Interaction of a small-size wave packet with a hadron target

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(Received 23 April 1996)

We calculate in QCD the cross section for the scattering of an energetic small-size wave packet off a hadron target. We use our results to study the small-σ behavior of $P_{\pi N}(\sigma)$, the distribution over the cross section for the pion-nucleon scattering, in the leading α_s order. [S0556-2821(97)01001-1]

PACS number(s): 12.38.Bx, 13.60.Hb

I. INTRODUCTION

Recently phenomena involving interactions of hadrons in small-size configurations have been intensively discussed both in relation with the phenomenon of color transparency and vector meson electroproduction observed at energies reached at the DESY ep collider HERA. There is also a deep relation between presence of the weakly interacting small-size configurations in hadrons and phenomenon of cross-section fluctuations in the interactions of the hadrons which manifests itself in the inelastic coherent diffraction processes: $h+N(A)\rightarrow X+N(A)$, see Ref. [1]. In this paper, we focus on the systematic derivation of the formulas for the interaction of the color singlet $q\overline{q}$ pair having a small transverse size with a hadron target. Then, we use these formulas to calculate the probability of the distribution for the interaction of a photon and a pion with a target for small interaction cross sections. Although some of equations deduced in the paper existed before, no derivations with analysis of their accuracy have been presented.

The paper is organized as follows. In Sec. II, we consider the virtual forward Compton amplitude in the small-x region where it is dominated by the photon-gluon scattering subprocess. We outline there a derivation of the basic formula expressing the total cross section $\sigma_{\gamma^* T}$ as a convolution of the gluon distribution amplitude $G_T(x,Q^2)$ and the γg scattering cross section. In Sec. III, we write down the γg cross section in terms of the $q\overline{q}$ light-cone wave functions of the virtual photons. In the next section, we calculate the cross-section distribution $P_{\gamma *}(\sigma)$ for the virtual photon. In Sec. V, we discuss the quark-hadron duality interplay between the perturbative free-quark results and contributions due to low-lying resonances. Finally, in Sec. VI, we calculate the cross-section distribution for the pion $P_\pi(\sigma)$ in the small cross-section limit where it is governed by $q\overline{q}$ configurations having small spatial size. Basing on QCD evolution equation we evaluate also the functional dependence of $P_\pi(\sigma)\rightarrow 0$ on σ and on the incident energy.

II. HARD $\gamma^* T$ TOTAL CROSS SECTION AND THE INTERACTION OF SMALL-SIZE CONFIGURATIONS

Let us consider a particular contribution into the $\gamma^* T$ cross section corresponding to a transformation of the virtual photon when γ^* converts into a $Q\overline{Q}$ pair with quarks having a large relative transverse momentum. Usually, this contribution is written as a convolution of the infinite momentum frame wave function of the target with the perturbative QCD (PQCD) calculable coefficient function describing the short-distance propagation of the particles between two virtual photon vertices. Our aim is to express the relevant coefficient function in terms of the light-cone wave functions of the virtual photons as viewed from the reference frame where the target is at rest. The contribution we are interested in is given by the sum of diagrams shown in Fig. 1.

The lower blob corresponds to the gluon distribution in the target. It is convenient to parametrize the gluon momentum k in terms of the Sudakov variables

$$k=-\alpha q'+\beta p'+k_t, \quad d^4k=\frac{s}{2}d\alpha d\beta d^2k_t. \quad (2.1)$$

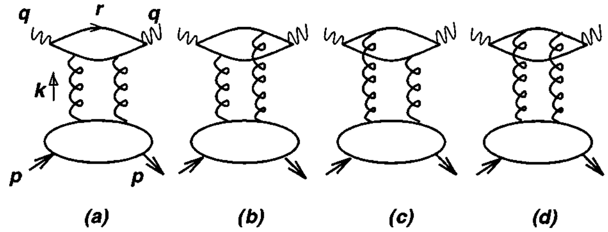

FIG. 1. Leading small-x contribution to the forward virtual Compton amplitude.

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Here, q' and p' are lightlike momenta related to p,q by

$$q=q'+\frac{q^2}{2(p'q')}p', \quad p=p'+\frac{p^2}{2(p'q')}q', \tag{2.2}$$

$$2(pq)=2(p'q')+\frac{q^2p^2}{2(p'q')}. \tag{2.3}$$

For our goals, the most interesting region is that of small values of the Bjorken parameter: $x=-q^2/2(pq)\to 0$, where we may safely approximate $2(p'q')=s$.

