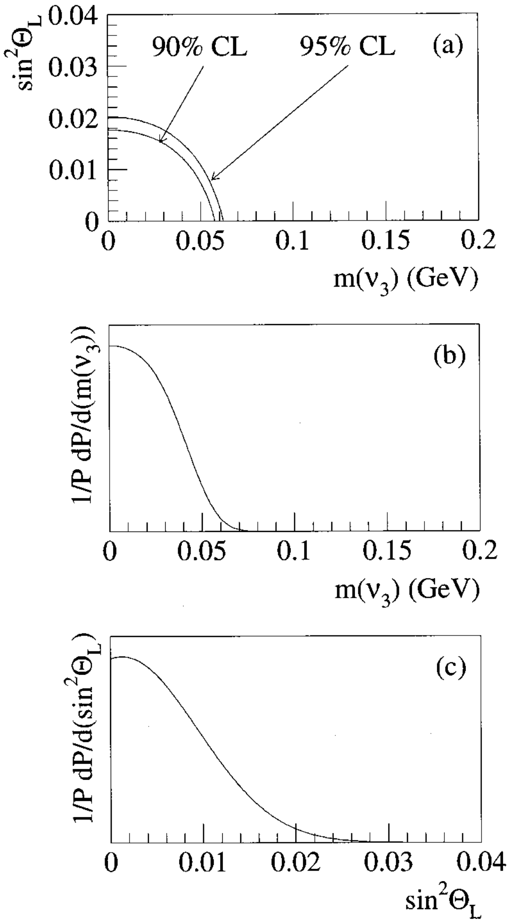

FIG. 2. Likelihood distributions for all τ decay channels combined, for (a) m_{ν_3} vs $\sin^2\theta_L$, (b) m_{ν_3}, integrated over $\sin^2\theta_L$, and (c) $\sin^2\theta_L$, integrated over m_{ν_3}.

body) hadronic decay modes compared to the (three-body) leptonic modes, despite the higher masses of the final state hadrons.

III. CONSTRAINTS ON ν_τ MASS AND MIXING

In the standard model of electroweak interactions, the three lepton generations interact in an identical way, apart from effects caused by their differing masses. In particular, they each couple with the same strength to the charged weak current. If this assumption of universality is relaxed for the τ, then the G_F^2 factor appearing in Eqs. (1) and (4) may be replaced by $G_F G_F^\tau$, where G_F^τ is not necessarily equal to G_F. If the measured values of G_F^τ, evaluated assuming massless neutrinos and no mixing, appeared to be significantly smaller than G_F it could indicate either new physics [13] or the suppression of decay rates due to nonzero neutrino masses or mixing.

We use the Particle Data Group values and errors [18] for the measured quantities: in particular,

$$\tau_\tau = (291.0 \pm 1.5)\ \text{fs}; \quad m_\tau = (1776.96^{+0.18+0.25}_{-0.21-0.17})\ \text{MeV};$$
$$\mathcal{B}_e = (17.80 \pm 0.08)\%; \quad \mathcal{B}_\mu = (17.30 \pm 0.10)\%;$$
$$\mathcal{B}_\pi = (11.07 \pm 0.18)\%; \quad \mathcal{B}_K = (0.71 \pm 0.05)\%.$$

TABLE I. Upper limits obtained on m_{ν_3} and $\sin^2\theta_L$, separately for each τ decay channel and for all channels combined.

	Upper limit on m_{ν_3} (MeV) at the 90% (95%) C.L.		Upper limit on $\sin^2\theta_L$ at the 90% (95%) C.L.	
$\tau^- \to e^- \overline{\nu_e} \nu_\tau$	63	(72)	0.016	(0.019)
$\tau^- \to \mu^- \overline{\nu_\mu} \nu_\tau$	66	(76)	0.018	(0.022)
$\tau^- \to \pi^- \nu_\tau$	141	(160)	0.028	(0.033)
$\tau^- \to K^- \nu_\tau$	333	(375)	0.036	(0.038)
All channels	42	(48)	0.014	(0.017)

