High temperature $\ln(T)$ contributions in thermal field theory

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We calculate, to one-loop order, the $\ln(T)$ contributions of three-point functions in the ϕ^3 and Yang-Mills theory at high temperature. We find that these terms are Lorentz invariant and have the same structure as the ultraviolet divergent contributions which occur at zero temperature. A simple argument, valid for all N-point Green functions, is given for this behavior. [S0556-2821(97)01210-1]

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I. INTRODUCTION

In thermal field theory one is often interested in the ''hard thermal'' loop contributions, meaning those terms which come from a region where the loop momenta are of the order of the temperature T, which is much larger than all external momenta. These Green functions, which have a leading behavior at high temperature proportional to T^2, where much studied in QCD [1–5]. They are an important tool in *resumming* the QCD thermal perturbation theory [6]. The hard thermal region is also relevant for the determination of the $\ln(T)$ contributions (unlike the terms linear in T which come also from soft loop momenta). There have been several investigations of the $\ln(T)$ terms associated with the two-point functions [1,7,8], but the corresponding calculations done in connection with the three-point functions have been thus far restricted to particular configurations of their external momenta [9–12].

This work intends to study the hard thermal $\ln(T)$ contributions associated with general *three-point Green functions* in thermal field theory. In order to discuss the logarithmic dependence, we use the analytically continued imaginary-time perturbation theory [7] and relate these functions to *forward scattering amplitudes* [5,13]. In Sec. II, we consider the three-point function in the ϕ^3 model in six dimensions, which has several similarities with the Yang-Mills theory and is useful to illustrate the main points in a simpler context. In Sec. III we study the hard thermal behavior of the three-point function in the Yang-Mills theory. We focus on the $\ln(T)$ terms, and find a Lorentz invariant result which is directly connected with the ultraviolet structure of the three-point function at *zero temperature*. This conclusion, obtained in the Feynman gauge, is in fact true in any gauge. (We have explicitly verified this statement in a general class of covariant gauges. Since this calculation requires a generalization of the method of forward scattering amplitudes, the corresponding analysis will be reported elsewhere.) In general, the dependence upon T for high T is not necessarily related to the ultraviolet divergence of the zero temperature amplitude. In the Yang-Mills theory, for example, the leading behavior at high temperatures for all N-gluon functions is proportional to T^2, although these functions are ultraviolet finite for $N>4$. Hence, the above connection between the $\ln(T)$ contributions and the ultraviolet behavior at zero temperature, which emerged after a rather involved calculation, may seem at first somewhat surprising. For this reason we present in Sec. IV a general argument concerning the $\ln(T)$ behavior of N-point gluon Green functions. This gives a simple explanation for the fact that the $\ln(T)$ contributions always appear with the same coefficient as the residue of the ultraviolet pole part of the zero-temperature amplitude.

II. THE THREE-POINT SCALAR FUNCTION

In order to exhibit the behavior of subleading hard thermal contributions in the simplest way, we consider the massless ϕ^3 model in six dimensions which is asymptotically free. The Feynman diagram associated with the thermal three-point function is shown in Fig. 1(a). The analytically continued imaginary time perturbation theory can be formulated [5,13] so as to express this function in terms of forward scattering amplitudes of on-shell particles, as illustrated in Fig. 1(b). Here $Q=(|\mathbf{Q}|,\mathbf{Q})$ is the four-momentum of the on-shell thermal particle. There are six diagrams such as this one, which are obtained by permutations of the external momenta k_i. These contributions can be written as

$$\Gamma_3=\frac{\lambda^3}{(2\pi)^5}\int\frac{d^5Q}{2|\mathbf{Q}|}\left\{\left[\frac{1}{2Q\cdot k_1+k_1^2}\,\frac{1}{-2Q\cdot k_2+k_2^2}+\text{permutations}\right]+Q\rightarrow -Q\right\}N(|\mathbf{Q}|), \tag{2.1}$$

where $N(|\mathbf{Q}|)$ is the Bose-Einstein distribution

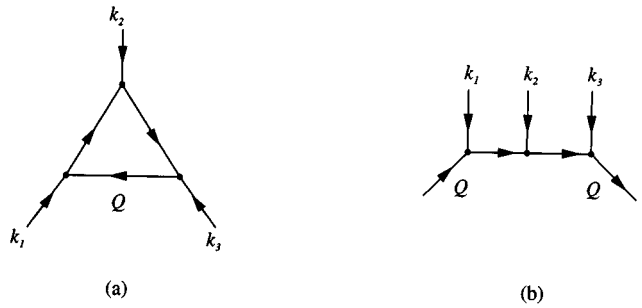

FIG. 1. (a) The three-scalar thermal loop diagram. Momentum and energy conservation is understood at each vertex. (b) An example of the forward scattering graph connected with diagram (a).