In the d^4k integral, the region $k_t^2\sim Q^2$ corresponds to the next-order α_s correction, so we will take into account only the contribution of the region $k_t^2\ll Q^2$. This corresponds to the leading $\alpha_s \ln Q^2/\Lambda^2$ approximation in which the $O(\alpha_s)$ corrections are neglected. In this kinematical region, the contribution of the diagrams shown in Fig. 1 can be considerably simplified. The essential region of integration is

$$|k^2|=|\alpha\beta s+k_t^2|\ll Q^2. \tag{2.4}$$

Note that $\beta s\sim Q^2\propto(\text{mass})^2$ of the $q\bar{q}$ state produced by γ^*. Hence,

$$\alpha\ll 1 \tag{2.5}$$

is the essential region of integration over α. It is convenient to write the propagator $d_{\mu\bar{\mu}}(k)/k^2$ of the exchanged gluon in the light-cone gauge $q'_\mu A^\mu=0$, in which

$$d_{\mu\bar{\mu}}(k)=-g_{\mu\bar{\mu}}+\frac{q'_\mu k_{\bar{\mu}}+k_\mu q'_{\bar{\mu}}}{(kq')}.$$

It can be shown (cf. [2]) that in our case the $k_\mu q'_{\bar{\mu}}$ part of the propagator dominates. Using Eq. (2.5), the dominant part of the gluon propagator can be further simplified:

$$d_{\mu\bar{\mu}}(k)\simeq\frac{p'_\mu q'_{\bar{\mu}}}{(p'q')}. \tag{2.6}$$

In other words, it is sufficient to take into account only the longitudinal polarization of the exchanged gluons. Indeed, let us estimate the contribution due to exchange of a transversely polarized gluon:

$$\delta\sigma\sim\frac{1}{(2\pi)^4}\int\frac{T^{\gamma^*}_{\mu_\perp\lambda_\perp}}{s}d(\beta s)\quad\int\frac{T^{T}_{\mu_\perp\lambda_\perp}}{s}d(\alpha s)\frac{d^2k_t}{(k^2)^2}. \tag{2.7}$$

Here, $T^{(\gamma^*)}_{\mu_\perp\lambda_\perp}$ is the imaginary part of the amplitude of the γ^* scattering off a gluon given by the lowest-order Feynman diagrams and $T^{T}_{\mu_\perp\lambda_\perp}$ is that for the gluon scattering off a target T. Using the fact that, at high energies, the Feynman amplitude of processes due to exchange by two elementary fermions tends to constant [3], we obtain

$$\int T^{\gamma^*}_{\mu_\perp\lambda_\perp}d(\beta s)\propto\frac{(\beta s)^2}{Q^2}\simeq(Q^2).$$

In this estimate we use also scaling over Q^2 in the box diagram. The amplitude due to the exchange by two-vector particles increases like s, and we have

$$\int T^{(T)}_{\mu_\perp\lambda_\perp}d\alpha s\propto(\alpha s)^{2+n}.$$

Here, $n>0$, since, according to the QCD evolution equations, the deep inelastic amplitudes increase with energy in region of applicability of the perturbative QCD.

As a result of this power counting estimate we obtain

$$\begin{aligned}\delta\sigma&\sim\int\frac{(\beta s)^2}{s}\frac{(\alpha s)^{2+n}}{s}\frac{d^2k_t}{(\alpha\beta s+k_t^2)^2}\theta(k_t^2<k_{t0}^2)\\&\quad\times\theta(\alpha\beta s\leqslant k_{t0}^2)\\&\approx\int\theta(k_t^2<k_{t0}^2)(\alpha s)^n\propto(\alpha s)^n.\end{aligned} \tag{2.8}$$

Here, we substituted $\alpha\beta s\sim k_t^2$. Thus, due to the presence of the factor α in Eq. (2.8), in the leading $\alpha_s \ln Q^2/\lambda^2$ approximation, the contribution due to the exchange of a transversely polarized gluon is negligible compared to the contribution of the longitudinal polarization specified by Eq. (2.6). We use here the observation that in QCD the power n characterizing the energy dependence of the amplitude is the same for scattering of transversely and longitudinally polarized gluons.

Using the gluon propagator in the form given by Eq. (2.6), we get the following expression for the total contribution of the diagrams shown in Fig. 1:

$$\mathrm{Im}M=\int\frac{d^4k}{(2\pi)^4 i}\frac{1}{(k^2)^2}2\,\mathrm{Im}T^{ab(P)}_{\mu\lambda}\mathrm{Im}T^{ab(T)}_{\bar{\mu}\bar{\lambda}}d_{\mu\bar{\mu}}(k)d_{\lambda\bar{\lambda}}(k). \tag{2.9}$$

Here, $T^{ab(P)}_{\mu\lambda}=T^{ab}_{\mu\lambda}(\gamma^* g\to Q\bar{Q})$ is the sum of the box diagrams describing the $\gamma^* g$ scattering and $T^{ab(T)}_{\bar{\mu}\bar{\lambda}}$ is the amplitude of the gluon scattering off the target T.

