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Electric dipole moment of the muon in a two Higgs doublet model

Vernon Barger
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

Ashok Das
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

Chung Kao
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

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The electric dipole moment of the muon (d_μ) is evaluated in a two Higgs doublet model with a softly broken discrete symmetry. The leading contributions from both one-loop and two-loop diagrams are considered. For $\tan\beta \equiv |v_2|/|v_1|$ close to one, contributions from two-loop diagrams involving the t quark and the W boson dominate, while for $\tan\beta \gtrsim 10$, contributions from two-loop diagrams involving the b quark and the τ lepton are dominant. In these two regions, $d_\mu \approx (m_\mu/m_e) d_e$. For $8 \gtrsim \tan\beta \gtrsim 4$, significant cancellation occurs among the contributions from two-loop diagrams and the one-loop contribution dominates for $\tan\beta \sim 7$. For $\tan\beta \gtrsim 15$, the calculated d_μ can be close to the reach of a recently proposed experiment at the Brookhaven National Laboratory. [S0556-2821(97)00911-9]

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I. INTRODUCTION

A non-self-conjugate particle with a nonzero spin can have a permanent electric dipole moment (EDM) if the combined transformation of charge conjugation (C) and parity (P) is not an exact symmetry. In the standard model (SM) of electroweak interactions, CP violation is generated by the Kobayashi-Maskawa (KM) phase. The electron EDM generated from the KM mechanism is about $8\times 10^{-41} e$ cm [1–3]. Similarly, the expected muon EDM in the SM is about $2\times 10^{-38} e$ cm. These values are more than 13 orders of magnitude below the current experimental limits. Therefore, precise measurements of the electron and the muon EDM's might reveal new sources of CP violation beyond the SM.

Several experiments have been carried out to search for a neutron EDM (d_n) and an electron EDM (d_e). At present, the experimental upper limits on d_n [4] and d_e [5] at 95% C.L. are $|d_n| < 11\times 10^{-26} e$ cm and $|d_e| < 6.2\times 10^{-27} e$ cm, respectively. The measurement of the muon EDM (d_μ) is not yet so precise as that for d_e and d_n. The experimental upper limit on the muon EDM at 95% C.L. is $|d_\mu| < 1.1\times 10^{-18} e$ cm [6]. Recently, a dedicated experiment has been proposed [7] to measure the electric dipole moment of the muon with the existing $g-2$ ring at the Brookhaven National Laboratory (BNL) [8]. This experiment may be able to improve the measurement of the muon EDM by at least 4 orders of magnitude.

A significant electric dipole moment for the electron or the neutron can be generated if CP violation is mediated by Higgs-boson exchange [9–11]. In a nonsupersymmetric model with multi-Higgs doublets, for flavor symmetry to be conserved naturally to a good degree, a discrete symmetry is usually required [12]. In a two Higgs doublet model, there will be no CP violation from the Higgs sector if the discrete symmetry enforcing natural flavor conservation is exact. If this symmetry is broken only by soft terms, CP violation can be introduced and the flavor-changing interaction can be kept at an acceptably low level as well [13,14].

The EDM of an elementary fermion, generated from Higgs boson exchange, has dominant contributions from one-loop or two-loop diagrams. Intuitively, the one-loop contributions are proportional to the product of two Yukawa couplings $(m/v)^2$; while the leading two-loop contributions are proportional to only one Yukawa coupling m/v, where m is the fermion mass and $v = 246$ GeV is the vacuum expectation value (VEV) of the SM Higgs field. The exact one-loop result actually goes like $m^3/m_0^2 \ln(m^2/m_0^2)$, where m_0 is the mass of the lightest neutral Higgs boson. The two-loop contributions dominate for the electron EDM because its one-loop contributions are greatly suppressed. For the muon EDM, the one-loop contributions could be comparable to those from two-loop diagrams for large $\tan\beta$.

Several studies have confirmed that a significant electron EDM can be generated if Higgs-boson exchange mediates CP violation in a two Higgs doublet model with Yukawa interactions of model II [15,16]. In this model, one Higgs doublet couples to down-type quarks and charged leptons while another doublet couples to up-type quarks and neutrinos. The electron EDM has important contributions from two-loop diagrams involving the top quark [11] and the W boson [17–19]. Diagrams with the charged Higgs boson ($H^\pm$) can also contribute to the electron EDM [20]. However, recent measurements on the decay rate of $b \to s\gamma$ by the CLEO Collaboration [21] constrain the mass of the $H^\pm$ in the model II to be at least three times larger than the W boson mass [22–27], and contributions from the $H^\pm$ are therefore not expected to be significant. For the muon EDM, the two-loop contributions involving the t quark as well as the W boson were found to be dominant and to satisfy a simple scaling relation $d_\mu/d_e \simeq m_\mu/m_e$ [28,29]. However, this conclusion did not take into account potentially important two-loop contributions involving the b quark and the τ lepton.