The sole exception is the use of the BES τ mass [20], obtained from the measurement of $\tau^+\tau^-$ production near threshold, since this has no dependence on m_{ν_τ}. Substituting in Eqs. (1) and (4) for the measured quantities, and assuming that $m_{\nu_\tau}=0$ and $\sin^2\theta_L=0$ we obtain

$$\frac{G_F^\tau}{G_F} = \begin{cases} 0.996 \pm 0.007 \ \ (-0.54\sigma) & \text{for } \tau^- \to e^- \overline{\nu_e} \nu_\tau, \\ 0.996 \pm 0.008 \ \ (-0.55\sigma) & \text{for } \tau^- \to \mu^- \overline{\nu_\mu} \nu_\tau, \\ 1.010 \pm 0.017 \ \ (+0.60\sigma) & \text{for } \tau^- \to \pi^- \nu_\tau, \\ 0.987 \pm 0.070 \ \ (-0.18\sigma) & \text{for } \tau^- \to K^- \nu_\tau. \end{cases} \tag{12}$$

where the number of standard deviations from the universality prediction (of unity) are shown in parentheses. These results, which are all consistent with unity, indicate that the lepton couplings are universal and show no indications for nonzero neutrino mass or mixing. We therefore *assume* universality holds and use the measured τ partial widths to constrain m_{ν_3} and $\sin^2\theta_L$.

We have mapped the likelihood of observing the measured τ partial widths, as a function of both m_{ν_3} and $\sin^2\theta_L$, by randomly sampling all the quantities used according to their experimental errors, allowing for the estimated 1% theoretical uncertainty on R_h. The CLEO measurement of the τ mass was used to further constrain m_{ν_3}. From an analysis of $\tau^+\tau^- \to (\pi^+ n \pi^0 \overline{\nu_\tau})$ events (with $n \leqslant 2$, $m \leqslant 2$, $1 \leqslant n+m \leqslant 3$), CLEO determined the τ mass to be $m_\tau = (1777.8 \pm 0.7 \pm 1.7) + [m_{\nu_3}\ (\text{MeV})]^2/1400$ MeV [21]. The likelihood for the CLEO and BES measurements to agree, as a function of m_{ν_3} is included in the global likelihood.

Figure 2(a) shows the 90% and 95% contours of the two-dimensional likelihood distribution combined for all four τ decay channels. No evidence is seen for a nonzero neutrino mass, nor for mixing. By integration of the two-dimensional likelihood over all values of $\sin^2\theta_L$, we obtain the one dimensional likelihood for m_{ν_3}, independent of the value of $\sin^2\theta_L$, as shown by the solid line of Fig. 2(b). We obtain the following upper limits: $m_{\nu_3} < 42(48)$ MeV at the 90(95)% confidence levels. The solid line of Fig. 2(c) shows the one-dimensional likelihood distribution for $\sin^2\theta_L$, integrated over all values of m_{ν_3}, from which we derive the upper limits: $\sin^2\theta_L < 0.014(0.017)$ at the 90(95)% confidence levels. Table I summarizes the upper limits of m_{ν_3} and $\sin^2\theta_L$, individually for each channel and for all channels combined.

Since the mixing of m_{ν_3} with other neutrinos is small, the limits derived for m_{ν_3} can be reasonably interpreted as limits on m_{ν_τ}.

Noninclusion of the constraint from the CLEO τ mass measurement results in the following limits: $m_{\nu_3} < 59(68)$ MeV and $\sin^2\theta_L < 0.013(0.016)$ at the 90(95)% confidence levels. The slight reduction in the $\sin^2\theta_L$ limit values, despite the increase in the m_{ν_3} limit, is due to the anticorrelation of the effects on the likelihood of nonzero values for m_{ν_3} and $\sin^2\theta_L$.

IV. CONCLUSIONS

We have derived the following constraints on the τ neutrino mass and fourth generation mixing from an analysis of the partial widths of tau lepton decays: $m_{\nu_\tau} < 42$ MeV and $\sin^2\theta_L < 0.014$ at the 90% confidence level. These results are statistically and systematically independent of traditional end-point analyses using multihadronic decays of the τ. Moreover we simultaneously consider mixing, and are insensitive to fortuitous or pathological events close to the kinematic limits, details of the resonant structure of multihadron τ decays, and the absolute energy scale of the detectors.

These results will improve with more precise measurements of the τ mass, lifetime, and branching fractions, for example at a τ-charm factory. Ultimately, we expect that this method will be limited by the uncertainty on the τ lifetime. The extension of this technique to include multihadronic τ decays, in conjunction with an improved theoretical description, should provide considerably more sensitivity due to the higher multiplicity of the final states.

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