Using the dominance of the longitudinal gluon polarization (2.6) and incorporating Eq. (2.1) we can rewrite Eq. (2.9) as

$$\frac{\mathrm{Im}M}{s}=\int\frac{s\,d\alpha\,d\beta\,d^2k_t}{2(2\pi)^4(k^2)^2}\frac{2\,\mathrm{Im}T^{ab(P)}_{\mu\lambda}p_\mu p_\lambda}{4(pq)^2}\frac{4\,\mathrm{Im}T^{ab(T)}_{\bar{\mu}\bar{\lambda}}q_{\bar{\mu}}q_{\bar{\lambda}}}{s}. \tag{2.10}$$

Now, we will use the fact that

$$T^{ab(P)}_{\mu\lambda}k_\mu=T^{ab(P)}_{\mu\lambda}k_\lambda=0, \tag{2.11}$$

since the box diagram contains no gluons and, therefore, the Ward identities in this approximation are the same as in an Abelian gauge theory (a,b are the color indices). From Eqs. (2.1) and (2.11) it follows that

$$\frac{\mathrm{Im}T^{ab(P)}_{\mu\lambda}p_\mu p_\lambda}{4(pq)^2}=\frac{\mathrm{Im}T^{ab(P)}_{\mu_\perp\lambda_\perp}k_t^\mu k_t^\lambda}{(\beta s)^2} \tag{2.12}$$

and

$$\frac{\mathrm{Im}M}{s}=\int d\beta s\frac{\frac{1}{2}\sum T^{ab(P)}_{\mu_\perp\mu_\perp}}{(\beta s)^2}\int\frac{d\alpha s d^2k_t}{(2\pi)^4(k^2)^2}\times\frac{4k_t^2\mathrm{Im}T^{ab(T)}_{\bar{\mu}\bar{\lambda}}q_{\bar{\mu}}q_{\bar{\lambda}}}{s^2}. \tag{2.13}$$

It is useful to define the cross section of γ^* scattering off a gluon g averaged over the gluon color:

$$\delta_{ab}\cdot s'\cdot\sigma(\gamma^*g\to q\bar{q})=\frac{1}{2}\sum_{\mu_\perp=1,2}\mathrm{Im}T^{ab(P)}_{\mu_\perp\mu_\perp}, \tag{2.14}$$

where s' is the invariant mass of the produced $q\bar{q}$ system: $s'=(k+q)^2\simeq\beta s-Q^2$. Thus,

$$\sigma_{\gamma^*T}=\frac{\mathrm{Im}M}{s}=\int\frac{d\beta}{\beta}\sigma(\gamma^*g\to\overline{qq})\int\frac{s d\alpha d^2k_t}{(2\pi)^4(k^2)^2}k_t^2\times\sum_a\frac{4\mathrm{Im}T^{a(T)}_{\bar{\mu}\bar{\lambda}}q_{\bar{\mu}}q_{\bar{\lambda}}}{s^2}. \tag{2.15}$$

In the leading $\alpha_s\ln Q^2$ approximation, we can substitute k^2 by k_t^2 Comparing our result with the QCD-improved parton model expression for the production of heavy quarks (see e.g., [4]), we observe that

$$\int\frac{s d\alpha d^2k_t}{(2\pi)^4k_t^2}\sum_a 4\mathrm{Im}T^{aa(T)}_{\bar{\mu}\bar{\lambda}}q_{\bar{\mu}}q_{\bar{\lambda}}=\beta G_T(\beta,Q^2), \tag{2.16}$$

where G_T is the gluon distribution in a target T. This gives

$$\sigma_{\gamma^*T}=\int\sigma_{\gamma^*g}\frac{d\beta}{\beta}[\beta G_T(\beta,Q^2)]. \tag{2.17}$$

The first argument of $G(\beta,Q^2)$ is $\beta=(Q^2+M^2)/s$. Here, M is the mass of the produced $q\bar{q}$ pair, which is typically of the order of Q. Hence, the essential region of integration is $\beta\sim x$. As the evolution scale for the gluon distribution function, we take Q^2. Of course, higher-order α_s corrections may change Q^2 by some numerical factor. This scale-fixing ambiguity is a usual feature of the leading $\alpha_s\ln Q^2$ approximation.

III. LIGHT-CONE WAVE FUNCTIONS AND σ_{γ^*g}

Now, let us express σ_{γ^*g} in terms of the light-cone wave functions of the virtual photon. To this end, we write down the four-momenta $r_1(r_2)$ of quark (antiquark) in the box in terms of the light-cone variables $r_1=\{r_1^+,r_1^-,r_t\}$ with $r_1^+=\eta q^+$ and take the integral over r_1^- by residue. Introducing the lowest-order perturbative $\overline{qq}$ light-cone wave functions of the virtual photon [5]

$$\psi_\mu=\frac{\overline{U(r_1)}\gamma_\mu U(-r_2)}{\frac{m^2+r_t^2}{\eta(1-\eta)}+Q^2}\frac{1}{\sqrt{\eta(1-\eta)}}, \tag{3.1}$$

we obtain the following expression for the sum of the box diagrams:

$$\int d\alpha s\frac{\mathrm{Im}\,T^{ab(p)}_{\mu\lambda}p_\mu p_\lambda}{s^2}=e^2g_s^2\int\frac{d\eta d^2r_t}{2(2\pi)^3}\pi\psi_\mu(\eta,r_t)\times\{2\psi_\mu(\eta,r_t)-\psi_\mu(\eta,r_t+k_t)-\psi_\mu(\eta,r_t-k_t)\}F_aF_b, \tag{3.2}$$

where g_s^2 is the QCD coupling constant and $F_a=\lambda_a/2$, λ_a being the Gell-Mann matrices of the $\mathrm{SU}(3)_c$ group in the fundamental representation.

