Effective vector-boson approximation applied to hadron-hadron collisions

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An improved effective vector-boson approximation is applied to hadron-hadron collisions. The effective vector-boson approximation in this form is accurate enough to reproduce the result of a complete perturbative calculation for the specific example of ZZ production within 10%. This is true even far away from a possible Higgs boson resonance and thus in a region where the transverse intermediate vector bosons give the dominant contribution. Simple approximate formulas which greatly reduce the calculational effort are derived. The full information about the kinematics is, however, lost in these approximations. A comparison between the improved, the approximate, and existing formulations is presented. [S0556-2821(97)03109-3]

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I. INTRODUCTION

The effective photon approximation (EPA) Weizsäcker-Williams approximation) of QED [1] has proved to be a useful tool in the study of photon-photon processes at e^+e^- colliders. With the prospect of high-energy hadron-hadron colliders, the possibility to study the scattering of *massive* vector bosons is given. Massive vector-boson scattering is of particular interest as the symmetry-breaking sector of the electroweak theory and the self-interactions of the vector bosons are directly tested. The method equivalent to the EPA applying to the scattering of massive vector bosons is the effective vector-boson approximation (EVBA) [2–5]. The EVBA can be applied to fermion-fermion scattering processes in which the final state consists of two fermions and a state Ξ which can be produced by the scattering of two vector bosons. The fermion-fermion cross section is written as a product of a probability distribution and the cross sections for vector boson scattering. The probability distribution describes the emission of vector bosons from fermions. The method is an approximate one which neglects Feynman diagrams of bremsstrahlung type. In general, the method is applicable if the fermion scattering energy is large against the masses of the electroweak vector bosons.

The possibility of an EVBA has been first noticed in connection with heavy Higgs boson production [6]. The Higgs boson can be produced via the diagram in Fig. 1(a), where a sum over all vector-boson pairs V_1, V_2, which can couple to the Higgs boson, is to be taken.

In this early application of the EVBA only the contribution from longitudinally polarized intermediate vector bosons $V_{1,L}V_{2,L}$ was calculated and the result was found to give a reasonable approximation to an exact perturbative calculation [7].

Subsequently, the EVBA was also applied to processes of the type $pp \to V_3V_4+X$, where two vector bosons are produced. The vector bosons V_3 and V_4 emerge as the decay products of a near-resonant heavy Higgs particle [2]. The scattering process was described by the diagram in Fig. 1(b). Also in this case, the inclusion of only longitudinal intermediate vector bosons was sufficient. It was noted that the production of heavy particles (Higgs bosons or fermions) is mainly due to the longitudinal intermediate states [4].

The concept of vector bosons as partons in quarks was further established and expressions for vector-boson distributions in quarks were derived [3,4]. The expressions were given for all polarizations of the intermediate vector bosons. By convolution with the quark distributions in a proton, numerical results were given for vector boson distributions in a proton [3]. For the production via two intermediate vector bosons it was assumed that convolutions of the distributions of single vector bosons could describe the emission probability of the vector-boson pair. The EVBA in this form gave reliable results for heavy Higgs boson production [3,4,7–9]

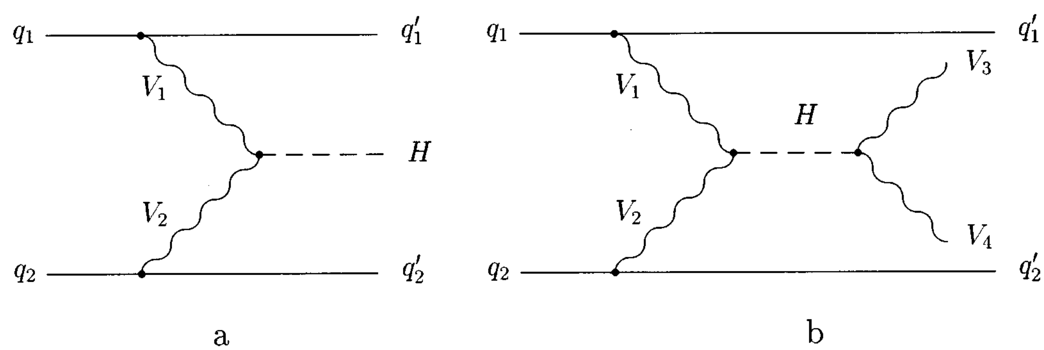

FIG. 1. (a) The diagram for Higgs boson production in quark-quark scattering $q_1q_2 \to q_1'q_2'H$. (b) The diagram for the production of a vector-boson pair V_3V_4 as the decay products of a heavy Higgs boson in q_1q_2 scattering.

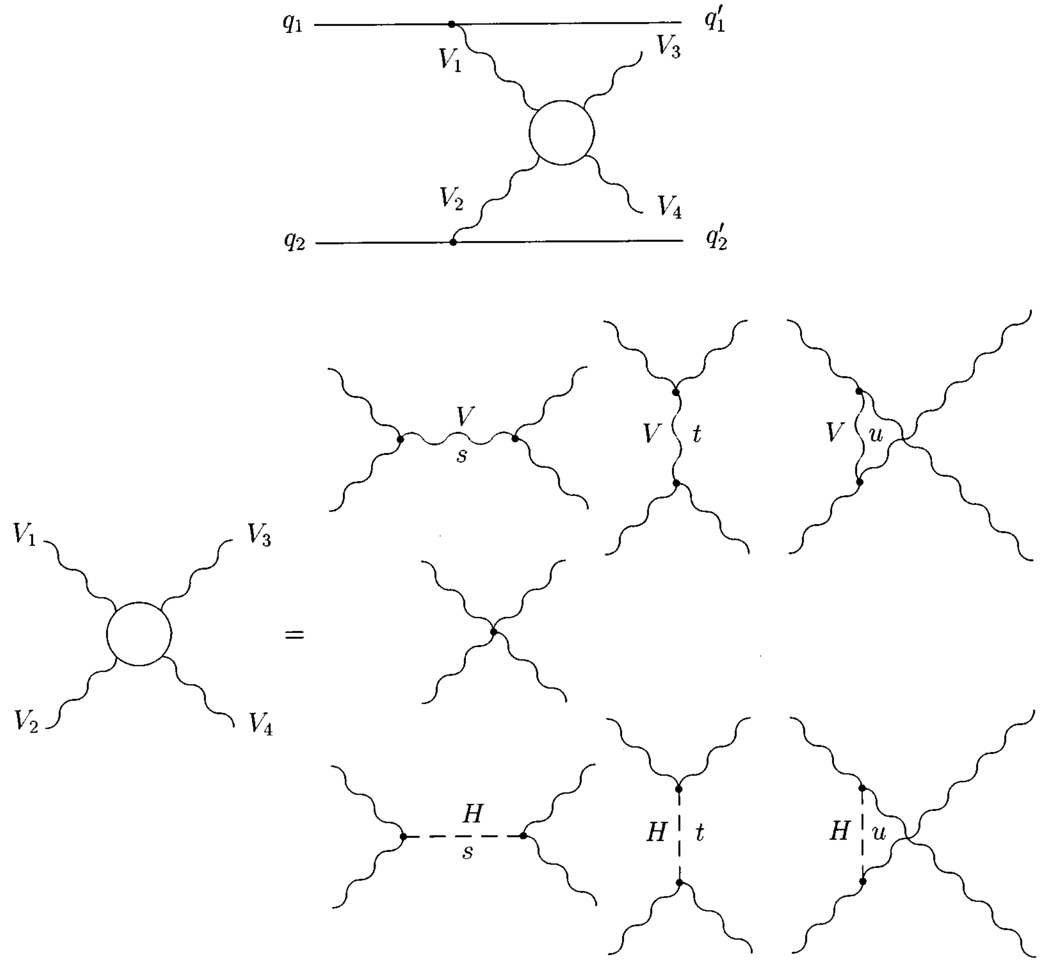

FIG. 2. The diagram for $q_1q_2 \rightarrow q_1'q_2'V_3V_4$ in the effective vector-boson approximation and the diagrams for vector-boson scattering.

and heavy fermion production [10].

The necessity to include all vector-boson scattering diagrams for $V_1V_2 \rightarrow V_3V_4$ in order to obtain EVBA predictions for the production of a vector-boson pair V_3V_4, not necessarily near a Higgs boson resonance, was first mentioned in [9] and [11]. The possible diagrams for these processes, $q_1q_2 \rightarrow q_1'q_2'V_3V_4$, where q_i, q_i' are quarks, are shown in Fig. 2.

It was further pointed out that the yield of V_3V_4 pairs from $q_1q_2 \rightarrow q_1'q_2'V_3V_4$ must be discussed together with the yield from the direct reaction $q_1q_2 \rightarrow V_3V_4$ (Drell-Yan reaction) unless a suitable analysis of the different proton remnants from the two production mechanisms allows one to separate the different production mechanisms.

In first applications to vector-boson scattering, again only the contribution from the longitudinal intermediate states was considered while the contribution from transverse states was neglected. This contribution was taken to be small against the $q_1q_2 \rightarrow V_3V_4$ contribution while the contribution from $V_{1,L}V_{2,L} \rightarrow V_3V_4$ could be large if the longitudinal vector bosons interact strongly. The interest in the strongly interacting scenario [12] was the original motivation to use the EVBA.

The EVBA has been used for vector-boson scattering in [2], [13–17]. In [14], the EVBA was used only for the longitudinal intermediate states. The transverse states were taken into account by a complete perturbative calculation (to lowest order in the coupling) of the process $q_1q_2 \rightarrow q_1'q_2'V_3V_4$. This calculation requires the evaluation of more diagrams than only the vector-boson scattering diagrams, as indicated in Fig. 3. To be precise, in [14] the EVBA was used only to calculate the difference between the cross sections in a strongly interacting model and in the standard model with a light Higgs boson. This difference shows an interesting behavior in a strongly interacting scenario and was therefore considered as a potential signal for strongly interacting vector bosons. The difference receives a contribution virtually only from the longitudinal states. It was found [14] that this calculation agrees with a complete perturbative calculation to about 10% (evidenced for $W^{\pm}Z$ and $W^{\pm}W^{\pm}$ production) if the standard model with a heavy Higgs boson is taken as the strongly interacting model. I note that for strong scattering a method has been recently described which does not make use of the EVBA [18].

In [13,16,17] the application of the EVBA was extended to the contributions from all intermediate polarization states. It was known, however, that the EVBA can overestimate results of complete perturbative calculations by a factor of 3 if the transverse helicities are important [19,20]. Other comparisons of results of complete calculations for $pp \rightarrow V_3V_4 + X$ with EVBA results [21,22] showed that the EVBA is always a good approximation on the Higgs boson

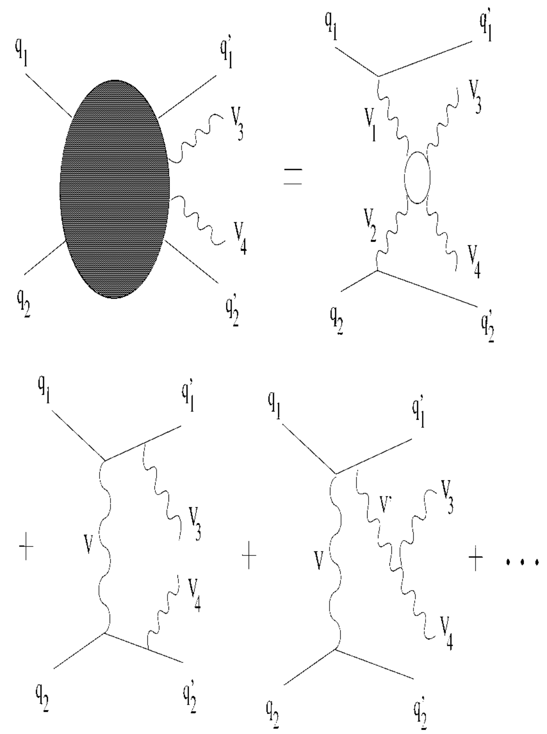

FIG. 3. Some of the Feynman diagrams for a process $q_1q_2 \to q_1'q_2'V_3V_4$ in a complete perturbative calculation. In the top row to the right is the diagram for vector-boson scattering, which is the only type of diagram which is considered in the effective vector-boson approximation. The bottom row shows diagrams of bremsstrahlung type.

resonance but in general overestimates the transverse contribution which is the important one for vector-boson scattering away from a Higgs boson resonance. The EVBA was found to be unreliable away from the Higgs boson resonance. Furthermore, the EVBA result depends strongly on the details of the approximations made in deriving the EVBA [19,20]. In particular, the frequently used leading logarithmic approximation can overestimate the transverse luminosity by an order of magnitude [23,24].

In this paper we will obtain exact luminosities for vector-boson pairs in a hadron pair from an improved formulation of the EVBA, previously introduced for fermion-fermion scattering in [25]. This formulation makes no approximation in the integration over the phase space of the two intermediate vector bosons. The only remaining assumption, necessary in an EVBA, concerns the off-shell behavior of vector-boson cross sections. The formulation, however, involves multiple numerical integrals and is thus not very practical in itself. However, we will apply suitable approximations in which some of the integrals can be carried out in closed form. The exact luminosities form a unique basis to test the quality of these approximations. We will present a test of the approximations here and also compare with other existing approximations. In addition, by means of a specific example, we will compare the EVBA, using the exact luminosities, with a calculation in which no EVBA has been applied.

