Primordial magnetic fields in false vacuum inflation

Anne-Christine Davis* and Konstantinos Dimopoulos†
*Department of Applied Mathematics and Theoretical Physics, University of Cambridge,
Silver Street, Cambridge, CB3 9EW, United Kingdom*
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We show that, during false vacuum inflation, a primordial magnetic field can be created sufficiently strong to seed the galactic dynamo and generate the observed galactic magnetic fields. Considering the inflaton-dominated regime, our field is produced by the Higgs-field gradients, resulting from a grand unified phase transition. The evolution of the field is followed from its creation through to the epoch of structure formation, subject to the relevant constraints. We find that it is possible to create a magnetic field of sufficient magnitude, provided the phase transition occurs during the final five e-foldings of the inflationary period. [S0556-2821(97)04310-5]

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I. INTRODUCTION

One of the most exciting astrophysical consequences of phase transitions in the early universe is the possible creation of primordial magnetic fields.

The existence of a primordial magnetic field could have significant effect on various astrophysical processes. Indeed, large scale magnetic fields are important in intercluster gas or rich clusters of galaxies, in quasistellar objects (QSO's), and in active galactic nuclei. The existence of a primordial field could influence the galaxy formation process and play a very important role in the resulting galactic spins [1]. A primordial field would also have an important effect on the fragmentation process of large scale structure and of the protogalaxies (by modifying the Jeans mass) and on the formation of population III stars [2]. But the most important consequence of the existence of a primordial magnetic field is that it can seed the observed galactic magnetic field.

The galactic field is also very important to the astrophysics of the galaxy. It influences the dynamics of the galaxy, the star formation process (by transferring angular momentum away from protostellar clouds [3,4] and by affecting the initial mass function of the star formation process [5]), the dynamics of compact stars (white dwarfs, neutron stars, and black holes), and the confining of cosmic rays, to name but some.

It is widely accepted that the galactic magnetic fields are generated through a dynamo mechanism usually referred to as *the galactic dynamo*, for which, though, there is no consistent mathematical model as yet [15]. The basic idea of the dynamo mechanism is that a weak seed field could be amplified by the turbulent motion of ionized gas, which follows the differential rotation of the galaxy [3,6]. The growth of the field is exponential and, thus, its strength can be increased several orders of magnitude in only a few e-foldings of amplification.

The currently observed magnetic field of the Milky Way and of nearby galaxies is of the order of a μGauss. If the e-folding time is no more than the galactic rotation period $\sim 10^8$ yr, then, considering the galactic age $\sim 10^{10}$ yr, the seed field needed to produce a field of the observed value is about $\sim 10^{-19}$ G [3,5] on a comoving scale of a protogalaxy.

Although, it has been argued by many authors that the seed field could be produced from stars via stellar winds or supernovae and other explosions [3], there is evidence that suggests that the seed field is more likely to be truly primordial. For example, the observed field of the Milky Way does not change sign with z (z being the galactic altitude) as it would if it was produced by the stars of the galactic disk [3].

Various attempts have been made to produce a primordial field in the early universe. A thorough investigation of the issue was attempted by Turner and Widrow [7], who incorporated inflation and created the field by explicitly breaking the conformal invariance of electromagnetism. This was done in a number of ways, such as coupling the photon to gravity through RA^2 and RF^2 terms (R being the curvature, A being the photon field, and F being the electromagnetic field strength), or with a scalar field ϕ, such as the axion, through a term of the form ϕF^2. It was, thus, shown that satisfactory results could be obtained only at the expense of gauge invariance. Garretson *et al.* [8] have generalized the effort of [7] by coupling the photon to an arbitrary pseudo Goldstone boson, rather than the QCD axion. They have showed, however, that, in all cases considered it was impossible to generate a primordial magnetic field of any astrophysical importance. Breaking the electromagnetic conformal invariance during inflation was a mechanism used also by a number of other authors, such as Ratra [9] and Dolgov [10]. Ratra *has* been successful in generating an adequately intense magnetic field. The field was generated by coupling the field strength with a scalar field Φ (the dilaton) through a term of the form $e^{\Phi}F^2$. Dolgov, however, did not introduce any extra coupling but considered photon production by external gravity by the quantum conformal anomaly. He produced a field of enough strength, but only in the case of a large number (over 30) of light charged bosons.

Another, more successful direction was using a phase transition for the creation of a primordial field. An early effort was made by Hogan [2], who considered the possibility of turbulence arising during the QCD transition. His treat-

*Electronic address: A.C.Davis@damtp.cam.ac.uk
†Electronic address: K.Dimopoulos@damtp.cam.ac.uk

ment, though, was based on a number of assumptions concerning equipartition of energy, which are of questionable validity. Much later, Vachaspati [11] proposed a mechanism to produce a marginally sufficient magnetic field during the electroweak transition. This has also been addressed, a bit more successfully, by Enqvist and Olesen [12]. The later have also considered a phase transition to a new, ferromagnetic ground state of the vacuum, which could also produce an adequately strong magnetic field [13]. Finally, the literature contains a number of other, more exotic mechanisms (such as, for example, the creation of a primordial magnetic field by the turbulent motion of infalling matter into wakes in the wiggly string scenario [14]). In most of the cases, though, the achieved field appeared to be too weak to seed the galactic dynamo.

In this paper we examine the production of a primordial magnetic field during false vacuum inflation. In false vacuum inflation a phase transition can occur during the inflationary period. As shown by Vachaspati [11], the existence of a horizon could result in the production of a magnetic field at the phase transition. Although our model incorporates the benefits of inflation, it does not require the breaking of the gauge or even the conformal invariance of electromagnetism. Neither does it involve the addition of any extra couplings between fields through the inclusion of peculiar, ‘‘by hand’’ terms in the Lagrangian. Our magnetic field is produced by the dynamic features of the grand unified theory (GUT) Higgs field, independently of specific GUT models. No additional fields are introduced in the problem and the results cover the most general case. By considering a GUT phase transition we find that, for some parameter space, the magnetic field produced is of enough strength to seed a galactic dynamo mechanism at the epoch of structure and galaxy formation.

Our results take into account any constraints imposed on the field during its evolution until the epoch of structure and galaxy formation. Before examining the behavior of the magnetic field, we give a detailed description of the model and of the mechanism, through which the original primordial magnetic field is created.

II. CREATION OF THE PRIMORDIAL MAGNETIC FIELD

Vachaspati [11] suggested that the existence of a horizon would result in the creation of a primordial magnetic field at a phase transition in the early Universe.

Consider a non-Abelian group G. The field strength of the gauge fields is

$$H^a_{\mu\nu}=\partial_\mu X^a_\nu-\partial_\nu X^a_\mu-g_o f^a_{bc}X^b_\mu X^c_\nu, \tag{1}$$

where f^a_{bc} are the structure constants of G and g_o is the gauge coupling. If the symmetry of the gauge group G is broken, leaving a residual symmetry corresponding to a subgroup H of G, then the gauge fields of the residual symmetry are given by

$$Y^b\equiv u^{ab}X^a, \tag{2}$$

where u^{ab} is a unitary matrix specifying the directions of the generators of the unbroken symmetry. Vachaspati argued that if the symmetry was spontaneously broken then the vacuum expectation value (VEV) of the Higgs field ψ^a would have been uncorrelated on superhorizon scales,[1] and hence, could not be ‘‘aligned’’ throughout all space with a gauge transformation. Therefore, the gradients of the Higgs field would, in general, be nonzero. Because of the coupling through the covariant derivative,

$$D_\mu\langle\psi\rangle=(\partial_\mu-ig\tau^b Y^b_\mu)\langle\psi\rangle, \tag{3}$$

where τ^a are the generators of the residual symmetry and g is the gauge coupling, the corresponding field strength $G^b_{\mu\nu}$ is nonzero. This can be seen explicitly by using the gauge-invariant generalization of ’t Hooft [16]:

$$\begin{aligned}G^b_{\mu\nu}&=u^{ab}[H^a_{\mu\nu}-g^{-1}\mu^{-2}f^a_{cd}D_\mu\psi^c D_\nu\psi^d]\\&=\partial_\mu Y^b_\nu-\partial_\nu Y^b_\mu-g^{-1}\mu^{-2}u^{ab}f^a_{cd}\partial_\mu\psi^c\partial_\nu\psi^d,\end{aligned} \tag{4}$$

where μ is the scale of the symmetry breaking. So, even if the gauge field Y^b_μ can be gauged away, the field strength is still nonzero:

$$G^b_{\mu\nu}=-\frac{1}{g\mu^2}u^{ab}f^a_{cd}\partial_\mu\psi^c\partial_\nu\psi^d. \tag{5}$$

Vachaspati applied the above in the case of the electroweak phase transition, taking $G_{\mu\nu}$ to be the field strength of electromagnetism.

In this paper we will follow a similar reasoning but for a GUT phase transition. G is now the GUT symmetry group and the residual symmetry is the electroweak. In order to get to electromagnetism we need to consider also the final electroweak phase transition. In analogy with the above, the electromagnetic gauge potential is given by

$$A_\mu=v^b Y^b_\mu\equiv\sin\theta_W n^a W^a_\mu+\cos\theta_W B^Y_\mu, \tag{6}$$

where v^b is a unit vector specifying the direction of the unbroken symmetry $U(1)_{em}$ generator, n^a are the SU(2) generators of the electroweak group $SU(2)\times U(1)_Y$, W^a_μ are the SU(2) gauge fields, B^Y_μ is the $U(1)_Y$ gauge field, and θ_W is the Weinberg angle. From (6) it is easy to see that

$$v^b\equiv(\sin\theta_W n^a,\cos\theta_W), \tag{7}$$

$$Y^b_\mu\equiv(W^a_\mu,B^Y_\mu), \tag{8}$$

with $b=1,\ldots,4$ and $a=1,\ldots,3$.

The contribution to the electromagnetic field strength $F_{\mu\nu}$ from the GUT transition is, therefore,

$$F_{\mu\nu}\equiv v^b G^b_{\mu\nu}, \tag{9}$$

where $v^b G^b_{\mu\nu}$ is

$$v^b G^b_{\mu\nu}\equiv\sin\theta_W n^a G^a_{\mu\nu}+\cos\theta_W G^Y_{\mu\nu}, \tag{10}$$

with $b=1,\ldots,4$, $a=1,\ldots,3$, and $G^Y_{\mu\nu}\equiv G^4_{\mu\nu}$.

The magnetic field produced by the GUT phase transition is, therefore,

[1]More precisely, on scales larger than the correlation length.

$$B_\mu \equiv \frac{1}{2}\varepsilon_{\mu\nu\lambda}F^{\nu\lambda}. \tag{11}$$

Thus, for an order of magnitude estimate, Eqs. (5), (9), and (11) suggest

$$|B_\mu| \sim |F_{\mu\nu}| \sim |G_{\mu\nu}| \sim \frac{1}{g\mu^2}(\partial_\mu\langle\psi\rangle)^2, \tag{12}$$

since v^a, u^{ab}, and f^a_{cd} are of unit magnitude. As far as the Higgs-field gradients are concerned, on dimensional grounds, we have

$$\partial_\mu\langle\psi\rangle \sim \frac{\mu}{\xi}, \tag{13}$$

where ξ is the correlation length of the Higgs-field configuration (see also Appendix).

