

## Confinement in $N=1$ SUSY gauge theories and model building tools

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We develop a systematic approach to confinement in  $N=1$  supersymmetric theories. We identify simple necessary conditions for theories to confine without chiral symmetry breaking and to generate a superpotential nonperturbatively ( $s$ -confine). Applying these conditions we identify all  $N=1$  theories with a single gauge group and no tree-level superpotential which  $s$ -confine. We give a complete list of the confined spectra and superpotentials. Some of these theories are of great interest for model building. We give several new examples of models which break supersymmetry dynamically. [S0556-2821(97)00712-1]

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### I. INTRODUCTION

The number of  $N=1$  supersymmetric gauge theories for which we know exact results on their vacuum structure has been growing steadily in the last two years. The great progress was sparked by Seiberg's conjectures about the infrared properties and phase structure of supersymmetric QCD [1]. Following in his footsteps, others have obtained results on a whole range of theories [2–13]. Most of the discovered phenomena follow similar patterns in the different theories, and one is tempted to ask if there is maybe a more general approach than the model-specific trial and error procedure that has been customary thus far.

Whereas a completely general approach that allows one to understand all the obtained results seems impossibly difficult to find, we can make much progress by focusing on the particular phenomenon of confinement. In fact, a frequently occurring and relatively easily identified infrared behavior is “ $s$ -confinement.” In a previous publication [13] we defined an  $s$ -confining theory as a theory for which all the degrees of freedom in the infrared are gauge-invariant composites of the fundamental fields. Furthermore, we demand that the infrared physics is described by a smooth effective theory in terms of these gauge invariants. This description should be valid everywhere on the moduli space of vacua, including the origin of field space. Finally, we also demand that an  $s$ -confining theory generates a dynamical superpotential. At the origin of moduli space all global symmetries of the theory are unbroken and the global anomalies of the microscopic theory are matched by the macroscopic gauge invariants of the effective theory.

The best-known example of a theory which has been conjectured to be  $s$ -confining is supersymmetric QCD (SQCD) with  $N$  colors and  $F=N+1$  flavors of fundamental and antifundamental matter  $Q$  and  $\bar{Q}$  [1,14]. The gauge-invariant confined degrees of freedom are mesons  $M=Q\bar{Q}$  and baryons  $B=Q^N$ ,  $\bar{B}=\bar{Q}^N$ . At the origin of moduli space, all com-

ponents of the mesons and baryons are massless, and they interact via the confining superpotential

$$W = \frac{1}{\Lambda^{2N-1}} (\det M - B M \bar{B}). \quad (1)$$

This description is also valid far from the origin of the moduli space where the large expectation values of the fields completely break the gauge group. In such a vacuum the theory is in the Higgs phase. A smooth gauge-invariant description of both the Higgs and confining vacua of the theory can only exist if there is no phase transition between the two regions in moduli space. In particular, there should be no gauge-invariant order parameter that distinguishes the two phases.

To understand this in the example of SQCD, note that the quarks transform in a faithful representation of the gauge group  $SU(N)$ . This implies that arbitrary test charges can be screened by the dynamical quarks because the vacuum can disgorge quark-antiquark pairs to screen charges transforming in any representation of the gauge group. Thus a Wilson loop will always obey a perimeter law because any charges we might want to use to define the Wilson loop can be screened. Our definition of  $s$ -confinement above necessitates that an  $s$ -confining theory is in such a “screening-confining” phase.

This situation should be contrasted with  $SU(N)$  with only adjoint matter or  $SO(N)$  with vector matter. In both these cases the matter does not transform in a faithful representation of the gauge group. Now there are charges that cannot be screened by the dynamical quarks, and a Wilson loop can serve as gauge-invariant order parameter to distinguish the Higgs and the confining phases. As a result, such theories cannot have a single smooth description of both the Higgs and confining phases of the theory, thus they are not  $s$ -confining.

In our previous publication [13], we identified two criteria which allow us to decide whether a given theory can be  $s$ -confining without having to know the explicit infrared de-

scription. If we limit our attention to theories with no tree-level superpotential and only one gauge group, then the symmetries completely determine the form of any nonperturbatively generated superpotential. Demanding that this superpotential is smooth everywhere on the moduli space yields the first of our two conditions. The other condition arises from studying the theory along some flat direction in which the gauge group is broken to a subgroup, and the theory may sufficiently simplify so that we can understand its infrared physics. If we find a result that cannot be smoothly connected to a confining phase, we know that the whole theory is not  $s$ -confining either. We discuss the arguments leading to these two conditions in Sec. II of this paper. In Sec. III we apply our conditions to identify all theories with a single gauge group and no tree-level superpotential which  $s$ -confine. We give a complete list of the confined spectra and superpotentials for all  $s$ -confining theories with an arbitrary SU, SO, Sp, or exceptional gauge group. Using the results for the  $s$ -confining theories, we then demonstrate in Sec. IV how one can generate many more exact solutions for other models by simply integrating out matter from the  $s$ -confining theories. The models which we obtain in this way display interesting dynamics: confinement with chiral symmetry breaking, nonperturbatively generated superpotentials which drive the vacuum to infinity, and confinement with noninteracting composites.

In Sec. V we turn to applications of our results to model building. We summarize the various known mechanisms of dynamical supersymmetry breaking and illustrate each of the mechanisms with a few examples which we construct using our results of Secs. III and IV. Finally, we comment on the possibility of using our models to construct composite models in the conclusions. We hope that our tables and superpotentials in Secs. III and IV together with the explicit examples of Sec. V will prove to be a valuable resource for model builders.

## II. NECESSARY CRITERIA FOR $s$ -CONFINEMENT

In this section we develop two necessary criteria which allow us to identify all  $s$ -confining theories with a simple gauge group and no tree-level superpotential. The first criterion follows from holomorphy of the dynamically generated superpotential, which can be determined using the global symmetries of the theory. This criterion allows us to reduce the number of theories that are candidates for  $s$ -confinement to a manageable set. Our second criterion follows from explorations of regions in moduli space which are easier to understand than the origin. As will be demonstrated in Sec. III, these two conditions combined are sufficient to identify all  $s$ -confining theories with a single gauge group and no tree-level superpotential.

### A. The index constraint

In this subsection, we derive a simple constraint on the matter content of  $s$ -confining theories which follows from the requirement of holomorphy of the confining superpotential. In theories with a simple gauge group  $G$  and no tree-level superpotential, the symmetries are sufficient to determine the form of any dynamically generated superpotential completely [15]. A simple way to prove this makes use of

nonanomalous  $R$  symmetries. Define a  $U(1)_R$  symmetry as follows: all chiral superfields, except for one arbitrarily chosen field  $\phi_i$ , are assigned zero  $R$  charge. The charge  $q$  of the remaining field is determined by requiring anomaly cancellation of the mixed  $G^2U(1)_R$  anomaly:

$$(q-1)\mu_i - \sum_{j \neq i} \mu_j + \mu_G = q\mu_i - \sum_{\text{all } j} \mu_j + \mu_G = 0, \quad (2)$$

where  $\mu_i$  is the Dynkin index<sup>1</sup> of the gauge representation of the field  $\phi_i$ , and  $(q-1)$  is the  $R$  charge of its fermion component. These three terms arise from the contributions of the fermion components of  $\phi_i$ , of all other matter superfields  $\phi_j$  with  $j \neq i$ , and of the gauge superfields, respectively. The  $\mu_j$  are the indices of the remaining matter representations, they are multiplied by the  $R$  charges  $-1$  of the fermion components of  $\phi_j$ , and finally  $\mu_G$  is the index of the adjoint representation of  $G$  multiplied by the  $R$  charge  $+1$  of the gauginos.  $R$  invariance of the supersymmetric Lagrangian requires the dynamically generated superpotential to have  $R$  charge 2. This uniquely fixes the dependence of the superpotential on the field  $\phi_i$ :

$$W \propto (\phi_i^{\mu_i})^{2/(\sum_j \mu_j - \mu_G)}. \quad (3)$$

To determine the functional dependence on the other superfields, we note that the global symmetries contain a corresponding  $U(1)_R$  symmetry for each of the matter superfields, and the superpotential has to have  $R$  charge 2 under each such  $R$  symmetry. Finally, the dependence on the dynamical scale  $\Lambda$  can be determined by dimensional analysis or using an anomalous  $R$  symmetry [2]. The result is

$$W \propto \Lambda^3 \left[ \prod_i \left( \frac{\phi_i}{\Lambda} \right)^{\mu_i} \right]^{2/(\sum_j \mu_j - \mu_G)}. \quad (4)$$

There may be several (or no) possible contractions of gauge indices, thus the superpotential can be a sum of several terms. We require the coefficient of this superpotential to be nonvanishing, then holomorphy at the origin implies that the exponents of all fields  $\phi_i$  are positive integers. Strictly speaking, we should require holomorphy in the confined degrees of freedom which would imply that the exponents of composites must be positive integers. Since we do not want to have to determine all gauge invariants for this argument, we settle for the weaker constraint on exponents of the fundamental fields. Therefore,<sup>2</sup>  $\sum_j \mu_j - \mu_G = 1$  or 2. However, in

<sup>1</sup>We normalize the index of the fundamental representations of SU and Sp to 1 and of the vector of SO to 2. This definition ensures invariance of the index when decomposing representations of  $SO(2N)$  under the  $SU(N)$  subgroup. This is relevant to the flows discussed in Sec. II B.

<sup>2</sup>Other solutions exist if all  $\mu_i$  have a common divisor  $d$ , then for  $\sum_j \mu_j - \mu_G = d$  or  $2d$  the superpotential Eq. (5) may be regular. We will argue at the end of Sec. III that these solutions generically do not yield  $s$ -confining theories. Another possibility is that the coefficient of the superpotential above vanishes. There are examples of confining theories with vanishing superpotentials in the literature [10].

our normalization of the index, anomaly cancelation further constrains this quantity to be even; thus,

$$\sum_j \mu_j - \mu_G = 2. \tag{5}$$

This formula summarizes our first necessary condition for  $s$ -confinement, which enables us to rule out most theories immediately. For example, for SQCD we find that the only candidate is the theory with  $F = N + 1$ . Unfortunately, Eq. (5) is not a sufficient condition. An example for a theory which satisfies Eq. (5) but does not  $s$ -confine is  $SU(N)$  with an adjoint superfield and one flavor. This theory is easily seen to be in an Abelian Coulomb phase for generic vacuum expectation values (VEV’s) of the adjoint scalars and vanishing VEV’s for the fundamentals. In the following section, we derive another necessary criterion which allows us to rule out theories that satisfy the “index constraint” but do not  $s$ -confine.

### B. Flows and $s$ -confinement

The second condition is obtained from studying different regions on the moduli space of the theory under consideration. A generic supersymmetric theory with vanishing tree-level superpotential has a large moduli space of vacua. By definition, an  $s$ -confining theory has a smooth description in terms of gauge invariants everywhere on this moduli space. There should be no singularities in the superpotential or the Kähler potential and there should be no massless gauge bosons anywhere.

Thus, we can test a given theory for  $s$ -confinement by expanding around points that are far out in moduli space where the theory simplifies. In the microscopic theory the gauge group gets broken to a subgroup when we go out in moduli space by giving large ( $\langle \phi \rangle \gg \Lambda$ ) expectation values to some fields. In this vacuum, the gauge superfields corresponding to broken symmetry generators get masses through the super-Higgs mechanism and the remaining matter fields decompose under the unbroken subgroup. This “reduced” theory has a smaller gauge group and may be easier to understand. If the original theory was  $s$ -confining then its confined description should be valid at this point in moduli space as well. Therefore, the reduced theory is  $s$ -confining if the original theory was. This statement can be applied in two directions.

Necessary condition: If the reduced theory does not have a smooth description with only gauge-invariant degrees of freedom, then the original theory cannot be  $s$ -confining. Suf-

ficient condition: If the original theory is known to be  $s$ -confining, then all possible reduced theories (with a remaining unbroken gauge group) which the original theory flows to are  $s$ -confining also. The confined spectrum and the confining superpotential of the reduced theories can be obtained by identifying the corresponding points in moduli space in the confined description of the original theory and integrating out all massive fields. In practice, this means identifying the correct gauge-invariant fields which have vacuum expectation values and integrating out fields which now have mass terms in the superpotential using their equations of motion.

The reduced theories will always contain some gauge-invariant fields in the high-energy description which originally transformed under the now broken gauge generators. These fields do not have any interactions and are irrelevant to the dynamics of the model. They can be removed from the theory. In the confined description the fields corresponding to these gauge singlets are only coupled through superpotential terms which scale to zero when the VEV’s are taken to infinity, or which are irrelevant in the infrared.