In Sec. II the improved EVBA is applied to hadron collisions and numerical results for the exact luminosities are

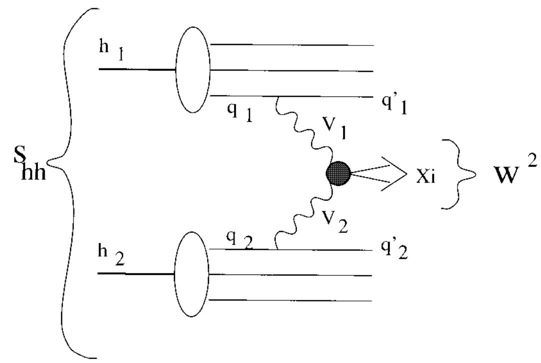

FIG. 4. The diagram for the hadron-hadron scattering process proceeding via two intermediate vector bosons $h_1h_2 \to q_1q_2 \to q_1'q_2'V_1V_2$ and $V_1V_2 \to \Xi$.

given. We derive useful approximations to the improved EVBA. A comparison with previous formulations of the EVBA is given. Section III contains a comparison of EVBA results with a complete perturbative calculation for $pp \to ZZ+X$. Details of various existing formulations of the EVBA are discussed in the Appendix.

II. LUMINOSITIES FOR VECTOR-BOSON PAIRS IN HADRON PAIRS

A. Improved effective vector-boson approximation

Applying the treatment of the improved EVBA [25], we present exact luminosities for finding a vector-boson pair in a hadron pair. The luminosities apply to the process shown in Fig. 4.

The cross section for a scattering process of two hadrons h_1 and h_2 with high energies, in which an arbitrary final state Ξ is produced, is given (in the quark-parton model) by a two-dimensional integral over a product of parton distribution functions and the cross sections for parton-parton scattering processes:

$$\sigma(h_1h_2 \to \Xi + X, s_{hh}) = \sum_{q_1,q_2} \int_0^1 d\xi_1 \int_0^1 d\xi_2\, f_{q_1}^{h_1}(\xi_1,\mu_1^2) f_{q_2}^{h_2}(\xi_2,\mu_2^2) \times \sigma(q_1q_2 \to \Xi + X', s_{qq}). \tag{1}$$

The sum in Eq. (1) extends over all partons (quarks, antiquarks, and gluons) q_1 in the hadron h_1 and q_2 in the hadron h_2. The variable ξ_i in Eq. (1) is the ratio of the momentum of the parton q_i and the hadron h_i. The quantities $f_{q_i}^{h_i}(\xi_i,\mu_i^2)$ are the parton distribution functions, evaluated at the momentum fractions ξ_i and the factorization scales μ_i^2. The scale is a characteristic energy of the process which is initiated by the parton q_i. The quantities $\sigma(q_1q_2 \to \Xi + X')$ in Eq. (1) are the cross sections for the parton-parton processes. In Eq. (1), s_{hh} is the square of the hadron-hadron scattering energy, related to the parton-parton scattering energy $\sqrt{s_{qq}}$ by

$$s_{qq}=\xi_1\xi_2 s_{hh}\,. \tag{2}$$

The symbols X and X' represent additional particles in the final state. In writing down Eq. (1), we assumed that the partons have no transverse momentum. We also neglected the masses of the hadrons and partons. These approximations may be made in any frame in which both hadrons are highly relativistic.

Expressed in terms of the ratio of the squared invariant masses $\tau\equiv s_{qq}/s_{hh}$ and the rapidity y_q of the motion of the center-of-mass of the parton pair in the center-of-mass system (c.m.s.) of the hadrons[1] the cross section (1) takes on the form

$$\begin{aligned}\sigma(h_1h_2\to\Xi+X,s_{hh}) &=\sum_{q_1,q_2}\int_0^1 d\tau\int_{(1/2)\ln(\tau)}^{-(1/2)\ln(\tau)}dy_q\\ &\quad\times f_{q_1}^{h_1}(\sqrt{\tau}e^{y_q},\mu_1^2)f_{q_2}^{h_2}(\sqrt{\tau}e^{-y_q},\mu_2^2)\\ &\quad\times\sigma(q_1q_2\to\Xi+X',s_{qq}=\tau s_{hh}).\end{aligned} \tag{3}$$

The relations between the variables ξ_1,ξ_2 in Eq. (1) and τ,y_q in Eq. (3) are given by $\tau\equiv\xi_1\xi_2$, $y_q\equiv\frac{1}{2}\ln(\xi_1/\xi_2)$ or, equivalently, $\xi_1\equiv\sqrt{\tau}e^{y_q}$ and $\xi_2\equiv\sqrt{\tau}e^{-y_q}$.

If the final state Ξ is produced via the vector-boson fusion mechanism $V_1V_2\to\Xi$ and the partons are quarks or antiquarks (we will simply call them quarks here) an expression for the parton-parton cross section is given in the EVBA by

$$\begin{aligned}\sigma(q_1q_2\to\Xi+X',s_{qq}) &=\sigma(q_1q_2\to q_1'q_2'\Xi,s_{qq})\\ &=\sum_{V_1,V_2}\sum_{\text{pol}}\int_0^1 d\hat{x}\mathcal{L}_{V_1,V_2,\text{pol}}^{q_1q_2}(\hat{x})\\ &\quad\times\sigma_{\text{pol}}(V_1V_2\to\Xi,\mathcal{W}^2=\hat{x}s_{qq}).\end{aligned} \tag{4}$$

In Eq. (4), the quantities $\mathcal{L}_{V_1,V_2,\text{pol}}^{q_1q_2}(\hat{x})$ are luminosities for vector-boson pairs in fermion pairs. The variable $\hat{x}$ is the ratio of the squared invariant mass $\mathcal{W}^2$ of the vector-boson pair and the one of the quark pair:

$$\hat{x}\equiv\frac{\mathcal{W}^2}{s_{qq}}. \tag{5}$$

The sum in Eq. (4) runs over all vector bosons V_1,V_2 which can produce the final state Ξ and over all their helicity states labeled by ''pol.'' Expressions for the luminosities $\mathcal{L}_{V_1,V_2,\text{pol}}^{q_1q_2}(\hat{x})$, using no other approximations than those inherent in the effective vector-boson method, have been given in [25]. The luminosities can be written in the form

$$\begin{aligned}\mathcal{L}_{V_1,V_2,\text{pol}}^{q_1q_2}(\hat{x})&=\left(\frac{\alpha}{2\pi}\right)^2\hat{x}c_{q_1(V_1)}^{\text{pol}}c_{q_2(V_2)}^{\text{pol}}\\ &\quad\times\int_{\hat{x}}^1\frac{d\hat{z}}{\hat{z}}\mathcal{L}_{\text{pol}}\left(\hat{x},\hat{z},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right).\end{aligned} \tag{6}$$

In Eq. (6), $\hat{z}$ is the ratio of the squared invariant mass M_Y^2 of a system consisting of V_1 and the quark q_2 and the squared invariant mass of the quark pair:

$$\hat{z}\equiv\frac{M_Y^2}{s_{qq}}. \tag{7}$$

The parameter α is the fine-structure constant and M_i are the vector-boson masses. The quantities $c_{q_i(V_i)}^{\text{pol}}$ are combinations of the vector and axial-vector couplings v_i and a_i, respectively, of V_i to q_i. They can be either $v_i^2+a_i^2$ or $2v_ia_i$, depending on pol. In Eq. (6), the quantities

$$\begin{aligned}&\mathcal{L}_{\text{pol}}\left(\hat{x},\hat{z},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right)\\ &\quad=\eta_0\int_{-s_{qq}(1-\hat{z})}^0 dk_1^2\int_{-s_{qq}\hat{z}(1-\hat{x}/\hat{z})}^0 dK_2^2\frac{1}{(k_1^2-M_1^2)^2}\\ &\qquad\times\frac{1}{(k_2^2-M_2^2)^2}\int_0^{2\pi}\frac{d\varphi_1}{2\pi}f_{\text{pol}}K_{\text{pol}}\end{aligned} \tag{8}$$

are ''amputated'' differential luminosities, which do not anymore contain the fermionic coupling constants. They depend only on the variables $\hat{x}$ and $\hat{z}$ and, since they are dimensionless, on the masses of the vector bosons via the ratios M_1^2/s_{qq} and M_2^2/s_{qq}. In Eq. (8), η_0 is the ratio of the on-shell flux factor for the process $V_1V_2\to\Xi$ and the flux factor for the same process evaluated for $M_i^2=0$:

$$\begin{aligned}&\eta_0\\ &=\sqrt{1+\left(\frac{M_1^2}{\mathcal{W}^2}\right)^2+\left(\frac{M_2^2}{\mathcal{W}^2}\right)^2-2\frac{M_1^2}{\mathcal{W}^2}-2\frac{M_2^2}{\mathcal{W}^2}-2\frac{M_1^2}{\mathcal{W}^2}\frac{M_2^2}{\mathcal{W}^2}}.\end{aligned} \tag{9}$$

We simply refer to η_0 as the on-shell flux factor. The k_i^2 are the squared four-momenta of the vector bosons and

$$K_2^2\equiv\frac{1}{1-(k_1^2/(\hat{z}s_{qq}))}k_2^2. \tag{10}$$

The quantities φ_1, f_{pol}, and K_{pol} have been defined in [25]. We note that if the momentum of V_1 is lightlike, $k_1^2=0$, the directions of motion of V_1 and q_1 are parallel. In this case, $\hat{z}$ is the ratio of the energy of V_1 and the energy of q_1. The variable $\hat{z}$ has in this case the same interpretation for the emission of a vector boson V_1 from a quark q_1 as ξ_i has for the emission of a quark q_i from a hadron h_i. The corresponding variable for the emission of V_2 from q_2 is $\hat{x}/\hat{z}$. The

[1]The rapidity is taken along the direction of motion of the hadron h_1.

vector bosons can be approximately treated as ''partons'' in the quarks. In analogy to ξ_1 and ξ_2 we introduce the two variables $\hat{z}_1=\hat{z}$ and $\hat{z}_2=\hat{x}/\hat{z}$.

Inserting Eq. (4) into Eq. (3) yields an expression for the cross section for the production of the state Ξ in the hadron-hadron process, proceeding via vector-boson fusion:

$$\begin{aligned}\sigma(h_1h_2\to q_1q_2\to q_1'q_2'\Xi,s_{hh})&\\ \equiv\sigma(h_1h_2\to V_1V_2\to\Xi,s_{hh})&\\ =\sum_{q_1,q_2}\sum_{V_1,V_2}\sum_{\text{pol}}\int_0^1 d\tau\int_{(1/2)\ln(\tau)}^{-(1/2)\ln(\tau)}dy_q\, f_{q_1}^{h_1}(\sqrt{\tau}e^{y_q},\mu_1^2)&\\ \times f_{q_2}^{h_2}(\sqrt{\tau}e^{-y_q},\mu_2^2)&\\ \times\int_0^1 d\hat{x}\mathcal{L}_{V_1,V_2,\text{pol}}^{q_1q_2}(\hat{x})\sigma(V_1V_2\to\Xi,\mathcal{W}^2=\tau\hat{x}s_{hh}).&\quad (11)\end{aligned}$$

The expression (11) allows one to define luminosities $\mathcal{L}_{(V_1V_2)_{\text{pol}}}^{h_1h_2}(x)$ of vector-boson pairs in a hadron-pair:

$$\begin{aligned}\sigma(h_1h_2\to V_1V_2\to\Xi,s_{hh})&\\ =\sum_{(V_1,V_2)_{\text{pol}}}\int_{x_{\min}}^1 dx\mathcal{L}_{(V_1V_2)_{\text{pol}}}^{h_1h_2}(x)&\\ \times\sigma_{\text{pol}}(V_1V_2\to\Xi,\mathcal{W}^2=xs_{hh}).&\quad (12)\end{aligned}$$