$$N(|\mathbf{Q}|)=\frac{1}{\exp(|\mathbf{Q}|/T)-1}. \tag{2.2}$$

Since in the hard thermal region we require large values of $|\mathbf{Q}|$, we may expand each denominator as

$$\frac{1}{2Q\cdot k_i+k_i^2}=\frac{1}{2Q\cdot k_i}-\frac{k_i^2}{(2Q\cdot k_i)^2}+\frac{(k_i^2)^2}{(2Q\cdot k_i)^3}+\cdots. \tag{2.3}$$

The first term has each denominator of the form $(Q\cdot k_i)^{-1}$. Such terms would give individually T^2 contributions, but these actually cancel by the eikonal identity

$$\frac{1}{Q\cdot k_1 Q\cdot k_2}+\frac{1}{Q\cdot k_1 Q\cdot k_3}+\frac{1}{Q\cdot k_2 Q\cdot k_3}=0. \tag{2.4}$$

The second term in Eq. (2.3) is down by one power of $k_i/|\mathbf{Q}|$ and would lead to individual T contributions, but such terms also cancel by symmetry under $Q\rightarrow -Q$. Hence, we are left with the following subleading contributions:

$$\Gamma_3=\frac{\lambda^3}{16(2\pi)^5}\int\frac{d^5Q}{|\mathbf{Q}|}N(|\mathbf{Q}|)\times\left\{\frac{1}{Q\cdot k_2 Q\cdot k_3}\left[\frac{k_1^2k_2^2}{Q\cdot k_1 Q\cdot k_2}-\frac{(k_1^2)^2}{(Q\cdot k_1)^2}-\frac{(k_2^2)^2}{(Q\cdot k_2)^2}\right]+\text{permutations}\right\}. \tag{2.5}$$

After some algebra, which makes use of the eikonal identity (2.4) and the overall momentum conservation of the external momenta, the above expression can be reduced to the form

$$\Gamma_3=\frac{\lambda^3}{8(2\pi)^5}\int\frac{d^5Q}{|\mathbf{Q}|}N(|\mathbf{Q}|)\left\{\frac{k_1^2k_2\cdot k_3}{(Q\cdot k_1)^2Q\cdot k_2Q\cdot k_3}+\text{permutations}\right\}, \tag{2.6}$$

which involves only homogeneous functions of zero degree in each of the external momenta. In order to perform this integral, it is convenient to rewrite Eq. (2.6) as

$$\Gamma_3=\frac{\lambda^3}{8(2\pi)^5}\int^\infty\frac{d|\mathbf{Q}|}{|\mathbf{Q}|}N(|\mathbf{Q}|)\int d\Omega\left\{\frac{k_1^2k_2\cdot k_3}{(\hat{Q}\cdot k_1)^2\hat{Q}\cdot k_2\hat{Q}\cdot k_3}+\text{permutations}\right\}, \tag{2.7}$$

where $\int d\Omega$ denotes angular integration of the five-dimensional unit vector $\hat{\mathbf{Q}}$ and $\hat{Q}\equiv(1,\hat{\mathbf{Q}})$.

We have evaluated the angular integral, using the approach discussed in Ref. [5]. From the fact that the integrand is a dimensionless function of zero degree in the external momenta, one finds after some calculation that the angular integration just gives a factor of $8\pi^2$. In the integration over $d|\mathbf{Q}|$, the lower limit must be chosen consistently with the range of validity of the expansion in Eq. (2.3). However, its value is immaterial for the determination of the ln(T) contribution, which comes from the region of high internal momenta $|\mathbf{Q}|\sim T\gg k_i$. Evaluating these terms, we then obtain for the logarithmic dependence on the temperature the simple Lorentz-invariant result

$$\tilde{\Gamma}_3=-\frac{\lambda^3}{64\pi^3}\ln(T). \tag{2.8}$$

This equation may be compared with the ultraviolet divergent contribution of the three-point function at zero temperature, evaluated in $6-2\epsilon$ dimensions:

$$\Gamma_3^{\mathrm{UV}}=\frac{\lambda^3}{64\pi^3}\frac{M^{2\epsilon}}{2\epsilon}\simeq\frac{1}{64\pi^3}\left[\frac{1}{2\epsilon}+\ln(M)\right], \tag{2.9}$$

where M is the renormalization scale. We see that the ln(T) contribution has the same structure as the ultraviolet divergent part at zero temperature, so that it may be naturally combined with the ln(M) term. The same behavior was previously noted in connection with the scalar self-energy function [8].