It is convenient to rewrite this formula in the impact parameter space:

$$\psi_\mu(x,r_t)=\int\psi_\mu(x,b)e^{ir_tb}d^2b. \tag{3.3}$$

Then,

$$\int d\alpha s\frac{\mathrm{Im}T^P_{\mu\lambda}p^\mu p^\lambda}{(2pq)^2}=\int\psi_\mu^2(x,b)\frac{dx d^2b}{4\pi}g_s^2\times\{\pi[2-e^{ik_tb}-e^{-ik_tb}]\mathrm{Tr}(F_aF_b)\}. \tag{3.4}$$

Within the leading $\alpha_s\ln Q^2$ approximation, to obtain Eq. (2.12), it is necessary to decompose exponent into a power series over (k_tb) and to keep terms up to the second order in k_t^2. Combining Eqs. (3.3), (2.10), and (2.16), we obtain

$$\sigma_{\gamma^*T}=e^2\int\psi_\mu^2(\eta,b)\frac{dz d^2b}{4\pi}N_c\left\{\frac{1}{N_c}g_s^2\pi\frac{(k_tb)^2}{k_t^2}\mathrm{Tr}\frac{F^2}{8}\right\}\times G_T(x,\lambda/b^2). \tag{3.5}$$

Here, factor λ can be estimated from analysis of $\sigma_L(\gamma^*N)$ cross section. Since the gluon density increases when x decreases, λ slowly increases with decrease of x [6]. For $x\sim10^{-3}$, $\lambda\approx9$.

It is instructive to represent σ_{γ^*T} in the form

$$\sigma_{\gamma^*T}=e^2\int\psi_\mu^2(\eta,b)\frac{d\eta d^2b}{4\pi}N_c\sigma_T^{q\bar{q}}(b^2). \tag{3.6}$$

Here, $\sigma_T^{q\bar{q}}$ is the cross section for the interaction of a colorless small transverse size $q\bar{q}$ pair with the target T:

$$\sigma_T^{q\bar{q}}=g_s^2\pi\frac{b^2}{2}\frac{1}{N_c}\mathrm{Tr}\left(\frac{F^2}{8}\right)xG_T(x,\lambda/b^2). \tag{3.7}$$

This expression was obtained originally in [7,8]. As usual, N_c is the number of colors, and the Casimir operator of the SU(3) group in the fundamental representation can be easily calculated:

$$\frac{1}{8}\frac{1}{N_c}\mathrm{Tr}F^2=\frac{1}{3}\mathrm{Tr}F_3^2=\frac{1}{6}. \tag{3.8}$$

Combining all the numbers together, we finally obtain:

$$\sigma_T^{q\bar{q}} = \frac{\pi^2}{3} b^2 [x G_T(x, \lambda/b^2)] \alpha_s(\lambda/b^2). \tag{3.9}$$

Here, $b=(b_q - b_{\bar{q}})$. This formula describes the essence of the color transparency (CT) phenomenon (cf. discussion in [9]): $q\bar{q}$ configuration of a small spatial size has a small interaction cross section. However, for sufficiently small x, the interaction becomes strong due to the formation of the soft gluon field. In this respect, Eq. (3.5) predicts the interaction of a small-size configuration which is qualitatively different from that of the models of Low [10] and of Gunion and Soper [11]. The fact that $\sigma_T^{q\bar{q}}$ is proportional to the gluon distribution in Eq. (3.7), increasing in the small-x region, has important experimental consequences, e.g., it makes it possible to observe the small-size quark configurations at HERA in the electroproduction of vector mesons at small x. In fact, Eq. (3.5) can be inferred from a formula derived in Ref. [12] within a model approximation to QCD. Using some simple tricks, one can also obtain Eq. (3.5) from a formula obtained in [13] within the leading $\alpha_s \ln x$ approximation of QCD combined with some bold assumptions concerning the parton model structure.

Using Eq. (3.9), we can calculate distribution over cross section for the fast photon or pion projectile for small σ (cf. [7]).

IV. DISTRIBUTION OF $P_{\gamma^* N}(\sigma)$ FOR THE PHOTON PROJECTILE

In the previous section we have derived Eq. (3.5) which expresses the $\sigma_{\gamma^* T}$ cross section in terms of the light-cone wave functions of the virtual photon γ^*. This formula gives us the possibility to calculate another useful quantity, distribution over cross section $P_{\gamma^* T}(\sigma)$. By definition, the differential probability that the virtual photon γ^* interacts with the target T with the cross section σ. In other words, the experimentally observable total cross section in terms of $P(\sigma)$ is given by

$$\sigma_{\gamma^* N} = \int P_{\gamma^* N}(\sigma) \sigma d\sigma. \tag{4.1}$$

In Refs. [14,15], it has been suggested to represent the cross section σ in terms of the eigenstates of the S matrix. In the case of small σ, as a result of color screening and asymptotic freedom, the scattering state is a $q\bar{q}$ pair. So, the contribution of small σ has the form of Eq. (3.6). Using Eq. (3.7), we can write

$$e\sigma_{\gamma^* N} = e^2 \int \psi_{\gamma^*}^2(\eta, b) \frac{d\nu}{4\pi} N_c \sigma \frac{\pi d b^2}{d\sigma} d\sigma\ . \tag{4.2}$$