In Eq. (12),

$$x\equiv\mathcal{W}^2/s_{hh} \quad (13)$$

is the ratio of the squares of the invariant masses of the vector-boson pair and of the hadron pair. The minimum value for x is given by $x_{\min}=(M_1+M_2)^2/s_{hh}$. The summation in Eq. (12) extends over all (unordered) vector-boson pairs (V_1V_2) which can produce the state Ξ and the luminosities are given by the expression

$$\begin{aligned}\mathcal{L}_{(V_1V_2)_{\text{pol}}}^{h_1h_2}(x)=C_{(12)}\left(\frac{\alpha}{2\pi}\right)^2 x\int_0^{-\ln(x)}\frac{d[\ln(1/\tau)]}{\tau}&\\ \times\{I_{y,\text{pol}}^{h_1h_2}(\tau)+I_{y,\text{pol}}^{h_2h_1}(\tau)\}&\\ \times\int_{(1/2)\ln(\hat{x})}^{-(1/2)\ln(\hat{x})}d\hat{y}\mathcal{L}_{\text{pol}}\left(\hat{x},\sqrt{\hat{x}}e^{\hat{y}},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right),&\\ &\quad (14)\end{aligned}$$

with

$$\begin{aligned}I_{y,\text{pol}}^{h_1h_2}(\tau)=\int_{(1/2)\ln(\tau)}^{-(1/2)\ln(\tau)}dy_q\left(\sum_{q_1(V_1)}c_{q_1(V_1)}^{\text{pol}}f_{q_1}^{h_1}(\sqrt{\tau}e^{y_q},\mu_1^2)\right)&\\ \times\left(\sum_{q_2(V_2)}c_{q_2(V_2)}^{\text{pol}}f_{q_2}^{h_2}(\sqrt{\tau}e^{-y_q},\mu_2^2)\right).&\quad (15)\end{aligned}$$

The luminosities (14) are exact in the sense that no approximation has been made on the kinematics of the two vector bosons. The only approximations which have been made are those inherent in the EVBA, thus the neglect of bremsstrahlung diagrams and the continuation from on-shell to off-shell vector-boson scattering. We call Eq. (14) with Eq. (8) the exact luminosities. In Eq. (14),

$$C_{(12)}\equiv\begin{cases}1 & \text{if}\quad V_1\neq V_2,\\ 1/2 & \text{if}\quad V_1=V_2,\end{cases} \quad (16)$$

is a combinatorial factor. We further introduced the variable

$$\hat{y}\equiv\frac{1}{2}\ln(\hat{z}^2/\hat{x})=\frac{1}{2}\ln\left(\frac{\hat{z}_1}{\hat{z}_2}\right). \quad (17)$$

In the case of lightlike momenta of the vector bosons V_1 and V_2, the variable $\hat{y}$ is the rapidity of the (V_1V_2) center-of-mass motion in the quark-quark c.m.s., taken along the direction of motion of the quark from which V_1 was emitted. The functions $I_{y,\text{pol}}^{h_1h_2}(\tau)$ contain all dependence on the type of the quarks $q_i(V_i)$, i.e., on the parton distribution functions and on the quark couplings to the vector bosons. The remaining part of the τ integral in Eq. (14) depends only on kinematical variables. The summations in Eq. (15) extend over all quarks q_1 and q_2 which can couple to the vector bosons V_1 and V_2, respectively. In the derivation of Eqs. (14), (15) we made use of the symmetry property of the luminosities for vector-boson pairs in fermion pairs:

$$\begin{aligned}\int d\hat{y}\mathcal{L}_{\text{pol}}\left(\hat{x},\sqrt{\hat{x}}e^{\hat{y}},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right)&\\ =\int d\hat{y}\mathcal{L}_{\overline{\text{pol}}}\left(\hat{x},\sqrt{\hat{x}}e^{\hat{y}},\frac{M_2^2}{s_{qq}},\frac{M_1^2}{s_{qq}}\right),&\quad (18)\end{aligned}$$

where $\overline{\text{pol}}$ is obtained from pol by exchanging the helicities of V_1 and V_2 (i.e. $TL\to LT$, $TT\to TT$, etc.).

Figure 5 shows the exact luminosities (14) with (8) for the vector-boson pairs W^+W^-, W^+Z, W^-Z, and ZZ in a proton pair of $\sqrt{s_{hh}}=14$ TeV for the diagonal helicity combinations as a function of x. The definition for the helicity combinations TT, $\overline{TT}$, TL, LT, and LL can be found, e.g., in [25]. The Martin-Roberts-Stirling set A [MRS(A)] parametrization [26] in the deep inelastic scattering (DIS) scheme was used for the parton distributions[2] $f_{q_i}^p(\xi_i,\mu_i^2)$. The electroweak parameters were $\alpha=1/128$, $M_W=80.17$ GeV, and $M_Z=91.19$ GeV.

B. Approximate luminosities

We give approximations to the exact luminosities which are obtained from the full expression (8). We stress, however, that these approximations do not contain the full information about the kinematic variables of V_1 and V_2 which is contained in Eq. (8) anymore. Therefore, these approximations might not be used for studies involving cuts on the kinematics of the outgoing quark jets as, e.g., carried out in

[2]We use $\mu_i^2=\xi_i s_{hh}$ unless explicitly stated otherwise. The sensitivity to the precise choice of the scale is small.

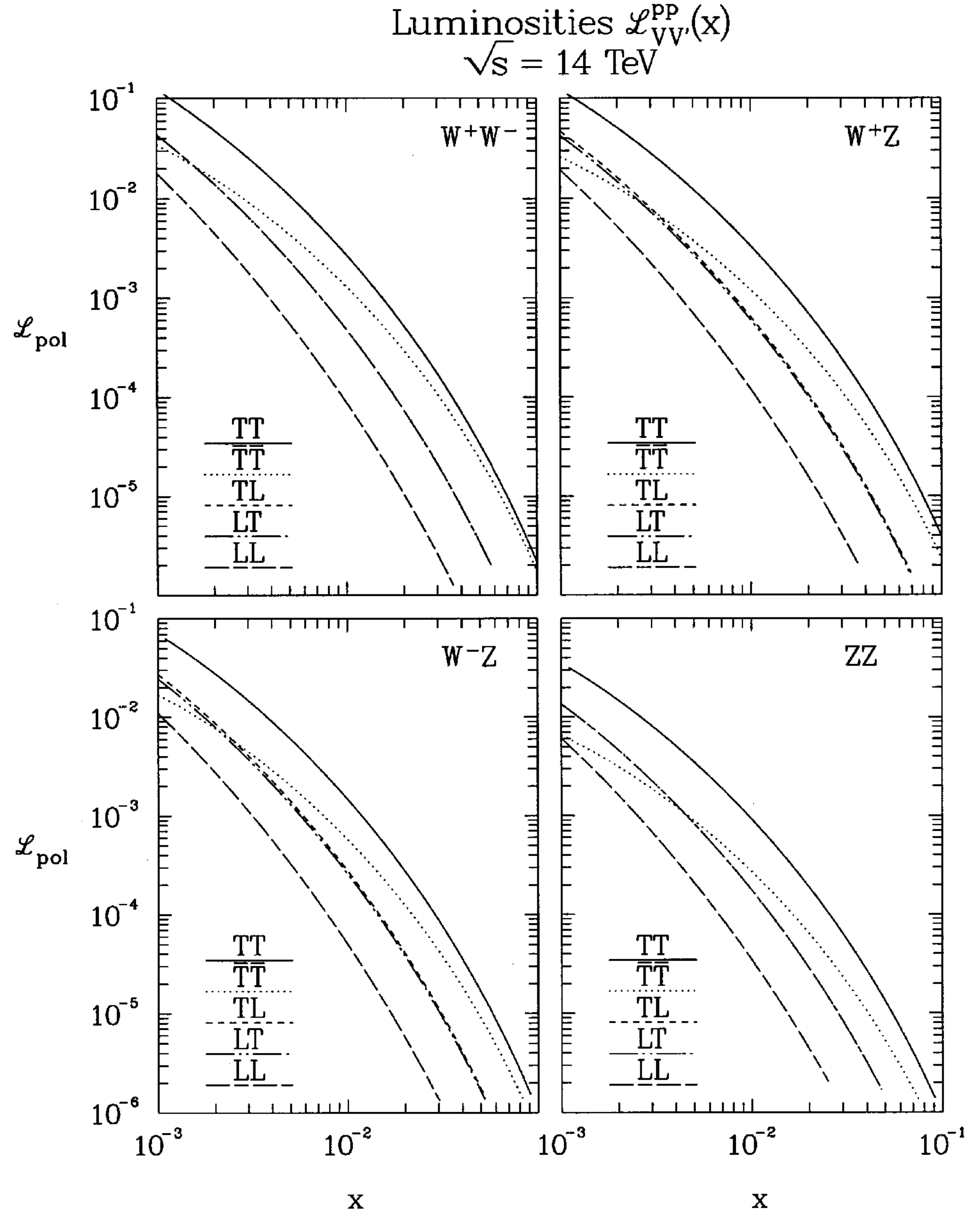

FIG. 5. The exact luminosities $\mathcal{L}^{h_1h_2}_{V_1V_2}(x)$, Eq. (14) using Eq. (8), for finding a vector-boson pair inside a proton pair of $\sqrt{s_{hh}}=14$ TeV for the diagonal helicity combinations of various vector-boson pairs as a function of the variable x.

[14]. The approximations are similar to luminosities which were commonly used in the literature. These luminosities were functions of $\hat{x}$ and $\hat{z}$ only. To obtain the approximations all kinematical variables other than $\hat{x}$ and $\hat{z}$ are integrated out. The integrations can be carried out analytically which results in simple formulas. The aim of this section is to compare the exact expressions (8) with approximations. If one is interested in distributions of other variables than $\hat{x}$ and $\hat{z}$ one has to go back to the exact expressions (8) and consider their differential forms. It is not meaningful to use the approximations in this case because in general no simplifications are obtained.

It has been shown in [25] how the expression (6) reduces to a convolution of vector-boson distributions if certain kinematical approximations are made. The approximate expression for (6) is given by

$$\mathcal{L}^{q_1q_2}_{V_1,V_2,\lambda_1\lambda_2}(\hat{x})=\eta_0\int_{\hat{x}}^{1}\frac{d\hat{z}}{\hat{z}}f^{q_1}_{V_1,\lambda_1}\left(\hat{z},\frac{M_1^2}{s_{qq}}\right)f^{q_2}_{V_2,\lambda_2}\left(\frac{\hat{x}}{\hat{z}},\frac{M_2^2}{\hat{z}s_{qq}}\right), \tag{19}$$

where the functions $f^q_{V_T}$, $f^q_{V_{\overline{T}}}$, and $f^q_{V_L}$ (we use $V=V_i$, $q=q_i$, etc., if no distinction between two different particles is necessary) are the distribution functions of vector bosons in fermions of [27][3]. The label $\lambda=T,\overline{T},L$ denotes the helicity

[3]No correction for the flux factor has to be applied to the distributions [27] appearing in Eq. (19) (as opposed to the prescription given in the Appendix). This is because the boson-boson flux factor already appears explicitly in front of the integral in Eq. (19). It does not have to be approximated as a product of boson-quark flux factors.

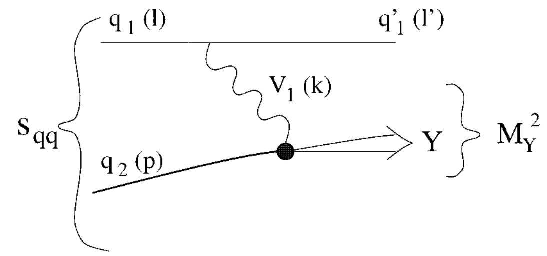

FIG. 6. The Feynman diagram in the effective vector-boson approximation for the scattering of a quark q_1 with a quark q_2 at a scattering energy $\sqrt{s_{qq}}$. A final state Y with the squared invariant mass M_Y^2 is produced. The particle V_1 is an exchanged vector boson. The four-momenta of the particles are denoted by l, l', k, and p.

of the vector boson. Distribution functions of vector bosons in fermions describe the process shown in Fig. 6. The cross section for the process in Fig. 6, averaged over the helicities of the quark q_1 and summed over the helicities of the quark q_1', is given in terms of $f^q_{V_\lambda}$ by the expression

$$\sigma(q_1q_2\to q_1'Y,s_{qq})=\sum_{V_1,\lambda_1}\int_0^1 d\hat{z}\; f^{q_1}_{V_{1,\lambda_1}}\left(\hat{z},\frac{M_1^2}{s_{qq}}\right)\times\sigma(V_{1,\lambda_1}q_2\to Y,M_Y^2=\hat{z}s_{qq}). \tag{20}$$

The distribution function $f^q_{V_\lambda}(\hat{z},M^2/s_{qq})$ is the probability density for the emission of a vector boson V with the helicity λ and mass M from a fermion q. Separating the fermionic couplings, the $f^q_{V_\lambda}$ can be written as

$$f^q_{V_\lambda}\left(\hat{z},\frac{M^2}{s_{qq}}\right)=\frac{\alpha}{2\pi}\hat{z}c^\lambda_{q(V)}h_\lambda\left(\hat{z},\frac{M^2}{s_{qq}}\right). \tag{21}$$

The quantities h_λ in Eq. (21) are ''amputated'' vector-boson distribution functions. They only depend on two dimensionless variables. Equivalent to Eq. (19), for the corresponding amputated differential luminosities one obtains the forms

$$\mathcal{L}_{\lambda_1\lambda_2}\left(\hat{x},\hat{z},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right)=\eta_0h_{\lambda_1}\left(\hat{z},\frac{M_1^2}{s_{qq}}\right)h_{\lambda_2}\left(\frac{\hat{x}}{\hat{z}},\frac{M_2^2}{\hat{z}s_{qq}}\right). \tag{22}$$

The amputated distribution functions[4] h_T, $h_{\overline{T}}$, and h_L have been given in closed form in [27]. We further approximate Eq. (22) by the expression

$$\mathcal{L}_{\lambda_1\lambda_2}\left(\hat{x},\hat{z},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right)\simeq\eta_0h_{\lambda_1}\left(\hat{z},\frac{M_1^2}{\sqrt{\hat{z}}s_{qq}}\right)h_{\lambda_2}\left(\frac{\hat{x}}{\hat{z}},\frac{M_2^2\sqrt{\hat{z}}}{\sqrt{\hat{x}}s_{qq}}\right). \tag{23}$$

The luminosity (23) is invariant under the simultaneous exchange of the vector bosons V_1 and V_2 (i.e., their masses,

[4]For numerical evaluation we require $\hat{z}>M_1^2/s_{qq}$ and $\hat{x}/\hat{z}>M_2^2/s_{qq}$, otherwise the functions are set to zero.