III. THE THREE-GLUON FUNCTION

It is known that the above connection between the ln(T) contributions at high temperature and the ultraviolet pole part at zero temperature is also exhibited by the two point gluon function in the Yang-Mills theory [1,8]. If this property continues to hold for the three-point gluon function, then, using the well-known result for the corresponding zero temperature amplitude, we could immediately write for the ln(T) contributions the following ansatz:

$$\tilde{\Gamma}_{3\,\mu_1\mu_2\mu_3}^{a_1a_2a_3}=\frac{1}{12\pi^2}g^2V_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}\ln(T), \tag{3.1}$$

where a_1,a_2,a_3 are color indices, μ_1,μ_2,μ_3 are Lorentz indices, and

$$V_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}=-igf^{a_1a_2a_3}[\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3}+\eta_{\mu_2\mu_3}(k_2-k_3)_{\mu_1}+\eta_{\mu_3\mu_1}(k_3-k_1)_{\mu_2}] \tag{3.2}$$

is the basic three gluon vertex. Equation (3.1) was obtained replacing $1/(2\epsilon)$ by $\ln(1/T)$ in the $T=0$ ultraviolet divergent three-gluon function, computed in the *Feynman gauge*. In what follows we will prove that this ansatz is indeed correct.

In the Feynman gauge the *pole structure* of the gluon propagator is very similar to the scalar case considered in the previous section; there are only *simple poles*. This is an important ingredient which makes it possible to formulate the imaginary time formalism in terms of *forward scattering amplitudes* [5,13]. It is then straightforward to write the following expression for the three-gluon function

$$\Gamma_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}(k_1,k_2,k_3)=\frac{g^3}{(2\pi)^3}\int\frac{d^3\mathbf{Q}}{2|\mathbf{Q}|}N(|\mathbf{Q}|)\times[\mathcal{S}_{\mu_1\mu_2\mu_3}^{a_1a_2a_3}(k_1,k_2,k_3,Q)+Q\rightarrow -Q], \tag{3.3}$$

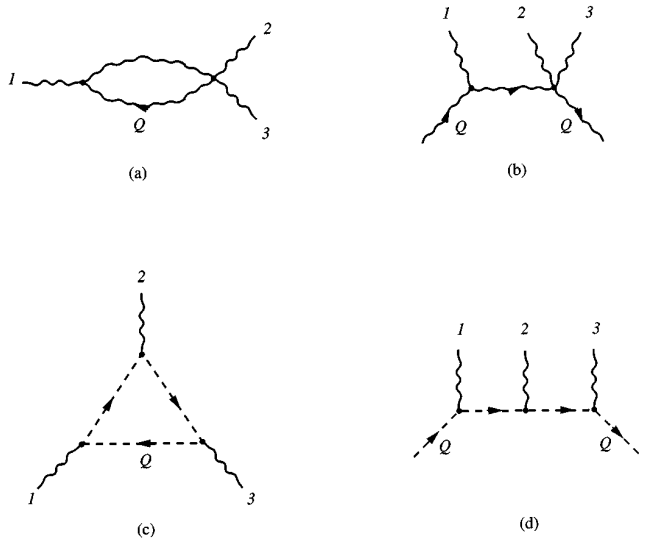

FIG. 2. The thermal three-gluon loop diagram. Wavy lines represent *gluons* and the broken lines in (c) and (d) denote *either* ghosts *or* internal gluons. The numbers 1, 2, and 3 represent a collective index for the momenta, Lorentz and color indices. Graphs (b) and (d) are examples of forward scattering amplitudes associated, respectively, with the diagrams (a) and (c). All external momenta are inward and $k_1+k_2+k_3=0$.

where $\mathcal{S}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,Q)$ is the forward scattering amplitude given by the sum of the diagrams shown in Figs. 2(b) and 2(d) and the graphs obtained by permutations of their external momenta and indices. We can now use a hard thermal expansion such as Eq. (2.3) for the denominators in $\mathcal{S}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,Q)$.

As in the scalar case, odd powers of Q will cancel when the $Q\rightarrow -Q$ terms are added. From the momentum dependence of the Yang-Mills vertices it is easy to see that the resulting terms in the hard thermal expansion of $\mathcal{S}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,Q)$ will be functions of decreasing degrees in Q starting from zero degree. Therefore, by naive power counting in Eq. (3.3) we conclude that the terms of degree 0 and -2 in $\mathcal{S}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,Q)$ will produce respectively the leading T^2 and the $\ln(T)$ contributions. The leading T^2 contribution does not vanish as in the ϕ^3 model of the previous section and the result is well known [4–6]. Using a similar procedure as the one in Eq. (2.7), we get for the $\ln(T)$ contributions an expression of the form

$$\widetilde{\Gamma}^{a_1a_2a_3}_{3\,\mu_1\mu_2\mu_3}(k_1,k_2,k_3) = -\frac{g^3}{4\pi^2}\ln(T)\int\frac{d\Omega}{4\pi}\mathcal{L}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q}), \qquad (3.4)$$

where $\int d\Omega$ denotes angular integration of the three-dimensional unit vector $\hat{\mathbf{Q}}$, $\hat{Q}\equiv(1,\hat{\mathbf{Q}})$ and $\mathcal{L}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})$ is a function of degree -2 in $\hat{Q}$.