Let us define

$$P_{\gamma^* N}(\sigma \to 0) = \int e^2 \psi_{\gamma^*}^2(\nu, b) \frac{d\nu}{4} N_c \frac{\pi d b^2}{d\sigma}, \tag{4.3}$$

where $\psi_{\gamma^*}(\eta, b)$ is given by Eqs. (3.1) and (3.3). It is implied here that the functional dependence of b on σ in Eq. (4.3) should be calculated from Eq. (3.7). Now, we can rewrite Eq. (4.2) in the form of Eq. (4.1). Though our derivation is applicable for the interactions with small σ, Eq. (4.1) has a more general nature. In fact, it has been understood long ago [14–16] that many features of the interaction of a fast projectile can be described in terms of distribution over cross section. An important advantage of such a quantity is that it accurately takes into account diffractive processes. Some properties of $P(\sigma)$ have been discussed in detail in [17]. However, for our purposes, it is sufficient to consider $P_{\gamma^* N}(\sigma)$ in the limit $\sigma \to 0$.

In general, Eq. (4.3) predicts a rather involved dependence of $P(\sigma)$ on $\ln\sigma$ at small σ. However, this dependence can be easily calculated using QCD evolution equations. The distinctive feature of Eq. (4.3) is that

$$P_{\gamma^* N}(\sigma \to 0)|_{\sigma \to 0} \sim \frac{1}{\sigma} \quad \text{up to } \ln(\sigma/\sigma_0) \text{ terms.} \tag{4.4}$$

V. TRANSITION TO MESONS

The perturbative version of the virtual photon wave function $\psi_\mu(\eta, r_t)$, Eq. (3.1), can be written through a dispersion integral

$$\psi_\mu(\eta, r_t) = \frac{1}{\pi} \int_0^\infty \psi_\mu^{q\bar{q}}(\kappa; \eta, r_t) \frac{d\kappa^2}{\kappa^2 + Q^2}, \tag{5.1}$$

where

$$\psi_\mu^{q\bar{q}}(\kappa; \eta, r_t) = \frac{\overline{U(\eta q_+)} \gamma_\mu U((1-\eta) q_+)}{\sqrt{\eta(1-\eta)}} \delta\left(\kappa^2 - \frac{m_q^2 + r_t^2}{\eta(1-\eta)}\right) \tag{5.2}$$

is the wave function of a noninteracting $q\bar{q}$ pair with invariant mass κ. The interaction between the quarks modifies the virtual photon wave function $\psi_\mu(\eta, r_t) \to \Psi_\mu(\eta, r_t)$, and the dispersion representation

$$\psi_\mu(\eta, r_t) = \frac{1}{\pi} \int_0^\infty \psi_\mu^{q\bar{q}}(\kappa; \eta, r_t) \frac{d\kappa^2}{\kappa^2 + Q^2} \tag{5.3}$$

for the "exact" wave function $\Psi_\mu(\eta, r_t)$ is in terms of the modified spectral density $\psi_\mu^{\text{hadr}}(\kappa; \eta, r_t)$ in which, instead of the free-quark approximation $\psi_\mu^{q\bar{q}}(\kappa; \eta, r_t)$, one has a sum over resonances, the ρ meson being the dominant feature in the low-κ region:

$$\psi_\mu^{q\bar{q}}(\kappa; \eta, r_t) \to \psi_\mu^{\text{hadr}}(\kappa; \eta, r_t) = g_\rho \psi_\mu^\rho(\eta, r_t) \delta(\kappa^2 - m_\rho^2) + \psi_\mu^{\text{higher states}}(\kappa; \eta, r_t), \tag{5.4}$$

where g_ρ is the magnitude of the ρ-state projection onto the electromagnetic current. At large κ, the resonances are wide, and their sum rapidly approaches the free-quark value, i.e., one has a perfect quark-hadron duality.[1] For sufficiently large Q^2, the dispersion integral (5.3) is dominated by higher

[1] Note, that since the large-κ behavior of $\psi_\mu^{\text{hadr}}(\kappa; \eta, r_t)$ coincides with that of $\psi_\mu^{q\bar{q}}(\kappa; \eta, r_t)$, the dispersion integral in Eq. (5.3) has the same convergence properties as those in Eq. (5.1), i.e., no subtractions are needed in Eq. (5.3).

states, and the free-quark approximation is completely justified. Decreasing Q^2, one would observe mismatch between the free-quark calculation and the dispersion integral over the resonances. Such a situation is well known from QCD sum rules: the difference between the resonance and free-quark spectra is described by power corrections $(1/Q^2)^N$. The usual procedure is to approximate the higher states by the free-quark contribution (''first resonance plus continuum'' model)

$$\psi_\mu^{\text{higher states}}(\kappa;\eta,r_t)=\theta(\kappa^2>s_0^\rho)\,\psi_\mu^{\bar{q}q}(\eta,r_t),$$

where s_0^ρ is the effective threshold for higher resonances in the ρ channel and then fix its value by the requirement of the best agreement between the two sides of the resulting sum rule

$$\frac{1}{\pi}\int_0^{s_0^\rho}[\pi g_\rho \psi_\mu^\rho(\eta,r_t)\,\delta(\kappa^2-m_\rho^2)-\psi_\mu^{\bar{q}q}(\kappa;\eta,r_t)]\frac{d\kappa^2}{\kappa^2+Q^2}$$
$$=\sum_{N=2}\frac{A_N}{(Q^2)^N}. \tag{5.5}$$

