Erratum and Addendum: Gravitational radiation from a particle in circular orbit around a black hole. VI. Accuracy of the post-Newtonian expansion

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(Received 11 February 1997)

A coding error was discovered in the computer program written to generate the results presented in Phys. Rev. D **52**, 5719 (1995). The corrected results are presented here, and the changes in the paper's original conclusions are discussed. Also, the results are updated so as to incorporate the recent post-Newtonian calculations of Tanaka, Tagoshi, and Sasaki. [S0556-2821(97)03712-0]

PACS number(s): 04.25.Nx, 04.30.Db, 97.60.Jd, 97.60.Lf

The quantity computed in Table I is

$$\mathcal{R}_n \equiv \frac{S/N|_{\rm actual}}{S/N|_{\rm max}} = \frac{|(h|h_n)|}{\sqrt{(h|h)(h_n|h_n)}},$$

the ratio of the actual signal-to-noise ratio obtained when matched filtering a gravitational-wave signal h with a post-Newtonian template h_n (n denotes the order of the approximation), to the maximum possible value. The precise definition of the inner product $(\cdot|\cdot)$, and all other technical details, are given in Ref. [1].

The reference signal h is constructed from exact expressions for both dE/dt, the rate at which energy is lost to gravitational waves, and dE/df, which provides the relation between orbital energy and gravitational-wave frequency. dE/dt is obtained by numerically integrating the Teukolsky equation for a small-mass object in circular motion around a Schwarzschild black hole. dE/df is obtained by integrating the geodesic equations for circular orbits.

On the other hand, the template h_n is constructed from post-Newtonian approximations to both dE/dt and dE/df, accurate through order v^n, where $v=(\pi M f)^{1/3}$ (with M the black-hole mass) is the orbital velocity. The expressions used are [2].

$$\begin{aligned}\frac{dE/dt}{(dE/dt)_N} \equiv P(v) = {}& 1-3.7113v^2+12.566v^3-4.9285v^4 \\ & -38.293v^5+(115.73-16.305\ln v)v^6 \\ & -101.51v^7+(-117.50+52.743\ln v)v^8 \\ & +(719.13-204.89\ln v)v^9+(-1216.9 \\ & +116.64\ln v)v^{10}+(958.93+473.62\ln v)v^{11} \\ & +\cdots,\end{aligned}$$

$$\begin{aligned}\frac{dE/df}{(dE/df)_N} \equiv Q(v) = {}& (1-6v^2)(1-3v^2)^{-3/2} \\ = {}& 1-\tfrac{3}{2}v^2-\tfrac{81}{8}v^4-\tfrac{675}{16}v^6-\tfrac{19845}{128}v^8 \\ & -\tfrac{137781}{256}v^{10}+\cdots=(1-6v^2)(1+\tfrac{9}{2}v^2+\tfrac{135}{8}v^4 \\ & +\tfrac{945}{16}v^6+\tfrac{25515}{128}v^8-\tfrac{168399}{256}v^{10}+\cdots),\end{aligned}$$

where $(dE/dt)_N$, $(dE/df)_N$ are the leading-order Newtonian

TABLE I. Reduction in signal-to-noise ratio incurred when matched filtering with approximate, post-Newtonian templates. For each of the considered binary systems, the first column lists the order n of the approximation, the second column lists $\mathcal{R}_n$ as calculated using the fully expanded expression for $Q(v)$, the third column lists $\mathcal{R}_n$ as calculated using the exact expression for $Q(v)$, and the fourth column lists $\mathcal{R}_n$ as calculated using the partially expanded expression for $Q(v)$.

n	Q fully exp.	Q exact	Q part. exp.
System A ($1.4M_\odot+1.4M_\odot$):			
4	0.6756	0.5852	0.5217
5	0.5565	0.6117	0.7157
6	0.9578	0.9451	0.9320
7	0.9777	0.9866	0.9802
8	0.9731	0.9772	0.9782
9	0.9848	0.9854	0.9858
10	0.9836	0.9837	0.9838
11	0.9836	0.9837	0.9838
System B ($1.4M_\odot+10M_\odot$):			
4	0.8580	0.4890	0.3772
5	0.6718	0.3788	0.4305
6	0.8271	0.7455	0.6859
7	0.8938	0.9977	0.8796
8	0.8762	0.8945	0.9111
9	0.9962	0.9650	0.9459
10	0.9897	0.9958	0.9984
11	0.9924	0.9977	0.9995
System C ($10M_\odot+10M_\odot$):			
4	0.8367	0.7610	0.5855
5	0.4433	0.5354	0.6684
6	0.9769	0.8500	0.7988
7	0.9038	0.9997	0.9469
8	0.9006	0.9314	0.9541
9	0.9998	0.9850	0.9669
10	0.9941	0.9983	0.9995
11	0.9959	0.9992	0.9999

expressions. The first form for $Q(v)$ is exact. The second form (fully expanded) is a Taylor expansion about $v=0$. The third form (partially expanded) forces $Q(v)$ to vanish at $v=6^{-1/2}$, the innermost stable circular orbit. The paper's original calculations used post-Newtonian expansions accurate through $O(v^8)$.

In calculating $\mathcal{R}_n$, the mass and arrival time parameters are assumed to be the same in h and h_n. In the original calculations, the initial-phase parameters were also assumed to be matched. Here we proceed differently: the overlap integral $(h|h_n)$ is *maximized* over all possible values of the difference in initial-phase parameters. While this obviously leads to larger results for $\mathcal{R}_n$, the improvement is not significant: at most 2% when $n \geqslant 6$.

The paper's conclusions are drastically affected by the coding error.

First, the values obtained for $\mathcal{R}_n$ are now much larger than those obtained before, and the table shows that $\mathcal{R}_n$ nicely converges to unity as the order of the post-Newtonian approximation increases. In particular, the table shows that a post-Newtonian template accurate through order v^6 would reproduce more than 95% of the signal-to-noise ratio. (System B is an exception.)

Second, there is no longer any evidence for the observation that an accurate knowledge of the location of the innermost stable circular orbit would significantly improve the performance of the post-Newtonian templates. Indeed, in most cases the numbers displayed in the fourth column of Table I, calculated using the partially expanded form for $Q(v)$, are not significantly larger than those displayed in the second column. (And in some cases they are significantly lower.)

Finally, it appears that the best results are obtained when using an $n=9$ template. In this case both dE/dt and dE/df are overestimated by the post-Newtonian approximations. However, since h_n depends on the ratio $df/dt=(dE/dt)/(dE/df)$, it appears that the individual discrepancies are best canceled with $O(v^9)$ expressions.

The author wishes to apologize for any inconvenience caused by his negligence. This work was supported by the Natural Sciences and Engineering Research Council of Canada.

[1] E. Poisson, Phys. Rev. D **52**, 5719 (1995).

[2] T. Tanaka, H. Tagoshi, and M. Sasaki, Prog. Theor. Phys. **96**, 1087 (1996).