All angular integrals of the form $\int d\Omega\,\mathcal{L}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}$ can be generated differentiating the following basic integral with respect to $k_{i\mu_l}$ [5]:

$$\int\frac{d\Omega}{4\pi}\frac{1}{k_i\cdot\hat{Q}k_j\cdot\hat{Q}} = \int_0^1 dx\int\frac{d\Omega}{4\pi}\{[xk_i+(1-x)k_j]\cdot\hat{Q}\}^{-2} = \int_0^1 dx[xk_i+(1-x)k_j]^{-2}. \qquad (3.5)$$

The right-hand side of Eq. (3.5) was obtained using standard *Feynman parametrization*. After an elementary angular integration, a manifest *Lorentz scalar* is unveiled in the resulting Feynman parameter integral which can be explicitly performed [5]. Thus, the *Lorentz covariance* of the logarithmic contributions is established. This remarkable property, which holds in spite of the fact that the angular integral is not a generally Lorentz-invariant process, was shown to be true in Ref. [5] for *any* angular integrand which is a function of degree -2 in $\hat{Q}$. We also note that, by power counting, the $\ln(T)$ contributions of *any* hard thermal N-point function will equally be a Lorentz-invariant quantity. In what follows we will use this property as an important tool for the explicit computation of the $\ln(T)$ contributions.

Our simple ansatz given by Eq. (3.1) is a very special case of the most general Lorentz-covariant expression. In general, for each set of color indices, there are 14 tensor structures which can be formed using 3 Lorentz indices and the 2 independent four-momenta. However, the explicit calculation of the diagrams in Fig. 2 shows that the angular integrals in Eq. (3.4) are such that the totally antisymmetric color factor $f^{a_1a_2a_3}$ *factorizes* and the result can be written as

$$\int\frac{d\Omega}{4\pi}\mathcal{L}^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q}) = -if^{a_1a_2a_3}\mathcal{A}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3). \qquad (3.6)$$

Therefore, the Lorentz-covariant tensor $\mathcal{A}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$ must be *antisymmetric* under any interchange of a pair of momenta and the corresponding Lorentz indices. A straightforward but quite involved calculation (we have used the *symbolic computer program* MAPLE) yields

$$\mathcal{A}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)\equiv\int\frac{d\Omega}{4\pi}\mathcal{L}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q}), \qquad (3.7)$$

where the tensors $\mathcal{L}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})$ are given in the Appendix.

We can perform the angular integrals for each individual term of these expressions in a straightforward way, using for instance the procedure of differentiation with respect to $k_{i\mu_l}$ as described above. In practice, we found easier to use a decomposition in terms of a set of tensors [14,15] built from the basic tensors

$$A^1_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3}, \qquad (3.8)$$

$$A^2_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=(k_1\cdot k_2\,\eta_{\mu_1\mu_2}-k_{1\mu_2}k_{2\mu_1})\times(k_1-k_2)_{\mu_3},$$

$$A^3_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=(k_1\cdot k_2\,\eta_{\mu_1\mu_2}-k_{1\mu_2}k_{2\mu_1})\times(k_{1\mu_3}k_2\cdot k_3-k_{2\mu_3}k_1\cdot k_3),$$

$$A^4_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=\eta_{\mu_1\mu_2}(k_{1\mu_3}k_2\cdot k_3-k_{2\mu_3}k_1\cdot k_3)+\tfrac{1}{3}(k_{1\mu_3}k_{2\mu_1}k_{3\mu_2}-k_{1\mu_2}k_{2\mu_3}k_{3\mu_1}),$$

$$S^1_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=\eta_{\mu_1\mu_2}(k_1+k_2)_{\mu_3},$$

$$S^2_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=k_{1\mu_3}k_{2\mu_1}k_{3\mu_2}+k_{1\mu_2}k_{2\mu_3}k_{3\mu_1}.$$

The *complete* set is generated from the above equations including new tensors obtained from $A^i_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$, $i=1,2,3,4$, and from $S^j_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$, $j=1,2$, by cyclic permutations of $(k_1,\mu_1),(k_2,\mu_2),(k_3,\mu_3)$. This gives a total of 16 tensors in terms of which we write the most general expression for $\mathcal{A}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)$. The coefficients of this expansion can be obtained in a straightforward way by simply solving a system of 16 equations. These equations are obtained using the expressions in the Appendix and performing the Lorentz indices contractions with each of the 16 tensors. The solution of this system will be, in general, scalar functions of k_1,k_2,k_3 involving angular integrals which can always be reduced to Eq. (3.5), or the special case of it when $k_i=k_j$, when one uses the momentum conservation $k_1+k_2+k_3=0$. In principle these scalars could have any kind of dependence on the external momenta involving Eq. (3.5) and rational functions. However, the explicit calculation shows that, after using the eikonal identity given by Eq. (2.4), all coefficients vanish, except for the coefficients of $A^1_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)=\eta_{\mu_1\mu_2}(k_1-k_2)_{\mu_3}$ and its cyclic permutations which simplify to give just $-1/3$. Inserting these terms into Eq (3.6) and using Eq. (3.4), we finally obtain the result stated in Eq. (3.1).