A nontrivial application of the sufficient condition is given by the flow from  $SU(4)$  with an antisymmetric tensor and four “flavors” of fundamentals and antifundamentals to  $Sp(4)$  with eight fundamentals. The  $SU(4)$  theory is known to  $s$ -confine [4]. By giving an expectation value to the antisymmetric tensor the gauge group is broken to  $Sp(4)$ . All components of the antisymmetric tensor field except for one singlet are “eaten” by the super-Higgs mechanism, and the four flavors of fundamentals and antifundamentals become eight fundamentals of  $Sp(4)$ . Applying our sufficient criterion, we conclude that the  $Sp$  theory is  $s$ -confining as well. Its confined spectrum and superpotential can be obtained from the spectrum and superpotential of the  $SU(4)$  theory.

A nontrivial example of a theory which can be shown not to  $s$ -confine is  $SU(4)$  with three antisymmetric tensors and two flavors. This theory satisfies our index condition, Eq. (5), and is therefore also a candidate for  $s$ -confinement. By giving a VEV to an antisymmetric tensor we can flow from this theory to  $Sp(4)$  with two antisymmetric tensors and four fundamentals. VEV’s for the other antisymmetric tensors let us flow further to  $SU(2)$  with eight fundamentals which is known to be at an interacting fixed point in the infrared. We conclude that the  $SU(4)$  with three tensors and  $Sp(4)$  with two tensors and all theories that flow to them cannot be  $s$ -confining either. This allows us to rule out the following chain of theories, all of which are gauge anomaly free and satisfy Eq. (5):

$$\begin{array}{ccccccc} SU(7) & \rightarrow & SU(6) & \rightarrow & SU(5) & \rightarrow & SU(4) & \rightarrow & Sp(4) \\ \square \square 2 \square 4 \square & & \square \square \square \square 3 \square & & 2 \square \square \square 2 \square & & 3 \square \square 2 \square 2 \square & & 2 \square \square 4 \square \end{array} \tag{6}$$

Note that a VEV for one of the quark flavors of the  $SU(4)$  theory lets us flow to an  $SU(3)$  theory with four flavors which is  $s$ -confining. We must therefore be careful: when we find a flow to an  $s$ -confining theory, it does not follow that the original theory is  $s$ -confining as well. The

flow is only a necessary condition. However, in all our examples we find that a theory with a single gauge group and no tree-level superpotential is  $s$ -confining if it is found to flow to  $s$ -confining theories in all directions of its moduli space.

III. ALL  $s$ -CONFINING THEORIES

In this section, we present our results which we obtained using the two conditions derived in Sec. II. We first created a list of all theories with a single gauge group and matter content satisfying the index constraint. Then we studied all possible flat directions of the individual theories and checked if they only flow to confining theories. We summarize these results in the first table of each subsection. In the first column we list all theories satisfying the index constraint. In the second column we indicate the result of the flows: theories which can be shown to have a branch with an unbroken Abelian gauge group we denote with ‘‘Coulomb branch,’’ for theories which can be shown to flow to a reduced theory with a non-Abelian gauge group which is not  $s$ -confining we indicate the gauge group of the reduced theory and its matter content, all other theories are  $s$ -confining.

After identifying all  $s$ -confining theories in this way, we explicitly construct the confined spectra for each  $s$ -confining theory. The group theory used to obtain these results can be found in Refs. [16–18]. We present our results in tables where we indicate the matter content of the ultraviolet theory in the upper part of the table, and the gauge-invariant infrared spectrum in the lower part. The gauge group and the Young tableaux of the representations of the matter fields are indicated in the first column. The other groups correspond to the global symmetries of the theory. In addition to the listed global symmetries, there is also a global  $U(1)$  with a  $G^2U(1)$  anomaly which is broken by instantons.

Finally, we also give the confining superpotentials when they are not too long. We denote gauge-invariant composites by their constituents in parenthesis. The relative coefficients of the different terms can be determined by demanding that the equations of motion following from this superpotential reproduce the classical constraints of the ultraviolet theory. This also constitutes an important consistency check: in the limit of large generic expectation values for fields  $\langle \phi \rangle \gg \Lambda$  the ultraviolet theory behaves classically and all its classical constraints need to be reproduced by the infrared description. Checking that all these constraints are reproduced and determining the coefficients is a very tedious exercise which we only performed for some theories. Since we have not determined the coefficients of the superpotential terms for several of the  $s$ -confining theories, it may turn out that some of the terms listed in the confining superpotentials have vanishing coefficients.

A more straightforward and also very powerful consistency check is provided by the ’t Hooft anomaly matching conditions. We explicitly checked that all global anomalies match between the microscopic and macroscopic degrees of freedom in every theory. Other consistency checks which we performed for a subset of the theories include explorations of the moduli spaces and adding masses for some matter fields and checking consistency of the results. More details on these techniques are described in Sec. IV.

A. The  $s$ -confining  $SU(N)$  theories

In this section, we present all  $s$ -confining theories based on  $SU(N)$  gauge groups. We normalize the Dynkin index and the anomaly coefficient of the fundamental representation to be one. With these conventions, the dimension, index

TABLE I. All  $SU$  theories satisfying  $\sum_j \mu_j - \mu_G = 2$ . This list is finite because the indices of higher index tensor representations grow very rapidly with the size of the gauge group. We list the gauge group and the field content of the theories in the first column. In the second column, we indicate which theories are  $s$ -confining. For the theories which do not  $s$ -confine we give the flows to non- $s$ -confining theories or indicate that there is a Coulomb branch on the moduli space.

$SU(N)$	$(N+1)(\square + \bar{\square})$	$s$ -confining
$SU(N)$	$\square + N\bar{\square} + 4\square$	$s$ -confining
$SU(N)$	$\square + \bar{\square} + 3(\square + \bar{\square})$	$s$ -confining
$SU(N)$	Adj $+\square + \bar{\square}$	Coulomb branch
$SU(4)$	Adj $+\square$	Coulomb branch
$SU(4)$	$3\square + 2(\square + \bar{\square})$	$SU(2)$ : $8\square$
$SU(4)$	$4\square + \square + \bar{\square}$	$SU(2)$ : $\square + 4\square$
$SU(4)$	$5\square$	Coulomb branch
$SU(5)$	$3(\square + \bar{\square})$	$s$ -confining
$SU(5)$	$2\square + 2\square + 4\bar{\square}$	$s$ -confining
$SU(5)$	$2(\square + \bar{\square})$	$Sp(4)$ : $3\square + 2\square$
$SU(5)$	$2\square + \bar{\square} + 2\square + \square$	$SU(4)$ : $3\square + 2(\square + \bar{\square})$
$SU(6)$	$2\square + 5\bar{\square} + \square$	$s$ -confining
$SU(6)$	$2\square + \bar{\square} + 2\bar{\square}$	$SU(4)$ : $3\square + 2(\square + \bar{\square})$
$SU(6)$	$\square + 4(\square + \bar{\square})$	$s$ -confining
$SU(6)$	$\square + \square + 3\bar{\square} + \square$	$SU(5)$ : $2\square + \bar{\square} + 2\bar{\square} + \square$
$SU(6)$	$\square + \square + \bar{\square}$	$Sp(6)$ : $\square + \square + \square$
$SU(6)$	$2\square + \square + \bar{\square}$	$SU(5)$ : $2(\square + \bar{\square})$
$SU(7)$	$2(\square + 3\bar{\square})$	$s$ -confining
$SU(7)$	$\square + 4\bar{\square} + 2\square$	$SU(6)$ : $\square + \square + 3\bar{\square} + \square$
$SU(7)$	$\square + \bar{\square} + \square$	$Sp(6)$ : $\square + \square + \square$

and anomaly coefficient of the smallest  $SU(N)$  representations are listed below:

Irrep	Dim	$\mu$	$A$
$\square$	$N$	1	1
Adj	$N^2 - 1$	$2N$	0
$\square$	$\frac{N(N-1)}{2}$	$N - 2$	$N - 4$
$\square$	$\frac{N(N+1)}{2}$	$N + 2$	$N + 4$
$\square$	$\frac{N(N-1)(N-2)}{6}$	$\frac{(N-3)(N-2)}{2}$	$\frac{(N-3)(N-6)}{2}$
$\square$	$\frac{N(N+1)(N+2)}{6}$	$\frac{(N+2)(N+3)}{2}$	$\frac{(N+3)(N+6)}{2}$
$\square$	$\frac{N(N-1)(N+1)}{3}$	$N^2 - 3$	$N^2 - 9$
$\square$	$\frac{N^2(N+1)(N-1)}{24}$	$\frac{N(N-2)(N+2)}{3}$	$\frac{N(N-4)(N+4)}{6}$
$\square$	$\frac{N(N+1)(N+2)(N+3)}{24}$	$\frac{(N+2)(N+3)(N+4)}{6}$	$\frac{(N+3)(N+4)(N+8)}{6}$
$\square$	$\frac{N(N+1)(N-1)(N-2)}{8}$	$\frac{(N-2)(N^2-N-4)}{2}$	$\frac{(N-4)(N^2-N-8)}{2}$

Because the index of a representation of  $SU(N)$  grows like  $N^{k-1}$  where  $k$  is the number of gauge indices, there are very few anomaly free representations which satisfy Eq. (5). These representations are listed in Table I. In the first column, we indicate the gauge group and the field content of the theory. In the second column we give the flows which allowed us to rule out  $s$ -confinement for a given theory. For those theories which do  $s$ -confine we then list the spectra and the confining superpotential in the following tables. For completeness, we also list those  $s$ -confining theories which are already known in the literature.

1.  $SU(N)$  with  $(N + 1)(\square + \bar{\square})$  (SUSY QCD) [1]

	$SU(N)$	$SU(N + 1)$	$SU(N + 1)$	$U(1)$	$U(1)_R$
$Q$	$\square$	$\square$	1	1	$\frac{1}{N+1}$
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	$\frac{1}{N+1}$
$Q\bar{Q}$		$\square$	$\square$	0	$\frac{2}{N+1}$
$Q^N$		$\bar{\square}$	1	$N$	$\frac{N}{N+1}$
$\bar{Q}^N$		1	$\bar{\square}$	$-N$	$\frac{N}{N+1}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^{2N-1}} [(Q\bar{Q})^{N+1} - (Q^N)(Q\bar{Q})(\bar{Q}^N)].$$

2.  $SU(2N)$  with  $\square + 2N \bar{\square} + 4 \square$  [4]

	$SU(2N)$	$SU(2N)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$A$	$\square$	1	1	0	$2N + 4$	0
$\bar{Q}$	$\bar{\square}$	$\square$	1	4	$-2N + 2$	0
$Q$	$\square$	1	$\square$	$-2N$	$-2N + 2$	$\frac{1}{2}$
$Q\bar{Q}$		$\square$	$\square$	$4 - 2N$	$-4N + 4$	$\frac{1}{2}$
$A\bar{Q}^2$		$\square$	1	8	$-2N + 8$	0
$A^N$		1	1	0	$2N^2 + 4N$	0
$A^{N-1}Q^2$		1	$\square$	$-4N$	$2N^2 - 2N$	1
$A^{N-2}Q^4$		1	1	$-8N$	$2N^2 - 8N$	2
$\bar{Q}^{2N}$		1	1	$8N$	$-4N^2 + 4N$	0

$$W_{\text{dyn}} = \frac{1}{\Lambda^{4N-1}} [(A^N)(Q\bar{Q})^4(A\bar{Q}^2)^{N-2} + (A^{N-1}Q^2)(Q\bar{Q})^2(A\bar{Q}^2)^{N-1} + (A^{N-2}Q^4)(A\bar{Q}^2)^N + (\bar{Q}^{2N})(A^N)(A^{N-2}Q^4) + (\bar{Q}^{2N})(A^{N-1}Q^2)^2].$$

3.  $SU(2N + 1)$  with  $\square + (2N + 1) \bar{\square} + 4 \square$  [4]

	$SU(2N + 1)$	$SU(2N + 1)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$A$	$\square$	1	1	0	$2N + 5$	0
$\bar{Q}$	$\bar{\square}$	$\square$	1	4	$-2N + 1$	0
$Q$	$\square$	1	$\square$	$-2N - 1$	$-2N + 1$	$\frac{1}{2}$
$Q\bar{Q}$		$\square$	$\square$	$3 - 2N$	$-4N + 2$	$\frac{1}{2}$
$A\bar{Q}^2$		$\square$	1	8	$-2N + 7$	0
$A^N Q$		1	$\square$	$-2N - 1$	$2N^2 + 3N + 1$	$\frac{1}{2}$
$A^{N-1}Q^3$		1	$\bar{\square}$	$-6N - 3$	$2N^2 - 3N - 2$	$\frac{3}{2}$
$\bar{Q}^{2N+1}$		1	1	$4(2N + 1)$	$-4N^2 + 1$	0

$$W_{\text{dyn}} = \frac{1}{\Lambda^{2N}} [(A^N Q)(Q\bar{Q})^3(A\bar{Q}^2)^{N-1} + (A^{N-1}Q^3)(Q\bar{Q})(A\bar{Q}^2)^N + (\bar{Q}^{2N+1})(A^N Q)(A^{N-1}Q^3)].$$