Also, at the GUT scale, $4\pi g^{-2} \simeq 40 \Rightarrow g^{-1} \sim 1$. Therefore, a dimensional estimate for the magnetic field is

$$B \equiv |B_\mu| \sim \xi^{-2}. \tag{14}$$

III. FALSE VACUUM INFLATION

In this section we review false vacuum inflation, a popular model of inflation corresponding to extensive literature [17–24].

In this model the inflaton field ϕ rolls down its potential towards the minimum, which does not correspond to the true vacuum, but is instead a false vacuum state. There are two distinct and quite different kinds of false vacuum inflation, depending on whether the energy density is dominated by the false vacuum energy density or by the potential energy density of the inflaton field. Full details are given by Copeland *et al.* in [24]. Unlike them, we concentrate on the inflaton-dominated case, as in [18–23]. In this case, the phase transition does not lead to the end of inflation as it does in the vacuum-dominated case.

A. The model

In this model the energy density is dominated by a potential with two scalar fields; the inflaton field ϕ and the Higgs field ψ. The latter is responsible for the phase transition. We should emphasize here that the Higgs field considered does not correspond to a specific GUT model and can have several components without this affecting the following analysis [24].

We take the form of the potential to be

$$V(\phi,\psi) = \frac{1}{4}\lambda(\psi^2-\mu^2)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}\lambda'\phi^2\psi^2. \tag{15}$$

The phase transition takes place at $\phi=\phi_0$, where

$$\phi_0^2 \equiv \frac{\lambda}{\lambda'}\mu^2. \tag{16}$$

This gives the effective scale of the symmetry breaking:

$$\mu_{\rm eff}^2 \equiv \mu^2\left(1-\frac{\phi^2}{\phi_0^2}\right). \tag{17}$$

Without loss of generality, we assume that ϕ is initially positive and rolls down the potential in such a way that $\dot\phi<0$. If there is sufficient inflation before the phase transition and $\lambda \gg \lambda'$, the Higgs field will have rolled to the minimum of its potential $\psi=0$ before the inflaton falls to its critical value ϕ_0. So, when $\phi>\phi_0$,

$$V(\phi,0) = \frac{1}{4}\lambda\mu^4 + \frac{1}{2}m^2\phi^2. \tag{18}$$

In the slow-roll approximation the dynamics of inflation are governed by the equations

$$H^2 \simeq \frac{8\pi}{3m_{\rm Pl}^2}V, \tag{19}$$

$$3H\dot\phi \simeq -V', \tag{20}$$

where the prime and the overdot denote derivatives with respect to ϕ and time, respectively, $H\equiv\dot a/a$ is the Hubble parameter, a is the scale factor of the Universe, and $m_{\rm Pl}$ is the Planck mass ($m_{\rm Pl}=1.22\times10^{19}$ GeV).

Thus,

$$H \simeq -\frac{8\pi}{m_{\rm Pl}^2}\frac{V}{V'}\dot\phi. \tag{21}$$

From Eq. (21) the number of e-foldings of expansion, which occur between the values ϕ_1 and ϕ_2 of the inflaton field, is given by

$$N(\phi_1,\phi_2) \equiv \ln\frac{a_2}{a_1} \simeq -\frac{8\pi}{m_{\rm Pl}^2}\int_{\phi_1}^{\phi_2}\frac{V}{V'}d\phi. \tag{22}$$

B. The inflaton-dominated regime

This is the case we are going to be interested in, since *the back reaction of the Higgs field ψ to the inflaton field ϕ is negligible* and so *the phase transition does not cause the end of inflation* [24]. If the opposite is true and inflation ends at the phase transition, then the effects of the transition are not too different from the usual, thermal phase transitions studied in the literature.

In the inflaton-dominated case the energy density of the inflaton field in Eq. (18) is much larger than the false vacuum energy density. Therefore,

$$V(\phi) \simeq \frac{1}{2}m^2\phi^2, \tag{23}$$

which is identical with chaotic inflation. Inflation ends at $\phi\sim\phi_\varepsilon$, where

$$\phi_\varepsilon \equiv \frac{m_{\rm Pl}}{\sqrt{4\pi}}. \tag{24}$$

To ensure inflaton domination until the end of inflation then

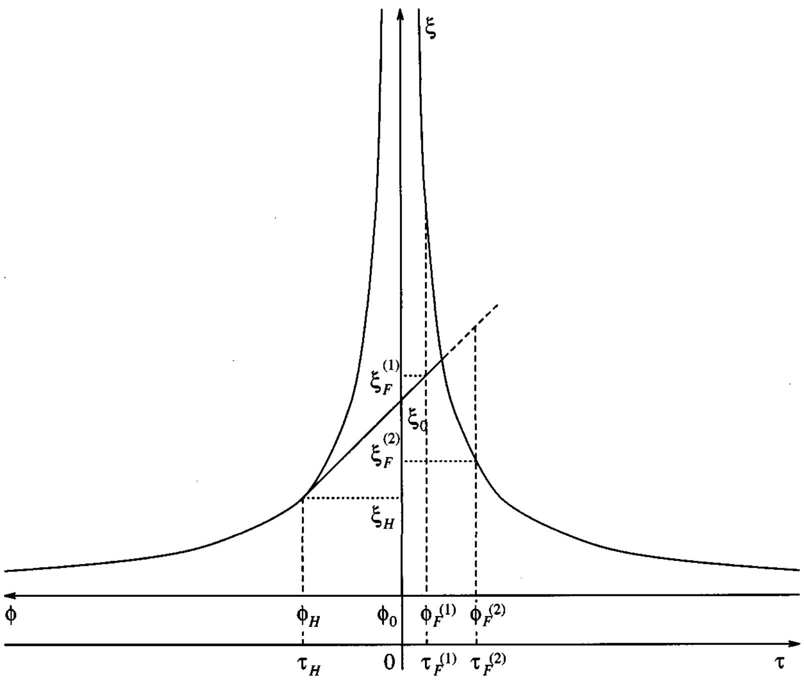

FIG. 1. Evolution of the correlation length ξ of the Higgs-field configuration near the phase transition. The linear growth of ξ starts at τ_H. ξ_0 is the correlation length at the time of the transition, $\xi_F^{(1)}$ is the correlation length at the time the Higgs-field configuration freezes, when this occurs during linear growth, and $\xi_F^{(2)}$ is the correlation length at freezing when this occurs after the end of the linear regime.

$$\frac{1}{2}m^2\phi_\varepsilon^2 \gg \frac{1}{4}\lambda\mu^4 \Rightarrow \frac{2\pi}{m_{\rm Pl}^2}\frac{\lambda\mu^4}{m^2} \ll 1. \tag{25}$$

This is the condition for inflaton domination. At this point it should be mentioned that *if the inflaton domination condition is strongly valid, the dynamics of inflation are not seriously affected by the phase transition, provided that ψ falls rapidly to its VEV*. Thus, Eqs. (19)–(21) can be used throughout the duration of inflation.

Thus, for the number of e-foldings between the phase transition and the end of inflation, in the inflaton-dominated case, we obtain

$$N \equiv N(\phi_0,0) = \frac{2\pi}{m_{\rm Pl}^2}\phi_0^2, \tag{26}$$

where we have used $\phi_0 \gg \phi_\varepsilon$.

Finally, for the roll down of the inflaton field, using Eqs. (19), (20), and (23), we obtain

$$\dot{\phi} = -\frac{m_{\rm Pl}\, m}{\sqrt{12\pi}}. \tag{27}$$

IV. THE CORRELATION LENGTH

A. Evolution of the correlation length

Through the use of the uncertainty principle, we can estimate the range of any interaction. Therefore, the physical correlation length for the Higgs field ψ is

$$\xi \equiv \frac{1}{|m_H|}, \tag{28}$$

where $m_H = m_H(t)$ is the mass of the Higgs particle:

$$m_H^2 = \frac{\partial^2 V(\phi,\psi)}{\partial\psi^2} = 3\lambda\psi^2 + \lambda'\phi^2 - \lambda\mu^2. \tag{29}$$

Before the phase transition $\psi = 0$ and thus, $m_H^2 = \lambda|\mu_{\rm eff}^2|$. Therefore, the physical correlation length is

$$\xi(\phi) = \frac{1}{\sqrt{\lambda}\,\mu}\frac{\phi_0}{\sqrt{\phi^2-\phi_0^2}}. \tag{30}$$

However, this is not valid as we approach the phase transition, $\phi \to \phi_0$. As $\dot{\xi} = 1$, the correlation length grows linearly with time as shown in Fig. 1.

Define

$$\tau \equiv t - t_0, \tag{31}$$

where t_0 is the time the transition occurs.

If at $\tau \equiv \tau_H$,

$$\frac{d}{d\tau}\left(\frac{1}{|m_H|}\right)\bigg|_H = 1. \tag{32}$$

Then, from τ_H until the transition $\tau = 0$, $d\xi/d\tau = 1$ and, therefore, the correlation length ξ_0 at the time of the transition is

$$\xi_0 = \xi_H - \tau_H, \tag{33}$$

where ξ_H is the correlation length at τ_H, and, in the linear regime,

$$\xi(\tau) = \xi_0 + \tau. \tag{34}$$

The linear growth of ξ continues until it hits the declining slope of $|m_H|^{-1}$ (Fig. 1). From then on, the correlation length is given again by Eq. (28).

The phase transition we are considering is not triggered by temperature fall, but by the roll down of the inflaton field. In that sense *it is not a thermal phase transition*. Also, since it occurs during inflation, the Universe is in a supercooled state with temperature $T\approx 0$ and so *there are no Ginzburg phenomena*. However, the configuration of the Higgs field ψ does not freeze at the moment of the phase transition because of long-wave quantum fluctuations that dominate the Higgs-field evolution immediately after the transition.

The long-wave fluctuations of ψ are determined by the behavior of the Higgs-field mass (29). Immediately after the phase transition, the Higgs field is still $\psi\simeq 0$ [20]. Then, since $\phi<\phi_0$, it follows from Eq. (29) that $m_H^2<0$, and quantum fluctuations grow until the evolution of ψ becomes potential dominated and the field starts falling to its new minima. At this stage the fluctuations of the Higgs field become impotent and the field configuration topology freezes. The fall of ψ is very rapid [20]. After $\psi=\mu_{\rm eff}/\sqrt{3}$, the mass of the field becomes positive again, Eq. (29). When the field reaches its minimum $\psi=\mu_{\rm eff}$, then, $m_H^2=2\lambda\mu_{\rm eff}$.