IV. DISCUSSION

To get a further understanding of the connection between the ln(T) contributions and the ultraviolet behavior of the Green functions at zero-temperature, let us consider the complete thermal amplitude, which includes the zero-temperature part. This can be written, for instance in the Yang-Mills case where we omit for simplicity the color indices, as follows [7,16]:

$$A_{\mu_1\cdots\mu_N}(k_{i0},\mathbf{k}_i,T)=M^{2\epsilon}T\sum_{Q_0=2\pi niT}\times\int d^{3-2\epsilon}\mathbf{Q}F_{\mu_1\cdots\mu_N}(Q_0,\mathbf{Q},k_{i0},\mathbf{k}_i).\tag{4.1}$$

Here M is the renormalization scale, $k_{i0}/2\pi iT$ are integers and n runs over all integers. For fixed n, the $\mathbf{Q}$ integral is ultraviolet finite, having no poles at $\epsilon=0$.

In order to determine those terms which can yield an ultraviolet pole when the summation over n is evaluated, let us examine a relevant contribution which is obtained after the $\mathbf{Q}$ integration has been performed. Making appropriate shifts in Q_0, one finds that such a term is proportional to a sum of the form

$$S=\sum_{Q_0=2\pi niT}\frac{1}{(Q_0^2+ak_0^2+b\mathbf{k}^2)^{(1/2)+\epsilon}},\tag{4.2}$$

where a, b are constants and k_0 is some linear combination of the external energies with integral coefficients. $\mathbf{k}$ is some linear combination of the external momenta which may be neglected in the high temperature limit, except when $Q_0^2+ak_0^2$ vanishes.

We now set $k_0=2\pi liT$, where l is some integer and consider the contributions to S from the regions $n^2<l^2$ and $n^2>l^2$. It turns out that the pole part arises from the summation over the domain where $|n|\gg|l|$, i.e., when $|Q_0|\gg|k_0|$. Expanding in this region (4.2) in powers of Q_0^{-1}, we find that the leading term gives a contribution involving the *zeta function* $\zeta(1+2\epsilon)$, which is defined in general as [17]

$$\zeta(z)=\sum_{n=1}^{\infty}\frac{1}{n^z}.\tag{4.3}$$

This function is analytic for all values of z, except near the point $z=1+2\epsilon$, where it has a simple pole $1/2\epsilon$.

At this stage, having performed the summation over the discrete frequencies $Q_0=2\pi niT$, we can analytically continue the external energies to continuous values of k_0. Identifying in the complete thermal amplitude all contributions which yield poles at $\epsilon=0$, and using the fact that the leading term in S is proportional to $T^{-1-2\epsilon}$, one obtains an expression of the form

$$A^{\epsilon}_{\mu_1\cdots\mu_N}(k_i,T)=\left(\frac{M}{T}\right)^{2\epsilon}\frac{1}{2\epsilon}\mathcal{R}_{\mu_1\cdots\mu_N}(k_i)\simeq\left(\frac{1}{2\epsilon}-\ln\frac{T}{M}\right)\mathcal{R}_{\mu_1\cdots\mu_N}(k_i).\tag{4.4}$$

where $\mathcal{R}_{\mu_1\cdots\mu_N}$ is the residue of the ultraviolet divergent part of the Green function at zero temperature. This has the same structure, because of the renormalizability of the theory, as the corresponding basic function appearing in the Yang-Mills Lagrangian.

The above equation shows that for general Green functions, the ln(T) contributions have the same form as the ultraviolet divergent terms which occur at zero temperature and combine in a simple way with the ln(M) terms. In particular, if the Green function at zero temperature is ultraviolet convergent, the residue $\mathcal{R}_{\mu_1\cdots\mu_N}$ must vanish and the ln(T) term should be absent at high temperature. This has been explicitly verified in the case of the electron-positron box diagram in thermal QED [18].