4.  $SU(2N+1)$  with  $\square + \bar{\square} + 3(\square + \bar{\square})$

	$SU(2N+1)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
$A$	$\square$	1	1	1	0	-3	0
$\bar{A}$	$\bar{\square}$	1	1	-1	0	-3	0
$Q$	$\square$	$\square$	1	0	1	$2N-1$	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	$\bar{\square}$	0	-1	$2N-1$	$\frac{1}{3}$
$M_k = Q(A\bar{A})^k \bar{Q}$		$\square$	$\bar{\square}$	0	0	$4N-2-6k$	$\frac{2}{3}$
$H_k = \bar{A}(A\bar{A})^k Q^2$		$\bar{\square}$	1	-1	2	$4N-5-6k$	$\frac{2}{3}$
$\bar{H}_k = A(A\bar{A})^k \bar{Q}^2$		1	$\bar{\square}$	1	-2	$4N-5-6k$	$\frac{2}{3}$
$B_1 = A^N Q$		$\square$	1	$N$	1	$-N-1$	$\frac{1}{3}$
$\bar{B}_1 = \bar{A}^N \bar{Q}$		1	$\bar{\square}$	$-N$	-1	$-N-1$	$\frac{1}{3}$
$B_3 = A^{N-1} Q^3$		1	1	$N-1$	3	$3N$	1
$\bar{B}_3 = \bar{A}^{N-1} \bar{Q}^3$		1	1	$-N+1$	-3	$3N$	1
$T_m = (A\bar{A})^m$		1	1	0	0	$-6m$	0

where  $k=0, \dots, N-1$  and  $m=1, \dots, N$ . The number of terms in the confining superpotential grows quickly with the size of the gauge group. Therefore we only present the superpotential for the  $SU(5)$  theory:

$$\begin{aligned}
 W_{\text{dyn}} = & \frac{1}{\Lambda^9} (M_0^3 T_1 T_2 + M_1^3 + T_2 B_3 \bar{B}_3 + T_2 H_0 \bar{H}_0 M_0 + T_2 M_1 M_0^2 + T_1^3 M_0^3 + T_1^2 B_3 \bar{B}_3 + T_1^2 H_0 \bar{H}_0 M_0 + T_1^2 M_1 M_0^2 + T_1 B_1 \bar{B}_1 M_0^2 \\
 & + T_1 H_0 \bar{H}_0 M_1 + B_1 \bar{B}_1 H_0 \bar{H}_0 + B_1 \bar{B}_1 M_1 M_0 + H_1 \bar{H}_1 M_0 + H_1 \bar{H}_0 M_0 T_1 + \bar{H}_1 H_0 M_0 T_1 + \bar{H}_1 \bar{B}_1 B_3 + H_1 B_1 \bar{B}_3 + H_0 B_1 \bar{B}_3 T_1 \\
 & + \bar{H}_0 \bar{B}_1 B_3 T_1 + H_1 \bar{H}_0 M_1 + \bar{H}_1 H_0 M_1).
 \end{aligned}$$

Note that the term  $T_1 M_1^2 M_0$  is allowed by all symmetries, however, its coefficient is 0, which can be verified by requiring that the equations of motion reproduce the classical constraints.

5.  $SU(2N)$  with  $\square + \bar{\square} + 3(\square + \bar{\square})$

	$SU(2N)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_R$
$A$	$\square$	1	1	1	0	-3	0
$\bar{A}$	$\bar{\square}$	1	1	-1	0	-3	0
$Q$	$\square$	$\square$	1	0	1	$2N-2$	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	$\bar{\square}$	0	-1	$2N-2$	$\frac{1}{3}$
$M_k = Q(A\bar{A})^k \bar{Q}$		$\square$	$\bar{\square}$	0	0	$4N-4-6k$	$\frac{2}{3}$
$H_m = \bar{A}(A\bar{A})^m Q^2$		$\bar{\square}$	1	-1	2	$4N-7-6m$	$\frac{2}{3}$
$\bar{H}_m = A(A\bar{A})^m \bar{Q}^2$		1	$\bar{\square}$	1	-2	$4N-7-6m$	$\frac{2}{3}$
$B_0 = A^N$		1	1	$N$	0	$-3N$	0
$\bar{B}_0 = \bar{A}^N$		1	1	$-N$	0	$-3N$	0
$B_2 = A^{N-1} Q^2$		$\bar{\square}$	1	$N-1$	2	$N-1$	$\frac{2}{3}$
$\bar{B}_2 = \bar{A}^{N-1} \bar{Q}^2$		1	$\bar{\square}$	$-N+1$	-2	$N-1$	$\frac{2}{3}$
$T_n = (A\bar{A})^n$		1	1	0	0	$-6n$	0

where  $k=0, \dots, N-1$ ,  $m=0, \dots, N-2$  and  $n=1, \dots, N-1$ . The case of  $SU(4)$  is different, because in  $SU(4)$  the two-index antisymmetric tensor is self-conjugate. Therefore there is an additional  $SU(2)$  global symmetry. The corresponding table is

	$SU(4)$	$SU(2)$	$SU(3)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$A$	$\square$	$\square$	1	1	0	-3	0
$Q$	$\square$	1	$\square$	1	1	2	$\frac{1}{3}$
$\bar{Q}$	$\bar{\square}$	1	1	$\bar{\square}$	-1	2	$\frac{1}{3}$
$M_0 = Q\bar{Q}$		1	$\square$	$\bar{\square}$	0	4	$\frac{2}{3}$
$M_2 = QA^2\bar{Q}$		1	$\square$	$\bar{\square}$	0	-2	$\frac{2}{3}$
$H = AQ^2$		$\square$	$\bar{\square}$	1	2	1	$\frac{2}{3}$
$\bar{H} = A\bar{Q}^2$		$\square$	1	$\bar{\square}$	-2	1	$\frac{2}{3}$
$T = A^2$		$\square$	1	1	0	-6	0

The superpotential for the  $SU(4)$  theory is

$$W_{\text{dyn}} = \frac{1}{\Lambda^7} (T^2 M_0^3 - 12TH\bar{H}M_0 - 24M_0M_2^2 - 24H\bar{H}M_2),$$

where the relative coefficients are fixed by requiring that the equations of motion reproduce the classical constraints.

6.  $SU(6)$  with  $\square + 4(\square + \bar{\square})$

	$SU(6)$	$SU(4)$	$SU(4)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$A$	$\square$	1	1	0	-4	-1
$Q$	$\square$	$\square$	1	1	3	1
$\bar{Q}$	$\bar{\square}$	1	$\square$	-1	3	1
<hr/>						
$M_0 = QQ$		$\square$	$\square$	0	6	2
$M_2 = QA^2\bar{Q}$		$\square$	$\square$	0	-2	0
$B_1 = AQ^3$		$\bar{\square}$	1	3	5	2
$\bar{B}_1 = A\bar{Q}^3$		1	$\bar{\square}$	-3	5	2
$B_3 = A^3Q^3$		$\bar{\square}$	1	3	-3	0
$\bar{B}_3 = A^3\bar{Q}^3$		1	$\bar{\square}$	-3	-3	0
$T = A^4$		1	1	0	-16	4

$$W_{\text{dyn}} = \frac{1}{\Lambda^{11}} (M_0 B_1 \bar{B}_1 T + B_3 \bar{B}_3 M_0 + M_2^3 M_0 + T M_2 M_0^3 + \bar{B}_1 B_3 M_2 + B_1 \bar{B}_3 M_2).$$

7.  $SU(5)$  with  $3(\square + \bar{\square})$

	$SU(5)$	$SU(3)$	$SU(3)$	$U(1)$	$U(1)_R$
$A$	$\square$	$\square$	1	1	0
$\bar{Q}$	$\bar{\square}$	1	$\square$	-3	$\frac{2}{3}$
<hr/>					
$A\bar{Q}^2$		$\square$	$\bar{\square}$	-5	$\frac{4}{3}$
$A^3\bar{Q}$		$\square$	$\square$	0	$\frac{2}{3}$
$A^5$		$\square$	1	5	0

$$W_{\text{dyn}} = \frac{1}{\Lambda^9} [(A^5)(A^3\bar{Q})(A\bar{Q}^2) + (A^3\bar{Q})^3].$$

8.  $SU(5)$  with  $2\square + 4\bar{\square} + 2\square$

	$SU(5)$	$SU(2)$	$SU(4)$	$SU(2)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$A$	$\square$	$\square$	1	1	0	-1	0
$\bar{Q}$	$\bar{\square}$	1	$\square$	1	1	1	$\frac{1}{3}$
$Q$	$\square$	1	1	$\square$	-2	1	$\frac{1}{3}$
<hr/>							
$QQ$		1	$\square$	$\square$	-1	2	$\frac{2}{3}$
$A\bar{Q}^2$		$\square$	$\square$	1	2	1	$\frac{2}{3}$
$A^2Q$		$\square$	1	$\square$	-2	-1	$\frac{1}{3}$
$A^3\bar{Q}$		$\square$	$\square$	1	1	-2	$\frac{1}{3}$
$A^2Q^2\bar{Q}$		1	$\square$	1	-3	1	1

$$W_{\text{dyn}} = \frac{1}{\Lambda^9} [(A^3\bar{Q})^2(Q\bar{Q})^2 + (A^3\bar{Q})(A^2Q^2\bar{Q})(A\bar{Q}^2) + (A^3\bar{Q})(A^2Q)(A\bar{Q}^2)(Q\bar{Q}) + (A^2Q)^2(A\bar{Q}^2)^2].$$

9,  $SU(6)$  with  $2\Box + 5\bar{\Box} + \square$

	$SU(6)$	$SU(2)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$A$	$\Box$	$\square$	1	0	3	$\frac{1}{4}$
$\bar{Q}$	$\bar{\Box}$	1	$\square$	1	-4	0
$Q$	$\square$	1	1	-5	-4	0
$Q\bar{Q}$		1	$\square$	-4	-8	0
$A\bar{Q}^2$		$\square$	$\bar{\Box}$	2	-5	$\frac{1}{4}$
$A^3$		$\Box\Box\Box$	1	0	9	$\frac{3}{4}$
$A^3Q\bar{Q}$		$\square$	$\square$	-4	1	$\frac{3}{4}$
$A^4\bar{Q}^2$		1	$\bar{\Box}$	2	4	1

$$W_{\text{dyn}} = \frac{1}{\Lambda^{11}} [(A^4\bar{Q}^2)^2(Q\bar{Q}) + (A^4\bar{Q}^2)(A^3Q\bar{Q})(A\bar{Q}^2) + (A^3)(A^3Q\bar{Q})(A\bar{Q}^2)^2 + (A^3)^2(A\bar{Q}^2)^2(Q\bar{Q})].$$

Note, that the term  $(A^4\bar{Q}^2)(A^3)(A\bar{Q}^2)(Q\bar{Q})$  is allowed by the  $U(1)$  symmetries but not by the non-Abelian global symmetries.

10,  $SU(7)$  with  $2\Box + 6\bar{\Box}$

	$SU(7)$	$SU(2)$	$SU(6)$	$U(1)$	$U(1)_R$
$A$	$\Box$	$\square$	1	3	0
$\bar{Q}$	$\bar{\Box}$	1	$\square$	-5	$\frac{1}{3}$
$H = A\bar{Q}^2$		$\square$	$\bar{\Box}$	-7	$\frac{2}{3}$
$N = A^4\bar{Q}$		$\Box\Box$	$\square$	7	$\frac{1}{3}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^{13}} N^2 H^2.$$

B. The  $s$ -confining  $Sp(2N)$  theories

We now discuss the  $s$ -confining  $Sp(2N)$  theories. First, we again summarize the group theoretical properties of the simplest  $Sp(2N)$  representations. Contrary to  $SU(N)$  groups there is no chiral anomaly for  $Sp(2N)$  groups. The only requirement on the field content is that there is no Witten anomaly, this is satisfied if the sum of the Dynkin indices of the matter fields is even.  $Sp(2N)$  is the subgroup of  $SU(2N)$  which leaves the tensor  $J^{\alpha\beta} = (\mathbf{1}_{N \times N} \otimes i\sigma_2)^{\alpha\beta}$  invariant. Irreducible tensors of  $Sp(2N)$  must be traceless with respect to  $J^{\alpha\beta}$ . One can obtain these irreducible representations by subtracting traces from the  $SU(2N)$  tensors. The properties of these representations are summarized in the table below. We use a normalization where the index of the fundamental is one. This normalization is consistent with the  $Sp(2N) \subset SU(2N)$  embedding, under which  $2N \rightarrow 2N$ . Thus with these conventions the index of the matter fields does not change under  $SU \rightarrow Sp$  decompositions. The adjoint of  $Sp(2N)$  is the two-index symmetric tensor:

Irrep	Dim	$\mu$
$\square$	$2N$	1
$\bar{\Box}$	$N(2N-1)-1$	$2N-2$
$\Box\Box$	$N(2N+1)$	$2N+2$
$\bar{\Box}\bar{\Box}$	$\frac{N(2N-1)(2N-2)}{3} - 2N$	$\frac{(2N-3)(2N-2)}{3} - 1$
$\Box\bar{\Box}$	$\frac{N(2N+1)(2N+2)}{3}$	$\frac{(2N+2)(2N+3)}{3}$
$\bar{\Box}\Box$	$\frac{2N(2N-1)(2N+1)}{3} - 2N$	$\frac{(2N)^2}{3} - 4$

With this knowledge one can again write down all anomaly-free theories for which the matter content satisfies Eq. (5). These theories are summarized in Table II. In the first column, we indicate the gauge group and the field content of the theory. The second column gives a possible flow to a non- $s$ -confining theory or if the theory is  $s$ -confining, we

TABLE II. All  $Sp$  theories satisfying  $\sum_j \mu_j - \mu_G = 2$ . This list is finite because the indices of higher index tensor representations grow very rapidly with the size of the gauge group. We list the gauge group and the field content of the theories in the first column. In the second column, we indicate which theories are  $s$ -confining. For the remaining ones we give the flows to nonconfining theories or indicate that there is a Coulomb branch on the moduli space.