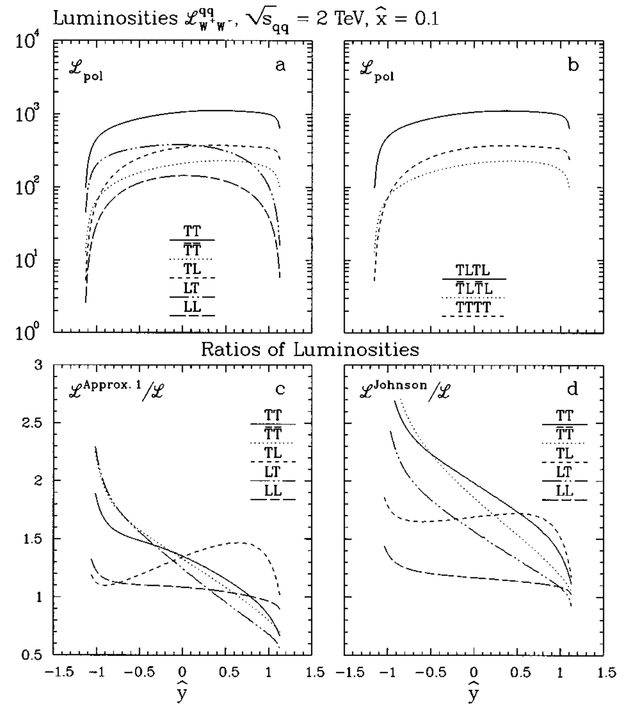

FIG. 7. (a) and (b): The amputated differential luminosities (8) for finding a W^+W^- pair with an invariant mass squared $\mathcal{W}^2=\hat{x}s_{qq}$, $\hat{x}=0.1$, in a fermion pair of $\sqrt{s_{qq}}=2$ TeV as a function of the rapidity $\hat{y}$. (a) shows the helicity combinations which are diagonal, (b) those which are nondiagonal in helicity space. (c) shows the ratios of the luminosities in Approximation 1, Eq. (23), and the luminosities (8). (d) shows the ratios of the luminosities (31) evaluated with the vector-boson distributions [27] and the luminosities (8).

fermionic couplings, and helicities λ_1 and λ_2) and the fermions q_1 and q_2. We refer to the luminosities (14) using Eq. (23) as Approximation 1.

Figure 7 addresses the quality of Approximation 1. Figures 7(a) and 7(b) show the amputated luminosities of the improved EVBA, Eq. (8), for finding a W^+W^- pair in a fermion pair as a function of the rapidity $\hat{y}$. We chose $\sqrt{s_{qq}}=2$ TeV and $\hat{x}=10^{-1}$ as typical values for a quark subprocess in pp collisions at $\sqrt{s_{pp}}=14$ TeV. Figure 7(c) shows the ratios of the luminosities calculated in Approximation 1, Eq. (23), and the luminosities (8). For the dominant TT helicity combination, the ratio decreases with growing $\hat{y}$. Figure 7(d) shows the ratios of the approximation of [27], Eq. (31), with the vector-boson distribution functions [27] (to be discussed in Sec. II C) and the luminosities (8). This latter approximation is clearly worse than Approximation 1. This result will be confirmed by the one obtained for the invariant mass distributions, shown in Figs. 8(a) and 8(b) (to be discussed below).

Figure 8(a) shows the ratios of the luminosities Eq. (14), evaluated with (23), and the exact luminosities (14) with (8), for the diagonal helicity combinations as a function of x. The region $10^{-3}\lesssim x\lesssim 2\times10^{-2}$ is particularly interesting for vector-boson pair production. It corresponds to invariant masses of 400 GeV $\lesssim M_{V_3V_4}\lesssim 2$ TeV. In this region, the dominant TT luminosity deviates by less than 30% from the

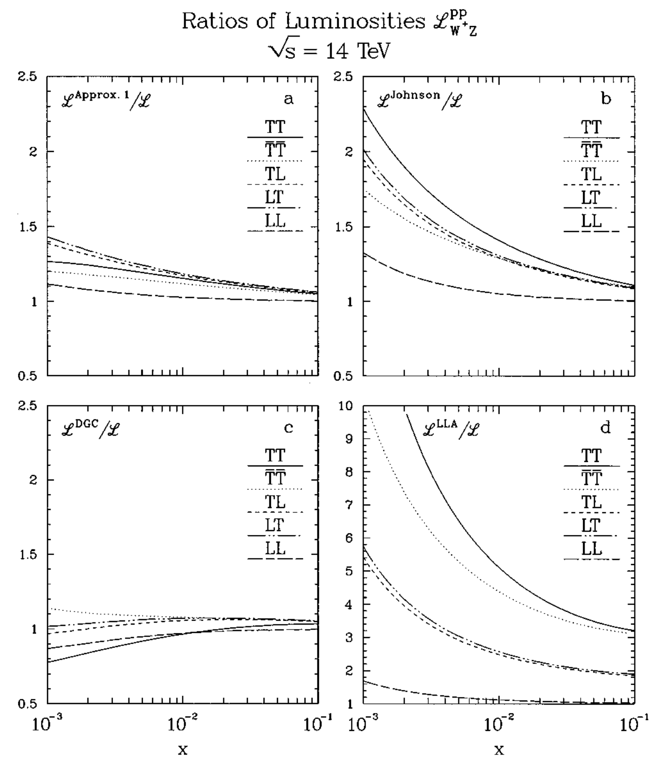

FIG. 8. The ratio of the luminosities using the approximations (23) or (31) and the exact luminosities, Eq. (8), for finding a W^+Z pair in a proton pair of $\sqrt{s_{hh}}=14$ TeV for the diagonal helicity combinations as a function of the variable x. In (a), the direct approximation, Eq. (23), was used. In (b), (c), and (d), the vector-boson distributions of [27], Eq. (A10) (DGC for Dawsen, Godbole, and Capdequi), and the leading logarithm approximation (LLA), respectively, were used to evaluate $\mathcal{L}_{\rm pol}$ according to Eq. (31). All luminosities have been calculated according to Eq. (14).

exact result. A comparison with results of other authors will be given in Sec. II C.

Further approximations may be applied. So far we have employed approximate expressions for the $\mathcal{L}^{q_1q_2}_{V_1,V_2,{\rm pol}}(\hat{x})$ but we used the correct expression for the quark-quark subenergy $s_{qq}\equiv\tau s_{hh}$. If approximate expressions for s_{qq} are used, the luminosity $\mathcal{L}^{h_1h_2}_{(V_1V_2)_{\rm pol}}(x)$ can be approximated as a convolution of vector-boson distribution functions $f^h_{V_\lambda}$ in hadrons. Luminosities have been previously obtained in this way [5,23]. The possibility of using vector-boson distributions in hadrons has already been mentioned in [3]. An equivalent expression for Eq. (14) is

$$\mathcal{L}^{h_1h_2}_{(V_1V_2)_{\rm pol}}(x)=C_{(12)}\left(\frac{\alpha}{2\pi}\right)^2 x\int_x^1\frac{dz}{z}\int_z^1\frac{d\xi_1}{\xi_1}\int_{x/z}^1\frac{d\xi_2}{\xi_2}\,\frac{1}{\xi_1\xi_2}$$
$$\times\Bigg[\Bigg(\sum_{q_1(V_1)}c^{\rm pol}_{q_1(V_1)}f^{h_1}_{q_1}(\xi_1,\mu_1^2)\Bigg)$$
$$\times\Bigg(\sum_{q_2(V_2)}c^{\rm pol}_{q_2(V_2)}f^{h_2}_{q_2}(\xi_2,\mu_2^2)\Bigg)+h_1\leftrightarrow h_2\Bigg]$$
$$\times\mathcal{L}_{\rm pol}\left(\hat{x}=\frac{x}{\xi_1\xi_2},\hat{z}=\frac{z}{\xi_1},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right),\qquad(24)$$

where we introduced the variable $z\equiv\xi_1\hat{z}_1$. The variable z describes the invariant mass squared M_Y^2 which is left for the reaction of the vector boson V_1 with the quark q_2. If V_1 is lightlike, z is equal to the ratio of the energy of V_1 and the energy of the hadron from which it was emitted. In analogy to $\hat{z}_1$ and $\hat{z}_2$ we introduce the variables $z_1=z$ and $z_2=x/z$. It follows that $z_i=\xi_i\hat{z}_i$. Inserting the convolutions (23) into (24) leads to the expression

$$\mathcal{L}^{h_1h_2}_{(V_1V_2)_{\lambda_1\lambda_2}}(x)=C_{(12)}\,\eta_0\int_x^1\frac{dz}{z}\int_z^1 d\xi_1\int_{x/z}^1 d\xi_2$$
$$\times\left[\frac{df^{h_1}_{V_{1,\lambda_1}}}{d\xi_1}(z)\frac{df^{h_2}_{V_{2,\lambda_2}}}{d\xi_2}\left(\frac{x}{z}\right)+h_1\leftrightarrow h_2\right].\qquad(25)$$

In Eq. (25), the quantities $df^h_{V_\lambda}/d\xi$ are differential distribution functions of a vector boson V_λ in a hadron h. They are given by

$$\xi\frac{df^h_{V_\lambda}}{d\xi}(z)=\frac{\alpha}{2\pi}\frac{z}{\xi}\sum_{q(V)}c^\lambda_{q(V)}f^h_q(\xi,\mu^2)h_\lambda\left(\frac{z}{\xi},\frac{M^2\sqrt{\xi}}{\sqrt{z}s_{qq}}\right),\qquad(26)$$

with $s_{qq}=\xi_1\xi_2 s_{hh}$.

The integrations over ξ_1 and ξ_2 in Eq. (25) cannot be carried out independently because s_{qq} in the differential distributions (26) depends on both ξ_1 and ξ_2. We may, however, approximate s_{qq} by $s_{qq}=\xi_i^2 s_{hh}$. It means that we assume the same energy, $E=\xi_i E_h$, for the quark which emits the vector boson V_i and the other quark which emits V_j. E_h is the hadron energy evaluated in the hadron-hadron c.m.s. $E_h=\sqrt{s_{hh}}/2$. Equivalently, it means that the parton-parton c.m.s. is approximated as the hadron-hadron c.m.s. With this approximation the vector-boson distributions in a hadron are given by

$$f^h_{V_\lambda}(z)=\int_z^1\frac{d\xi}{\xi}\sum_{q(V)}f^h_q(\xi,\mu^2)f^q_{V_\lambda}\left(\frac{z}{\xi},\frac{M^2\sqrt{\xi}}{\sqrt{z}s_{qq}}\right),\qquad(27)$$

with $s_{qq}=\xi^2 s_{hh}$ and $f^q_{V_\lambda}(\hat{z}=z/\xi)$ from Eq. (21). We require $z/\xi>M^2/s_{qq}$, i.e., $\xi_{\rm min}=\max[z,M^2/(zs_{hh})]$ as the lower limit of integration in Eq. (27). Again, as in Eq. (19), the functions (27) do not contain a flux factor since the boson-boson flux factor η_0 already appears explicitly in front of the integral in Eq. (25).