In this paper, we present the first complete calculation for the muon EDM in a two Higgs doublet model with CP violation generated from Higgs boson exchange and Yukawa couplings of model II. We evaluate the leading contributions to the EDM for the muon from two-loop diagrams involving the W boson, the t quark, the b quark and the τ lepton, and the one-loop diagrams. We find that (i) for $\tan\beta \gtrsim 10$ contributions from two-loop diagrams involving the b quark and the τ lepton are dominant [30], and (ii) for $8 \gtrsim \tan\beta \gtrsim 4$, significant cancellation occurs among the two-loop diagrams and the one-loop contribution dominates for $\tan\beta \sim 7$.

II. *CP* VIOLATION FROM HIGGS EXCHANGE

In a two Higgs doublet model with a discrete symmetry softly broken, the Higgs potential can have the form [10]

$$V[\phi_1,\phi_2] = m_1\phi_1^\dagger\phi_1 + m_2\phi_2^\dagger\phi_2 + \eta\phi_1^\dagger\phi_2 + \eta^*\phi_2^\dagger\phi_1 + \frac{1}{2}g_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}g_2(\phi_2^\dagger\phi_2)^2 + g(\phi_1^\dagger\phi_1)\times(\phi_2^\dagger\phi_2) + g'(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}h(\phi_1^\dagger\phi_2)^2 + \frac{1}{2}h^*(\phi_2^\dagger\phi_1)^2. \tag{1}$$

This potential respects a discrete symmetry $\phi_1 \to -\phi_1$, and $\phi_2 \to +\phi_2$, except for the soft terms $\phi_1^\dagger\phi_2$ and $\phi_1^\dagger\phi_2$. We can rewrite the Higgs potential as

$$V[\phi_1,\phi_2] = +\frac{1}{2}g_1\left(\phi_1^\dagger\phi_1 - \frac{|v_1|^2}{2}\right)^2 + \frac{1}{2}g_2\left(\phi_2^\dagger\phi_2 - \frac{|v_2|^2}{2}\right)^2 + g\left(\phi_1^\dagger\phi_1 - \frac{|v_1|^2}{2}\right)\left(\phi_2^\dagger\phi_2 - \frac{|v_2|^2}{2}\right) + g'\left|\phi_1^\dagger\phi_2 - \frac{v_1^* v_2}{2}\right|^2 + \frac{1}{2}h\left(\phi_1^\dagger\phi_2 - \frac{v_1^* v_2}{2}\right)^2 + \frac{1}{2}h^*\left(\phi_2^\dagger\phi_1 - \frac{v_2^* v_1}{2}\right)^2 + \xi\left(\frac{\phi_1^\dagger}{v_1^*} - \frac{\phi_2^\dagger}{v_2^*}\right)\left(\frac{\phi_1}{v_1} - \frac{\phi_2}{v_2}\right), \tag{2}$$

where all the coupling constants are real, with the possible exception of h. CP is violated if $hv_1^{*2}v_2^2$ has an imaginary part. The minimum of this potential occurs at

$$\langle\phi_1\rangle = \frac{v_1}{\sqrt{2}}, \quad \langle\phi_2\rangle = \frac{v_2}{\sqrt{2}}, \tag{3}$$

where $v_1/\sqrt{2}$ and $v_2/\sqrt{2}$ are the vacuum expectation values (VEV's) of ϕ_1 and ϕ_2. Both VEV's can be complex. Without loss of generality, we will take $\langle\phi_1^0\rangle = v_1/\sqrt{2}$ and $\langle\phi_2^0\rangle = v_2 e^{i\theta}/\sqrt{2}$ with v_1 and v_2 real and $\tan\beta \equiv v_2/v_1$. The two VEV's satisfy $\sqrt{v_1^2+v_2^2} = v$, where v is the VEV of the SM Higgs field and $v^2 = (\sqrt{2}G_F)^{-1}$.

Introducing a transformation, which takes the two Higgs doublets to the eigenstates Φ_1 and Φ_2, such that $\langle\Phi_1^0\rangle = v/\sqrt{2}$ and $\langle\Phi_2^0\rangle = 0$, we have

$$\phi_1 = \cos\beta\Phi_1 - \sin\beta\Phi_2, \quad \phi_2 = (\sin\beta\Phi_1 + \cos\beta\Phi_2)e^{i\theta},$$
$$\Phi_1 = \begin{pmatrix} G^+ \\ \dfrac{v + H_1 + iG^0}{\sqrt{2}} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} H^+ \\ \dfrac{H_2 + iA}{\sqrt{2}} \end{pmatrix}, \tag{4}$$

where $G^\pm$ and G^0 are Goldstone bosons, $H^\pm$ are singly charged Higgs bosons, H_1 and H_2 are CP-even scalars, and A is a CP-odd pseudoscalar.