The magnetic field is formed at the freezing of the Higgs-field configuration. This occurs when $|m_H^2|\simeq H^2$ [18,19].[2] The correlation length ξ_F at the time of freezing τ_F, is either given by Eq. (28) or by Eq. (34), depending on whether or not we are still in the linear regime.

B. Computation of ξ_0 and ξ_F

We assume that ϕ_H is very close to ϕ_0 or, equivalently, that the time τ_H, when the growth of ξ reaches the speed of light, is very close to the time $\tau=0$ of the phase transition. Therefore,

$$\Lambda\equiv\frac{\phi_H^2-\phi_0^2}{\phi_0^2}\ll 1. \tag{35}$$

This will be verified when we introduce specific values for the parameters.

Using Eqs. (27) and (30), we find

$$\frac{d}{d\tau}\left(\frac{1}{|m_H|}\right)\Bigg|_H=\frac{m_{\rm Pl}m}{\sqrt{12\pi}}\frac{1}{\sqrt{\lambda'}}\frac{\phi_0}{[\phi_H^2-\phi_0^2]^{3/2}}. \tag{36}$$

Using Eq. (32), we obtain

$$\phi_H^2-\phi_0^2=\left[\frac{m_{\rm Pl}m\phi_0}{\sqrt{12\pi\lambda'}}\right]^{2/3}. \tag{37}$$

From Eq. (27) we have

$$\phi_H=\phi_0-\frac{m_{\rm Pl}m}{\sqrt{12\pi}}\tau_H. \tag{38}$$

Thus, solving for τ_H, we obtain

$$\tau_H=-\sqrt{3\pi}(12\pi\lambda' m_{\rm Pl}m\phi_0)^{-1/3} \tag{39}$$

and thus,

$$\xi_H=\frac{1}{\sqrt{\lambda'}}\left[\frac{m_{\rm Pl}m\phi_0}{\sqrt{12\pi\lambda'}}\right]^{-1/3}=2\sqrt{3\pi}(12\pi\lambda' m_{\rm Pl}m\phi_0)^{-1/3}. \tag{40}$$

Using Eq. (33) and the above equation (40), we obtain

$$\xi_0=3\sqrt{3\pi}(12\pi\lambda' m_{\rm Pl}m\phi_0)^{-1/3}. \tag{41}$$

Let us compute, now, the correlation length at the time when the field configuration freezes, i.e., when $|m_H^2|\simeq H^2$. From Eqs. (19), (23), and (29), we obtain

$$\phi_0^2-\phi_F^2\simeq\frac{4\pi}{3}\frac{m^2\phi_0^2}{\lambda' m_{\rm Pl}^2}, \tag{42}$$

where ϕ_F is the magnitude of the inflaton at the time of freezing τ_F, for which we find

$$\tau_F\simeq\sqrt{3\pi}\frac{(\phi_0^2-\phi_F^2)}{m_{\rm Pl}m\phi_0}\simeq\sqrt{\frac{16\pi^3}{3}\frac{m\phi_0}{\lambda' m_{\rm Pl}^3}}, \tag{43}$$

where we have used Eq. (27) and the assumption (35).

Using Eqs. (22), (23), and (42), we find that the number of e-foldings of inflation between the phase transition and the freezing of the field configuration is given by

$$\Delta N_F\simeq\frac{8\pi^2}{3}\frac{m^2\phi_0^2}{\lambda' m_{\rm Pl}^4}. \tag{44}$$

Only during this time are the quantum fluctuations of the Higgs field important (for more details on quantum fluctuations, see also Appendix).

If $\Delta N_F\leqslant 1$, then the transition proceeds rapidly [19].

If we are still in the regime of linear growth of the correlation length, the value of it at the time of freezing is simply given by Eq. (34),

$$\xi_F^{(1)}=\xi_0+\tau_F. \tag{45}$$

If, however, the linear growth of ξ has ended before τ_F, then it is given by Eq. (28):

$$\xi_F^{(2)}\simeq\sqrt{\frac{3}{8\pi}\frac{m_{\rm Pl}}{m\phi_0}}, \tag{46}$$

where we have used that $m_H^2\simeq\lambda'(\phi_0^2-\phi_F^2)$ and Eq. (42).

Therefore, the initial correlation length of the Higgs-field configuration is given by

$$\xi_F=\min(\xi_F^{(1)},\xi_F^{(2)}). \tag{47}$$

V. EVOLUTION OF THE MAGNETIC FIELD

A. During inflation

From Eqs. (47) and (14), we estimate the initial value of the magnetic field produced just after the phase transition to be

[2] Nagasawa and Yokoyama [22] suggest that the freezing of the field occurs a bit later. However, with the set of parameters used (see Sec. VIII A), the corresponding difference in the correlation length is less than an order of magnitude.

$$B_0 \sim \xi_F^{-2}. \tag{48}$$

Of course, after the GUT phase transition we still have electroweak unification. Therefore, Eq. (48) represents, in fact, an ''electroweak'' magnetic field. However, since the residual, electromagnetic symmetry generator is just a projection of the electroweak generators (through the Weinberg angle), the residual ''electromagnetic'' magnetic field will be of the same order of magnitude as the one given by Eq. (48). Therefore, from now on, we will ignore the electroweak transition and treat the above magnetic field as ''electromagnetic.''

During inflation the magnetic field is *not* frozen into the supercooled plasma [7] but still it scales as a^{-2}, since it remains coupled to the Higgs field ψ and, thus, Eq. (14) is still valid. The configuration of the Higgs-field remains comovingly frozen during inflation because the initially correlated volumes expand exponentially, faster than causal correlations. This is not the case after inflation ends. The scale factor, then, grows slower than the causal correlations and the comoving picture of the Higgs-field configuration starts changing as the field becomes correlated over larger and larger comoving volumes. However, after the end of inflation, the magnetic field gets frozen into the reheated plasma[3] and decouples from the Higgs field.

B. The rms magnetic field

In order to estimate the magnetic field on scales larger than the typical dimensions of the correlated volumes, we have to introduce a statistical method to do so.

A thorough treatment by Enqvist and Olesen [12] suggests that, in all cases that the Higgs-field gradients are a diminishing function of n (number of correlated domains), the root-mean-square value of the field would behave as

$$B_{\rm rms} \equiv \sqrt{\langle B^2 \rangle} = \frac{1}{\sqrt{n}} B_{\rm CD}, \tag{49}$$

where $B_{\rm CD}$ is the field inside a correlated domain and n is the number of correlation length scales, over which the field is averaged. In their treatment Enqvist and Olesen choose the Higgs-field gradients as the stochastic variables and also assume that their distribution is Gaussian and isotropic. Choosing the magnetic field itself as the stochastic variable, Enqvist and Olesen reached the same result [Eq. (49)].

At this point it should be mentioned that in the above treatment the rms value of the field has been computed as a line average, that is an average over all the possible curves in space between the points that fix the length scale, over which the field is averaged. The above result may be sensitive to the averaging procedure. One argument in favor of line averaging is that the current galactic magnetic field has been measured using the Faraday rotation of light spectra, which is also a line (line of sight) computation. If we assume that the ratio of the seed field for the galactic dynamo and the currently observed galactic field is independent of the averaging procedure then this would suggest that line averaging is required for the computation of the primordial field. However, the nonlinearity of the dynamo process as well as the rather poor knowledge we have for galaxy formation make such an assumption nontrivial. In any case, apart from the above, there seem to be no other argument in favor of a particular averaging procedure. Therefore, using line averaging could be the safest choice. Here it is important to point out that *line averaging just gives an estimate of the rms field and does not correspond to any physical process*.

Suppose that we are interested in calculating the rms field at a time t over a physical length scale $L = L(t)$. Then,

$$n(t) = \frac{L}{\xi}, \tag{50}$$

where ξ is the correlation length. This scale is equal to the correlation length ξ_0 at the time of the phase transition.

In this paper we are mainly interested in the value of the magnetic field at $t_{\rm eq}$, the time of equal matter and radiation densities, when structure formation begins. The scale of interest is the typical intergalactic distance, since $t_{\rm eq}$ is preceding the gravitational collapse of the galaxies (see also Sec. VII B). At $t_{\rm eq}$, the corresponding scale is found to be

$$L_{\rm eq} \sim \left(\frac{t_{\rm eq}}{t_p} \right)^{2/3} L_p \sim 10 \ \ {\rm pc}, \tag{51}$$

where $t_p \sim 10^{18}$ sec is the present time and $L_p \sim 1$ Mpc is taken as the typical intergalactic scale at present.

From Eq. (50) the number of correlated domains at $t_{\rm eq}$ is,

$$n \equiv n_{\rm eq} = \frac{L_{\rm eq}}{\xi_{\rm eq}}, \tag{52}$$

where $\xi_{\rm eq}$ is the correlation length at $t_{\rm eq}$.

Therefore, the rms value of the magnetic field over the scale of a protogalaxy is

$$B_{\rm rms}^{\rm eq} \sim \frac{1}{\sqrt{n}} B_{\rm CD}^{\rm eq}, \tag{53}$$

where $B_{\rm CD}^{\rm eq}$ is the value of the field inside a correlated domain at $t_{\rm eq}$.

C. Growth of the correlated domains

It is clear that, in order to calculate the rms field over the galactic scale at $t_{\rm eq}$, we need to estimate the correlation length $\xi_{\rm eq}$, i.e., the size of the correlated domains at that time. Therefore, we have to follow carefully the evolution of the correlated domains throughout the whole radiation era.[4]

During inflation, as explained already, ξ scales as the scale factor a. However, after the end of inflation, it grows faster. This is because, when two initially uncorrelated neighboring domains come into causal contact, the magnetic

[3] See also Sec. VII E.

[4] The correlated domains should not be pictured as attached bubbles of coherent magnetic field, but as regions around any given point in space in which the orientation of the field is influenced by its orientation at this point.

field around the interface is expected to untangle and smooth, in order to avoid the creation of energetically unfavored magnetic domain walls. In time the field inside both domains ''aligns'' itself and becomes coherent over the total volume. The velocity v, with which such a reorientation occurs, is determined by the plasma, which carries the field and has to reorientate its motion for that purpose.[5]

Thus, the evolution of the correlation length is given by

$$\frac{d\xi}{dt}=H\xi+v, \tag{54}$$

where v is the peculiar, bulk velocity, determined, in principle, by the state of the plasma.

From Eq. (54) it is apparent that the correlated domains could grow faster than the Hubble expansion. Therefore, *the magnetic field configuration is not necessarily comovingly frozen* and the domains could expand much faster than the Universe, resulting in large correlations of the field and high coherency.

In order to describe the evolution of the correlated domains one has to determine the peculiar velocity v. This primarily depends on the opacity of the plasma.