In conclusion, we mention that the above property allows us to include in a simple way the ln(T) contributions into the *running coupling constant* $g(T)$ at high temperature. Several investigations on this important topic [19–22] have exposed

ambiguities which are related, at least in part, to the fact that the thermal contributions to $g(T)$ are not generally Lorentz-invariant functions. On the other hand, the $\ln(T)$ terms are Lorentz invariant, being directly related to the ultraviolet behavior of the Green functions at zero temperature. It is well known that the effective coupling $\overline{g(\kappa/M)}$ at zero temperature, where κ is a typical external momentum, involves a logarithmic dependence of the form $\ln(\kappa/M)$. The κ dependence in this term must be canceled by a corresponding dependence in the $\ln(T/\kappa)$ term at high temperature, since the complete thermal amplitude contains only a combination of the form $\ln(T/M)$. We thus obtain a gauge and Lorentz-invariant quantity $\overline{g(T/M)}$ which is relevant, for example, in the calculation of the pressure in thermal field theories [23–25].

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APPENDIX

In this appendix we present the results for the integrand of the angular integrals corresponding to the diagrams of Fig. 2. In terms of the individual contributions of each diagram, we can write

$$\mathcal{L}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})=\mathcal{L}^{\text{tadpole}}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})+\mathcal{L}^{\text{ghost}}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})+\mathcal{L}^{\text{gluon}}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q}), \tag{A1}$$

where

$$\mathcal{L}^{\text{tadpole}}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})=-\frac{9\,k_3\cdot k_3 k_{3\mu_1}\eta_{\mu_2\mu_3}}{8\,(\hat{Q}\cdot k_3)^2}+\frac{9\,k_3\cdot k_3\,\eta_{\mu_1\mu_3}k_{3\mu_2}}{8\,(\hat{Q}\cdot k_3)^2}+\{\text{cyclic permutations of } (k_1,\mu_1),(k_2,\mu_2),(k_3,\mu_3)\}, \tag{A2}$$

$$\begin{aligned}\mathcal{L}^{\text{ghost}}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})=&-\frac{k_{1\mu_3}k_{2\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{k_{2\mu_2}\hat{Q}_{\mu_1}\hat{Q}_{\mu_3}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}-\frac{\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)^2}\\&+\frac{\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}k_2\cdot k_2(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^3}-\frac{k_{2\mu_3}k_{1\mu_1}\hat{Q}_{\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{k_{2\mu_3}k_{1\mu_1}\hat{Q}_{\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}\\&-\frac{k_{2\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}+\frac{k_{2\mu_2}\hat{Q}_{\mu_1}\hat{Q}_{\mu_3}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}-\frac{k_{2\mu_2}\hat{Q}_{\mu_1}\hat{Q}_{\mu_3}(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}\\&-\frac{k_{2\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}+\frac{k_{2\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}-\frac{\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}(k_1\cdot k_1)^3}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^4}\\&+\frac{k_{1\mu_1}\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}+\frac{k_{1\mu_1}\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}(k_1\cdot k_1)^2}{32(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}-\frac{k_{1\mu_1}\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}\\&+\frac{k_{1\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}-\frac{k_{1\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}+\frac{k_{1\mu_3}\hat{Q}\_\mu_2\hat{Q}_{\mu_1}(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}\\&+\frac{\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}(k_2\cdot k_2)^3}{32\,(\hat{Q}\cdot k_2)^4(\hat{Q}\cdot k_1)}+\frac{k_{1\mu_3}k_{2\mu_2}\hat{Q}_{\mu_1}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}\\&+\{\text{cyclic permutations of } (k_1,\mu_1),(k_2,\mu_2),(k_3,\mu_3)\},\end{aligned} \tag{A3}$$

$$\begin{aligned}\mathcal{L}^{\text{gluon}}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3,\hat{Q})=&+\frac{k_{2\mu_2}\eta_{\mu_1\mu_3}k_2\cdot k_2}{4\,(\hat{Q}\cdot k_2)^2}-\frac{k_1\cdot k_2k_{1\mu_2}\eta_{\mu_1\mu_3}}{2\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}+\frac{5\,(k_1\cdot k_1)^2\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{k_{1\mu_2}\eta_{\mu_1\mu_3}k_1\cdot k_1}{2\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)}\\&+\frac{3\,k_{2\mu_2}\eta_{\mu_1\mu_3}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_1)^2}+\frac{\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}(k_1\cdot k_1)^2}{16\,(\hat{Q}\cdot k_1)^3}+\frac{\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}(k_2\cdot k_2)^2}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)^2}+\frac{k_{1\mu_2}\eta_{\mu_1\mu_3}k_2\cdot k_2}{2\,(\hat{Q}\cdot k_2)^2}\\&+\frac{k_{1\mu_2}\eta_{\mu_1\mu_3}k_2\cdot k_2}{4\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)}-\frac{3\,k_{2\mu_2}\eta_{\mu_1\mu_3}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)}+\frac{3\,k_{2\mu_2}\eta_{\mu_1\mu_3}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)}-\frac{5\,k_1\cdot k_2\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)^2}\end{aligned} \tag{A4}$$