$Sp(2N)$	$(2N+4)\square$	$s$ -confining
$Sp(2N)$	$\bar{\Box} + 6\square$	$s$ -confining
$Sp(2N)$	$\Box\Box + 2\square$	Coulomb branch
$Sp(4)$	$2\bar{\Box} + 4\square$	$SU(2)$ : $8\square$
$Sp(4)$	$3\bar{\Box} + 2\square$	$SU(2)$ : $\Box\Box + 4\square$
$Sp(4)$	$4\bar{\Box}$	$SU(2)$ : $2\Box\Box$
$Sp(6)$	$2\bar{\Box} + 2\square$	$Sp(4)$ : $2\bar{\Box} + 4\square$
$Sp(6)$	$\bar{\Box} + 5\square$	$Sp(4)$ : $2\bar{\Box} + 4\square$
$Sp(6)$	$\bar{\Box} + \bar{\Box} + \square$	$SU(2)$ : $\Box\Box + 4\square$
$Sp(6)$	$2\bar{\Box}$	$SU(3)$ : $\Box\Box + \bar{\Box}\bar{\Box}$
$Sp(8)$	$2\bar{\Box}$	$Sp(4)$ : $5\bar{\Box}$



state that in the second column. The only  $s$ -confining theories based on  $Sp(2N)$  groups are the two sequences that are already known in the literature. We give the spectra and dynamically generated superpotentials of these theories in the tables below.

1.  $Sp(2N)$  with  $(2N+4) \square [6]$

	$Sp(2N)$	$SU(2N+4)$	$U(1)_R$
$Q$	$\square$	$\square$	$\frac{1}{N+2}$
$Q^2$		$\square$	$\frac{2}{N+2}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^{2N+1}} (Q^2)^{N+2}.$$

2.  $Sp(2N)$  with  $\square + 6 \square [11, 12]$

	$Sp(2N)$	$SU(6)$	$U(1)$	$U(1)_R$
$A$	$\square$	1	-3	0
$Q$	$\square$	$\square$	$N-1$	$\frac{1}{3}$
$A^k$		1	$-3k$	0
$QA^mQ$		$\square$	$2(N-1) - 3k$	$\frac{2}{3}$

Here  $k=2,3, \dots, N$  and  $m=0,1, \dots, N-1$ . The number of terms in the superpotential grows quickly with  $N$ . For  $Sp(4)$  the superpotential is

$$W_{\text{dyn}} = \frac{1}{\Lambda^5} [(A^2)(Q^2)^3 + (Q^2)(QAQ)^2].$$

C. The  $s$ -confining  $SO(N)$  theories

$SO(N)$  theories<sup>3</sup> are distinct from the  $SU$  and  $Sp$  theories because contrary to those groups  $SO(N)$  has representations which cannot be obtained from products of the vector representations. These are the spinorial representations. A theory can be  $s$ -confining only if all possible test charges can be screened by the matter fields. Spinors cannot be screened by matter in the vector representation of  $SO$ . Thus, theories without spinorial matter cannot be  $s$ -confining. This restricts the number of possible  $s$ -confining  $SO(N)$  theories, because the Dynkin index of the spinor representation grows exponentially with the size of the gauge group. The biggest group for which Eq. (5) can be satisfied with matter including spinor representations is  $SO(14)$ .

$SO(N)$  theories (for  $N > 6$ ) do not have either chiral or Witten anomalies. We do not consider the  $N \leq 6$  theories because they can be obtained from our previous results by using the following isomorphisms:  $SO(6) \sim SU(4)$ ,  $SO(5) \sim Sp(4)$ ,  $SO(4) \sim SU(2) \times SU(2)$ ,  $SO(3) \sim SU(2)$ ,  $SO(2) \sim U(1)$ .

The spinor representations of  $SO(N)$  have different properties depending on whether  $N$  is even or odd. For odd  $N$ , there is just one spinor representation, while for even  $N$  there are two inequivalent spinors. For  $N=4k$  the two spinors are self-conjugate while for  $N=4k+2$  the two spinors are complex conjugate to each other.

<sup>3</sup>We do not distinguish between  $SO(N)$  and its covering group  $\text{Spin}(N)$ .

We use a normalization where the index of the vector of  $SO(N)$  is 2. The reason is that under the embedding  $SO(2N) \supset SU(N)$  the vector of  $SO(2N)$  decomposes as  $2N \rightarrow N + \bar{N}$ . If we do not want the index of the matter fields to change under this decomposition we need to normalize the index of the vector to two. The fundamental properties of the smallest  $SO(N)$  representations are summarized in the tables below. The adjoint of  $SO(N)$  is the two-index antisymmetric tensor:

$SO(2N+1)$		
Irrep	Dim	$\mu$
$\square$	$2N+1$	2
$S$	$2^N$	$2^{N-2}$
$\square$	$N(2N+1)$	$4N-2$
$\square$	$(N+1)(2N+1)-1$	$4N+6$

$SO(2N)$		
Irrep	Dim	$\mu$
$\square$	$2N$	2
$S$	$2^{N-1}$	$2^{N-3}$
$\bar{S}, (S')$	$2^{N-1}$	$2^{N-3}$
$\square$	$N(2N-1)$	$4N-4$
$\square$	$N(2N+1)-1$	$4N+4$

Since the vector and the spinors are the only representations that potentially have smaller index than the adjoint, it is clear that candidates for  $s$ -confining theories contain only vectors and spinors. For odd  $N$  we denote the field content by  $(s, v)$ , where  $s$  is the number of spinors and  $v$  is the number of vectors. For even  $N$  we use the notation  $(s, s', v)$ , where  $s$  and  $s'$  are the numbers of matter fields in the two inequivalent spinor representations and  $v$  is the number of vectors.

The  $SO(8)$  group requires special attention. The reason is that there is a group automorphism which permutes the two spinor and the vector representations. Therefore only relative labelings of the representations are meaningful. For example  $(4,3,0)$  and  $(0,3,4)$  in  $SO(8)$  are equivalent.

With this knowledge of group theory we can write down all theories which satisfy Eq. (5). These theories are listed in Table III. Almost all of these theories are  $s$ -confining. The only spectrum that has been given in the literature [9] is for  $SO(7)$  with  $(5,1)$ . Below we list the spectra and the confining superpotentials for the  $s$ -confining  $SO(N)$  theories. Most of the confining superpotentials are very complicated. We only list those where the number of terms in the superpotential is reasonably small.

1.  $SO(14)$  with  $(1,0,5)$

	$SO(14)$	$SU(5)$	$U(1)$	$U(1)_R$
$S$	64	1	5	$\frac{1}{8}$
$Q$	$\square$	$\square$	-8	0
$Q^2$		$\square$	-16	0
$S^2Q^3$		$\square$	-14	$\frac{1}{4}$
$S^4Q^2$		$\square$	4	$\frac{1}{2}$
$S^4Q^4$		$\square$	-12	$\frac{1}{2}$
$S^6Q^3$		$\square$	6	$\frac{3}{4}$
$S^8$		1	40	1
$S^8Q^4$		$\square$	8	1

TABLE III. All  $SO(N)$  theories which contain at least one spinor and satisfy  $\sum_j \mu_j - \mu_G = 2$ . This list is finite because the index of the spinor representations grows exponentially with  $N$ . We list the gauge group of the theory in the first column and the matter content in the second column. As explained in the text, for odd  $N$  ( $s, v$ ) denotes the number of spinors and the number of vectors, while for even  $N$  ( $s, s', v$ ) denotes the numbers of the two inequivalent spinors and vectors. In the third column, we indicate which theories are  $s$ -confining. For the remaining ones we give the flows to nonconfining theories or indicate that there is a Coulomb branch on the moduli space.

$SO(14)$	(1, 0, 5)	$s$ -confining
$SO(13)$	(1, 4)	$s$ -confining
$SO(12)$	(1, 0, 7)	$s$ -confining
$SO(12)$	(2, 0, 3)	$s$ -confining
$SO(12)$	(1, 1, 3)	$s$ -confining
$SO(11)$	(1, 6)	$s$ -confining
$SO(11)$	(2, 2)	$s$ -confining
$SO(10)$	(4, 0, 1)	$s$ -confining
$SO(10)$	(3, 0, 3)	$s$ -confining
$SO(10)$	(2, 0, 5)	$s$ -confining
$SO(10)$	(3, 1, 1)	$s$ -confining
$SO(10)$	(2, 1, 3)	$s$ -confining
$SO(10)$	(1, 1, 5)	$s$ -confining
$SO(10)$	(2, 2, 1)	$s$ -confining
$SO(10)$	(1, 0, 7)	$SU(4)$ with $3 \square + 2 (\square + \bar{\square})$
$SO(9)$	(4, 0)	$s$ -confining
$SO(9)$	(3, 2)	$s$ -confining
$SO(9)$	(2, 4)	$s$ -confining
$SO(9)$	(1, 6)	$SU(4)$ with $3 \square + 2 (\square + \bar{\square})$
$SO(8)$	(7, 0, 0)	Coulomb branch
$SO(8)$	(6, 1, 0)	Coulomb branch
$SO(8)$	(5, 2, 0)	$SU(4)$ with $3 \square + 2 (\square + \bar{\square})$
$SO(8)$	(5, 1, 1)	$SU(4)$ with $3 \square + 2 (\square + \bar{\square})$
$SO(8)$	(4, 3, 0)	$s$ -confining
$SO(8)$	(4, 2, 1)	$s$ -confining
$SO(8)$	(3, 3, 1)	$s$ -confining
$SO(8)$	(3, 2, 2)	$s$ -confining
$SO(7)$	(6, 0)	$s$ -confining
$SO(7)$	(5, 1)	$s$ -confining
$SO(7)$	(4, 2)	$s$ -confining
$SO(7)$	(3, 3)	$s$ -confining
$SO(7)$	(2, 4)	$SU(4)$ with $3 \square + 2 (\square + \bar{\square})$
$SO(7)$	(1, 5)	Coulomb branch

$$\begin{aligned}
W_{\text{dyn}} = & \frac{1}{\Lambda^{23}} [(S^8 Q^4)^2 (Q^2) + (S^8 Q^4) (S^6 Q^3) (S^2 Q^3) \\
& + (S^8 Q^4) (S^4 Q^4) (S^4 Q^2) + (S^8)^2 (Q^2)^5 \\
& + (S^8) (S^6 Q^3) (S^2 Q^3) (Q^2)^2 + (S^4 Q^2)^4 (Q^2) \\
& + (S^6 Q^3)^2 (S^4 Q^2) (Q^2) + (S^8) (S^4 Q^4)^2 (Q^2) \\
& + (S^8) (S^4 Q^2)^2 (Q^2)^3 + (S^6 Q^3) (S^2 Q^3) (S^4 Q^2)^2 \\
& + (S^6 Q^3)^2 (S^4 Q^4)].
\end{aligned}$$

Note that several terms allowed by  $U(1)$  symmetries are not allowed by the full set of global symmetries. For example, the  $SU(5)$  contraction in the term  $(S^8 Q^4) (S^8) (Q^2)^3$  vanishes, since it is not possible to make an  $SU(5)$  invariant from the third power of a symmetric tensor and one field in the antifundamental representation. There are more examples of such terms prohibited by non-Abelian global symmetries in other theories in this section.