If the plasma is opaque on the scale of a correlated domain, then radiation cannot penetrate this scale and is blocked inside the plasma volume. Consequently, the plasma is subject to the total magnetic pressure of the magnetic field gradient energy. Therefore, this energy dissipates through coherent magnetohydrodynamic oscillations, i.e., Alfvén waves. In this case, the peculiar velocity of the magnetic field reorientation is the well-known Alfvén velocity [2][6]:

$$v_A\equiv\frac{B_{\rm CD}}{\sqrt{\rho}}, \tag{55}$$

where $B_{\rm CD}$ is the magnitude of the magnetic field inside a correlated domain and ρ is the *total* energy density of the Universe, since, before $t_{\rm eq}$, matter and radiation are strongly coupled.[7]

If the plasma is not opaque over the scale ξ of a correlated domain, then radiation can penetrate this scale and carry away momentum, extracted from the plasma through Thomson scattering of the photons. This subtraction of momentum is equivalent to an effective drag force, $F\sim\rho\sigma_T v_T n_e$ [2]. Balancing this force with the magnetic force determines the ''Thomson'' velocity over the scale ξ:

$$v_T\equiv\frac{v_A^2}{\xi n_e\sigma_T}, \tag{56}$$

[5]Note that the plasma does not have to be carried from one domain to another or get somehow mixed. Also, conservation of flux is not violated with the field's rearrangements, since *the field always remains frozen into the plasma*, which is carried along.

[6]Unless explicitly specified, natural units are being used $(\hbar=c=1)$. In natural units, $G=m_{\rm Pl}^{-2}$

[7]This coupling implies that any reorientation of the momentum of matter has to drag radiation along with it. This increases the inertia of the plasma, that balances the magnetic pressure.

where v_A is the Alfvén velocity, n_e is the electron number density, and σ_T is the Thomson cross section.

Hence, for a nonopaque plasma the peculiar velocity of the plasma reorientation is given by [2]

$$v=\min(v_A,v_T). \tag{57}$$

In order to explore the behavior of the opaqueness of the plasma, we need to compare the mean free path of the photon $l_T\sim(n_e\sigma_T)^{-1}$ to the scale ξ of the correlated domains. For realistic models, the correlated domains remain opaque at least until the epoch $t_{\rm anh}\sim 0.1$ sec of electron-positron annihilation ($T\sim 1$ MeV). The reason for this can be easily understood by calculating l_T before and after pair annihilation.

For $T>1$ MeV, instead of the usual Thomson cross section σ_T, we have the Klein-Nishina cross section [25]

$$\sigma_{\rm KN}\simeq\frac{3}{8}\sigma_T\left(\frac{m_e}{T}\right)\left[\ln\frac{2T}{m_e}+\frac{1}{2}\right]\simeq 2.7\left(\frac{\rm GeV}{T}\right)\ln\left[\frac{T}{\rm GeV}\right]\ {\rm GeV}^{-2}, \tag{58}$$

where $m_e\simeq 0.5$ GeV is the electron mass and $\sigma_T\simeq 6.65\times 10^{-25}\ {\rm cm}^2\simeq 1707.8\ {\rm GeV}^{-2}$. The electron number density is given by [26]

$$n_e\simeq\frac{3}{4}\frac{\zeta(3)}{\pi^2}g_eT^3, \tag{59}$$

where $\zeta(3)\simeq 1.20206$ and $g_e=4$ are the internal degrees of freedom of electrons and positrons.

From Eqs. (58) and (59), we find

$$l_T\sim\frac{0.1\ {\rm GeV}}{T^2}\quad {\rm for}\quad T>1\ {\rm MeV}, \tag{60}$$

which at annihilation gives, $l_T(t_{\rm anh})\sim 10^5\ {\rm GeV}^{-1}$.

After annihilation the electron number density is given by [26]

$$n_e\simeq 6\times 10^{-10}n_\gamma\simeq 1.44\times 10^{-10}T^3, \tag{61}$$

where n_γ is the photon number density given by

$$n_\gamma\simeq\frac{\zeta(3)}{\pi^2}g_\gamma T^3, \tag{62}$$

where $g_\gamma=2$ are internal degrees of freedom of the photon.

With the usual value for σ_T, we obtain

$$l_T\sim\frac{10^6\ {\rm GeV}^2}{T^3}\quad {\rm for}\quad T<1\ {\rm MeV}. \tag{63}$$

At annihilation the above gives, $l_T(t_{\rm anh})\sim 10^{15}\ {\rm GeV}^{-1}$.

Hence, the mean free path of the photon at the time of pair annihilation is enlarged by a factor of 10^{10}. As a result, l_T is very likely to become larger than ξ after $t_{\rm anh}$. If this is so, the Thomson dragging effect has to be taken into account and the peculiar velocity of the plasma reorientation is given by Eq. (57).

In order to calculate the peculiar velocity it is necessary to compute the Alfvén velocity, which requires the knowledge of the magnetic field value $B_{\rm CD}$ inside a correlated domain.

To estimate this we assume that the magnetic flux, on scales larger than the sizes of the correlated domains, is conserved, as implied by the frozen-in condition.

Consider a closed curve C in space, of length scale $L>\xi$, encircling an area A. Conservation of flux suggests that the flux-averaged mean magnetic field inside A scales as a^{-2}. This implies that for the field inside a correlated domain we have[8] $B_{\rm CD}(L/\xi)^{-1}\propto a^{-2}$. Since C follows the Universe expansion $L\propto a$, with $a\propto t^{1/2}$. Thus, for the radiation era, we obtain

$$B_{\rm CD}t^{1/2}\xi=K\Rightarrow B_{\rm CD}=\frac{K}{t^{1/2}\xi}, \tag{64}$$

where K is a constant to be evaluated at any convenient time. Since, the correlation length grows at least as fast as the Universe expands, the magnetic field inside a correlated domain dilutes at least as rapidly as a^{-2} for the radiation era.

Substituting the above into Eq. (55), we find

$$v_A\sim 10\frac{K}{m_{\rm Pl}}\frac{t^{1/2}}{\xi}. \tag{65}$$

Solving the evolution equation (54) with $a\propto t^{1/2}$ in the case that $v=v_A$, gives

$$\xi(t)^2=\left(\frac{t}{t_i}\right)\xi_i^2+4v_A(t)\;\xi(t)t\left(1-\sqrt{\frac{t_i}{t}}\right), \tag{66}$$

where ξ_i is the correlation length of the field at the time t_i. The first term of Eq. (66) is because of the Hubble expansion, whereas the second term is because of the peculiar velocity.

In the case of $v=v_T$, for $t>t_{\rm anh}$, using Eqs. (61) and (62) and the usual value of σ_T, Eq. (56) gives,

$$v_T=D\frac{t^{5/2}}{\xi^3}, \tag{67}$$

where

$$D\sim 10^{-57}K^2\;\;{\rm GeV}^{-3/2}. \tag{68}$$

Using Eq. (67), the evolution equation (54) gives

$$\xi(t)^4=\left(\frac{t}{t_i}\right)^2\xi_i^4+\frac{8}{3}v_T(t)\;\xi^3t\left[1-\left(\frac{t_i}{t}\right)^{3/2}\right]. \tag{69}$$

The evolution of the correlation length of the magnetic field configuration is described initially by the Alfvén expansion equation (66) until the moment when $\xi\sim l_T$. From then on, the growth of ξ continues according either to Eq. (66) or to Eq. (69), depending on the relative magnitudes of the velocities v_A and v_T. Using the above, we can calculate the scale $\xi_{\rm eq}$ of the correlated domains at $t_{\rm eq}$ and, thus, calculate the rms magnetic field from Eq. (53).

[8]Note that the flux averaging of the field on scales larger than the correlation length, *corresponds to a physical process,* that of the field untangling, and is so in order to preserve flux conservation on scales that the magnetic field is frozen into the plasma. This should not be confused with the line-averaging procedure which we use to estimate the rms field, and does *not* correspond to a physical process.

D. Diffusion

An important issue, which should be considered, is the diffusion length of the freezing of the field. Indeed, the assumption that the field is frozen into the plasma corresponds to neglecting the diffusion term of the magnetohydrodynamical induction equation [27]

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B})+\sigma^{-1}\nabla^2\mathbf{B}, \tag{70}$$

where $\mathbf{v}$ is the plasma velocity and σ is the conductivity. In the limit of infinite conductivity the diffusion term of Eq. (70) vanishes and the field is frozen into the plasma on all scales. However, if σ is finite then spatial variations of the magnetic field of length scale l will decay in a diffusion time $\tau\simeq\sigma l^2$ [27]. Thus, the field at a given time t can be considered frozen into the plasma only over the diffusion scale

$$l_d\sim\sqrt{\frac{t}{\sigma}}. \tag{71}$$

If $l_d>\xi$, the magnetic field configuration is expected, in less than a Hubble time, to become smooth on scales smaller than $l_d(t)$. Thus, in this case, it is more realistic to consider a field configuration with coherence length l_d and magnitude of the coherent magnetic field $\overline{B_{\rm CD}}$, where $\overline{B_{\rm CD}}=B_{\rm CD}/n_d$ is the flux-averaged initial magnetic field over $n_d\equiv l_d/\xi_i$ number of domains.

An estimate of the plasma conductivity is necessary to determine the diffusion length. The current density in the plasma is given by $\mathbf{J}=ne\mathbf{v}$, where n is the number density of the charged particles. The velocity $\mathbf{v}$ acquired by the particles because of the electric field $\mathbf{E}$ can be estimated as $\mathbf{v}\simeq e\mathbf{E}\tau_c/m$, where m is the particle mass and $\tau_c=l_{mfp}/v$ is the time scale of collisions. Since the mean free path of the particles is given by, $l_{mfp}\simeq 1/n\sigma_c$, the current density is $\mathbf{J}\simeq e^2\mathbf{E}/mv\sigma_c$, where σ_c is the collision cross section of the plasma particles. Comparing with Ohm's law gives, for the conductivity [27,28],

$$\sigma\simeq\frac{e^2}{mv\sigma_c}. \tag{72}$$

The collision cross section is given by the Coulomb formula [28]

$$\sigma_c\simeq\frac{e^4}{T^2}\ln\Lambda, \tag{73}$$

where $\ln\Lambda\simeq\ln(e^{-3}\sqrt{T^3/n})$ is the Coulomb logarithm. Thus, the behavior of the conductivity depends crucially on the temperature.