$$-\frac{5\,k_1\cdot k_1\,\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}k_2\cdot k_2}{8\,\hat{Q}\cdot k_1(\hat{Q}\cdot k_2)^2}+\frac{5\,k_1\cdot k_2\,\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{k_1\cdot k_1\,\eta_{\mu_1\mu_3}\hat{Q}_{\mu_2}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$+\frac{k_{1\mu_3}\eta_{\mu_1\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_1)^2}-\frac{3\,k_{1\mu_3}k_{1\mu_1}\hat{Q}_{\mu_2}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}+\frac{9\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}(k_1\cdot k_1)^3}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^4}+\frac{9\,k_{2\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}$$

$$+\frac{3\,k_{1\mu_3}\eta_{\mu_1\mu_2}k_1\cdot k_2}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}-\frac{3\,k_{2\mu_1}k_{2\mu_2}k_{1\mu_3}}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}-\frac{3\,k_{2\mu_3}\eta_{\mu_1\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}+\frac{5\,\eta_{\mu_2\mu_3}k_2\cdot k_2\hat{Q}_{\mu_1}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$+\frac{\eta_{\mu_2\mu_3}k_2\cdot k_2\hat{Q}_{\mu_1}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{13\,k_{2\mu_1}k_{1\mu_3}\hat{Q}_{\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{5\,k_1\cdot k_2\,\eta_{\mu_2\mu_3}\hat{Q}_{\mu_1}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{3\,k_{2\mu_3}k_{2\,\mu_2}\hat{Q}_{\mu_1}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$-\frac{3\,k_{1\mu_1}\eta_{\mu_2\mu_3}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2}+\frac{5\,k_1\cdot k_2\,\eta_{\mu_2\mu_3}\hat{Q}_{\mu_1}k_1\cdot k_1}{8\,\hat{Q}\cdot k\_2(\hat{Q}\cdot k_1)^2}+\frac{13\,k_{2\mu_3}k_{1\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}$$

$$+\frac{9\,k_{1\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}-\frac{\eta_{\mu_1\mu_2}k_2\cdot k_2\hat{Q}_{\mu_3}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{3\,k_{1\,\mu_1}k_{1\mu_2}\hat{Q}_{\mu_3}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}$$

$$+\frac{3\,k_{1\mu_3}k_{1\mu_1}\hat{Q}_{\mu_2}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{\eta_{\mu_2\mu_3}k_{2\mu_1}k_2\cdot k_2}{2\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}+\frac{3\,k_{2\mu_3}\eta_{\mu_1\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_1)^2}-\frac{k_{1\mu_1}\eta_{\mu_2\mu_3}k_1\cdot k_1}{4\,(\hat{Q}\cdot k_1)^2}$$

$$-\frac{3\,k_{2\mu_1}k_{2\mu_2}\hat{Q}_{\mu_3}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{9\,k_{2\mu_2}\hat{Q}_{\mu_1}\hat{Q}_{\mu_3}(k_1\cdot k_1)^2}{32(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}+\frac{13\,k_{2\mu_1}k_{1\mu_3}\hat{Q}_{\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}$$

$$+\frac{\eta_{\mu_1\mu_2}\hat{Q}_{\mu_3}(k_1\cdot k_1)^2}{16\,(\hat{Q}\cdot k_1)^3}-\frac{\eta_{\mu_2\mu_3}(k_1\cdot k_1)^2\hat{Q}_{\mu_1}}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}+\frac{k_{1\mu_3}k_{2\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{7\,k_1\cdot k_2\,\eta_{\mu_1\mu_2}\hat{Q}_{\mu_3}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$+\frac{7\,k_1\cdot k_2\,\eta_{\mu_1\mu_2}\hat{Q}_{\mu_3}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)^2\hat{Q}\cdot k_1}-\frac{9\,k_{2\mu_2}\hat{Q}_{\mu_1}\hat{Q}_{\mu_3}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}+\frac{9\,k_{2\mu_2}\hat{Q}_{\mu_1}\hat{Q}_{\mu_3}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}$$

$$-\frac{9\,k_{1\,\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}+\frac{k_{2\mu_1}\eta_{\mu_2\mu_3}k_1\cdot k_2}{2\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}-\frac{k_{2\mu_1}k_{1\mu_2}\hat{Q}_{\mu_3}k_2\cdot k_2}{(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{3\,k_{1\mu_3}\eta_{\mu_1\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}$$

$$-\frac{3\,k_{1\mu_3}\eta_{\mu_1\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2}-\frac{\eta_{\mu_2\mu_3}k_{2\mu_1}k_1\cdot k_1}{2\,(\hat{Q}\cdot k_1)^2}+\frac{\eta_{\mu_1\mu_2}(k_2\cdot k_2)^2\hat{Q}_{\mu_3}}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{3\,k_{2\mu_3}k_{2\mu_2}k_{1\mu_1}}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}$$