## 2. $SO(13)$ with (1,4)

	$SO(13)$	$SU(4)$	$U(1)$	$U(1)_R$
$S$	64	1	1	$\frac{1}{8}$
$Q$	$\square$	$\square$	-2	0
$Q^2$		$\square$	-4	0
$S^2 Q^3$		$\bar{\square}$	-4	$\frac{1}{4}$
$S^2 Q^2$		$\square$	-2	$\frac{1}{4}$
$S^4 Q^4$		1	-4	$\frac{1}{2}$
$S^4 Q^3$		$\bar{\square}$	-2	$\frac{1}{2}$
$S^4 Q^2$		$\square$	0	$\frac{1}{2}$
$S^4 Q$		$\square$	2	$\frac{1}{2}$
$S^4$		1	4	$\frac{1}{2}$
$S^6 Q^3$		$\bar{\square}$	0	$\frac{3}{4}$
$S^6 Q^2$		$\square$	2	$\frac{3}{4}$
$S^8 Q^3$		$\bar{\square}$	2	1
$S^8$		1	8	1

Note, that one could add the operator  $S^8 Q^4$  to the above list without affecting anomaly matching. However, there is a mass term allowed for this operator, and by flowing to this theory from  $SO(14)$  with (1,0,5) one finds that this mass term is generated. Thus  $S^8 Q^4$  is not in the IR spectrum. Similar operators appear in many other  $s$ -confining  $SO(N)$  theories. Since a mass term is always generated for such operators, we do not include them in any of the forthcoming  $s$ -confining spectra.

## 3. $SO(12)$ with (1,0,7)

	$SO(12)$	$SU(7)$	$U(1)$	$U(1)_R$
$S$	32	1	7	$\frac{1}{4}$
$Q$	$\square$	$\square$	-4	0
$Q^2$		$\square$	-8	0
$S^2 Q^2$		$\square$	6	$\frac{1}{2}$
$S^2 Q^6$		$\bar{\square}$	-10	$\frac{1}{2}$
$S^4$		1	28	1
$S^4 Q^6$		$\bar{\square}$	4	1

$$\begin{aligned}
W_{\text{dyn}} = & \frac{1}{\Lambda^{19}} [(S^4 Q^6)^2 (Q^2) + (S^4 Q^6) (S^2 Q^6) (S^2 Q^2) \\
& + (S^4) (S^2 Q^2)^2 (Q^2)^5 + (S^4) (S^2 Q^6)^2 (Q^2) \\
& + (S^2 Q^2)^4 (Q^2)^3 + (Q^2)^7 (S^4)^2].
\end{aligned}$$

## 4. $SO(12)$ with (2,0,3)

	$SO(12)$	$SU(2)$	$SU(3)$	$U(1)$	$U(1)_R$
$S$	32	$\square$	1	3	$\frac{1}{8}$
$Q$	$\square$	1	$\square$	-8	0
$Q^2$		1	$\square$	-16	0
$S^2$		1	1	6	$\frac{1}{4}$
$S^2 Q^2$		$\square$	$\bar{\square}$	-10	$\frac{1}{4}$
$S^4$		$\square$	1	12	$\frac{1}{2}$
$S^4 Q^2$		1	$\square$	-4	$\frac{1}{2}$
$S^4 Q^{2'}$		$\square$	$\bar{\square}$	-4	$\frac{1}{2}$
$S^6$		1	1	18	$\frac{3}{4}$
$S^6 Q^2$		$\square$	$\bar{\square}$	2	$\frac{3}{4}$
$S^8 Q^2$		1	$\square$	8	1

5. SO(12) with (1,1,3)

	SO(12)	SU(3)	U(1) <sub>1</sub>	U(1) <sub>2</sub>	U(1) <sub>R</sub>
S	32	1	1	3	$\frac{1}{8}$
S'	32'	1	-1	3	$-\frac{1}{8}$
Q	$\square$	$\square$	0	-8	0
Q <sup>2</sup>		$\square\square$	0	-16	0
S S' Q <sup>3</sup>		1	0	-18	$\frac{1}{4}$
S <sup>2</sup> Q <sup>2</sup>		$\bar{\square}$	2	-10	$\frac{1}{4}$
S <sup>2</sup> Q <sup>2</sup>		$\bar{\square}$	-2	-10	$\frac{1}{4}$
S S' Q		$\square$	0	-2	$\frac{1}{4}$
S <sup>4</sup>		1	4	12	$\frac{1}{2}$
S <sup>4</sup>		1	-4	12	$\frac{1}{2}$
S <sup>2</sup> S <sup>2</sup>		1	0	12	$\frac{1}{2}$
S <sup>3</sup> S' Q <sup>3</sup>		1	2	-12	$\frac{1}{2}$
S <sup>3</sup> S Q <sup>3</sup>		1	-2	-12	$\frac{1}{2}$
S <sup>2</sup> S <sup>2</sup> Q <sup>2</sup>		$\square\square$	0	-4	$\frac{1}{2}$
S <sup>2</sup> S <sup>2</sup> Q <sup>2</sup>		$\bar{\square}$	0	-4	$\frac{1}{2}$
S <sup>3</sup> S' Q		$\square$	2	4	$\frac{1}{2}$
S <sup>3</sup> S Q		$\square$	-2	4	$\frac{1}{2}$
S <sup>3</sup> S <sup>3</sup> Q <sup>3</sup>		1	0	-6	$\frac{3}{4}$
S <sup>3</sup> S <sup>3</sup> Q <sup>3</sup>		$\square$	0	10	$\frac{3}{4}$
S <sup>4</sup> S <sup>2</sup> Q <sup>2</sup>		$\bar{\square}$	2	2	$\frac{3}{4}$
S <sup>4</sup> S <sup>2</sup> Q <sup>2</sup>		$\bar{\square}$	-2	2	$\frac{3}{4}$
S <sup>4</sup> S <sup>4</sup>		1	0	24	1
S <sup>4</sup> S <sup>4</sup> Q <sup>2</sup>		$\bar{\square}$	0	8	1

6. SO(11) with (1,6)

	SO(11)	SU(6)	U(1)	U(1) <sub>R</sub>
S	32	1	3	$\frac{1}{4}$
Q	$\square$	$\square$	-2	0
Q <sup>2</sup>		$\square\square$	-4	0
S <sup>2</sup> Q <sup>2</sup>		$\square$	2	$\frac{1}{2}$
S <sup>2</sup> Q <sup>5</sup>		$\bar{\square}$	-4	$\frac{1}{2}$
S <sup>4</sup>		1	12	1
S <sup>4</sup> Q <sup>5</sup>		$\bar{\square}$	2	1
S <sup>2</sup> Q		$\square$	4	$\frac{1}{2}$
S <sup>2</sup> Q <sup>6</sup>		1	-6	$\frac{1}{2}$

7. SO(11) with (2,2)

	SO(11)	SU(2)	SU(2)	U(1)	U(1) <sub>R</sub>
S	32	$\square$	1	1	0
Q	$\square$	1	$\square$	-4	$\frac{1}{2}$
Q <sup>2</sup>		1	$\square\square$	-8	1
S <sup>2</sup> Q <sup>2</sup>		$\square\square$	1	-6	1
S <sup>2</sup> Q		$\square$	$\square$	-2	$\frac{1}{2}$
S <sup>2</sup>		1	1	2	0
S <sup>4</sup>		$\square\square\square$	1	4	0
S <sup>4</sup>		1	1	4	0
S <sup>4</sup> Q <sup>2</sup>		1	$\square\square$	-4	1
S <sup>4</sup> Q <sup>2</sup>		$\square\square$	1	-4	1
S <sup>4</sup> Q		$\square$	$\square$	0	$\frac{1}{2}$
S <sup>6</sup> Q <sup>2</sup>		$\square$	1	-2	1
S <sup>6</sup> Q		$\square$	$\square$	2	$\frac{1}{2}$
S <sup>8</sup>		1	1	8	0
S <sup>8</sup> Q		1	$\square$	4	$\frac{1}{2}$
S <sup>4</sup> Q		1	$\square$	0	$\frac{1}{2}$
S <sup>6</sup>		1	1	6	0

8. SO(10) with (4,0,1)

	SO(10)	SU(4)	U(1)	U(1) <sub>R</sub>
S	16	$\square$	1	0
Q	$\square$	1	-8	1
Q <sup>2</sup>		1	-16	2
S <sup>2</sup> Q		$\square$	-6	1
S <sup>4</sup>		$\square\square$	4	0
S <sup>6</sup> Q		$\square$	-2	1

$$W_{\text{dyn}} = \frac{1}{\Lambda^{15}} [(S^6 Q)^2 (S^4) + (S^6 Q)(S^2 Q)(S^4)^2 + (S^2 Q)^2 (S^4)^3 + (S^4)^4 (Q^2)].$$

9. SO(10) with (3,0,3)

	SO(10)	SU(3)	SU(3)	U(1)	U(1) <sub>R</sub>
S	16	$\square$	1	1	0
Q	$\square$	1	$\square$	-2	$\frac{1}{3}$
Q <sup>2</sup>		1	$\square\square$	-4	$\frac{2}{3}$
S <sup>2</sup> Q		$\square$	$\square$	0	$\frac{1}{3}$
S <sup>2</sup> Q <sup>3</sup>		$\bar{\square}$	1	-4	1
S <sup>4</sup>		$\square$	1	4	0
S <sup>4</sup> Q <sup>2</sup>		$\square$	$\bar{\square}$	0	$\frac{2}{3}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^{15}} [(S^4 Q^2)^3 + (S^4 Q^2)^2 (S^2 Q)^2 + (S^4 Q^2)^2 (S^4)(Q^2) + (S^2 Q^3)^2 (S^4)^2 + (S^2 Q)^2 (Q^2)^2 (S^4)^2 + (S^2 Q)^4 (Q^2)(S^4) + (Q^2)^3 (S^4)^3 + (S^2 Q)^6 + (S^4)(S^2 Q^3)(S^4 Q^2)(S^2 Q) + (S^4 Q^2)(S^4)(S^2 Q)^2 (Q^2) + (S^4 Q^2)(S^4 Q)^4 + (S^2 Q^3)(S^2 Q)^3 (S^4)].$$

10. SO(10) with (2,0,5)

	SO(10)	SU(2)	SU(5)	U(1)	U(1) <sub>R</sub>
S	16	$\square$	1	5	$\frac{1}{4}$
Q	$\square$	1	$\square$	-4	0
Q <sup>2</sup>		1	$\square$	-8	0
S <sup>2</sup> Q		$\square$	$\square$	6	$\frac{1}{2}$
S <sup>2</sup> Q <sup>3</sup>		1	$\bar{\square}$	-2	$\frac{1}{2}$
S <sup>2</sup> Q <sup>5</sup>		$\square$	1	-10	$\frac{1}{2}$
S <sup>4</sup>		1	1	20	1
S <sup>4</sup> Q <sup>4</sup>		1	$\bar{\square}$	4	1

11.  $SO(10)$  with  $(3,1,1)$ 

	$SO(10)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$S$	16	$\square$	1	0	0
$\bar{S}$	$\bar{16}$	1	-3	1	0
$Q$	$\square$	1	0	-2	1
$Q^2$		1	0	-4	2
$S^2Q$		$\square$	2	-2	1
$S\bar{S}$		$\square$	-2	1	0
$S^3\bar{S}Q$		$\square$	0	-1	1
$S^2\bar{S}^2$		$\square$	-4	2	0
$S^4$		$\square$	4	0	0
$S^5\bar{S}$		$\square$	2	1	0
$S^4\bar{S}^2Q$		$\square$	-2	0	1
$\bar{S}^2Q$		1	-6	0	1
$S^3\bar{S}^3Q^2$		1	-6	-1	2

12.  $SO(10)$  with  $(2,1,3)$ 

	$SO(10)$	$SU(2)$	$SU(3)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$S$	16	$\square$	1	1	1	0
$\bar{S}$	$\bar{16}$	1	1	-2	1	$\frac{1}{2}$
$Q$	$\square$	1	$\square$	0	-2	0
$Q^2$		1	$\square$	0	-4	0
$S^2Q$		$\square$	$\square$	2	0	0
$\bar{S}^2Q$		1	$\square$	-4	0	1
$S\bar{S}$		$\square$	1	-1	2	$\frac{1}{2}$
$S^2\bar{S}^2$		$\square$	1	-2	4	1
$S^2Q^3$		1	1	2	-4	0
$S^3\bar{S}Q$		$\square$	$\square$	1	2	$\frac{1}{2}$
$S^4$		1	1	4	4	0
$S\bar{S}Q^2$		$\square$	$\square$	-1	-2	$\frac{1}{2}$
$S^2\bar{S}^2Q^2$		1	$\square$	-2	0	1
$S^3\bar{S}Q^3$		$\square$	1	1	-2	$\frac{1}{2}$

13.  $SO(10)$  with  $(1,1,5)$ 

	$SO(10)$	$SU(5)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$S$	16	1	1	5	$\frac{1}{4}$
$\bar{S}$	$\bar{16}$	1	-1	5	$\frac{1}{4}$
$Q$	$\square$	$\square$	0	-4	0
$Q^2$		$\square$	0	-8	0
$S^2Q$		$\square$	2	6	$\frac{1}{2}$
$\bar{S}^2Q$		$\square$	-2	6	$\frac{1}{2}$
$S\bar{S}$		1	0	10	$\frac{1}{2}$
$S^2Q^5$		1	2	-10	$\frac{1}{2}$
$\bar{S}^2Q^5$		1	-2	-10	$\frac{1}{2}$
$S\bar{S}Q^2$		$\square$	0	2	$\frac{1}{2}$
$S\bar{S}Q^4$		$\square$	0	-6	$\frac{1}{2}$
$S^2\bar{S}^2$		1	0	20	1
$S^2\bar{S}^2Q^4$		$\square$	0	4	1