The luminosities $\mathcal{L}^{h_1h_2}_{(V_1V_2)}(x)$ which one obtains by using the functions (27) in Eq. (25) are given by

$$\mathcal{L}^{h_1h_2}_{(V_1V_2)_{\lambda_1\lambda_2}}(x)=C_{(12)}\,\eta_0\int_{(1/2)\ln(x)}^{-(1/2)\ln(x)}dy[f^{h_1}_{V_{1,\lambda_1}}(\sqrt{x}e^y)$$
$$\times f^{h_2}_{V_{2,\lambda_2}}(\sqrt{x}e^{-y})+h_1\leftrightarrow h_2],\qquad(28)$$

where we introduced the variable

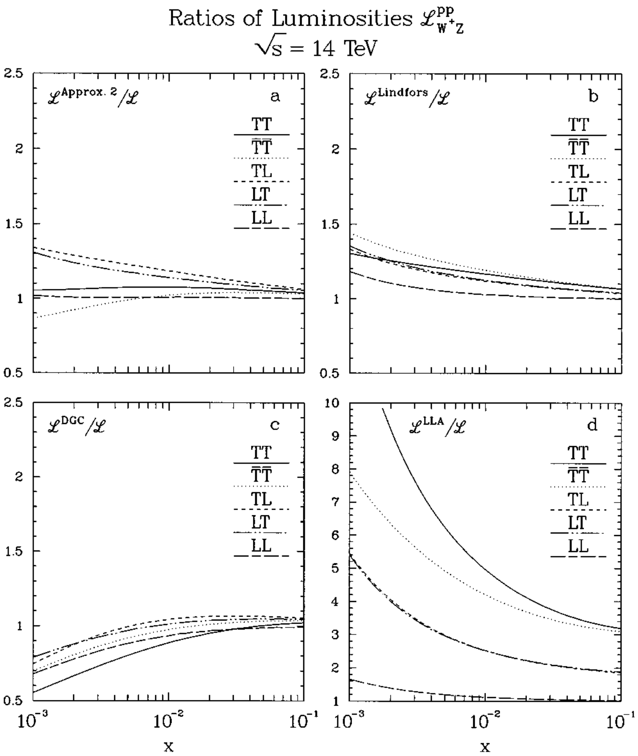

FIG. 9. The ratio of the luminosities approximated as convolutions of vector-boson distribution functions $f^p_{V_\lambda}$ and the exact luminosities for finding a W^+Z pair in a proton pair of $\sqrt{s_{hh}}=14$ TeV for the diagonal helicity combinations as a function of the variable x. In (a), Eq. (28) was used to evaluate $f^q_{V_\lambda}$ in Eq. (32). In (b), (c), and (d), Eq. (35) was used and the vector-boson distributions of [5], Eq. (A10) (DGC), and the LLA, respectively, were used.

$$y \equiv \frac{1}{2}\ln(z^2/x) = \frac{1}{2}\ln\left(\frac{z_1}{z_2}\right) = y_q + \hat{y}. \tag{29}$$

If the vector bosons V_1, V_2 are lightlike, y is the rapidity of the V_1V_2 center-of-mass motion taken along the direction of motion of the hadron which emitted V_1. The formula (28) has been derived with only the mentioned approximations [using the factorized forms (23) and approximating s_{qq} by $s_{qq} \simeq \xi^2 s_{hh}$] from the exact luminosities for a vector-boson pair in a proton pair, Eq. (14) with Eq. (8). We refer to Eq. (28) as Approximation 2.

Figure 9(a) shows the ratios of the luminosities in Approximation 2, Eq. (28), and the exact luminosities for finding a W^+Z pair in a proton pair of $\sqrt{s_{hh}}=14$ TeV for the diagonal helicity combinations as a function of x. The Approximation 2 is in excellent agreement with the improved EVBA. The results of other authors will be discussed in the following section.

C. Comparison with the literature

The exact luminosities (14) with (8) may be compared with results presented in the literature [3,5,23]. In contrast to the approximations (23) and (28), these results do not use the exact expression as a starting point. Instead, they use the *ad hoc* assumption that the luminosities $\mathcal{L}^{q_1q_2}_{V_1,V_2,\text{pol}}(\hat{x})$ can be obtained by convolutions of vector-boson distribution functions. The convolution is similar to Eq. (19) and is given by

$$\mathcal{L}^{q_1q_2}_{V_1,V_2,\lambda_1\lambda_2}(\hat{x}) = \int_{\hat{x}}^1 \frac{d\hat{z}}{\hat{z}} f^{q_1}_{V_{1,\lambda_1}}\left(\hat{z}, \frac{M_1^2}{s_{qq}}\right) f^{q_2}_{V_{2,\lambda_2}}\left(\frac{\hat{x}}{\hat{z}}, \frac{M_2^2}{s_{qq}}\right). \tag{30}$$

Instead of the particular functions $f^q_{V_\lambda}$ of [27], various different functions have been used. Equivalent to the approximation (30), the amputated differential luminosities are written as a product of amputated distribution functions:

$$\mathcal{L}_{\lambda_1\lambda_2}\left(\hat{x}, \hat{z}, \frac{M_1^2}{s_{qq}}, \frac{M_2^2}{s_{qq}}\right) = h_{\lambda_1}\left(\hat{z}, \frac{M_1^2}{s_{qq}}\right) h_{\lambda_2}\left(\frac{\hat{x}}{\hat{z}}, \frac{M_2^2}{s_{qq}}\right). \tag{31}$$

Vector-boson distribution functions have been derived by several authors [3–5,20,22,23,27,28]. In most cases more assumptions than only the ones inherent in the EVBA, i.e., that the reaction proceeds via the exchange of vector bosons and that the vector-boson scattering cross sections for off-shell vector bosons must be known (or an assumption has to be made), were made in the derivation. The differences of various derivations are discussed in the Appendix. In the Appendix we also specify the vector-boson distribution functions $f^q_{V_\lambda}$ which we use for our numerical examples.

Figures 8(b), 8(c), and 8(d) show the ratios of the approximated luminosities, Eq. (14) evaluated with Eq. (31), and the exact luminosities, Eq. (14) with Eq. (8), using for h_λ the distributions [27], the distributions (A10), and the LLA,[5] respectively, for the diagonal helicity combinations as a function of x. The LLA overestimates the improved EVBA by an order of magnitude at small x if both polarizations are transverse. Using Eq. (A10) or [27], instead, greatly diminishes the deviation of the approximation from the improved EVBA. We note that the better agreement of the distributions (A10) than the one of the distributions [27] with the improved EVBA is accidental since the distributions (A10) involve additional approximations as compared to the distributions [27] (see the Appendix). One sees that the use of Eq. (23) [Fig. 8(a)] further improves the agreement between approximated and exact luminsoities, at least compared to the convolutions of [27]. In particular, the agreement is substantially improved for the dominating TT luminosity.

Similarly to Approximation 2, vector-boson distribution functions in hadrons have been used. They were derived in order to describe the process shown in Fig. 10. The distribution functions were obtained as convolutions of the quark distributions in hadrons and the vector-boson distributions in quarks:

$$f^h_{V_\lambda}(z) = \int_z^1 \frac{d\xi}{\xi} \sum_{q(V)} f^h_q(\xi, \mu_i^2) f^q_{V_\lambda}\left(\frac{z}{\xi}, \frac{M^2}{s_{qq}}\right). \tag{32}$$

[5] We use s_{qq}/M^2 as the arguments of the logarithms. This is the simplest choice. Other choices and next-to-leading forms have been used, e.g., in [29].

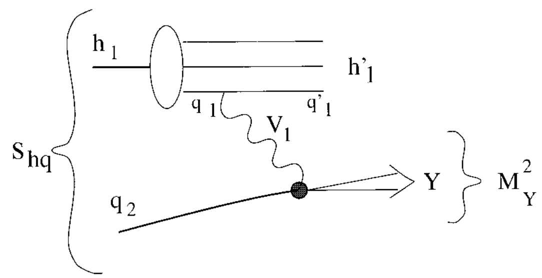

FIG. 10. The diagram for the scattering of a hadron h_1 and a quark q_2 proceeding via the exchange of a single vector boson V_1 originating from the hadron. A final state Y of invariant mass M_Y is produced.

The functions $f^h_{V_\lambda}$ describe the emission probability of a vector boson V with helicity λ and mass M from a hadron h. The sum in (32) extends over all quarks and antiquarks which can couple to V. Equation (32) is similar to Eq. (27), however, in Eq. (27) the specific distributions [27] have to be used. In addition, in Eq. (27) we included the square roots introduced in (23). The definition of z in terms of s_{hq} (defined in Fig. 10) and M_Y^2 is given by

$$z \equiv M_Y^2/s_{hq}. \tag{33}$$

The cross section for the process shown in Fig. 10 is given by

$$\sigma(h_1 q_2 \to h_1' Y, s_{hq}) = \sum_{V_1,\lambda_1} \int_0^1 dz f^{h_1}_{V_{1,\lambda_1}}\left(z, \frac{M_1^2}{s_{qq}}\right) \times \sigma(V_{1,\lambda_1} q_2 \to Y, M_Y^2). \tag{34}$$

In writing down Eq. (34) no other assumptions than those inherent in the EVBA have been made.

The quark-quark energy s_{qq} is in principle unknown if only the energy E_h of the hadron or the hadron-hadron scattering energy s_{hh} is known. Thus, an approximation for s_{qq} has to be made. We will again use $s_{qq} \simeq \xi^2 s_{hh}$, as above.

In [29], a variable Q^2, which was defined by $Q^2 \equiv M_Y^2$ was used. The approximation $s_{qq} \simeq \xi^2 s_{hh}$ applied to Q^2 is $Q^2 = \hat{z} s_{qq} = z/(\xi s_{qq}) \simeq z \xi s_{hh}$.

The luminosities $\mathcal{L}^{h_1 h_2}_{(V_1 V_2)_{\lambda_1 \lambda_2}}(x)$ were approximately expressed as convolutions of the vector-boson distribution functions (32):

$$\mathcal{L}^{h_1 h_2}_{(V_1 V_2)_{\lambda_1 \lambda_2}}(x) = C_{(12)} \int_{(1/2)\ln(x)}^{-(1/2)\ln(x)} dy [f^{h_1}_{V_{1,\lambda_1}}(\sqrt{x} e^{y}) \times f^{h_2}_{V_{2,\lambda_2}}(\sqrt{x} e^{-y}) + h_1 \leftrightarrow h_2]. \tag{35}$$

The approximation (35) has often been used in the past and numerical results for the luminosities can be found in [23,29].

We note that in [29] an excellent approximation was made. As discussed above, instead of the variable s_{qq} two variables Q_1^2 and Q_2^2, which appear in $f^{h_1}_{V_{1,\lambda_1}}$ and $f^{h_2}_{V_{2,\lambda_2}}$, respectively, were used. The Q_i^2 are the squared invariant masses of a vector boson and a quark which are defined in terms of s_{qq} or, alternatively, in terms of s_{hh} by

$$Q_i^2 \equiv \hat{z}_i s_{qq} = \frac{z_i}{\xi_i} s_{qq} = x \frac{\xi_j}{z_j} s_{hh}, \quad i \neq j, \quad i = 1,2, \tag{36}$$

where we have only used the exact relations $s_{qq} = \xi_1 \xi_2 s_{hh}$ and $z_1 z_2 = x$. Clearly, again, factorization does not occur using the exact expressions (36). Instead of using the approximation $s_{qq} \simeq \xi_i^2 s_{hh}$, another approximation for the Q_i^2 has been made in [29] when luminosities were calculated, namely, the simple approximate choice $Q_i^2 \simeq x s_{hh} = \mathcal{W}^2$. Thus, the quark vector-boson invariant masses Q_i^2 have been approximated by the vector-boson–vector-boson invariant mass. This choice always underestimates[6] the exact values of Q_i^2. However, we know that reducing the invariant masses involving quarks will in general improve the agreement with the improved EVBA. We will therefore use this choice of Q_i^2 in our numerical example below. It leads to an excellent agreement with the improved EVBA. For the other distributions $f^h_{V_\lambda}$, we use $s_{qq} \simeq \xi_i^2 s_{hh}$ as before.

Figures 9(b), 9(c), and 9(d) show the ratios of the luminosities calculated according to Eq. (35) and the exact luminosities for finding a W^+Z pair in a proton pair of $\sqrt{s_{hh}} = 14$ TeV for the diagonal helicity combinations as a function of x. To evaluate Eq. (35), the distributions of [5] (using $Q_i^2 = \mathcal{W}^2$), the distributions (A10), and the LLA were used. The LLA overestimates the exact luminosities by far. The distributions (A10) yield slightly low values at low x. The distributions [5] are an excellent approximation to the improved EVBA. For the dominant TT luminosity, the direct approximation [Fig. 9(a)] is better than the distributions [5].

We finally present numerical results for the vector-boson distribution functions in hadrons. In this numerical example we approximate the boson-boson flux factor in Eq. (25) by a product of boson-quark flux factors,

$$\eta_0 \simeq \left(1 - \frac{M_1^2}{\hat{z}_1 s_{qq}}\right)\left(1 - \frac{M_2^2}{\hat{z}_2 s_{qq}}\right). \tag{37}$$

One of the boson-quark flux factors, $[1 - (M^2/\hat{z} s_{qq})]$, is then included in the $f^q_{V_\lambda}(\hat{z} = z/\xi)$ in Eq. (27). No changes are made in Eq. (32). Figure 11 shows the distributions functions for a W^+ in a proton of $E_h = \sqrt{s_{hh}}/2 = 7$ TeV for the various helicity combinations of the W^+ as a function of z. The distribution functions have been calculated according to Eq. (27) or Eq. (32). To evaluate Eq. (32), the LLA, the distributions (A10), and those of [5] and [27] have been used for $f^q_{V_\lambda}$. We note that we used the complete expressions [5] instead of the next-to leading forms [29].

For the T and $\bar{T}$ polarization, the LLA overestimates any of the other distributions by far. The distributions [27] are larger than those of Eq. (27). The distributions (A10) yield

[6]We see from Eq. (36) that the approximated values for the Q_i^2 are smaller by the factors $z_j/\xi_j < 1$ than the exact values. One should note that the variable ξ_i runs in the limits $z_i < \xi_i < 1$.