In the new eigenstates of Eq. (4) for the Higgs fields, the Higgs potential becomes [20]

$$V[\Phi_1,\Phi_2] = \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1 - \frac{v^2}{2}\right)\Phi_2^\dagger\Phi_2 + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) + \lambda_5\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2 - \frac{v^2}{2}\right) \times(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + (\lambda_6\Phi_1^\dagger\Phi_2 + \lambda_6^*\Phi_2^\dagger\Phi_1)\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2 - \frac{v^2}{2}\right) + \frac{1}{2}\lambda_7(\Phi_1^\dagger\Phi_2)^2 + \frac{1}{2}\lambda_7^*(\Phi_2^\dagger\Phi_1)^2 + \rho(\Phi_2^\dagger\Phi_2), \tag{5}$$

where the parameters ρ, v, and λ_i, $i=1$–5, are all real; λ_6 and λ_7 can be complex. CP is violated if the imaginary part of λ_6 or λ_7 is nonvanishing. There are 11 parameters, but only 10 of them are independent. One of the relations among the 11 parameters is

$$\mathrm{Im}(\lambda_6)=-\frac{\lambda_5}{2(\lambda_1-\lambda_2)}\mathrm{Im}(\lambda_7). \tag{6}$$

In the above parametrization, $\tan\beta$ is given by

$$\tan\beta=\frac{\lambda_1-\lambda_2}{\lambda_5}+\sqrt{1+\frac{(\lambda_1-\lambda_2)^2}{\lambda_5^2}}. \tag{7}$$

In model II [15,16], the down-type quarks and charged leptons have bilinear couplings with the doublet ϕ_1, while all the up-type quarks and neutrinos have bilinear couplings with the doublet ϕ_2. The Lagrangian density of Yukawa interactions has the form

$$\mathcal{L}_Y=-\sum_{m,n=1}^{3}\overline{L_L^m}\phi_1 E_{mn} l_R^n-\sum_{m,n=1}^{3}\overline{Q_L^m}\phi_1 F_{mn} d_R^n -\sum_{m,n=1}^{3}\overline{Q_L^m}\tilde{\phi}_2 G_{mn} u_R^n+\text{H.c.}, \tag{8}$$

where

$$\phi_\alpha=\begin{pmatrix}\phi_\alpha^+\\ \dfrac{v_\alpha+\phi_\alpha^0}{\sqrt{2}}\end{pmatrix},\quad \tilde{\phi}_\alpha=\begin{pmatrix}\dfrac{v_\alpha^*+\phi_\alpha^{0*}}{\sqrt{2}}\\ -\phi_\alpha^-\end{pmatrix},\quad \phi_\alpha^-=\phi_\alpha^{+*},$$

$$\alpha=1,2, \tag{9}$$

and

$$L_L^m=\begin{pmatrix}\nu_l\\ l\end{pmatrix}_L^m,\quad Q_L^m=\begin{pmatrix}u\\ d\end{pmatrix}_L^m,\quad m=1,2,3, \tag{10}$$

l^m, d^m, and u^m are the leptons, the down-type quarks and the up-type quarks in the gauge eigenstates. This Lagrangian respects a discrete symmetry,

$$\begin{aligned}&\phi_1\to-\phi_1,\quad \phi_2\to+\phi_2,\\ &l_R^m\to-l_R^m,\quad d_R^m\to-d_R^m,\\ &L_L^m\to+L_L^m,\quad Q_L^m\to+Q_L^m,\quad u_R^m\to+u_R^m,\end{aligned} \tag{11}$$

with $m=1,2,3$ and $\alpha=1,2$.

In the new eigenstates of Eq. (4) for the Higgs fields, the neutral Yukawa interactions of the quarks become

$$\begin{aligned}\mathcal{L}_Y^N=&-\sum_{l=e,\mu,\tau}\frac{m_l}{v}\overline{l}l(H_1-\tan\beta H_2)-i\sum_{l=e,\mu,\tau}\frac{m_l}{v}\overline{l}\gamma_5 l(G^0-\tan\beta A)-\sum_{d=d,s,b}\frac{m_d}{v}\overline{d}d(H_1-\tan\beta H_2)\\ &-i\sum_{d=d,s,b}\frac{m_d}{v}\overline{d}\gamma_5 d(G^0-\tan\beta A)-\sum_{u=u,c,t}\frac{m_u}{v}\overline{u}u[H_1+\cot\beta H_2]+i\frac{m_u}{v}\overline{u}\gamma_5 u[G^0+\cot\beta A],\end{aligned} \tag{12}$$

where the quarks and leptons are in the mass eigenstates.