After fixing s_0^ρ from the magnitude of the power corrections $A_N/(Q^2)^N$, one can take the limit $Q^2\to\infty$ to get the local duality relation

$$\pi g_\rho \psi_\mu^\rho(\eta,r_t)=\int_0^{s_0^\rho}\psi_\mu^{\bar{q}q}(\kappa;\eta,r_t)\;d\kappa^2. \tag{5.6}$$

In other words, the ρ-meson wave function in such an approach is dual to the free-quark wave functions integrated over the duality interval $0\leqslant\kappa^2\leqslant s_0^\rho$.

For the forward virtual Compton amplitude, the dispersion representation can be applied both for the initial and ''final'' virtual photon. However, taking only the ρ-meson contribution in the dispersion integral for the final state, one naturally obtains the amplitude for the $\gamma^*T\to\rho T$ transition considered in Ref. [9]. Furthermore, by picking out the ρ-meson contribution in both dispersion integrals, one would get the amplitude for the $\rho T\to\rho T$ scattering. This idea can be also used to study the pion diffractive electroproduction and the pion diffractive scattering.

VI. CALCULATION OF $P_{\pi N}(\sigma\to 0)$

To analyze the pion scattering, we substitute the electromagnetic current by the axial current in the original amplitude, i.e., simply add γ_5 in the current vertices. For massless quarks, the final result has the same structure as that for the vector current. Of course, the $\bar{q}q$-pair wave function would have an extra γ_5, and the vertex factor analogous to that in Eq. (5.2) is

$$\frac{\overline{U(xP_+)}\gamma^\mu\gamma_5 V((1-x)P_+)}{\sqrt{x(1-x)}}=P_+^\mu, \tag{6.1}$$

where P is the four-momentum associated with the axial current.

The projection of a single-pion state onto the axial current is specified by the $\pi\to\mu\nu$ decay constant f_π:

$$\langle 0|J_\mu^A|\pi,P\rangle=\sqrt{2}f_\pi P_\mu. \tag{6.2}$$

Hence, we should extract the amplitude $\sim P_\mu P_\nu$ corresponding to the longitudinal polarization of the axial current. Again, the transition from the virtual amplitude for the currents to that involving the pion can be understood in terms of the dispersion representation and quark-hadron duality. In other words, below the effective higher-state threshold s_0^π, one should substitute the free-quark contribution by that due to the pion pole:

$$\psi_{5\mu}^{\bar{q}q}(\kappa;\eta,r_t)\to\Psi_{5\mu}^{\text{hadr}}(\kappa;\eta,r_t)$$
$$=q_\mu[f_\pi\psi_\pi(\eta,r_t)\,\delta(\kappa^2-m_\pi^2)$$
$$+\theta(\kappa^2>s_0^\pi)\,\psi^{\bar{q}q}(\kappa;\eta,r_t)]. \tag{6.3}$$

The local duality prescription gives a correctly normalized wave function provided that $s_0^\pi=16\pi^2f_\pi^2\approx 0.67\ \text{GeV}^2$. Of course, one can use a pion wave function different from that given by the local duality. However, the duality considerations justify the use of the effective two-body wave function (see [18]).

The actual calculation consists of the same steps as those leading to Eq. (3.6). For a small-size configuration, we get the following contribution $\delta\sigma_{\pi N}$ into the scattering cross section:

$$\delta\sigma_{\pi N}=\int|\psi_\pi(\eta,b)|^2\frac{d\eta d^2b}{4\pi}N_c\sigma_N^{\bar{q}q}(b^2). \tag{6.4}$$

Effectively, the vertex $e\psi_{\gamma^*}\sqrt{N_c}$ is substituted by the pion wave function. Rewriting $\delta\sigma_{\pi N}$ as

$$\delta\sigma_{\pi N}=P_{\pi N}(\sigma)d\sigma=\frac{db^2}{d\sigma}\int|\psi_\pi(\eta,b)|^2\frac{d\eta}{4}\sigma d\sigma\ , \tag{6.5}$$

we obtain

$$P_{\pi N}(\sigma\to 0)=\frac{db^2}{d\sigma}\int|\psi_\pi(\eta,b\to 0)|^2\frac{d\eta}{4}\ , \tag{6.6}$$

where $\sigma(b^2)$ is given by Eq. (3.9).