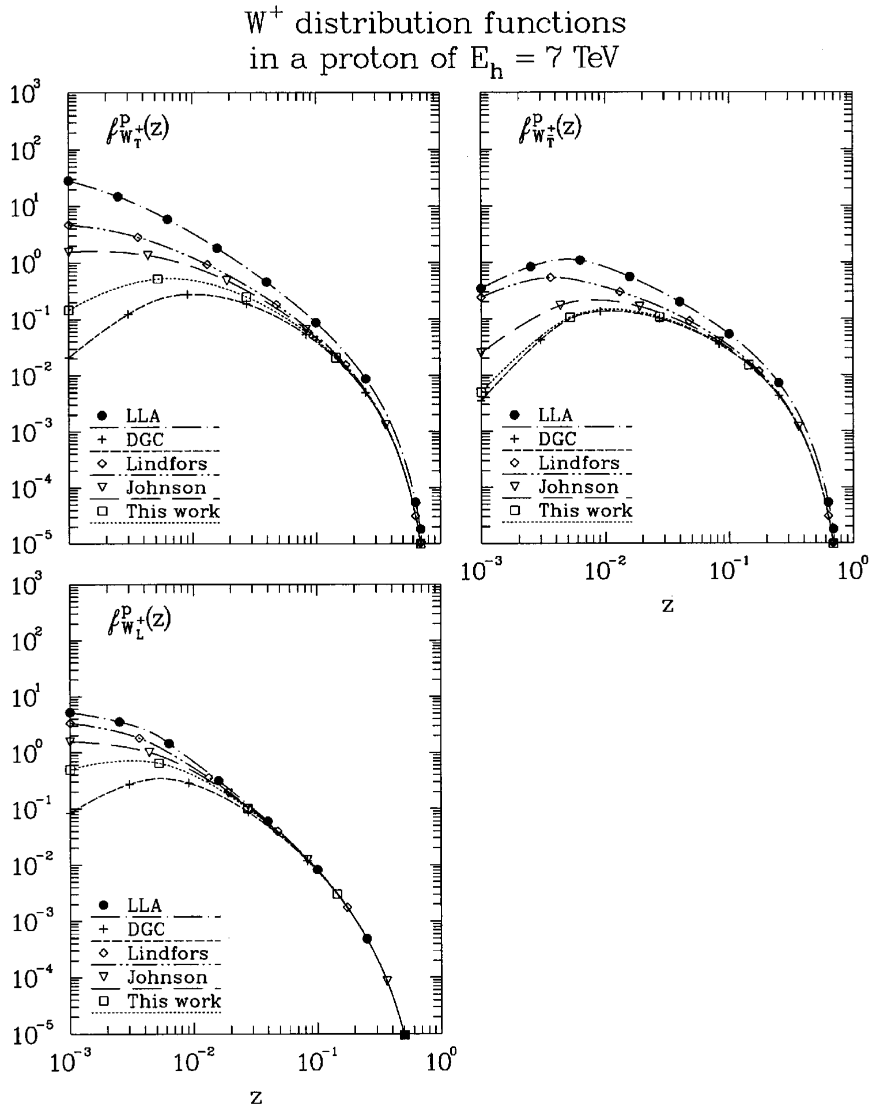

FIG. 11. The distribution functions of a W^+ boson in a proton of $E_h = \sqrt{s_{hh}}/2 = 7$ TeV, Eq. (27) (this work), or Eq. (32) (all others), for the helicity combinations T, $\bar{T}$, and L as a function of z. The LLA, Eq. (A10) (DGC), and the distributions [5] and [27] were used to evaluate $f^q_{V_\lambda}$ in Eq. (32).

rather low values. The differences between the distributions increase at small z. For the L polarization, the differences between the models only manifest themselves at low values of z.

We note that a typical value for z is $z = \sqrt{x} \simeq 7 \times 10^{-2}$ if a final state Ξ of mass 1 TeV is produced in pp collisions at $\sqrt{s_{hh}} = 14$ TeV. However, all values of z in the range $x < z < 1$ contribute to the integral in Eq. (28) or (35). For $\mathcal{W} = 450$ GeV, which is still a large energy compared to the vector-boson masses, z becomes as small as $z = x \simeq 10^{-3}$ and the whole range of z which is shown in Fig. 11 contributes to the luminosity.

III. COMPARISON WITH A COMPLETE PERTURBATIVE CALCULATION

We are now going to present a numerical comparison of the EVBA with a complete perturbative calculation. The complete perturbative calculation includes the contribution from bremsstrahlung diagrams as shown in Fig. 3. As an example for a vector-boson pair production process, we choose the process $pp \to ZZ + X$, for which complete results are available in the literature.

The complete perturbative calculation uses in Eq. (3) the complete (lowest order) cross section of the process on the quark level $q_1 q_2 \to q_1' q_2' ZZ$. Numerical results of the complete calculation for $\sqrt{s} = 40$ TeV can be found in [30] and [22,31]. Only the production via W^+W^--pairs, $pp \to W^+W^- \to ZZ$, was considered.[7]

In the EVBA one has to calculate the cross sections for

[7]A separation into a contribution from intermediate W^+W^- pairs and a contribution from intermediate ZZ pairs is also possible in the complete calculation (in a very good approximation) [30]. The diagrams of the complete calculation can be grouped into two classes. One class contains the W^+W^- diagrams of the EVBA and additional bremsstrahlung-type diagrams, the other class contains the ZZ diagrams and also additional bremsstrahlung diagrams. Both classes are a gauge-invariant subset. The interference term between the two classes, which arises when the amplitude is squared, is very small.

$W^+W^-\to ZZ$. In the Born approximation there are four diagrams which contribute to these processes. Two diagrams describe the exchange of massive vector bosons, one diagram a four-particle point interaction, and one diagram Higgs boson exchange. An analytical expression for the helicity amplitudes has been given in [32].

As in [22], we apply a rapidity cut on the produced vector bosons $V_3V_4=ZZ$ in the hadron-hadron c.m.s. frame. We treat this cut approximately assuming that the vector bosons V_1,V_2 move collinearly to the hadron beam direction. The rapidities of the vector bosons V_3 and V_4 in the V_1V_2 c.m.s. frame, taken along the direction of motion of the hadron from which V_1 is emitted, are

$$y_3^*=\operatorname{arctanh}\left(\frac{q\cos\theta}{\sqrt{q^2+M_3^2}}\right),\quad y_4^*=\operatorname{arctanh}\left(\frac{-q\cos\theta}{\sqrt{q^2+M_4^2}}\right). \tag{38}$$

In Eq. (38), θ is the angle between the directions of motion of V_1 and V_3 evaluated in the (V_1V_2) center-of-mass system. The variable q is the magnitude of the spacelike momentum of the vector boson V_3 in this system:

$$q=\frac{\sqrt{\mathcal{W}^2}}{2}\sqrt{1-\frac{2}{\mathcal{W}^2}(M_3^2+M_4^2)+\frac{1}{\mathcal{W}^4}(M_3^2-M_4^2)^2}. \tag{39}$$

The rapidities y_3,y_4 of V_3,V_4 in the hadron-hadron c.m.s. frame are approximately obtained by addition, $y_3\simeq y+y_3^*$ and $y_4\simeq y+y_4^*$, where the equality holds strictly if both V_1 and V_2 are lightlike. We apply a rapidity cut Y to both produced vector bosons:

$$|y_3|<Y$$

and

$$|y_4|<Y. \tag{40}$$

Following from Eq. (12) and Eq. (14) with Eq. (15), we obtain the expression for the cross section for $h_1h_2\to V_1V_2\to V_3V_4$ with a rapidity cut:

$$\begin{aligned}
&\frac{d\sigma}{dx}(h_1h_2\to V_1V_2\to V_3V_4,s_{hh})\,\theta(Y-|y_3|)\,\theta(Y-|y_4|)\\
&=\left(\frac{\alpha}{2\pi}\right)^2 x\int_{-y_{\max}}^{y_{\max}}dy\int_0^{\ln(1/x)}\frac{d\ln(1/\tau)}{\tau}\int_{\max[-(1/2)\ln(1/\tau),-(1/2)\ln(\tau/x)+y]}^{\min[(1/2)\ln(1/\tau),(1/2)\ln(\tau/x)+y]}dy_q\sum_{(V_1V_2)}C_{(12)}\eta_0\sum_{\mathrm{pol}}\\
&\times\left[\left(\sum_{q_1(V_1)}f_{q_1}^{h_1}(\sqrt{\tau}e^{y_q},\mu_1^2)c^{\mathrm{pol}}_{q_1(V_1)}\right)\left(\sum_{q_2(V_2)}f_{q_2}^{h_2}(\sqrt{\tau}e^{-y_q},\mu_2^2)c^{\mathrm{pol}}_{q_2(V_2)}\right)\mathcal{L}_{\mathrm{pol}}\left(\hat{x},\sqrt{\frac{x}{\tau}}e^{y-y_q},\frac{M_1^2}{s_{qq}},\frac{M_2^2}{s_{qq}}\right)+h_1\leftrightarrow h_2\right]\\
&\times\int_{z_{\min}(y)}^{z_{\max}(y)}d\cos\theta\frac{d\sigma}{d\cos\theta}[(V_1V_2)_{\mathrm{pol}}\to V_3V_4,\mathcal{W}^2],
\end{aligned} \tag{41}$$

where the integration limits are determined by the rapidity cut,

$$y_{\max}=\min\left[Y,\frac{1}{2}\ln\left(\frac{1}{x}\right)\right],$$

$$z_{\min}(y)=\max\left[\frac{-\tanh(Y+y)}{\beta(M_3^2,M_4^2)},\frac{-\tanh(Y-y)}{\beta(M_4^2,M_3^2)},-\cos\theta_{\min}\right],$$

$$z_{\max}(y)=\min\left[\frac{\tanh(Y-y)}{\beta(M_3^2,M_4^2)},\frac{\tanh(Y+y)}{\beta(M_4^2,M_3^2)},\cos\theta_{\min}\right], \tag{42}$$

with

$$\begin{aligned}
\beta(M^2,M'^2)&\\
&\equiv\frac{\sqrt{1-(2/\mathcal{W}^2)(M^2+M'^2)+(1/\mathcal{W}^4)(M^2-M'^2)^2}}{1+((M^2-M'^2)\mathcal{W}^2)}\\
&=\frac{q}{\sqrt{q^2+M^2}},
\end{aligned} \tag{43}$$

and $\cos\theta_{\min}=1$. On the left-hand side of the equality sign in Eq. (41) θ is the Heaviside function. In the vicinity of the threshold for the production of the pair V_3V_4, the rapidity cut has no effect anymore, i.e., $z_{\max}(y)$ and $z_{\min}(y)$ are determined by $\cos\theta_{\min}=1$.

If the masses of the vector bosons V_3 and V_4 are equal or only slightly different, $M_3^2\simeq M_4^2$, or if the momenta of the bosons are large against their masses, $q^2\gg\max(M_3^2,M_4^2)$, the expressions (42) for $z_{\min}$ and $z_{\max}$ simplify to give $z_{\max}=-z_{\min}=z_0$, where

$$z_0=\min\left[\frac{\tanh(Y-|y|)}{\beta(M_3^2,M_4^2)},\cos\theta_{\min}\right]. \tag{44}$$

If one applies a rapidity cut to the expression for convolutions of vector-boson distributions, Eq. (28) with (27), one obtains the expression