For low temperatures, $T<m_e\simeq 1$ MeV (i.e., after $t_{\rm anh}$), the velocity of the electrons is $v\sim\sqrt{T/m_e}$. Thus, from Eqs. (72) and (73), the conductivity is given by

$$\sigma \sim \frac{1}{e^2}\sqrt{\frac{T^3}{m_e}}\frac{1}{\ln\Lambda}. \tag{74}$$

For high temperatures $T \gg m_e$, Eq. (59) suggests that $\ln\Lambda \sim 1$. Also, the mass of the plasma particles is dominated by thermal corrections, i.e., $m \sim T$, and $v \sim 1$. Consequently, in this case, Eqs. (72) and (73) give, for the conductivity,

$$\sigma \sim \frac{T}{e^2}. \tag{75}$$

Using the above results we can estimate the diffusion length. Indeed, from Eqs. (71), (74), and (75), we obtain

$$l_d \sim \begin{cases} 10^8 \ \mathrm{GeV}^{1/2} T^{-3/2}, & T \geq 1 \ \mathrm{MeV}, \\ 10^8 \ \mathrm{GeV}^{3/4} T^{-7/4}, & T < 1 \ \mathrm{MeV}. \end{cases} \tag{76}$$

An important point to stress is that the diffusion length is also increasing with time. *If $l_d > \xi$, then the size of the correlated domains is actually determined by the diffusion length and it is the growth of the latter that drives the evolution of the magnetic field configuration.*

At this point we could briefly discuss the behavior of any *electric* field, produced by the phase transition. As can be seen by Eqs. (5) and (12), the electric field $E^\mu \equiv F^{\mu 0}$ is determined by the time derivative of the Higgs field VEV. Thus, strong currents are expected to arise at the time of the transition, when the VEV of the Higgs field falls rapidly from zero to μ. These are the currents that accompany the creation of the magnetic field [29]. However, after the transition and during inflation, the VEV of the Higgs field, at any point in space, is more or less fixed and constant in time, since the field configuration is comovingly frozen. Therefore, *there should not be any significant electric field surviving the transition*. After inflation this comoving picture begins to change but the magnetic field decouples from the Higgs field and, thus, any electric field produced, by shifting of the magnetic field lines, is related to plasma motion phenomena. Since such reorientations occur, we expect small electric fields to be present in the form of electromagnetic waves, which will diffuse and thermalize the gradient energy of the magnetic field, that is reduced by its reorientation and alignment.

VI. AT THE END OF INFLATION

A. The reheating temperature

The time $t_{\rm end}$ when inflation ends could be determined by the reheating temperature $T_{\rm reh}$ with the use of the well-known relation

$$t_{\rm end} \simeq 0.3 \ g_*^{-1/2}\left(\frac{m_{\rm Pl}}{T_{\rm reh}^2}\right), \tag{77}$$

where g_* is the number of particle degrees of freedom which, in most models, is of order 10^2 (e.g., in the standard model it is 106.75 whereas in the minimal supersymmetric standard model it is 229).

The reheating temperature is usually estimated by [30]

$$\frac{T_{\rm reh}}{m_{\rm Pl}} \approx 0.78 \ \alpha^{1/4} g_*^{-1/4}\left(\frac{H_{\rm end}}{m_{\rm Pl}}\right)^{1/2}, \tag{78}$$

where $H_{\rm end}$ is the Hubble parameter at the end of inflation and α is the reheating efficiency, which determines how much of the inflaton's energy is going to be thermalized. Using Eqs. (19) and (24), we obtain

$$H_{\rm end} \simeq \frac{m}{\sqrt{3}}. \tag{79}$$

Substituting to Eqs. (77) and (78), we find

$$T_{\rm reh}^2 \approx 0.35 \ \alpha^{1/2} g_*^{-1/2} m \ m_{\rm Pl}. \tag{80}$$

In most inflationary models reheating is prompt, it is completed quickly, and $\alpha \approx 1$. In case of a quadratic inflaton potential, however, as in false vacuum inflation, the reheating process could be incomplete and extremely inefficient [31]. However, the magnitude of the reheating inefficiency is still an open question. Kofman *et al.* [31] suggest that the reheating temperature would be of the order $T_{\rm reh} \sim 10^{-2}\sqrt{m m_{\rm Pl}}$ which, compared to Eq. (80), implies that $\alpha \sim 10^{-4}$. Shtanov, Traschen and Brandenberger [32] make a lower estimate $T_{\rm reh} \sim m$.

B. Thermal fluctuations

The Higgs field, through the Higgs mechanism, provides the masses of the particles after the GUT phase transition. Thus, it is in that way coupled to the thermal bath of the particles. Therefore, at reheating, this coupling introduces thermal corrections to the effective potential of the Higgs field. Consequently, if the reheating temperature is high enough, the configuration of the Higgs field may be destroyed because of excessive thermal fluctuations. This will erase any magnetic field if the later has not been frozen into the plasma already. The above will occur if the temperature exceeds the well-known Ginzburg temperature T_G. Moreover, if the temperature exceeds a critical value T_c, there is a danger of thermal restoration of the GUT symmetry itself.

The Ginzburg and the critical temperatures are simply related [33]:

$$T_c - T_G \sim \lambda T_c. \tag{81}$$

Thus, for $\lambda \leq 1$,

$$T_G \sim T_c \sim \sqrt{\lambda}\mu. \tag{82}$$

Therefore, it is very important to see if the temperatures during the reheating process could exceed T_c. At this point it should be noted that *the reheating temperature is not the highest temperature achieved during the reheating process*. Indeed, as soon as the field begins its coherent oscillations, the temperature rises rapidly and assumes its maximum value [7,26]:

$$T_{\rm max} \simeq (V_{\rm end}^{1/4} T_{\rm reh})^{1/2}, \tag{83}$$

where $V_{\rm end}$ is the energy density of the inflaton at the end of inflation. From Eqs. (23) and (24), we obtain

$$V_{end}\sim 0.1 m^2 m_{Pl}^2. \tag{84}$$

Thus,

$$T_{max}\sim(\sqrt{m m_{Pl}} T_{reh})^{1/2}. \tag{85}$$

Therefore, in order to avoid symmetry restoration and any Ginzburg phenomena, we should have

$$T_{max}<T_c \Rightarrow T_{reh}<\frac{\lambda \mu^2}{\sqrt{m m_{Pl}}}. \tag{86}$$

If the reheating temperature exceeds the above value then the magnetic field is thermally unstable and we are in danger of restoring the GUT symmetry. However, if the field survives then its stability is ensured [29].

After reaching its highest value T_{max}, the temperature slowly decreases during the matter-dominated era of the coherent inflaton oscillations, until it falls to the value T_{reh} when the Universe becomes radiation dominated.[9]

VII. CONSTRAINTS

A. Constraints on the parameters

If we assume that the observed density perturbations are because of inflation, then we have, from the Cosmic Background Explorer (COBE) [24],

$$\frac{\sqrt{8\pi}}{m_{Pl}} m = 5.5\times 10^{-6}, \tag{87}$$

which yields

$$m\sim 10^{13}\ \text{GeV}. \tag{88}$$

Other restrictions of the model imposed on μ, λ, and λ' are [24]

$$0<\lambda,\lambda'\leqslant 1, \quad \mu\leqslant\frac{m_{Pl}}{\sqrt{8\pi}}\sim 10^{18}\ \text{GeV}. \tag{89}$$

Additional constraints for the λ are established by the inflaton domination condition (25). Also the ratio of the λ's can be determined by Eqs. (16) and (26) with the reasonable assumption that the number of e-foldings of inflation after the phase transition is of order $N\sim 10$,

$$\frac{\lambda}{\lambda'}\sim\left(\frac{m_{Pl}}{\mu}\right)^2. \tag{90}$$

B. The galactic dynamo constraint

From the present understanding of the galactic dynamo process [3], it follows that, in order for a primordial magnetic field to be the seed for the currently observed galactic magnetic field, it should be stronger than 10^{-19} G at the time of galaxy formation, on a comoving scale of a protogalaxy ($\sim$100 kpc).

Since the gravitational collapse of the protogalaxies enhances their frozen-in magnetic field by a factor of $(\rho_G/\rho_c)^{2/3}\sim 10^3$ (where $\rho_G\sim 10^{-24}$ g cm^{-3} is the typical mass density of a galaxy and $\rho_c\simeq 2\times 10^{-29}\Omega h^2$ g cm^{-3} is the current cosmic mass density), the above seed field corresponds to an field of the order of $\sim 10^{-22}$ G over the comoving scale of $\sim$1 Mpc. With the assumption that the rms field scales as a^{-2} with the expansion of the Universe ($a\propto t^{2/3}$ for the matter era), we find that the required magnitude of the seed field at t_{eq} is $\sim 10^{-22}$ G$\times(t_{GC}/t_{eq})^{4/3}\sim 10^{-20}$ G, where $t_{GC}\sim 10^{15}$ sec is the time of the gravitational collapse of the galaxies.

The above justify our choice to calculate the magnetic field at t_{eq} over the comoving scale of 1 Mpc and consider the constraint

$$B^{eq}\geqslant 10^{-20}\ \text{G}. \tag{91}$$

From recombination onwards, the nonlinear nature of structure formation is very difficult to follow. Indeed, there exists a numerous collection of different models. A strong primordial magnetic field could influence in various ways some of these models, possibly with a positive rather than a negative effect.

C. The nucleosynthesis constraint

One upper bound to be placed on the magnetic field at t_{eq} is coming from nucleosynthesis. This has been studied in detail by Cheng *et al.* [34]. They conclude that, at $t_{nuc}\sim 1$ sec, the magnetic field should not be stronger than

$$B^{nuc}\leqslant 10^{11}\ \text{G} \tag{92}$$

on a scale larger than $\sim 10^4$ cm. A more recent treatment by Kernan *et al.* [35] relaxes the bound by about an order of magnitude, $B^{nuc}\leqslant e^{-1}(T_\nu^{nuc})^2\sim 10^{12}$ G, where T_ν is the neutrino temperature and e is the electric charge. This bound is valid over all scales. Similar results are also reached by Grasso and Rubinstein [36].

D. Energy density constraints

Constraints are also induced by ensuring that the energy density of the magnetic field is less than the energy density of the Universe. During the inflationary period, because of inflaton domination, the energy density of the Universe is mainly in the inflaton field. However, after reheating and until t_{eq}, the energy density of the Universe is just the radiation energy density.

Thus, for the inflationary period we should verify that

$$\frac{\rho_B}{\rho_{inf}}\ll 1, \tag{93}$$

where $\rho_B\equiv B_{CD}^2/8\pi$ and $\rho_{inf}\equiv V(\phi)$ are the energy densities of the magnetic and inflaton fields, respectively.

The highest value of the above ratio corresponds to the time of the phase transition since the magnetic field is rapidly diluted during inflation, whereas the inflaton's potential

[9] We should mention that this small period of matter domination is not taken into account in our treatment because of the fact that its duration is very small compared to the time scales considered and so we choose to ignore it for the sake of simplicity.

energy remains almost unchanged. Using Eqs. (48) and (23), we find the first energy density constraint:

$$\sqrt{m\phi_0}>\xi_F^{-1}. \tag{94}$$

After reheating, the expansion of the Universe dilutes the energy density ρ_B of the magnetic field, inside a correlated domain more effectively than the radiation density, which scales as a^{-4}. Therefore, it is sufficient to ensure that $\rho_B(t)$ is less than the energy density $\rho(t)$ of radiation at the time t_i of the formation of the magnetic field configuration. That is,

$$\rho_B(t_i)\leqslant\rho(t_i)\Rightarrow B_{\rm CD}^i\leqslant\frac{\sqrt{3}}{2}\frac{m_{\rm Pl}}{t_i}, \tag{95}$$

which is the second energy density constraint.