$$-\frac{3\,k_{1\mu_1}\eta_{\mu_2\mu_3}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}-\frac{3\,k_{2\mu_3}k_1\cdot k_2\,\eta_{\mu_1\mu_2}}{8\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}+\frac{3\,k_{2\mu_3}\eta_{\mu_1\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}+\frac{3\,k_{2\mu_3}k_{1\mu_2}k_{1\mu_1}}{8\,\hat{Q}\cdot k_1(\hat{Q}\cdot k_2)}$$

$$-\frac{\eta_{\mu_2\mu_3}k_{2\mu_1}k_1\cdot k_1}{4\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}-\frac{3\,k_{1\mu_3}\eta_{\mu_1\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_1)(\hat{Q}\cdot k_2)}-\frac{3\,k_{1\mu_3}k_{2\mu_2}k_{1\mu_1}}{8\,\hat{Q}\cdot k\_1(\hat{Q}\cdot k_2)}-\frac{k_{2\mu_3}k_{1\mu_1}\hat{Q}_{\mu_2}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$+\frac{k_{2\mu_3}k_{1\mu_1}\hat{Q}_{\mu_2}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{3\,k_{1\mu_1}k_{1\mu_2}\hat{Q}_{\mu_3}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}+\frac{3\,k_{2\mu_1}k_{2\mu_2}\hat{Q}_{\mu_3}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{\eta_{\mu_1\mu_2}(k_1\cdot k_1)^2\hat{Q}_{\mu_3}}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$+\frac{\eta_{\mu_1\mu_2}k_1\cdot k_1\hat{Q}_{\mu_3}k_2\cdot k_2}{8(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{k_{2\mu_1}k_{1\mu_2}\hat{Q}_{\mu_3}k_1\cdot k_1}{(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{5\,\eta_{\mu_2\mu_3}(k_2\cdot k_2)^2\hat{Q}_{\mu_1}}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{13\,k_{2\mu_3}k_{1\mu_2}\hat{Q}_{\mu_1}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}$$

$$-\frac{k_{1\mu_3}k_{2\mu_2}\hat{Q}_{\mu_1}k_1\cdot k_1}{8\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{k_{2\mu_3}\eta_{\mu_1\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2}+\frac{9\,k_{2\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}-\frac{9\,k_{2\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}$$

$$-\frac{9\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}k_2\cdot k_2(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^3}+\frac{9\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)^2}-\frac{9\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}\hat{Q}_{\mu_1}(k_2\cdot k_2)^3}{32\,(\hat{Q}\cdot k_2)^4(\hat{Q}\cdot k_1)}$$

$$+\frac{3\,k_{1\mu_3}k_{1\mu_2}\hat{Q}_{\mu_1}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{3\,k_{1\mu_3}k_{1\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{\eta_{\mu_1\mu_2}\hat{Q}_{\mu_3(k_2\cdot k_2)^2}}{16\,(\hat{Q}\cdot k_2)^3}$$

$$-\frac{9\,k_{1\mu_1}\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}+\frac{9\,k_{1\mu_1}\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}k_2\cdot k_2k_1\cdot k_1}{32\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)^2}-\frac{9\,k_{1\mu_1}\hat{Q}_{\mu_2}\hat{Q}_{\mu_3}(k_2\cdot k_2)^2}{32\,(\hat{Q}\cdot k_2)^3(\hat{Q}\cdot k_1)}$$

$$-\frac{k_{2\mu_2}k_{1\mu_1}\hat{Q}_{\mu_3}k_1\cdot k_1}{4\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}+\frac{k_{2\mu_2}k_{1\mu_1}\hat{Q}_{\mu_3}k_2\cdot k_2}{4\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}-\frac{9\,k_{1\mu_3}\hat{Q}_{\mu_2}\hat{Q}_{\mu_1}(k_1\cdot k_1)^2}{32\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^3}-\frac{\eta_{\mu_2\mu_3}\hat{Q}_{\mu_1}(k_2\cdot k_2)^2}{16\,(\hat{Q}\cdot k_2)^3}$$

$$+\frac{3\,k_{1\mu_1}\eta_{\mu_2\mu_3}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)}+\frac{3\,k_{2\mu_3}k_{2\mu_2}\hat{Q}_{\mu_1}k_2\cdot k_2}{8\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}+\frac{3\,k_{2\mu_3}k_{2\mu_1}\hat{Q}_{\mu_2}k_1\cdot k_1}{16\,(\hat{Q}\cdot k_2)(\hat{Q}\cdot k_1)^2}-\frac{3\,k_{2\mu_3}k_{2\mu_1}\hat{Q}_{\mu_2}k_2\cdot k_2}{16\,(\hat{Q}\cdot k_2)^2(\hat{Q}\cdot k_1)}$$

$$+\{\text{cyclic permutations of } (k_1,\mu_1),(k_2,\mu_2),(k_3,\mu_3)\}.$$

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