Thus, $P_{\pi N}(\sigma\to 0)$ is determined by the pion wave function at the origin of the impact parameter space, or, which is the same, by the integral of the momentum wave function $\psi_\pi(\eta,r_t)$ over all transverse momenta r_t. This integral formally gives the pion distribution amplitude

$$\varphi_\pi(\eta)=\frac{\sqrt{3}}{(2\pi)^3}\int\psi_\pi(\eta,r_t)d^2r_t.$$

However, in QCD (and in any theory with dimensionless coupling constant), this integral diverges. The standard procedure is to supplement the integral with some renormalization prescription characterized by a cutoff parameter μ, i.e., $\varphi_\pi(\eta)\to\varphi_\pi(\eta,\mu)$. In fact, the Fourier transformation from the momentum to the impact parameter space

$$\psi_\pi(\eta,b)=\int\psi_\pi(\eta,r_t)\frac{d^2r_t}{(2\pi)^2}$$

for small b can also be treated as a particular cutoff prescription with $1/b$ playing the role of the renormalization parameter μ. In the $b\rightarrow 0$ limit, one encounters the singular $\ln b^2$ terms. It is exactly the logarithms which generate the evolution of the pion distribution amplitude. Summing the logarithms by the renormalization group methods gives, for small b:

$$\psi_\pi(\eta,b)=\eta(1-\eta)\sum_{n=0} a_n C_n^{3/2}(2\eta-1)\left(\frac{\ln b_0^2\Lambda^2}{\ln b^2\Lambda^2}\right)^{\gamma_n/2\beta_0}, \tag{6.7}$$

where $\eta(1-\eta)C_n^{3/2}(2\eta-1)$ are the eigenfunctions of the evolution kernel [$C_n^{3/2}(2\eta-1)$ being the Gegenbauer polynomials], the anomalous dimensions γ_n are its eigenvalues and β_0 is the one-loop QCD β-function coefficient. The b_0 parameter characterizes the effective onset of the perturbative evolution. The coefficients a_n are the Gegenbauer moments of the pion wave function at this scale. Note that the anomalous dimension of the axial current vanishes ($\gamma_0=0$) and all other γ_n's are positive. Hence, after the renormalization group improvement, the limit $b\rightarrow 0$ is well defined in this case and

$$\psi_\pi(\eta,b=0)=\sqrt{48}\pi f_\pi\eta(1-\eta), \tag{6.8}$$

where $f_\pi=92$ MeV. The absolute normalization of the pion wave function for $b=0$ is fixed by the matrix element of the axial-vector current,

$$\int \psi_\pi(\eta,r_t)\,\frac{d\eta d^2r_t}{8\pi^3}=\frac{f_\pi}{\sqrt{N_c}}, \tag{6.9}$$

or in the impact parameter space [see Eq. (3.3)]:

$$\int \psi_\pi(\eta,b=0)\,\frac{d\eta}{2\pi}=\frac{f_\pi}{\sqrt{N_c}}. \tag{6.10}$$

In other words, for the pion, the singular $\ln b$ terms sum into harmless $(1/\ln b^2\Lambda^2)^{\gamma_n/2\beta_0}$ factors vanishing in the $b\rightarrow 0$ limit. As a result, the η dependence of the pion wave function $\psi_\pi(\eta,b)$ in the formal $b\rightarrow 0$ limit always assumes its asymptotic form $\psi_\pi(\eta,b)\sim\eta(1-\eta)$, irrespective of its shape at the scale b_0. It is natural to expect that b_0 is related to the scale characterizing the magnitude of the nonperturbative momentum distribution in the pion. The momentum scale $\mu_0=\sqrt{s_0^\pi}\approx 0.8$ GeV suggested by the local duality is rather large, and there may exist a transitional region of distances $b\sim b_0$ small compared to the pion size but not small enough to produce sizable perturbative evolution effects. In this case, one can try the η dependences of $\psi_\pi(\eta,b)$ different from the asymptotic form. In fact, the integral

$$I\equiv\int|\psi_\pi(\eta,b)|^2\frac{d\eta}{4} \tag{6.11}$$

is rather insensitive to the evolution effects. If we take the asymptotic wave function (6.8), then

$$P_\pi(\sigma\rightarrow 0)=\frac{2}{5}\pi^2 f_\pi^2\frac{db^2}{d\sigma}. \tag{6.12}$$

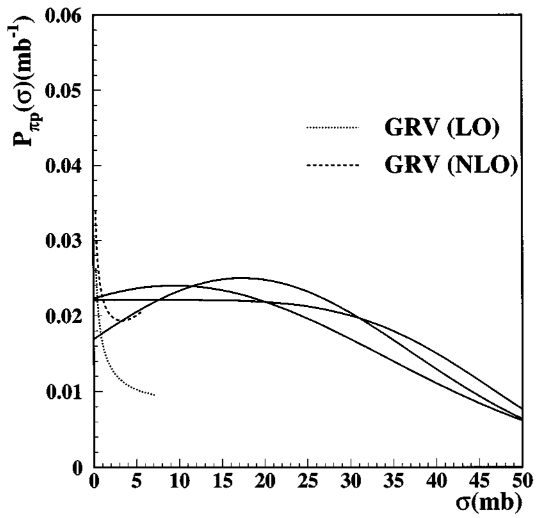

FIG. 2. Comparison of $P_{\pi p}(\sigma)$ calculated in PQCD using Eq. (6.14) and GRV parametrizations [20] of the gluon density and fits based on the analysis of the soft diffraction data [7].