E. The non-Abelian constraint

During the electroweak era, the freezing of the magnetic field into the electroweak plasma is not at all trivial to assume. Indeed, before the electroweak transition, since the electroweak symmetry group $SU(2)\times U(1)_Y$ is still unbroken, there are four apparent ''magnetic'' fields, three of which are non-Abelian.

It would be more precise, then, to refer only to the Abelian (hypercharge) part of the magnetic field, which satisfies the same magnetohydrodynamical equations as the Maxwell field of electromagnetism. The non-Abelian part of the field may not influence the motion of the plasma because of the existence of a temperature-dependent magnetic mass, $m_B\approx 0.28g^2T$ (see, for example, [37,38]), which could screen the field over the relevant length scales.

The condition for this screening to be effective can be obtained by comparing the screening length $r_S\sim m_B^{-1}$ of the non-Abelian magnetic fields with the Larmor radius of the plasma motion $r_L\sim mv/gB$, where $m\sim\sqrt{\alpha}\,T$ ($\alpha=g^2/4\pi$) is the temperature-induced physical mass of the plasma particles, $g\simeq 0.3$ is the gauge coupling (charge), and v is the plasma particle velocity. If we assume thermal velocity distribution, i.e., $mv^2\sim T$, we find

$$R\equiv\frac{r_L}{r_S}\sim 10^{-2}\,\frac{T^2}{B_{\rm CD}}. \tag{96}$$

If $R\geqslant 1$, then our restriction to the Abelian (hypercharge) part of the magnetic field is well justified. This restriction will not cause any significant change to our results since, at the electroweak transition, the hypercharge field projects onto the photon through the Weinberg angle (6), $\cos\theta_W\approx 0.88$. If, however, $R<1$ then the non-Abelian fields do affect the plasma motion, and should be taken into account. Since $T\propto a^{-1}$ and $B_{\rm CD}$ falls at least as rapid as a^{-2}, R is, in general, an increasing function of time. Thus, the constraint has to be evaluated at reheating.

F. The monopole constraints

Unfortunately, the mechanism, which we use to generate the primordial magnetic field, could also produce stable magnetic monopoles. Since these monopoles should not dominate the energy density of the Universe, we require that the the fraction Ω_M of the critical density, contributed by the monopoles, to be less than unity, that is [26],

$$\Omega_M\,h^2\simeq 10^{24}\left(\frac{n_M}{s}\right)\left(\frac{M}{10^{16}\ \rm GeV}\right)\leqslant 1, \tag{97}$$

where $M=4\pi\mu g^{-1}\sim 10\,\mu$ is the monopole mass, n_M is the monopole number density, s is the entropy density of the Universe, and h is the Hubble constant in units of 100 km/sec/Mpc. The ratio n_M/s is a constant[10] and can be evaluated at the end of inflation $t_{\rm end}$. Taking $n_M\sim\xi_{\rm end}^{-3}$, where $\xi_{\rm end}$ is the correlation length at that time, we have

$$\frac{n_M}{s}\simeq\frac{10^2}{\zeta^3}\left(\frac{T_{\rm reh}}{m_{\rm Pl}}\right)^3, \tag{98}$$

where $\zeta\equiv\xi_{\rm end}H_{\rm end}$ gives the correlation length as a fraction of the Hubble radius. From Eqs. (97) and (98) we find the first monopole constraint [24]:

$$\zeta^3\geqslant 10^{11}\left(\frac{T_{\rm reh}}{10^{14}\ \rm GeV}\right)^3\left(\frac{\mu}{10^{15}\ \rm GeV}\right). \tag{99}$$

Apart from the above, mass density constraint, another constraint is the well-known ''Parker bound'' [39], which considers the effect of galactic magnetic fields onto the magnetic monopole motion. The flux of the monopoles is [26]

$$\Phi_M=\frac{1}{4\pi}n_Mv_M\sim 10^{10}\left(\frac{n_M}{s}\right)\left(\frac{v_M}{10^{-3}}\right)\ \rm cm^{-2}\,sr^{-1}\,sec^{-1}, \tag{100}$$

where v_M is the monopole velocity. The monopoles are accelerated by the galactic magnetic field $B_g\sim 10^{-6}$ G to velocity:

$$v_M\simeq\left(\frac{2h_MB_gl}{M}\right)^{1/2}\sim 10^{-3}\left(\frac{10^{16}\ \rm GeV}{M}\right)^{1/2}, \tag{101}$$

where $l\sim 1$ kpc is the coherence length of the magnetic field and $h_M\sim e^{-1}$ is the magnetic charge of the monopole.

The magnetic field ejects the monopoles from the galaxy, while providing them with kinetic energy $E_K\simeq hB_gl\sim 10^{11}$ GeV. Demanding that the monopoles do not drain the field energy in shorter times than the dynamo time scale, i.e., the galactic rotation period $\tau\sim 10^8$ yr, we find the constraint

$$\frac{B_g^2/2}{\Phi_ME_Kd}\geqslant\tau\Rightarrow\Phi_M\leqslant 10^{-15}\ \rm cm^{-2}\,sr^{-1}\,sec^{-1}, \tag{102}$$

where $d\simeq 30$ kpc is the size of the galactic magnetic field region. Using Eq. (98) and from Eqs. (100) and (102), we find the second monopole constraint,

$$\zeta^3\geqslant 10^{12}\left(\frac{T_{\rm reh}}{10^{14}\ \rm GeV}\right)^3\left(\frac{10^{15}\ \rm GeV}{\mu}\right)^{1/2}. \tag{103}$$

[10] We can ignore monopole annihilations (see [26]).

G. Additional constraints and considerations

Finally, we have to make sure that at the time the magnetic field is formed the correlation length given by Eq. (47) is still inside the horizon, that is

$$H_F^{-1} \geqslant \xi_F, \tag{104}$$

where H_F is the Hubble parameter at the time of the formation of the magnetic field. With great accuracy $H_F \simeq H_0$, where H_0 is the Hubble parameter at the time of the phase transition. H_0 can be easily computed using Eqs. (16), (20), and (27) for the potential (23).

VIII. EVALUATING

In order to be consistent with our assumptions (e.g., inflaton domination), we will consider the phase transition to take place at the latest at $N \simeq 1$. We first choose a set of typical model parameters.

A. Choosing the values of the parameters

As already mentioned, the mass of the inflaton field m is determined by COBE:

$$m \sim 10^{13} \text{ GeV}. \tag{105}$$

For the self-coupling of the Higgs field λ we choose the usual value

$$\lambda \sim 1. \tag{106}$$

Inserting the above values into the inflaton domination condition (25), we find that the maximum value of μ is

$$\mu \sim 10^{15} \text{ GeV}. \tag{107}$$

Finally, the coupling λ' between the Higgs field and the inflaton can be determined with the use of Eq. (90):

$$\lambda' \sim 10^{-8}. \tag{108}$$

B. For all N

Now that the parameters of the model are chosen, the only parameter still to be determined is the number N of e-foldings of inflation, which remain after the phase transition. We will treat this as a free parameter, link it with the resulting magnetic field, and then try, with the use of the constraints, to establish its extreme values. In that way we will be able to fully examine the corresponding behavior of the field at t_{eq}.

We begin by extracting some direct, N-independent results from the, previously chosen, values of the model parameters. From Eq. (86) we find that the upper bound for the reheating temperature is estimated to be

$$T_{\text{reh}} \sim 10^{14} \text{ GeV}, \tag{109}$$

which is in agreement with the estimates of Kofman *et al.* [31] and higher than the estimates of Shtanov *et al.* [32].

Using Eq. (77), this gives the time when inflation ends:

$$t_{\text{end}} \sim 10^{-35} \text{ sec}. \tag{110}$$

With the use of Eqs. (16) and (37), the assumption (35) is easily verified:

$$\Lambda \sim 10^{-2} \ll 1. \tag{111}$$

Now, for the correlation length, from Eq. (41) we find

$$\xi_0 \sim 10^{-14} \text{ GeV}^{-1}. \tag{112}$$

From Eqs. (16) and (43), we obtain

$$\tau_F \sim 10^{-16} \text{ GeV}^{-1} \ll \xi_0. \tag{113}$$

Thus, from Eq. (45),

$$\xi_F^{(1)} \sim 10^{-14} \text{ GeV}^{-1}. \tag{114}$$

Using Eq. (46), we also find,

$$\xi_F^{(2)} \sim 10^{-14} \text{ GeV}^{-1}. \tag{115}$$

Therefore, from Eq. (47) and the above, we have

$$\xi_F \sim \xi_0 \sim 10^{-14} \text{ GeV}^{-1} \sim 10^{-28} \text{ cm} \sim 10^{-47} \text{ pc}. \tag{116}$$

Now, from Eq. (44), we find that

$$\Delta N_F \sim 10^{-3} \ll 1 \tag{117}$$

and, therefore, the phase transition is very rapid.

We can, now, check on the horizon constraint (104). The value of H_F is found to be

$$H_F \simeq H_0 \sim 10^{13} \text{ GeV}. \tag{118}$$

Comparing with Eq. (116), we see that the constraint is satisfied. By using Eq. (116) into Eq. (94), we find that the first energy density constraint is also satisfied.

The initial magnetic field is found from Eqs. (48) and (116) to be

$$B_0 \sim 10^{47} \text{ G}. \tag{119}$$

Let us now evaluate the N-dependent quantities.

The correlation length at the end of inflation is

$$\xi_{\text{end}} = \frac{a_{\text{end}}}{a_0} \xi_F \sim 10^{-14} e^N \text{ GeV}^{-1}, \tag{120}$$

where we have used Eqs. (22), (26), and (116).

From the above and considering also the fact that, during inflation, the magnetic field configuration is comovingly frozen, we find that the magnitude of the magnetic field inside a correlated domain is given by

$$B_{\text{CD}}^{\text{end}} \sim 10^{47} e^{-2N} \text{ G}. \tag{121}$$

Evaluating Eq. (96) at the end of inflation, we find

$$R \sim 0.1\, e^{2N} \geqslant 1 \quad \text{for } N \geqslant 1 \tag{122}$$

and the non-Abelian constraint is satisfied for all N.

Using Eqs. (110) and (121) we can show from Eq. (95) that the second energy density constraint is also satisfied for all N.

From Eq. (64), we find

$$K \sim 10^8 e^{-N} \ \mathrm{GeV}^{1/2}. \tag{123}$$

We evaluated the above also at the end of inflation, using Eqs. (14), (110), and (120).

At early times the correlated domains are opaque to radiation and, thus, their growth is determined by Eq. (66) with $t_i \to t_{\mathrm{end}}$. The domains remain opaque at least until the time of the electron pair annihilation.