14.  $SO(10)$  with  $(2,2,1)$ 

	$SO(10)$	$SU(2)$	$SU(2)$	$U(1)_1$	$U(1)_2$	$U(1)_R$
$S$	16	$\square$	1	1	1	0
$\bar{S}$	$\bar{16}$	1	$\square$	-1	1	0
$Q$	$\square$	1	1	0	-8	1
$Q^2$		1	1	0	-16	2
$S^2Q$		$\square$	1	2	-6	1
$\bar{S}^2Q$		1	$\square$	-2	-6	1
$S\bar{S}$		$\square$	$\square$	0	2	0
$S^4$		1	1	4	4	0
$\bar{S}^4$		1	1	-4	4	0
$S^2\bar{S}^2$		$\square$	$\square$	0	4	0
$S^3\bar{S}Q$		$\square$	$\square$	2	-4	1
$\bar{S}^3SQ$		$\square$	$\square$	-2	-4	1
$S^2\bar{S}^2Q^2$		1	1	0	-12	2
$S^4\bar{S}^2Q$		$\square$	1	2	-2	1
$\bar{S}^4S^2Q$		1	$\square$	-2	-2	1
$S^3\bar{S}^3$		$\square$	$\square$	0	6	0
$S^6\bar{S}^2$		1	1	4	8	0
$\bar{S}^6S^2$		1	1	-4	8	0

15.  $SO(9)$  with  $(4,0)$ 

	$SO(9)$	$SU(4)$	$U(1)_R$
$S$	16	$\square$	$\frac{1}{8}$
$S^2$		$\square$	$\frac{1}{4}$
$S^4$		$\square$	$\frac{1}{2}$
$S^6$		$\square$	$\frac{3}{4}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^{13}} [(S^6)^2(S^4) + (S^6)(S^4)^2(S^2) + (S^4)^4 + (S^4)^3(S^2)^2].$$

16.  $SO(9)$  with  $(3,2)$ 

	$SO(9)$	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
$S$	16	$\square$	1	1	0
$Q$	$\square$	1	$\square$	-3	$\frac{1}{2}$
$Q^2$		1	$\square$	-6	1
$S^2Q$		$\square$	$\square$	-1	$\frac{1}{2}$
$S^2$		$\square$	1	2	0
$S^4$		$\square$	1	4	0
$S^2Q^2$		$\square$	1	-4	1
$S^4Q^2$		$\square$	1	-2	1
$S^4Q$		$\square$	$\square$	1	$\frac{1}{2}$

17. SO(9) with (2,4)

	SO(9)	SU(2)	SU(4)	U(1)	U(1) <sub>R</sub>
<i>S</i>	16	□	1	1	$\frac{1}{4}$
<i>Q</i>	□	1	□	-1	0
<i>Q</i> <sup>2</sup>		1	□□	-2	0
<i>S</i> <sup>2</sup> <i>Q</i>		□□	□	1	$\frac{1}{2}$
<i>S</i> <sup>2</sup>		□□	1	2	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>Q</i> <sup>3</sup>		1	□	-1	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>Q</i> <sup>2</sup>		1	□	0	$\frac{1}{2}$
<i>S</i> <sup>4</sup> <i>Q</i> <sup>3</sup>		1	□	1	1
<i>S</i> <sup>2</sup> <i>Q</i> <sup>4</sup>		□□	1	-2	$\frac{1}{2}$
<i>S</i> <sup>4</sup>		1	1	4	1

18. SO(8) with (3,0,4)

	SO(8)	SU(4)	SU(3)	U(1)	U(1) <sub>R</sub>
<i>Q</i>	8 <sub>v</sub>	□	1	3	$\frac{1}{4}$
<i>S</i>	8 <sub>s</sub>	1	□	-4	0
<i>Q</i> <sup>2</sup>		□□	1	6	$\frac{1}{2}$
<i>S</i> <sup>2</sup>		1	□□	-8	0
<i>S</i> <sup>2</sup> <i>Q</i> <sup>2</sup>		□	□	-2	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>Q</i> <sup>4</sup>		1	□□	4	1

$$W_{\text{dyn}} = \frac{1}{\Lambda^{11}} [(S^2 Q^4)^2 (S^2) + (S^2 Q^4) (S^2 Q^2)^2 + (S^2 Q^2)^3 (Q^2) + (S^2)^3 (Q^2)^4 + (S^2 Q^2)^2 (S^2) (Q^2)^2].$$

19. SO(8) with (2,1,4)

	SO(8)	SU(4)	SU(2)	U(1)	U(1)	U(1) <sub>R</sub>
<i>Q</i>	8 <sub>v</sub>	□	1	1	0	$\frac{1}{4}$
<i>S</i>	8 <sub>s</sub>	1	□	-2	1	0
<i>S</i> '	8 <sub>c</sub>	1	1	0	-2	0
<i>Q</i> <sup>2</sup>		□□	1	2	0	$\frac{1}{2}$
<i>S</i> <sup>2</sup>		1	□□	-4	2	0
<i>S</i> ' <sup>2</sup>		1	1	0	-4	0
<i>S</i> <sup>2</sup> <i>Q</i> <sup>2</sup>		□	1	-2	2	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>Q</i> <sup>4</sup>		1	□□	0	2	1
<i>S</i> ' <sup>2</sup> <i>Q</i> <sup>4</sup>		1	1	4	-4	1
<i>SS</i> ' <i>Q</i>		□	□	-1	-1	$\frac{1}{4}$
<i>SS</i> ' <i>Q</i> <sup>3</sup>		□	□	1	-1	$\frac{3}{4}$

20. SO(8) with (3,3,1)

	SO(8)	SU(3)	SU(3)	U(1) <sub>1</sub>	U(1) <sub>2</sub>	U(1) <sub>R</sub>
<i>Q</i>	8 <sub>v</sub>	1	1	0	6	1
<i>S</i>	8 <sub>s</sub>	□	1	1	-1	0
<i>S</i> '	8 <sub>c</sub>	1	□	-1	-1	0
<i>Q</i> <sup>2</sup>		1	1	0	12	2
<i>S</i> <sup>2</sup>		□□	1	2	-2	0
<i>S</i> ' <sup>2</sup>		1	□□	-2	-2	0
<i>SS</i> ' <i>Q</i>		□	□	0	4	1
<i>S</i> <sup>3</sup> <i>S</i> ' <i>Q</i>		1	□	2	2	1
<i>S</i> ' <sup>3</sup> <i>S</i> <i>Q</i>		□	1	-2	2	1
<i>S</i> <sup>2</sup> <i>S</i> ' <sup>2</sup>		□	□	0	-4	0

21. SO(8) with (2,2,3)

	SO(8)	SU(3)	SU(2)	SU(2)	U(1) <sub>1</sub>	U(1) <sub>2</sub>	U(1) <sub>R</sub>
<i>Q</i>	8 <sub>v</sub>	□	1	1	0	4	0
<i>S</i>	8 <sub>s</sub>	1	□	1	1	-3	$\frac{1}{4}$
<i>S</i> '	8 <sub>c</sub>	1	1	□	-1	-3	$\frac{1}{4}$
<i>Q</i> <sup>2</sup>		□□	1	1	0	8	0
<i>S</i> <sup>2</sup>		1	□□	1	2	-6	$\frac{1}{2}$
<i>S</i> ' <sup>2</sup>		1	1	□□	-2	-6	$\frac{1}{2}$
<i>SS</i> ' <i>Q</i>		□	□	□	0	-2	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>Q</i> <sup>2</sup>		□	1	1	2	2	$\frac{1}{2}$
<i>S</i> ' <sup>2</sup> <i>Q</i> <sup>2</sup>		□	1	1	-2	2	$\frac{1}{2}$
<i>SS</i> ' <i>Q</i> <sup>3</sup>		1	□	□	0	6	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>S</i> ' <sup>2</sup>		1	1	1	0	-12	1
<i>S</i> <sup>2</sup> <i>S</i> ' <sup>2</sup> <i>Q</i> <sup>2</sup>		□	1	1	0	-4	1

22. SO(7) with (6,0)

	SO(7)	SU(6)	U(1) <sub>R</sub>
<i>S</i>	8	□	$\frac{1}{6}$
<i>S</i> <sup>2</sup>		□□	$\frac{1}{3}$
<i>S</i> <sup>4</sup>		□□	$\frac{2}{3}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^9} [(S^4)^3 + (S^4)^2 (S^2)^2 + (S^2)^6].$$

23. SO(7) with (5,1) [9]

	SO(7)	SU(5)	U(1)	U(1) <sub>R</sub>
<i>S</i>	8	□	1	0
<i>Q</i>	□	1	-5	1
<i>Q</i> <sup>2</sup>		1	-10	2
<i>S</i> <sup>2</sup>		□□	2	0
<i>S</i> <sup>4</sup>		□	4	0
<i>S</i> <sup>2</sup> <i>Q</i>		□	-3	1
<i>S</i> <sup>4</sup> <i>Q</i>		□	-1	1

$$W_{\text{dyn}} = \frac{1}{\Lambda^9} [(S^4 Q)^2 (S^2) + (S^4 Q) (S^2 Q) (S^4) + (S^2 Q)^2 (S^4) \times (S^2) + (Q^2) (S^2) (S^4)^2 + (S^2)^5 (Q^2)].$$

24. SO(7) with (4,2)

	SO(7)	SU(4)	SU(2)	U(1)	U(1) <sub>R</sub>
<i>S</i>	8	□	1	1	0
<i>Q</i>	□	1	□	-2	$\frac{1}{2}$
<i>Q</i> <sup>2</sup>		1	□□	-4	1
<i>S</i> <sup>2</sup>		□□	1	2	0
<i>S</i> <sup>2</sup> <i>Q</i>		□	□	0	$\frac{1}{2}$
<i>S</i> <sup>2</sup> <i>Q</i> <sup>2</sup>		□	1	-2	1
<i>S</i> <sup>4</sup>		1	1	4	0
<i>S</i> <sup>4</sup> <i>Q</i>		1	□	2	$\frac{1}{2}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^9} [(S^4 Q)^2 (Q^2) + (S^4 Q) (S^2 Q) (S^2 Q^2) + (S^2 Q)^2 (S^2 Q^2) (S^2) + (S^4) (S^2 Q^2)^2 + (S^2 Q)^2 (S^2)^2 (Q^2) + (S^2 Q^2)^2 (S^2)^2 + (S^2)^4 (Q^2)^2].$$

25.  $SO(7)$  with (3,3)

	$SO(7)$	$SU(3)$	$SU(3)$	$U(1)$	$U(1)_R$
$S$	8	$\square$	1	1	0
$Q$	$\square$	1	$\square$	-1	$\frac{1}{3}$
$Q^2$		1	$\square\square$	-2	$\frac{2}{3}$
$S^2$		$\square\square$	1	2	0
$S^2Q$		$\bar{\square}$	$\square$	1	$\frac{1}{3}$
$S^2Q^2$		$\bar{\square}$	$\bar{\square}$	0	$\frac{2}{3}$
$S^2Q^3$		$\square\square$	1	-1	1

$$W_{\text{dyn}} = \frac{1}{\Lambda^9} [(S^2Q^3)^2(S^2) + (S^2Q^3)(S^2Q^2)(S^2Q) + (S^2Q^2)^3 + (S^2)^3(Q^2)^3 + (S^2Q^2)^2(S^2)(Q^2) + (S^2Q)^2(S^2)(Q^2)^2 + (S^2Q)^2(S^2Q^2)(Q^2)].$$

26. The  $SO(N)$  theories with  $\sum \mu_i - \mu_G = 4$ 

Our normalization for the indices of SO groups is somewhat nonstandard. It follows from demanding that the index is invariant under flows from  $SO(2N)$  groups to their  $SU(N)$  subgroups. In the normalization where the index of the vector is 1 rather than 2, it is obvious that one can obtain a superpotential that is regular at the origin for  $\sum \mu_i - \mu_G = 1$  or 2. In our normalization, this corresponds to  $\sum \mu_i - \mu_G = 2$  or 4. We have explicitly checked that none of the  $\sum \mu_i - \mu_G = 4$  theories are  $s$ -confining by identifying flows to non- $s$ -confining theories.

The  $\sum \mu_i - \mu_G = 4$   $SO(N)$  theories are examples of the special case where the confining superpotential can be holomorphic at the origin without Eq. (5) being satisfied. This can only happen when  $\mu_G$  and all  $\mu_i$  have a common divisor. Just like the previously mentioned  $\sum \mu_i - \mu_G = 4$   $SO(N)$  theories, such theories are unlikely to  $s$ -confine. The reason is that while Eq. (5) is preserved under most flows along flat directions, the property that  $\mu_G$  and all  $\mu_i$  have a common divisor is not. Thus for most such theories one should be able to find a flow to a non- $s$ -confining theory. We expect that none of these ‘‘common divisor’’ theories  $s$ -confine.