Assuming that, at the scale $b=b_0$, the η dependence of the pion wave function corresponds to the Chernyak-Zhitnitsky [19] ansatz

$$\psi_\pi^{\mathrm{CZ}}(\eta,b=b_0)=5\sqrt{48}\pi f_\pi\quad\eta(1-\eta)(1-2\eta)^2, \tag{6.13}$$

we obtain

$$P_\pi(\sigma,b=b_0)=\frac{10}{21}\pi^2 f_\pi^2\frac{db^2}{d\sigma}. \tag{6.14}$$

Thus, in this case the evolution would decrease the integral I by $\sim 20\%$ when b changes from b_0 to 0. Taking the asymptotic result, we get

$$P_\pi(\sigma\rightarrow 0)=\frac{6}{5}\,\frac{f_\pi^2}{\alpha_s x G_N(x,\lambda/b^2)}. \tag{6.15}$$

Distribution $P_{\pi N}(\sigma)$ was determined in Ref. [7] from the analysis of the soft diffractive processes for $E_\pi\approx 200$ GeV, see solid curves in Fig. 2. In the limit $\sigma\ll\langle\sigma\rangle$, we can compare this result with Eq. (6.14). The applicability region of this equation is restricted by several conditions. First, x_{eff} should be small enough so that the average longitudinal distances in the scattering process $1/2m_N x$ are larger than the nucleon size, which corresponds to $x\lesssim 0.05$. Furthermore, the virtualities in the process should be large enough so that one can apply PQCD which corresponds to the requirement $Q_{\mathrm{eff}}^2\gtrsim 1-2$ GeV^2. In our analysis we also neglect the b dependence of the wave function of the $q\bar{q}$ component at large b (this is a higher twist effect), which restricts consideration to $b\lesssim 0.5$ fm. In the numerical calculation, we use the Gluck-Reya-Vogt (GRV) parametrization [20] since it describes well the parton distributions down to $Q^2\sim 1.5$ GeV^2. We present results both for the leading order (LO) and next-to-leading order (NLO) GRV parametrizations, see dashed curves in Fig. 2. Difference between LO and NLO results illustrates range of uncertainties of the current analysis. One

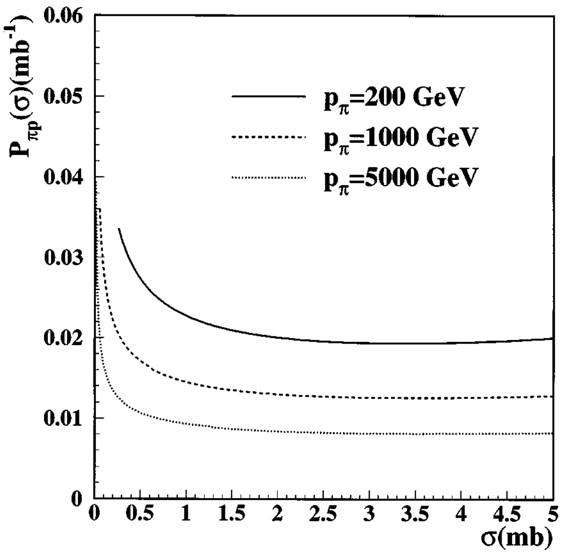

FIG. 3. Incident momentum dependence of $P_{\pi p}(\sigma)$ for small σ calculated using Eq. (6.14) and GRV NLO parametrization [20] of the gluon density.

can see that the results of our calculations are in qualitative agreement with the phenomenological results of [7].

Another interesting feature of our results is a substantial energy dependence of $P(\sigma<\langle\sigma\rangle)$ on the incident energy due to a fast increase of $xG_N(x,Q^2)$ with the decrease of x, see Fig. 3. This reflects the fact that the probability of pointlike configurations in hadrons decreases with the increase of energy. Further diffractive data (preferably at higher energies) are necessary to get better information about $P_{\pi N}(\sigma)$.

Since the existence of configurations with small spatial size has been confirmed experimentally in the energy dependence and absolute value of cross section of electroproduction of vector mesons, we consider the above result as a reflection of soft matching between nonperturbative and PQCD regimes.

VII. SUMMARY AND CONCLUSIONS

In this paper, we applied a PQCD approach to describe the basic features of the high-energy interactions of a small-size $\overline{q}q$ configurations with a hadron target. This interaction is proportional to the gluon distribution function $G_T(x,Q^2)$ of the target and, hence, the cross section is enhanced in the small-x region. The $\overline{q}q$ configuration can be described by the wave functions whose particular form is determined by the projection of the initial particle (γ^*, ρ, or π) onto the $\overline{q}q$ component. For small σ, we calculated the cross-section distribution $P_\pi(\sigma)$ for the pion and demonstrated that it is rather insensitive to the specific form of the pion distribution amplitude.

ACKNOWLEDGMENTS

This work was partially supported by the BSF Grant No. 9200126 and by the U.S. Department of Energy under Contract Nos. DE-FG02-93ER40771 and DE-AC05-84ER40150. Two of us (A.R. and M.S.) thank the DOE's Institute for Nuclear Theory at the University of Washington for its hospitality and support during the workshop ''Quark and Gluon Structure of Nucleons and Nuclei.''

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