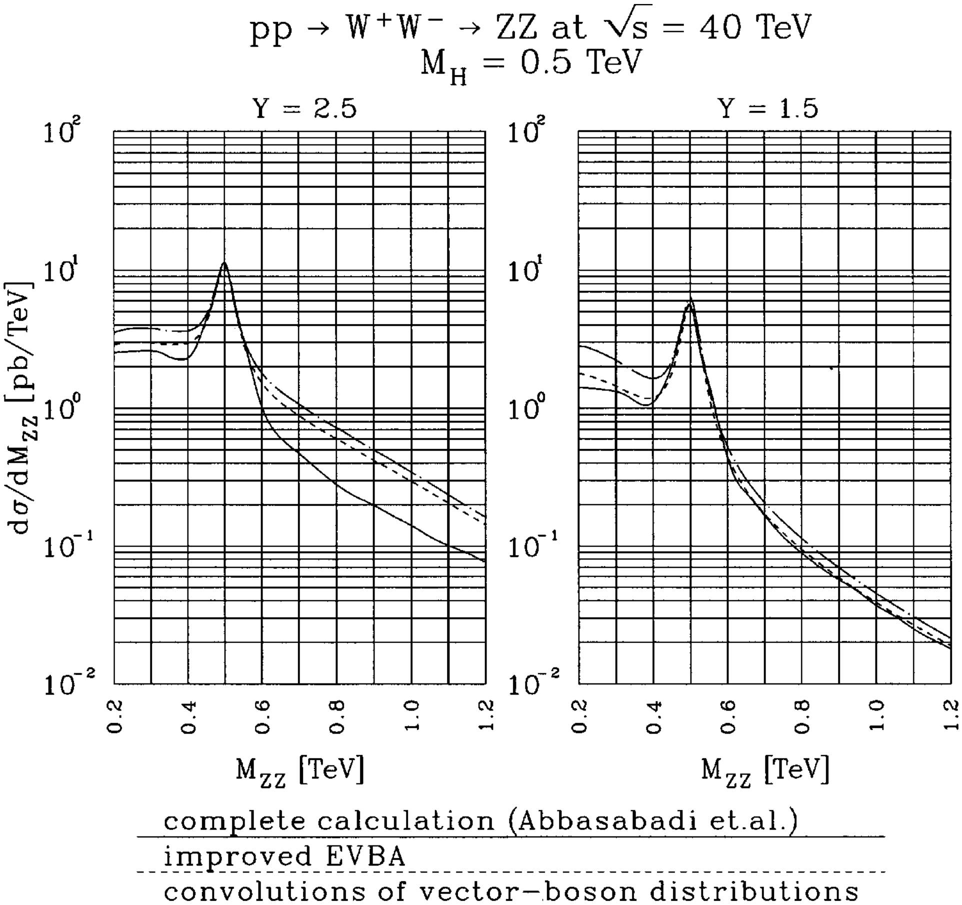

FIG. 12. The cross section for $pp\to ZZ+X$ via W^+W^- scattering as a function of the invariant mass M_{ZZ} at $\sqrt{s_{hh}}=40$ TeV. A rapidity cut of $Y=2.5$ and $Y=1.5$ was applied. Shown is the result of the complete perturbative calculation [22], the improved EVBA (41), and the result of the convolutions of vector-boson distributions (45).

$$\frac{d\sigma}{dx}(h_1h_2\to V_1V_2\to V_3V_4,s_{hh})|_{\text{cut}}$$
$$=\sum_{(V_1V_2)}C_{(12)}\eta_0\sum_{\text{pol}=\lambda_1\lambda_2}\int_{-y_{\max}}^{y_{\max}}dy[f_{V_{1,\lambda_1}}^{h_1}(\sqrt{x}e^y,\mu_1^2)$$
$$\times f_{V_{2,\lambda_2}}^{h_2}(\sqrt{x}e^{-y},\mu_2^2)+h_1\leftrightarrow h_2]\int_{z_{\min}(y)}^{z_{\max}(y)}d\cos\theta$$
$$\times\frac{d\sigma}{d\cos\theta}[(V_1V_2)_{\lambda_1\lambda_2}\to V_3V_4,\mathcal{W}^2],\qquad(45)$$

with $y_{\max}$, $z_{\min}(y)$, and $z_{\max}(y)$ from Eq. (42).

We calculate the differential cross section $d\sigma/dM_{ZZ}$ from Eq. (41) with the luminosities of the improved EVBA. As in [22], the quark distributions of EHLQ [33], set 2, are used and the electroweak parameters are $\alpha=1/128$, $s_W^2=0.22$, $M_W=80$ GeV, $M_H=0.5$ TeV, and $\Gamma_H=51.5$ GeV. For the scales μ_i^2 in the quark distributions, $\mu_i^2=s_{qq}/4$ is chosen. We also carry out a calculation with the convolutions of vector-boson distributions from Eq. (45).[8]

Figure 12 shows the cross section for $pp\to W^+W^-\to ZZ$ at a scattering energy of $\sqrt{s_{hh}}=40$ TeV as a function of the invariant mass M_{ZZ} of the ZZ pair for rapidity cuts of $Y=2.5$ and $Y=1.5$ as a result of the improved EVBA calculation and the calculation with convolutions of vector-boson distributions together with the complete result from [22]. For $Y=2.5$, the cross section of the improved EVBA deviates by a factor of 2 from the complete result at $M_{ZZ}\gtrsim 0.7$ TeV. The result obtained with the convolutions deviates by 13% ($M_{ZZ}=1.2$ TeV) and 18% ($M_{ZZ}=0.6$ TeV) from the improved EVBA result, independently of the magnitude of the cut. For $Y=1.5$, a good agreement between the improved EVBA and the complete calculation is found. The EVBA deviates by less than 10% from the exact result for $M_{ZZ}>0.4$ TeV.

An explanation for the different results for $Y=2.5$ and $Y=1.5$ is that the bremsstrahlung-type diagrams in Fig. 3 begin to play a role if the angle between the produced vector boson and the hadron beam direction is small. This is the case for $Y=2.5$. In contrast, the bremsstrahlung diagrams might be neglected if only large angles are involved. This is the case for $Y=1.5$. For a cut of $Y=2.5$ the smallest allowed angle is $\theta_{\min}=9.4°$, while the smallest angle for $Y=1.5$ is $\theta_{\min}=25.2°$.

In summary, we have seen that the improved EVBA deviates by only $\sim 10\%$ from the result of a complete perturbative calculation for a cut of $Y=1.5$. This result was found for $pp\to ZZ+X$ at $\sqrt{s_{hh}}=40$ TeV and invariant masses of

[8]The value of μ_i^2 in $f_{q_i}^h(\xi_i,\mu_i^2)$ was again $\mu_i^2=\xi_i s_{hh}$.

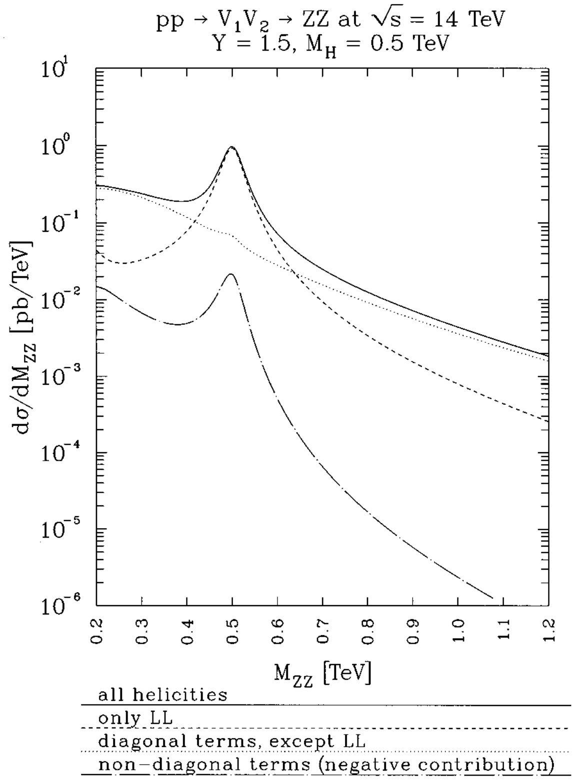

FIG. 13. The cross section for $pp \to (W^+W^- + ZZ) \to ZZ$ in the improved EVBA, Eq. (41), for a scattering energy of $\sqrt{s_{hh}} = 14$ TeV with a rapidity cut of $Y = 1.5$ as a function of the invariant mass M_{ZZ}. The contribution from the LL-diagonal, the diagonal (without LL), the nondiagonal, and the sum of all helicity combinations are shown separately.

$\sqrt{\mathcal{W}^2} > 0.4$ TeV. There is, however, no reason that a similar conclusion could not also be drawn for the production of other vector-boson pairs $pp \to V_3V_4 + X$. We expect this because the EVBA only pertains to the process-independent vector boson luminosities. The use of convolutions instead of the improved EVBA leads to an additional error of $< 20\%$ for $\sqrt{\mathcal{W}^2} \gtrsim 0.5$ TeV (10% at $\sqrt{\mathcal{W}^2} = 2$ TeV).

We finally present another result which is of interest in connection with the EVBA. It concerns the magnitude of the off-diagonal terms in the helicities of V_1 and V_2, denoted by $TTTT$, $TLTL$, and $\overline{TL}TL$ in [25]. Figure 13 shows the contributions of the LL-diagonal, the other diagonal, the nondiagonal, and the sum of all helicity combinations for the cross section for $pp \to (W^+W^- + ZZ) \to ZZ$ at $\sqrt{s_{hh}} = 14$ TeV as a function of the invariant mass M_{ZZ} for a rapidity cut of $Y = 1.5$. The parameters and parton distributions were chosen as in Sec. II and the parameters for the Higgs boson were $M_H = 500$ GeV and $\Gamma_H = 51.5$ GeV. The sum of the nondiagonal helicity contributions $TTTT$, $TLTL$, and $\overline{TL}TL$ is negative and very small compared to the diagonal helicity combinations. The nondiagonal terms can therefore be safely neglected for this process. The longitudinal helicity combination LL only plays a role near the Higgs resonance and is otherwise also small. For the production of vector-boson pairs with large invariant masses, the transverse helicities are important.

IV. CONCLUSION

We have given exact results for luminosities of vector-boson pairs in a proton pair. In contrast to previous results, our treatment of the effective vector-boson method made no approximation in the integration over the phase space of the two intermediate vector bosons. The full calculation is involved but we have shown that approximate expressions exist which reproduce the exact luminosities to a fairly good degree. Identifying in detail the approximations leading to the simple formalism of convolutions of vector-boson distributions in hadrons we have given a direct approximation to the exact luminosities. For one of the phenomenologically interesting processes of vector-boson pair production in high-energy proton-proton collisions we have shown that the direct approximation deviates by less than 20% from the result obtained with the exact luminosities.

In a numerical comparison of the improved EVBA with a complete perturbative calculation for the process $pp \to ZZ + X$ we have shown that the improved EVBA can reproduce the complete result to $\sim 10\%$ if a rapidity cut of large enough strength is applied. This is true not only on the Higgs boson resonance but also far away from it. The improved EVBA thus gives a good approximation not only for longitudinal but also for transverse vector-boson scattering. If a light Higgs boson exists, this latter process is the dominating production mechanism of high-energy vector-boson pairs in pp collisions at LHC energies.

We further investigated previous formulations of the EVBA. These formulations always used the approximation of convolutions of distribution functions of single vector bosons. We investigated in detail the approximations which were made and discussed the differences between various existing derivations. Only some of the derivations use no other approximations than those inherent in the EVBA. We numerically addressed the deviation of existing formulations from the exact luminosities. The deviations are in general larger than those of the direct approximation given here.

ACKNOWLEDGMENTS

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APPENDIX: DIFFERENCES BETWEEN VECTOR-BOSON DISTRIBUTION FUNCTIONS IN THE LITERATURE

In the main text I gave some results obtained by using vector-boson distribution functions in fermions $f^q_{V_\lambda}(\hat{z})$. In this appendix I specify the explicit forms which I used for the functions and briefly discuss the differences of various functions which were derived in the literature.

Vector-boson distribution functions have been derived by several authors [3–5,20,22,23,27,28]. All distributions describe the emission of a vector boson as shown in Fig. 6 according to Eq. (20). In general the functions differ from each other because different approximations and assumptions were made. A discussion of the differences can be found in [34]. We repeat the main points. In [4,22], kinematic approximations concerning the transverse momentum $k_\perp$ of the vector boson were made. It was assumed that $k_\perp^2 \ll s_{qq}$. These approximations were removed in [28]. Also in [3,20,23], no approximations of this kind were made. The distributions [3,20,23] are all very similar to each other. They have in common that the scale variable $\hat{z}$ was defined as the ratio of the vector boson's energy and the energy of the quark q $\hat{z} \equiv k^0/E$. For clarity, we define a single set of distribution functions instead of using any particluar one of the parametrizations [3,20,23] or [28]. The three parametrizations [3,20,23] all agree if they are written in the form

$$f^q_{V_\lambda}(\hat{z}) = \frac{\alpha}{2\pi}\int_{-4E^2(1-\hat{z})}^{0} dk^2 \frac{T_\lambda}{(k^2-M^2)^2}\frac{F_{Vp}(M^2)}{F_{lp}} \times \frac{|\hat{\mathcal{M}}|^2_\lambda(k^2)}{|\hat{\mathcal{M}}|^2_\lambda(M^2)}. \tag{A1}$$

In Eq. (A1), T_λ is the fermionic trace tensor contracted with the polarization vectors $\epsilon(h)$:

$$T_T = (v^2+a^2)\sum_{h=+,-}[l\cdot\epsilon^*(h)l'\cdot\epsilon(h) + l\cdot\epsilon(h)l'\cdot\epsilon^*(h) + l\cdot l'],$$

$$T_{\overline{T}} = (2va)\sum_{h=+,-}(-1)^h i\epsilon^{\mu\mu'\rho\sigma}l_\rho l'_\sigma \epsilon^*_\mu(h)\epsilon_{\mu'}(h),$$

$$T_L = (v^2+a^2)[2l\cdot\epsilon(0)l'\cdot\epsilon(0) - l\cdot l'\epsilon(0)\cdot\epsilon(0)]. \tag{A2}$$

The index h is the helicity of the vector boson and we used the four-momenta defined in Fig. 6. $F_{Vp}(M^2)$ and F_{lp} are the on-shell flux factors for the scattering of the vector boson with the quark q_2 and for the scattering of the two quarks with each other, respectively. In terms of the particles' four-momenta the flux factors are given by

$$F_{Vp}(k^2) = \sqrt{(k\cdot p)^2 + k^2p^2}, \quad F_{lp} = \sqrt{(l\cdot p)^2 + l^2p^2}. \tag{A3}$$

The quantities $|\hat{\mathcal{M}}|^2_\lambda(M^2)$ and $|\hat{\mathcal{M}}|^2_\lambda(k^2)$ are the on-shell and off-shell, respectively, squared matrix elements for the scattering of the vector boson with the quark q_2.