## D. Exceptional groups

The analysis for exceptional groups  $G_2, F_4, E_6, E_7,$  and  $E_8$  is surprisingly simple. The  $s$ -confined spectrum of a  $G_2$  gauge theory with 5 fundamentals has already been worked out in Refs. [8,9]. The representations of  $G_2$  are real, thus the invariant tensors include the two index symmetric tensor. Furthermore, there are two totally antisymmetric tensors with three and four indices, respectively. The confined spectrum is given in the following table:

1.  $G_2$  with 5  $\square$  [8]

	$G_2$	$SU(5)$	$U(1)_R$
$Q$	7	$\square$	$\frac{1}{5}$
$M = Q^2$		$\square\square$	$\frac{2}{5}$
$A = Q^3$		$\bar{\square}$	$\frac{3}{5}$
$B = Q^4$		$\bar{\square}$	$\frac{4}{5}$

$$W_{\text{dyn}} = \frac{1}{\Lambda^7} [M^5 + M^2A^2 + MB^2 + A^2B].$$

2. The  $F_4, E_6, E_7,$  and  $E_8$  theories

Theories based on any of the other exceptional gauge groups can be shown to flow to theories which are not  $s$ -confining. This is derived most easily by starting with the real group  $F_4$ . The lowest dimensional representations of  $F_4$  are the 26-dimensional fundamental representation and the 52-dimensional adjoint. Since any theory with adjoint matter has a Coulomb branch on its moduli space, we can restrict our attention to theories with only fundamentals. By giving an expectation value to a fundamental one can break  $F_4$  to its maximal subgroup  $SO(9)$ . Under  $SO(9)$  the representations decompose as follows:  $26 \rightarrow 1 + 9 + 16$  and  $52 \rightarrow 16 + 36$ . The 9, 16, 36 are the fundamental, spinor, and adjoint of  $SO(9)$ . When giving an expectation value to a fundamental of  $F_4$ , the spinor component of its  $SO(9)$  decomposition is eaten. Thus an  $F_4$  theory with  $N_f$  fundamentals flows to an  $SO(9)$  theory with  $N_f$  fundamentals and  $N_f - 1$  spinors. For no  $N_f$  is this  $SO(9)$  theory  $s$ -confining, therefore no  $F_4$  theory  $s$ -confining.

Using this result, it is easy to show that none of the groups  $E_6, E_7,$  and  $E_8$   $s$ -confine. The lowest dimensional representations of  $E_6$  are the (complex) fundamental and the adjoint. By giving an expectation value to a fundamental, one can flow to  $F_4$ , whereas expectation values for an adjoint lead to a Coulomb branch. Thus,  $E_6$  theories cannot be  $s$ -confining either.

By giving an expectation value to a field in the 56-dimensional fundamental representation of  $E_7$  one can flow to  $E_6$ , while an expectation value for the adjoint again yields a Coulomb branch. For  $E_8$  the lowest dimensional representation is the adjoint, again leading to a Coulomb branch. Thus none of the  $E_{6,7,8}$  groups with arbitrary matter are  $s$ -confining.

## IV. OBTAINING NEW MODELS BY INTEGRATING OUT MATTER

In the previous chapter we obtained a low-energy description for many theories which satisfy  $\sum \mu_i - \mu_G = 2$ . Since a number of these theories contain matter in vectorlike representations one can easily derive descriptions for theories with smaller matter content by integrating out fields. In this way we obtain confining theories with a quantum modified constraint, theories with dynamically generated superpotentials and theories with multiple branches.

## A. Theories with quantum-deformed moduli spaces

In these theories a classical constraint of the form  $\Sigma(\Pi_i X_i) = 0$  (where  $X_i$  are gauge-invariant operators) is modified quantum mechanically to  $\Sigma(\Pi_i X_i) = \Lambda^p \Pi_j X_j$ . Here, the  $X_j$  are some other combination of the gauge-invariant operators, including the possibility that the quantum modification is just  $\Lambda^p$ . The power  $p$  must necessarily be positive to reproduce the correct classical limit. Such a modification of the classical constraint is only possible in theories where  $\sum \mu_i - \mu_G = 0$ . To show this, consider assigning  $R$  charge 0 to every chiral superfield. This  $R$  symmetry is anomalous and the anomaly has to be compensated by assigning  $R$  charge  $\sum \mu_i - \mu_G$  to the scale of the gauge group raised to the power of its one loop  $\beta$  function coefficient

$\Lambda^{(3\mu_G - \sum\mu_i)/2}$  [2]. Since the constraints have to respect this  $R$  symmetry one immediately sees that  $\Lambda$  can only appear in a constraint if it has vanishing  $R$  charge. Therefore, we conclude that only theories with  $\sum\mu_i - \mu_G = 0$  may exhibit quantum deformed moduli spaces.

We can find all theories satisfying  $\sum\mu_i - \mu_G = 0$  by simply leaving out a flavor from the matter contents listed in Tables I and II for SU and Sp theories and by leaving out a vector from Table III for SO theories. The theories obtained from the  $s$ -confining ones are all confining with a quantum modified constraint. It follows from the procedure of integrating out a flavor that the form of the quantum modified constraint is  $\Sigma(\Pi_i X_i) = \Lambda^{\sum\mu_i}$ .

In those cases where the  $s$ -confining theory contains several meson type fields (e.g.,  $Q\bar{Q}$ ,  $QA^2\bar{Q}$ , etc.) there will be additional constraints which are not modified quantum mechanically [11,12]. All constraints can be implemented by adding them to the superpotential with Lagrange multipliers. Here, we list only those SU theories which were not previously known in the literature. Similar results can be obtained from the  $s$ -confining SO theories. In the case of  $SO(N)$  theories there is always one quantum modified constraint, while the total number of constraints equals the number of operators containing exactly two vectors  $Q$  in a symmetric representation of the  $Q$ -flavor symmetry.

In the following superpotentials we denote Lagrange multipliers by Greek letters, the notation for the confined fields is defined in the corresponding tables in Sec. III.

### 1. SU(4) with $2\Box + 2(\Box + \bar{\Box})$

$$W = \lambda(3T^2 M_0^2 - 12TH\bar{H} - 24M_2^2 - \Lambda^8) + \mu(2M_0 M_2 + H\bar{H}).$$

### 2. SU(5) with $\Box + \bar{\Box} + 2(\Box + \bar{\Box})$

$$\begin{aligned} W = & \lambda(3M_0^2 T_1 T_2 + T_2 H_0 \bar{H}_0 + 2T_2 M_0 M_1 + 3T_1^3 M_0^2 \\ & + T_1^2 H_0 \bar{H}_0 + 2T_1^2 M_0 M_1 + 2T_1 B_1 \bar{B}_1 M_0 + B_1 \bar{B}_1 M_1 \\ & + H_1 \bar{H}_1 + \bar{H}_0 H_1 T_1 + H_0 \bar{H}_1 T_1 - \Lambda^{10}) + \mu(3M_1^2 + T_2 M_0^2 \\ & + T_1^2 M_0^2 + T_1 H_0 \bar{H}_0 + B_1 \bar{B}_1 M_0 + H_0 \bar{H}_1 + \bar{H}_0 H_1). \end{aligned}$$

### 3. SU(5) with $2\Box + \Box + 3\bar{\Box}$

$$\begin{aligned} W = & \lambda[(A^3 \bar{Q})^2 (Q\bar{Q}) + (A^3 \bar{Q})(A^2 Q)(A\bar{Q}^2) - \Lambda^{10}] \\ & + \mu(A^3 \bar{Q})(A\bar{Q}^2). \end{aligned}$$

### 4. SU(6) with $2\Box + 4\bar{\Box}$

$$\begin{aligned} W = & \lambda[(A^4 \bar{Q}^2)^2 + (A^3)^2 (A\bar{Q}^2)^2 - \Lambda^{12}] \\ & + \mu[(A^4 \bar{Q}^2)(A\bar{Q}^2) + (A^3)(A\bar{Q}^2)^2]. \end{aligned}$$

### 5. SU(6) with $\Box + 3(\Box + \bar{\Box})$

$$\begin{aligned} W = & \lambda(B_1 \bar{B}_1 T + B_3 \bar{B}_3 + M_2^3 + TM_2 M_0^2 - \Lambda^{12}) \\ & + \mu(M_2^2 M_0 + TM_0^3 + \bar{B}_1 B_3 + B_1 \bar{B}_3). \end{aligned}$$

We have seen that all  $s$ -confining  $\sum\mu_i - \mu_G = 2$  theories result in confining  $\sum\mu_i - \mu_G = 0$  theories with a quantum modified constraint after integrating out a flavor. This does not imply that  $\sum\mu_i - \mu_G = 2$  theories which do not  $s$ -confine cannot result in confining theories with quantum modified constraints after eliminating one flavor.

As an example we consider SU(4) with  $3\Box + \Box + \bar{\Box}$ . The theory with an additional flavor is not  $s$ -confining, it flows to SU(2) with  $8\Box$ . Moreover, one can explicitly construct a dual description for SU(4) with  $3\Box + 2(\Box + \bar{\Box})$  by noting that this theory is equivalent to an SO(6) theory with  $3\Box + 2(S + \bar{S})$ , where  $S$  and  $\bar{S}$  denote a spinor and its conjugate. This dual can be obtained from the dual of SO(10) with one spinor and seven vectors [9]. The confining description with one less flavor is obtained from the SO(8) theory with a spinor and five vectors [9]. The confining spectrum is given in the table below:

	SU(4)	SU(3)	U(1) <sub>1</sub>	U(1) <sub>2</sub>	U(1) <sub>R</sub>
$A$	$\Box$	$\Box$	0	1	0
$Q$	$\Box$	1	1	-3	0
$\bar{Q}$	$\bar{\Box}$	1	-1	-3	0
$A^2$		$\Box$	0	2	0
$QA^2\bar{Q}$		$\bar{\Box}$	0	-4	0
$Q\bar{Q}$		1	0	-6	0
$A^3 Q^2$		1	2	-3	0
$A^3 \bar{Q}^2$		1	-2	-3	0

The quantum modified constraint is

$$\begin{aligned} W = & \lambda[(Q\bar{Q})^2 (A^2)^3 + (A^2)(QA^2\bar{Q})^2 \\ & + (A^3 Q^2)(A^3 \bar{Q}^2) - \Lambda^8 (Q\bar{Q})]. \end{aligned}$$

Note that one can eliminate the field  $(Q\bar{Q})$  from the theory by solving the quantum modified constraint. The remaining fields match all anomalies of the ultraviolet theory. It would be interesting to determine which of the remaining  $\sum\mu_i - \mu_G = 0$  theories are confining with a quantum modified constraint.

## B. Dynamically generated runaway superpotentials

Starting from the confining theories with a quantum deformed moduli space one obtains theories with dynamically generated runaway superpotentials by integrating out more flavors. Here we only list the dynamical superpotentials which one finds by starting with the  $s$ -confining SU theories and which are not already in the literature. It is straightforward to obtain similar results from the  $s$ -confining SO theories by integrating out vectors. Our notation for the composites in the following superpotentials is defined in the corresponding tables in Sec. III.

### 1. SU(4) with $2\Box + F(\Box + \bar{\Box})$

$$\begin{aligned} W_{F=1} = & \frac{\Lambda^9 M_0}{6T^2 M_0^2 + 48M_2^2}, \\ W_{F=0} = & 0 \quad \text{or} \quad W_{F=0} = \frac{\Lambda^5}{\sqrt{T^2}}. \end{aligned}$$

### 2. SU(5) with $\square + \bar{\square} + F(\square + \bar{\square})$

$$W_{F=1} = (\Lambda^{11} M_1) / [M_1 M_0 T_1 T_2 + T_2 M_1^2 + T_1^3 M_0 M_1 + T_1^2 M_1^2 + T_1 B_1 \bar{B}_1 M_1 - (T_2 M_0 + T_1^2 M_0 + B_1 \bar{B}_1)^2],$$

$$W_{F=0} = \pm \frac{\Lambda^6}{\sqrt{(T_2 + T_1^2)(T_1 \pm \sqrt{T_2 + T_1^2})}}.$$

### 3. SU(5) with $2\square + 2\bar{\square}$ [23]

$$W = \frac{\Lambda^{11}}{(A^3 \bar{Q})^2}.$$

### 4. SU(6) with $\square + F(\square + \bar{\square})$

$$W_{F=2} = \frac{\Lambda^{13} M_2 M_0}{T(M_2 M_0)^2 - (M_2^2 + T M_0^2)^2},$$

$$W_{F=1} = \pm \frac{\Lambda^3 \sqrt{x_{\pm}}}{x_{\pm}^2 x_{\pm} T^2 / M_0^2 z^3 y^2 + x_{\pm} / z},$$

$$W_{F=0} = 0 \quad \text{or} \quad W_{F=0} = \frac{\Lambda^5}{\sqrt{T}},$$

where

$$x = \frac{8M_2^5}{T} - 10M_0^2 M_2^3 + 2M_0^4 M_2 T, \quad y = M_2^2 - M_0^2 T,$$

$$z = 4M_2^2 - M_0^2 T, \quad x_{\pm} = x \pm yz^{3/2}/T.$$

### C. Theories with multiple branches

When integrating out flavors from a few of the  $s$ -confining theories we find that there are multiple possible solutions for the superpotential: one or more solutions with a dynamically generated term, and a solution with vanishing superpotential. This indicates that such theories have several branches of vacua. There is not only a moduli space with a smooth continuous parametrization but there is also a discrete parameter distinguishing a discrete set of vacua. In our examples there are two sets of vacua which are characterized by  $W=0$  with a nontrivial moduli space, and  $W \propto 1/\text{fields}$  without a stable vacuum [5,11,12]. A consistency check on the assumption that the branch with vanishing superpotential describes a confining theory is that the 't Hooft anomaly matching conditions are satisfied. In addition to the previously described SU(4) with  $2\square$  SU(6) with  $\square$ , also SO(14) with one spinor field has multiple branches.