At annihilation, $t_{\mathrm{anh}} \sim 0.1$ sec, Eq. (66) gives

$$\xi_{\mathrm{anh}} \sim \begin{cases} 10^3 e^N \ \mathrm{GeV}^{-1}, & N > 15, \\ 10^{13} e^{-N/2} \ \mathrm{GeV}^{-1}, & 1 \leqslant N \leqslant 15. \end{cases} \tag{124}$$

Comparing with the photon mean free path, it is evident that $\xi_{\mathrm{anh}} > l_T \sim 10^5 \ \mathrm{GeV}^{-1}$ for all N.

However, after annihilation, l_T increases drastically in size, $l_T(t_{\mathrm{anh}}) \sim 10^{15} \ \mathrm{GeV}^{-1}$. Comparing this value with Eq. (124), we find that, at $T \sim 1$ MeV, the correlated domains become transparent to radiation for $N < 28$. Thus, for $N \geqslant 28$, the Alfvén expansion continues after pair annihilation, whereas for $N < 28$, the Thomson scattering effect has to be taken into account.

***CASE 1:** For* **1≤*N*<28.**

If $N < 28$ then, after t_{anh}, the Thomson effect has to be taken into account.

The Alfvén velocity v_A at t_{anh} is found by Eq. (65) with the use of Eq. (123):

$$v_A(t_{\mathrm{anh}}) \sim \begin{cases} 0.1 e^{-2N}, & N > 15, \\ 10^{-11} e^{-N/2}, & 1 \leqslant N \leqslant 15. \end{cases} \tag{125}$$

Similarly, the Thomson velocity v_T at t_{anh} is found by Eq. (67) with the use of Eqs. (64) and (68):

$$v_T(t_{\mathrm{anh}}) \sim \begin{cases} 10^{10} e^{-5N}, & N > 15, \\ 10^{-20} e^{-N/2}, & 1 \leqslant N \leqslant 15. \end{cases} \tag{126}$$

From the above it is straightforward that, for $N < 28$, $v_T(t_{\mathrm{anh}}) \leqslant v_A(t_{\mathrm{anh}})$. Therefore, after t_{anh}, the evolution of the correlated domains is determined by the Thomson effect. If we assume that the Alfvén expansion does not take over again until t_{eq}, then the correlation length at that time can be obtained by Eq. (69),

$$\xi_{\mathrm{eq}} \sim \begin{cases} 10^9 e^N \ \mathrm{GeV}^{-1}, & 19 \leqslant N < 28, \\ 10^{21} e^{-N/2} \ \mathrm{GeV}^{-1}, & 1 \leqslant N < 19. \end{cases} \tag{127}$$

Using this value we can verify that the Thomson velocity remains always smaller than the Alfvén velocity until t_{eq}. Physically, Eq. (127) implies that, if $N \geqslant 19$, the damping of the growth of the correlated domains after t_{anh} is so effective that the Hubble term dominates their evolution. However, for $1 \leqslant N < 19$, the Thomson velocity, although small, is still capable of outshining the Hubble term.

***CASE 2:** For* **$N \geqslant$28.**

For high values of N the magnetic field is so much diluted by inflation that the Alfvén or Thomson expansions are insignificant. The growth of the correlated domains is driven solely by the Hubble expansion and, thus,

$$\xi_{\mathrm{eq}} \simeq \sqrt{\frac{t_{\mathrm{eq}}}{t_{\mathrm{anh}}}} \xi_{\mathrm{anh}} \sim 10^9 e^N \ \mathrm{GeV}^{-1}, \quad N \geqslant 28. \tag{128}$$

In total, Eqs. (127) and (128) suggest the following behavior for the correlation length at t_{eq}:

$$\xi_{\mathrm{eq}} \sim \begin{cases} 10^9 e^N \ \mathrm{GeV}^{-1}, & N \geqslant 19, \\ 10^{21} e^{-N/2} \ \mathrm{GeV}^{-1}, & 1 \leqslant N < 19. \end{cases} \tag{129}$$

From Eq. (76), we find that at t_{eq} the diffusion length is, $l_d^{\mathrm{eq}} \sim 10^{23} \ \mathrm{GeV}^{-1}$. Comparing with the above we see that $l_d^{\mathrm{eq}} > \xi_{\mathrm{eq}}$ for $N \leqslant 32$. Thus, the dimensions of the correlated domains at t_{eq} are actually given by

$$\xi_{\mathrm{eq}} \sim \begin{cases} 10^9 e^N \ \mathrm{GeV}^{-1}, & N > 32, \\ l_d^{\mathrm{eq}} \sim 10^{23} \ \mathrm{GeV}^{-1}, & 1 \leqslant N \leqslant 32. \end{cases} \tag{130}$$

C. The magnetic field's range of values

We are now in the position to calculate the magnetic field strength at t_{eq}. From Eqs. (64) and (130), we have

$$B_{\mathrm{CD}}^{\mathrm{eq}} \sim \begin{cases} 10 \, e^{-2N} \ \mathrm{G}, & N > 32, \\ 10^{-13} e^{-N} \ \mathrm{G}, & 1 \leqslant N \leqslant 32. \end{cases} \tag{131}$$

Also, from Eqs. (52) and (130), we get

$$n \sim \begin{cases} 10^{24} e^{-N}, & N > 32, \\ 10^{10}, & 1 \leqslant N \leqslant 32. \end{cases} \tag{132}$$

With the use of the above, in view also of Eq. (53), we can immediately find the rms value of the field at t_{eq} for a given N:

$$B_{\mathrm{rms}}^{\mathrm{eq}} \sim \begin{cases} 10^{-11} e^{-3N/2} \ \mathrm{G}, & N > 32, \\ 10^{-18} e^{-N} \ \mathrm{G}, & 1 \leqslant N \leqslant 32. \end{cases} \tag{133}$$

As can be seen from Eq. (133), the maximum rms value of the field at t_{eq} corresponds to $N = 1$:

$$(B_{\mathrm{rms}}^{\mathrm{eq}})_{\mathrm{max}} \sim 10^{-18} \ \mathrm{G}. \tag{134}$$

For the minimum value of the field we just employ the galactic dynamo constraint (91). This gives

$$N_{\mathrm{max}} \simeq 5. \tag{135}$$

Thus, the range of values of the magnetic field is

$$5 \geqslant N \geqslant 1,$$

$$10^{-20} \ \mathrm{G} \leqslant B^{\mathrm{eq}} \leqslant 10^{-18} \ \mathrm{G}. \tag{136}$$

The above results, however, are still subject to the nucleosynthesis and monopole constraints.

D. Nucleosynthesis and monopole constraints on N

Since nucleosynthesis occurs very near the electron pair annihilation we will assume, for simplicity, that the correlation length $\xi_{\rm nuc}$ at $t_{\rm nuc}\sim 1$ sec is approximately equal to the one at annihilation,[11] i.e., $\xi_{\rm nuc}\sim\xi_{\rm anh}$.

The diffusion length at $t_{\rm anh}$ is found by Eq. (76) to be, $l_d^{\rm anh}\sim 10^{13}$ GeV^{-1}. Thus, from Eq. (124) we have

$$\xi_{\rm nuc}\sim\xi_{\rm anh}\sim\begin{cases}10^3e^N\ \ {\rm GeV}^{-1}, & N>23,\\ l_d^{\rm anh}\sim 10^{13}\ \ {\rm GeV}^{-1}, & 1\leqslant N\leqslant 23.\end{cases}\tag{137}$$

Inserting the above into Eq. (64) and with the use of Eq. (123), we find

$$B_{\rm CD}^{\rm nuc}\sim\begin{cases}10^{13}e^{-2N}\ \ {\rm G}, & N>23,\\ 10^3\ \ {\rm G}, & 1\leqslant N\leqslant 23.\end{cases}\tag{138}$$

The maximum value of the magnetic field is $B_{\rm CD}^{\rm nuc}(N=1)\sim 10^2$ G. Comparing with Eq. (92), we see that the maximum value of the field is well below the nucleosynthesis constraint and, therefore, the constraint is not violated for any value of N.

Let us consider the monopole constraints. Given the values of the model parameters and the assumed reheating temperature, both of the monopole constraints (99) and (103) reduce to $\zeta\geqslant 10^4$. Using Eqs. (79) and (120), we find that

$$\zeta=\xi_{\rm end}H_{\rm end}\sim 0.1\ e^N\tag{139}$$

and the constraints are satisfied only if $N\geqslant 11$. Thus, a magnetic field strong enough to seed the galactic dynamo, would violate the monopole constraints.

One way to overcome the monopole problem is to consider GUT models which do not admit monopole solutions, such as ''flipped'' SU(5), i.e., the semisimple group $SU(5)\times U(1)$.

IX. CONCLUSIONS

We have analyzed the creation and evolution of a primordial magnetic field in false vacuum inflation. We have shown that, in GUT theories that do not produce monopoles, a sufficiently strong primordial magnetic field can be generated, provided that the phase transition takes place no earlier than five e-foldings before the end of inflation. Although the magnetic field produced is strong enough to seed the dynamo process in galaxies, it does not violate any of the numerous constraints imposed (apart from the monopole constraint, if applicable).

Our results are sensitive to the reheating efficiency. Indeed, if reheating is efficient, then the time of the end of inflation is earlier and the resulting field diluted by the expansion of the Universe. More importantly, though, if the reheating temperature is of the order of the critical temperature or the Ginzburg temperature, then the magnetic field will be erased. Fortunately, this does not appear to be the case.

Finally, the strength of the magnetic field produced by our mechanism relies on the value of N, i.e., on the moment that the phase transition occurs. In turn, this depends on the exact values of the model parameters. Observational data on the primordial magnetic field could determine, or constraint, these parameters. Experiments to detect such a field have occasionally been suggested (see, for example, [40 or 41]). Not merely would the observation of a primordial field yield information on false vacuum inflation, but it would also improve our understanding of the galactic dynamo and of nonlinear astrophysical processes in general.

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APPENDIX: QUANTUM FLUCTUATIONS

In this appendix we treat in some detail the quantum fluctuations of the Higgs and the inflaton fields at the time of the phase transition. We show that, although the amplitude of the generated density perturbations is larger than the usual scale-invariant perturbations of the inflaton, their scale is so small that they do not contradict with observations since their angular size is unobservable on the microwave sky and, also, because they are heavily damped after reentering the horizon.

By studying the quantum fluctuations of the above fields we also justify the Higgs-field configuration picture, described in Sec. IV, as well as the estimate of the Higgs-field gradients given in Eq. (13).

1. Scale and amplitude of density perturbations

One can calculate the isocurvature fluctuations of the Higgs field as follows. The fluctuations peak when $m_H\sim H$ [18,19]. According to Sec. IV B this corresponds to $\phi=\phi_F$, where ϕ_F is the the inflaton at the time the Higgs field reaches the minima manifold. From Eq. (42) with $\phi_0\simeq m_{\rm Pl}$, we obtain,[12]

$$(\phi_0^2-\phi_F^2)\simeq\frac{4\pi}{3}\frac{m^2}{\lambda'}.\tag{A1}$$

At that moment, the above and Eq. (16) suggest that the Higgs field is

$$\psi_F^2=\mu_{\rm eff}^2(\phi_F)=\mu^2-\frac{\lambda'}{\lambda}\phi_F^2\simeq\frac{4\pi}{3}\frac{m^2}{\lambda}.\tag{A2}$$

[11]The magnitude of $\xi_{\rm nuc}$ does not affect the results when the Hubble term in the evolution equations is subdominant. In the opposite case, our assumption perturbs the results by an order of magnitude in the values of $\xi_{\rm eq}$ and n but less than an order of magnitude in the value of the rms magnetic field, since the later depends on $1/\sqrt{n}$.