The polarization vectors were defined in a system in which the vector boson has a three-momentum $\vec{k}$ of magnitude K along a particular direction in space $\vec{k} = K\hat{e}_z$. We have $K^2 = \hat{z}^2E^2 - k^2$. Inserting the polarization vectors one obtains

$$T_T = (v^2+a^2)\frac{(-k^2)[1+(1-\hat{z})^2 - (k^2/(2E^2))]}{\hat{z}^2 - (k^2/E^2)},$$

$$T_{\overline{T}} = (2va)\frac{E}{K}(2-\hat{z})(-k^2). \tag{A4}$$

To evaluate T_L, the polarization vector $\epsilon(0)$ for an on-shell vector boson was used in [3,20] while $\epsilon(0)$ for a vector boson of arbitrary k^2 was used in [23]. Since one has to integrate over k^2, we use the latter choice, leading to

$$T_L = 2(v^2+a^2)E^2(-k^2)\frac{1-\hat{z}+(k^2/(4E^2))}{K^2}. \tag{A5}$$

So far, no reference has been made to a specific frame. In all distributions [3,20,23] the flux factor ratio in (A1) was evaluated in the laboratory system of the quark q_2, thus,

$$F_{Vp}(M^2)/F_{lp} = \sqrt{\hat{z}^2 - M^2/E^2}. \tag{A6}$$

In this frame, however, the relation $M_Y^2/s_{qq} = \hat{z}$ is only an approximate one. It is really given by $M_Y^2/s_{qq} = \hat{z} + k^2/(2Em_q)$, where m_q is the mass of the quark q_2. I note that since the integration variable $|k^2|$ becomes as large as $4E^2(1-\hat{z})$, the desired connection between $\hat{z}$ and M_Y^2/s_{qq} becomes completely disturbed even if $|k^2|$ is not even very large. It is therefore not meaningful to carry out the integration over k^2 in the laboratory frame. The relation $\hat{z} = M_Y^2/s_{qq}$, however, holds exactly in the c.m.s. of q_1 and q_2. We therefore evaluate the flux factor ratio in the c.m.s.,

$$F_{Vp}(M^2)/F_{lp} = \hat{z} - \frac{M^2}{4E^2}, \tag{A7}$$

and we have $4E^2 = s_{qq}$. The remaining task in evaluating Eq. (A1) is to make a model assumption about the k^2 dependence of the $|\hat{\mathcal{M}}|^2_\lambda$. The most simple assumption,

$$\frac{|\hat{\mathcal{M}}|^2_\lambda(k^2)}{|\hat{\mathcal{M}}|^2_\lambda(M^2)} = 1, \tag{A8}$$

was made for all λ in [3,20]. In [23], more refined assumptions were made. These led to the same simple relation (A8) for $\lambda = T$ and different relations for $\lambda = \overline{T}$ and $\lambda = L$. We will adopt here the following minimal model assumptions:

$$\frac{|\hat{\mathcal{M}}|^2_\lambda(k^2)}{|\hat{\mathcal{M}}|^2_\lambda(M^2)} = 1, \quad \lambda = T, \overline{T},$$

$$\frac{|\hat{\mathcal{M}}|^2_\lambda(k^2)}{|\hat{\mathcal{M}}|^2_\lambda(M^2)} = \frac{M^2}{-k^2}, \quad \lambda = L. \tag{A9}$$

The same assumptions have been made in [27]. They amount to taking into account the k^2 dependence of the polarization vectors and assuming no k^2 dependence otherwise. The distribution functions are now given by

$$f_{V_T}^q(\hat{z})=\frac{\alpha}{2\pi}(v^2+a^2)\left(\hat{z}-\frac{M^2}{4E^2}\right)\int_{-4E^2(1-\hat{z})}^{0}dk^2\frac{(-k^2)[1+(1-\hat{z})^2-(k^2/(2E^2))]}{(k^2-M^2)^2[\hat{z}^2-(k^2/E^2)]},$$

$$f_{V_{\overline{T}}}^q(\hat{z})=\frac{\alpha}{2\pi}(2va)\left(\hat{z}-\frac{M^2}{4E^2}\right)\int_{-4E^2(1-\hat{z})}^{0}dk^2\frac{(-k^2)(2-\hat{z})}{(k^2-M^2)^2\sqrt{\hat{z}^2-(k^2/E^2)}},$$

$$f_{V_L}^q(\hat{z})=\frac{\alpha}{\pi}(v^2+a^2)\left(\hat{z}-\frac{M^2}{4E^2}\right)M^2\int_{-4E^2(1-\hat{z})}^{0}dk^2\frac{1-\hat{z}+(k^2/(4E^2))}{(k^2-M^2)^2[\hat{z}^2-(k^2/E^2)]}. \tag{A10}$$

The distribution function $f_{V_T}^q$ in Eq. (A10) is the one given (in integrated form) in [3,20] (and it is also the one in [23], but there are errors in the formulas given there) provided one divides these latter functions by the flux factor ratio in the laboratory frame and multiplies by the flux factor ratio in the c.m.s., thus,

$$f_{V_\lambda}^q=\frac{1-(M^2/(4\hat{z}E^2))}{\sqrt{1-(M^2/(\hat{z}^2E^2))}}f_{V_\lambda}^q\Big|_{\text{literature}}. \tag{A11}$$

The distribution function $f_{V_{\overline{T}}}^q$ in Eq. (A10) is the one given in [20] provided one applies the same multiplication (A11). The function $f_{V_{\overline{T}}}^q$ has only been given for $\hat{z}>M/E$ in [20]. For arbitrary values of $\hat{z}$ [in the allowed range $M^2/(4E^2)<\hat{z}<1$] it is given by

$$f_{V_{\overline{T}}}^q(\hat{z})=\frac{\alpha}{2\pi}(2va)\left(\hat{z}-\frac{M^2}{4E^2}\right)(2-\hat{z})I_3. \tag{A12}$$

The integral I_3 in Eq. (A12) is defined by

$$I_3=\int_{-4E^2(1-\hat{z})}^{0}\frac{dk^2(-k^2)}{(k^2-M^2)^2\sqrt{\hat{z}^2-(k^2/E^2)}}. \tag{A13}$$

For $\hat{z}<M/E$, the result of the integration is

$$I_3=-\frac{1}{\hat{z}^2-(M^2/E^2)}\left(\hat{z}-\frac{M^2(2-\hat{z})}{4E^2(1-\hat{z})+M^2}\right)+\frac{2\hat{z}^2E^2-M^2}{2(\hat{z}^2E^2-M^2)}\frac{2}{\sqrt{M^2/E^2-\hat{z}^2}}\times\left[\arctan\left(\frac{2-\hat{z}}{\sqrt{M^2/E^2-\hat{z}^2}}\right)-\arctan\left(\frac{\hat{z}}{\sqrt{M^2/E^2-\hat{z}^2}}\right)\right]. \tag{A14}$$

This result thus continues the result given in [20] into the region $\hat{z}<M/E$. The distribution function $f_{V_L}^q$ in Eq. (A10) is different from any one of those in [3,20,23]. It is given by

$$f_{V_L}^q(\hat{z})=\frac{\alpha}{\pi}(v^2+a^2)\left(\hat{z}-\frac{M^2}{4E^2}\right)\left[(1-\hat{z})I_4-\frac{M^2}{4E^2}I_1\right], \tag{A15}$$

with the integrals

$$I_1=E^2\int_{-4E^2(1-\hat{z})}^{0}\frac{dk^2(-k^2)}{(k^2-M^2)^2(\hat{z}^2E^2-k^2)}=\frac{\hat{z}^2E^4}{(\hat{z}^2E^2-M^2)^2}\left[\ln\left(\frac{4E^2(1-\hat{z})+M^2}{M^2}\right)-2\ln\frac{2-\hat{z}}{\hat{z}}\right]-\frac{E^2}{\hat{z}^2E^2-M^2}\left\{1-\frac{M^2}{4E^2(1-\hat{z})+M^2}\right\}, \tag{A16}$$

$$I_4=M^2E^2\int_{-4E^2(1-\hat{z})}^{0}\frac{dk^2}{(k^2-M^2)^2(\hat{z}^2E^2-k^2)}=\frac{E^2}{\hat{z}^2E^2-M^2}\left(1-\frac{M^2}{4E^2(1-\hat{z})+M^2}\right)-\frac{M^2E^2}{(\hat{z}^2E^2-M^2)^2}\left[\ln\left(\frac{4E^2(1-\hat{z})+M^2}{M^2}\right)-2\ln\frac{2-\hat{z}}{\hat{z}}\right]. \tag{A17}$$

The distribution functions (A10) are defined for all values of $\hat{z}$ in the range

$$M^2/s_{qq}<\hat{z}<1, \tag{A18}$$

where $s_{qq}=4E^2$, and they are zero otherwise. The lower limit in Eq. (A18) is meaningful because the cross sections for on-shell vector-boson scattering vanish for $\hat{z}s_{qq}=M_Y^2<M^2$. We will use the distributions (A10) in the main text.

Having evaluated the functions (A1) in the center-of-mass frame of the quarks we have induced an additional approximation, namely that the helicities $h=0,\pm1$ are not well defined. To the order $k_\perp^2/E^2$, there appears mixing between the helicity states. In particular, the transverse and longitudinal helicity states mix. To see this we note that the on-shell cross section $\sigma(V_{1,\lambda_1}q_2\to Y,M_Y^2)$ appearing in Eq. (20) must be evaluated for definite values of the components of the four-vectors k and p since one has to use specific polarization vectors. In particular, the components cannot depend on the integration variable k^2 appearing in Eq. (A10). Of course, for a given value of the integration variable, a Lorentz transformation into a frame in which k and p have given components may be applied. However, this transformation in general changes the helicity of the vector boson. Only in frames which are related to each other by a boost in the direction of motion of the vector boson is the helicity the same. Therefore, in the frame in which the helicity is defined, the transverse components of p with respect to k must be the same for all values of the integration variable k^2. For the distributions (A10) evaluated in the q_1q_2 center-of-mass frame this is not the case. I note that the helicity could have been defined without an approximation in the laboratory frame. It thus seems that with the distributions (A10) we can choose between either having mixing of the helicity states or a violation of the relation (7).

The above-mentioned approximations were avoided in the derivations [5,27]. By defining $\hat{z}$ directly as $\hat{z} \equiv M_Y^2/s_{qq}$ (i.e., not as a ratio of energies) and defining the vector-boson helicity in its Breit frame no approximations of kinematic origin were made. The only remaining necessary (in the framework of the EVBA) assumption concerned the continuation of the vector-boson cross sections into the region of virtual vector bosons. In [5], the specific assumption that the final state Y couples like a fermion to the intermediate vector boson was made. In [27], the minimal assumptions (A9) were used. Concerning [5], I note that the expression for the integral $I_2(\hat{z})$ given there is not correct. This expression would lead to vector-boson distribution functions which become infinite as $\hat{z} \to M^2/s_{qq}$. The expression must be replaced by

$$I_2(\hat{z}) = [a^2 + 2\hat{z}r(1-r)a]\ln\left(\frac{a}{\hat{z}r}\right) + \ln(\hat{z}) + b(1-2a), \tag{A19}$$

where I used the variables r, a, and b defined in [5]. Concerning [27], I note that the flux factor F_{Vp} was evaluated at $k^2=0$ but it should be evaluated at $k^2=M^2$ [since it is the cross section for on-shell vector bosons which appears in Eq. (20)]. I therefore multiplied the distributions of [27] by the flux factor ratio $F_{Vp}(M^2)/F_{Vp}(0) = [1 - M^2/(\hat{z}s_{qq})]$ before using them for numerical examples.[9] Like the distributions (A10), the distributions [5,27] are defined for all values of $\hat{z}$ in the range (A18) and they are zero otherwise.

All distribution functions reduce to the same analytical expressions if a crude approximation is made. This approximation is obtained by retaining only the leading terms in the limit of vanishing vector-boson masses $M^2 \ll \hat{z}s_{qq}$ and $M^2 \ll (1-\hat{z})\hat{z}s_{qq}$. This approximation has been frequently used in the literature and has been called the leading logarithmic approximation (LLA).[10] Expressions for the $f_{V_\lambda}^q$ in the LLA can be found, e.g., in [3,23,29]. We use the lower limit for $\hat{z}$, $\hat{z} > M^2/s_{qq}$, also for the LLA distributions.

[9]The flux factor ratio was not included when the distributions [27] were used to evaluate Approximation 1, Eq. (23), or Approximation 2, Eq. (28), because these approximations use the exact boson-boson flux factor η_0.

[10]It should be noted that not all leading terms are of logarithmic type.

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