## V. DYNAMICAL SUPERSYMMETRY BREAKING

Our new results on the low-energy behavior of supersymmetric theories can be used to construct new models of dy-

namical supersymmetry breaking. We begin by reviewing the various mechanisms of dynamical supersymmetry breaking and then present new models which illustrate these possibilities.

A sufficient set of conditions for dynamical supersymmetry breaking is that there are no classical flat directions and that there is a spontaneously broken global symmetry [15]. In a nutshell, the argument can be summarized as follows. A spontaneously broken global symmetry implies the presence of a Goldstone boson. Since there are no noncompact flat directions, there is no massless scalar which could combine with the Goldstone boson into a supersymmetric multiplet. Therefore, supersymmetry must be broken.

The spontaneous breaking of a global symmetry can be achieved by one of three known mechanisms: by a dynamically generated superpotential [15], by a quantum modified constraint [19], or by confining dynamics [10]. In the supersymmetry breaking models based on confining dynamics a suitably chosen tree-level superpotential combines with the dynamically generated potential to give an effective O'Raifeartaigh model. We will give examples of all three mechanisms of dynamical supersymmetry breaking using our new results presented in the previous sections. More complicated mechanisms of dynamical supersymmetry breaking appear in product group theories, where an interplay of the strong gauge dynamics and the presence of tree-level Yukawa couplings results in dynamical supersymmetry breaking [20].

### A. Confining dynamics

The best known example of this type of model is an SU(2) theory with a field  $Q$  in the three-index symmetric representation [10]. It has been argued that this SU(2) theory confines without generating a superpotential for the confined field  $T = Q^4$ . In order to lift the only classical flat direction, the superpotential term  $W = \lambda Q^4$  is added. This tree-level superpotential becomes a linear term after confinement and breaks supersymmetry. At low energies the theory is effectively an O'Raifeartaigh model.

A similar example based on confining dynamics can be found using the  $s$ -confining SU(7) theory with  $2\square + 6\bar{\square}$ .<sup>4</sup> The main difference compared to the ISS model is that the confining SU(7) gauge group generates a superpotential for the confined fields. The field content, the confined degrees of freedom and the confining superpotential for this theory have been described in Sec. III A. In order to lift the flat directions we add the following renormalizable tree-level superpotential:

$$W_{\text{tree}} = A^1 \bar{Q}_1 \bar{Q}_2 + A^1 \bar{Q}_3 \bar{Q}_4 + A^1 \bar{Q}_5 \bar{Q}_6 + A^2 \bar{Q}_2 \bar{Q}_3 + A^2 \bar{Q}_4 \bar{Q}_5 + A^2 \bar{Q}_6 \bar{Q}_1.$$

A detailed analysis shows that this superpotential lifts all flat directions but preserves a  $U(1) \times U(1)_R$  global symmetry. After confinement of the SU(7) gauge group the superpotential is

<sup>4</sup>This model has been obtained independently by Nelson and Thomas [21].



$$W = H_{12}^1 + H_{34}^1 + H_{56}^1 + H_{23}^2 + H_{45}^2 + H_{61}^2 + \frac{1}{\Lambda^{13}} H^2 N^2.$$

The equations of motion with respect to the fields  $H_{12}^1$ ,  $H_{34}^1$ ,  $H_{56}^1$ ,  $H_{23}^2$ ,  $H_{45}^2$ , and  $H_{61}^2$  force nonzero VEV's for some of the  $H$  and  $N$  fields. This results in spontaneous breaking of at least one of the global  $U(1)$ 's. Therefore, supersymmetry must be broken as well.

### B. Models with a quantum deformed moduli space

A well-known model of dynamical supersymmetry breaking based on a theory with a quantum deformed moduli space is the  $SU(2)$  theory with four doublets  $Q_i$  and six singlets  $S^{ij}$  [19]. This supersymmetry breaking theory also has a tree-level superpotential  $W = \lambda S^{ij} Q_i Q_j$ . Confining  $SU(2)$  dynamics results in a quantum modified constraint  $\text{Pf}M = \Lambda^4$ , where  $M_{ij} = Q_i Q_j$ . The equations of motion with respect to the singlets  $S^{ij}$  give  $M_{ij} = 0$ . This point is not on the quantum deformed moduli space, so supersymmetry is broken.

In this theory the flat directions corresponding to the singlets  $S^{ij}$  are not lifted by the tree-level superpotential. After including the quantum corrections to the Kähler potential, the  $S^{ij}$  directions are no longer flat [22]. This theory is non-chiral, but it nevertheless breaks supersymmetry. The theory avoids Witten's no-go theorem for vectorlike theories because the Witten index of the theory changes along the ‘pseudoflat’ direction  $S^{ij}$  [19].

Similar models can be built using any theory which has a quantum modified constraint. One can introduce a singlet for every confined degree of freedom and a tree-level superpotential  $W = \Sigma S^i M_i$ . Here, the  $S^i$ 's are the singlets and the  $M_i$ 's are the gauge invariant operators. This superpotential lifts all flat directions except for the ones corresponding to the gauge singlet fields. Since the equations of motion with respect to the  $S^i$  set the VEV's of all gauge-invariant operators to 0, the quantum modified constraint cannot be obeyed and supersymmetry is broken. This mechanism can be applied to any of our theories with quantum deformed moduli space, whether or not the theory is chiral.

As an explicit example consider an  $SO(7)$  theory with five spinors. The table of symmetries and invariants is

	$SO(7)$	$SU(5)$	$U(1)_R$
$S$	8	$\square$	0
$S^2$		$\square\square$	0
$S^4$		$\bar{\square}$	0

The quantum modified constraint is  $(S^2)^5 + (S^2)(S^4)^2 = \Lambda^{10}$ . We need to introduce the  $SO(7)$  gauge singlets  $A_{ij}$  and  $B^i$ , where  $A$  transforms as a conjugate symmetric tensor of  $SU(5)$ , while  $B$  as a fundamental of  $SU(5)$ . The superpotential which sets all  $SO(7)$  invariants containing spinors to 0 is

$$W_{\text{tree}} = A_{ij} S^{2,ij} + B^i S^4_i.$$

The full superpotential after confinement is

$$W_{\text{tree}} = A_{ij} (S^2)^{ij} + B^i (S^4)_i + \lambda [(S^2)^5 + (S^2)(S^4)^2 - \Lambda^{10}],$$

where  $\lambda$  is a Lagrange multiplier enforcing the constraint. The equations of motion with respect to the singlets are incompatible with the quantum modified constraint, hence supersymmetry is broken.

### C. Theories with a dynamically generated superpotential

Most of the examples of models of dynamical supersymmetry breaking are based on theories with a dynamically generated superpotential. Here we show two new examples using our theories analyzed in the previous sections. We can summarize our method for finding these models as follows: we start with a nonchiral theory which has a dynamically generated superpotential. We gauge a global  $U(1)$  symmetry which makes the theory chiral, and we include some singlets which have nonzero charges under the  $U(1)$ . The tree-level superpotential together with the  $U(1)$   $D$  term lifts all flat directions and supersymmetry is seen to be broken after the dynamically generated superpotential is added.

The first example is based on an  $SO(12) \times U(1)$  gauge group with matter content

	$SO(12)$	$U(1)$
$S$	32	1
$Q$	$\square$	-4
$A$	1	8
$B$	1	2
$C$	1	6

The independent  $SO(12)$  invariant operators and their  $U(1)$  charges are

	$U(1)$
$Q^2$	-8
$S^4$	4
$A$	8
$B$	2
$C$	6

The tree-level superpotential

$$W_{\text{tree}} = A Q^2$$

sets the  $Q^2$  operator to zero. Since the remaining  $SO(12)$  invariants all have positive  $U(1)$  charges, all flat directions are lifted by the  $U(1)$   $D$  term. The  $SO(12)$  gauge group generates a dynamical superpotential

$$W_{\text{dyn}} = \frac{\Lambda^5}{[Q^2 (S^4)^2]^{1/5}},$$

and the full superpotential is

$$W = A Q^2 + \frac{\Lambda^5}{[Q^2(S^4)^2]^{1/5}}.$$

The equations of motion cannot be satisfied, so we conclude that this theory breaks supersymmetry. Note that the fields  $B$  and  $C$  are only needed to cancel the  $U(1)$  anomalies.

A similar model can be obtained by using the  $SU(6)$  theory with a three-index antisymmetric tensor. The field content is

	$SU(6)$	$U(1)$	$SU(3)$
$A$	$\square$	1	1
$Q$	$\square$	-3	1
$\bar{Q}$	$\bar{\square}$	-3	1
$S_1$	1	6	1
$S_2$	1	4	1
$S'$	1	2	$\square$

The tree-level superpotential

$$W = S_1(Q\bar{Q}) + S_2(QA^2\bar{Q})$$

again lifts all flat directions, and the presence of the dynamically generated superpotential of Sec. IV B breaks supersymmetry dynamically.

Clearly, there are other possibilities for constructing similar models. One can use theories that are chiral without gauging a  $U(1)$  symmetry, such as the  $SU(5)$  model with 2  $\square$  and 2  $\bar{\square}$  [23]. Or one can make theories chiral by gauging a larger subgroup of the global symmetries, an example is the well-known 3-2 model [15].

## VI. CONCLUSIONS

Determining the phase structure of  $N=1$  supersymmetric theories with arbitrary matter content is a very difficult problem. We have shown that it is possible to identify all theories which belong to a certain class of confining theories. A salient feature of these  $s$ -confining theories is that the massless degrees of freedom are given by the independent gauge-invariant chiral operators. They describe the theory everywhere on the moduli space including the origin. Another important characteristic is that there is a nonvanishing superpotential for the confined degrees of freedom.

We have given two necessary conditions for a theory to be  $s$ -confining. Using these conditions and the requirement of 't Hooft anomaly matching we determined all  $s$ -confining theories with a single gauge group. We listed several new examples of  $s$ -confining theories with  $SU(N)$  gauge groups. The  $SU(N)$  theory with  $\square + \bar{\square} + 3(\square + \bar{\square})$  is  $s$ -confining for any  $N$ , while other new examples  $s$ -confine only for particular  $N$ . There are no new examples of  $s$ -confinement with  $Sp(N)$  gauge group.  $s$ -confinement in  $SO(N)$  groups requires the presence of at least one spinorial representation, which restricts  $N \leq 14$ . It turns out that most of the  $SO(N)$  theories which satisfy our index condition are  $s$ -confining.

The quantity  $\sum \mu_i - \mu_G$  which appears in our index formula is very useful for determining the dynamics of a given theory. For example, all  $s$ -confining theories satisfy  $\sum \mu_i - \mu_G = 2$ , all theories which confine with a quantum modified constraint satisfy  $\sum \mu_i - \mu_G = 0$ , and for  $\sum \mu_i - \mu_G = -2$  the dynamically generated superpotential has the correct  $\Lambda$  dependence to be generated by single instantons.

An interesting possible application of our results on  $s$ -confinement is to composite model building. Recently, several examples of models with quark-lepton compositeness have been given [12,24,25]. All these models rely on the recent exact results for the infrared spectra of  $s$ -confining theories. In these models the dynamically generated superpotentials can be used to give a natural explanation of the hierarchy between the top and bottom quark mass [24]. A toy model based on  $Sp(6)$  with an antisymmetric tensor [12] has the interesting feature that it generates three generations of quarks with a hierarchical structure for the Yukawa couplings dynamically. We hope that the wealth of new  $s$ -confining theories listed in this paper can be applied to build further interesting and realistic models of compositeness.

Our results can also be applied to dynamical supersymmetry breaking. We have shown several new examples of supersymmetry breaking models which illustrate different dynamical mechanisms. These models use either  $s$ -confining theories, or theories obtained from them by integrating out flavors. Many other new models can be built using our exact results.

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