[12]E.g., for $N=5$, Eq. (26) gives $\phi_0\simeq 0.89\ m_{\rm Pl}$.

Thus, the perturbation of the energy density of the Higgs field corresponding to the comoving scale k that exits the horizon when $\phi=\phi_F$, is

$$\frac{\delta\rho_\psi}{\rho_\psi}(k)\sim\frac{\psi_F\,\delta\psi_F}{\mu_{\rm eff}^2}\sim\sqrt{\lambda}\sim 1, \tag{A3}$$

where $\rho_\psi\sim\lambda\mu_{\rm eff}(\phi_F)^4$ and $\delta\rho_\psi\simeq(\partial V/\partial\psi)\delta\psi_F\sim\lambda\mu_{\rm eff}(\phi_F)^2\psi_F\delta\psi_F$. We have also used that, during inflation, $H\simeq\sqrt{(4\pi/3)}(\phi/m_{\rm Pl})m$, as suggested by Eqs. (19) and (23), and that [18]

$$\delta\psi_F\simeq H/\sqrt{2}\;\pi. \tag{A4}$$

The inflaton scale-invariant adiabatic perturbations are similarly found to be

$$\frac{\delta\rho}{\rho}\sim\frac{\left(\dfrac{dV}{d\phi}\delta\phi\right)}{V}\sim\frac{\delta\phi}{\phi}\sim\frac{m}{m_{\rm Pl}}\sim 10^{-6}, \tag{A5}$$

where $\rho=\rho_\phi\sim m^2m_{\rm Pl}^2$.

Thus, the Higgs-field density perturbations are much larger than the inflaton's scale-invariant adiabatic perturbations. However, since the energy density of the Universe during inflation is dominated by the inflaton field, the isocurvature perturbations cannot dominate the inflaton ones:

$$\frac{\delta\rho_\psi}{\rho}(k)=\frac{\delta\rho_\psi}{\rho_\psi}\frac{\rho_\psi}{\rho}\sim 10^{-12}. \tag{A6}$$

Note that both fields decay as matter ($\rho\propto a^{-3}$) during reheating, in contrast with the quartic chaotic inflation case when the inflaton field decays faster, as radiation ($\rho\propto a^{-4}$). In that way the ratio of their perturbations is conserved.

However, the growth of the Higgs-field perturbations results in a similar growth of the adiabatic perturbations of the inflaton [20,21]. Since these adiabatic perturbations of the inflaton are caused by the isocurvature perturbations, we should have, for the scale k,

$$\frac{\delta\rho_\phi}{\rho_\phi}(k)\sim\frac{\delta\rho_\psi}{\rho_\psi}(k)\sim 1. \tag{A7}$$

The above perturbations are not scale invariant. Instead, they form a mountain on the spectrum of the scale-invariant perturbations [20,21]. These perturbations are of much larger amplitude than the usual, scale-invariant, inflaton perturbations, given by Eq. (A5), and would produce unacceptably high density inhomogeneities and CMB temperature anisotropies, *if they corresponded to observable scales*.

However, the scale corresponding to the peak of the mountain is set by the horizon scale at the time of the transition. Indeed, in [20,21] it is clearly shown that the peak of the mountain of the adiabatic fluctuations corresponds to the scale

$$k=(Ha)\exp\left[-\frac{2\pi\lambda\mu^2}{\lambda' m_{\rm pl}^2}\right]=(Ha)\exp\left[-\frac{2\pi\phi_0^2}{m_{\rm Pl}^2}\right]$$
$$\Rightarrow a_{\rm end}\,k_{\rm peak}^{-1}=e^NH_0^{-1}, \tag{A8}$$

where k is the comoving wave number of the fluctuation scale, a is the scale factor normalized at the present day, and H is the Hubble parameter, for which, during inflation, $H\simeq{\rm const}\simeq m$. Thus, the wavelength of the peak of the spectrum mountain corresponds to the scale of the horizon at the phase transition. The width of the mountain can be estimated by (see also [18])

$$\Delta\ln k=\frac{\Delta a}{a}=\Delta N_F, \tag{A9}$$

where ΔN_F is the e-folding interval when $m_H^2<0$ given by Eq. (44). In the model considered we have shown that $\Delta N_F\sim 10^{-3}$ and, therefore, the mountain is very sharp. As a consequence, if we move towards larger than the horizon (at the transition) scales, the amplitude of the perturbations in expected to be falling rapidly. Thus, *we do not expect significant perturbations on comoving scales a lot larger than the scale that leaves the horizon at the time of the phase transition*.

A more thorough investigation of the issue by Nagasawa and Yokoyama [22] does not change the picture significantly. Following their treatment (applied to our model), we found that the exponential growth of the perturbations lasts a little longer and ΔN_F is given by,

$$\Delta N_F=\frac{4\pi}{\sqrt{2\lambda'}}\frac{m\phi_0}{m_{\rm Pl}^2}\sim 0.1, \tag{A10}$$

which is again fairly small. Numerical simulations attempted by Salopek *et al.* [21] relax the above number even more and suggest, in their example, that the mass squared of the field can remain negative for almost four e-foldings. Their mountain peaks very near the scale of the horizon at the transition. In their paper they stress the crucial role of the timing of the transition not only with respect to the scales where the mountain occurs but also to its other characteristics (width and height).

Our phase transition has to occur at the very late stages of the inflationary period, no earlier than five e-foldings before its end. The corresponding scales are so small that in order for the mountain to have any effect on observable scales it would have to be extremely wide ($\Delta N_F\geqslant 30$). The narrow window of the observable scales is well described in [21] (see Fig. 4), where it is pointed out that these scales correspond to inflaton values for which $N\simeq 60$, for standard chaotic inflation (as in our model).

The comoving scale of the mountain peak for our model is given by Eq. (A8):

$$k^{-1}\sim 10^{15}e^N\;\;{\rm GeV}^{-1}, \tag{A11}$$

with $H_0\sim 10^{13}$ GeV. Thus, the largest peak-scale peak would be $k^{-1}(N=5)\sim 1m$.

The fluctuations freeze after they exit the horizon. When they reenter the horizon they begin oscillating (since the Jeans length is about the horizon size before $t_{\rm eq}$). The reentry of the above fluctuations occurs at t_x where

$$k=\left(\frac{t_{pr}}{t_{\rm eq}}\right)^{2/3}\left(\frac{t_{\rm eq}}{t_x}\right)^{1/2}\left[H_{pr}^{-1}\left(\frac{t_x}{t_{pr}}\right)\right]\Rightarrow t_x$$
$$=(t_{pr}^2 t_{\rm eq})^{1/3}\left(\frac{k}{H_{pr}^{-1}}\right)^2\Rightarrow t_x\sim 10^{-35}e^{2N}\ \ \text{sec},\qquad \text{(A12)}$$

where $t_{pr}\sim 10^{18}$ sec is the present time, $t_{\rm eq}\sim 10^{11}$ sec is the time of equal matter and radiation densities, and $H_{pr}^{-1}\sim 10^{27}$ cm$\sim 10^{41}$ GeV^{-1} is the present size of the horizon. For $N=5$, we find that these fluctuations can reenter the horizon at the latest at $t_x\sim 10^{-31}$ sec$\ll t_{\rm eq}$.

Density perturbations of a comoving scale which reenters the horizon that early, will, most probably, be erased by the various damping processes (e.g., Silk damping, free-streaming damping, etc.). But even if they survived, their imprint on the microwave background would be too small to be observable. Indeed, the relevant angular scale is given by [26]

$$\theta\simeq 34.4\ (\Omega_0 h)\left(\frac{k}{\rm Mpc}\right)\ \ \text{arcsec},\qquad \text{(A13)}$$

which, in our model, corresponds to a maximum of $\theta\sim 10^{-21}$ arcsec, i.e., entirely unobservable.

2. The Higgs-field gradients

As shown in Sec. IV, the freezing of the Higgs-field configuration occurs when $\phi=\phi_F$ and the field has rolled down to the vacuum manifold. The Higgs-field *radial* fluctuation at that time can be found by Eqs. (A2) and (A4):

$$\left.\frac{\delta\psi}{\psi}\right|_F\sim\frac{\sqrt{\lambda}}{\sqrt{2}\ \pi}\sim 0.1.\qquad \text{(A14)}$$

Thus, the magnitude of the Higgs-field radial fluctuations at freeze-out is smaller than its expectation value, $\Delta|\psi|<|\psi|\sim\mu_{\rm eff}$. This inequality is strengthened with time since $\psi\rightarrow\mu\gg m\sim H\sim\delta\psi$. Consequently, after freeze-out, the field evolves classically and *its VEV is much larger than its radial fluctuations*. Also, in this case, the typical *phase* fluctuation is given by [19], $\delta\chi\simeq H/2\pi\mu_{\rm eff}(\phi_F)\sim\sqrt{\lambda}/2\pi\sim 0.1<1$ at freeze-out. The inequality is again strengthened with time and, therefore, we can safely consider that, after the topology of the field configuration freezes, *its quantum fluctuations are too small to perturb the direction of the field* (in its inner space) at any given point in space. What really happens is that the time of freeze-out is the *last moment* that the quantum fluctuations could be important. The strengthening of the inequality $\Delta\psi<\psi$ in the radial and the azimuthal component of the field signifies just that. As a result, at freeze-out, the importance of the quantum fluctuations is terminated and the correlation length in later times is determined only by causality as shown in Sec. IV B.

The reason for having, then, nonzero Higgs-field gradients is only the fact that, at two points in space, we would expect the field inner space orientation (phase) to be different, if the distance between the points is larger than the correlation length of the configuration. Thus, *the existence of nonzero gradients is an entirely geometrical effect, based on the fact that over some distance scale the field's phases are uncorrelated* and has nothing to do with any fluctuations, which, in any case, after freeze-out, do not have any effect on the field configuration.

The stochastic distribution of the phase of the field suggests that, over distances larger than the correlation length, the phase of the field may vary between 0 and π and, consequently, $\Delta\psi$ would be between 0 and 2μ. Thus, a reasonable estimate is $\Delta\psi\sim\mu$ over length scales of the order of the correlation length. Therefore, the gradients of the field would be,

$$\partial_x\langle\psi\rangle\sim\frac{\Delta\psi}{\Delta x}\sim\frac{\mu}{\xi}\qquad \text{(A15)}$$

as suggested also in Eq. (13).

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