Adopting Weinberg's parametrization [10], we can write the following neutral Higgs boson exchange propagators as

$$\begin{aligned}\langle H_1A\rangle_q&=\frac{1}{2}\sum_n\frac{\sin2\beta\,\mathrm{Im}Z_{0n}}{q^2-m_n^2}=\frac{1}{2}\sum_n\frac{-\cos^2\beta\cot\beta\,\mathrm{Im}\tilde{Z}_{1n}+\sin^2\beta\tan\beta\,\mathrm{Im}\tilde{Z}_{2n}}{q^2-m_n^2},\\ \langle H_2A\rangle_q&=\frac{1}{2}\sum_n\frac{\cos2\beta\,\mathrm{Im}Z_{0n}-\mathrm{Im}\tilde{Z}_{0n}}{q^2-m_n^2}=\frac{1}{2}\sum_n\frac{\cos^2\beta\,\mathrm{Im}\tilde{Z}_{1n}+\sin^2\beta\,\mathrm{Im}\tilde{Z}_{2n}}{q^2-m_n^2},\end{aligned} \tag{13}$$

where the summation is over all the mass eigenstates of neutral Higgs bosons. We approximate the above expressions by assuming that the sums are dominated by the lightest neutral Higgs boson of mass m_0, and drop the sums and indices n in Eq. (13) hereafter. There are relations among the CP violation parameters:

$$\begin{aligned}\mathrm{Im}Z_0+\mathrm{Im}\tilde{Z}_0&=-\cot^2\beta\,\mathrm{Im}\tilde{Z}_1,\\ \mathrm{Im}Z_0-\mathrm{Im}\tilde{Z}_0&=+\tan^2\beta\,\mathrm{Im}\tilde{Z}_2.\end{aligned} \tag{14}$$

Employing unitarity, Weinberg has shown [10] that $|\mathrm{Im}\tilde{Z}_1|\leq(1/2)\tan\beta(1+\tan^2\beta)^{1/2}$ and $|\mathrm{Im}\tilde{Z}_2|\leq(1/2)\cot\beta(1+\cot^2\beta)^{1/2}$. Therefore, we have

$$\begin{aligned}|\mathrm{Im}Z_0+\mathrm{Im}\tilde{Z}_0|&\leq(1/2)\cot\beta(1+\tan^2\beta)^{1/2},\\ |\mathrm{Im}Z_0-\mathrm{Im}\tilde{Z}_0|&\leq(1/2)\tan\beta(1+\cot^2\beta)^{1/2}.\end{aligned} \tag{15}$$

III. MUON ELECTRIC DIOPLE MOMENT

A. Two-loop diagrams

Two loop diagrams for fermion loops contributing to the muon EDM are illustrated in Figs. 1(c) and 1(d). The diagrams with the intermediate Z boson are highly suppressed by the vector part of the $Z\mu^+\mu^-$ couplings. Therefore, we consider only the diagrams involving an intermediate γ.

The muon EDM generated from two-loop diagrams with the top quark is [11]

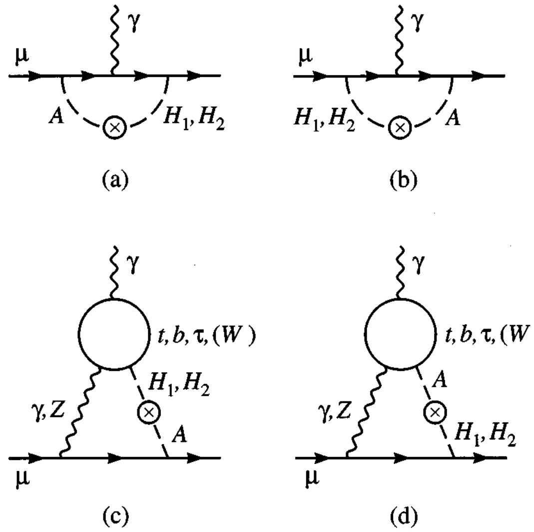

FIG. 1. Feynman diagrams for leading contributions to the muon EDM from one-loop diagrams [(a) and (b)] and two-loop diagrams with heavy fermions and the W boson [(c) and (d)]. There are many more diagrams involving the W boson [17–19] that are not shown in this figure.

$$d_\mu^{t\text{ loop}} = -\frac{16}{3}\frac{em\alpha\sqrt{2}G_F}{(4\pi)^3}\{[f(\rho_t)+g(\rho_t)]\mathrm{Im}Z_0 + [g(\rho_t)-f(\rho_t)]\mathrm{Im}\widetilde{Z}_0\}, \tag{16}$$

where $m=m_\mu$, $\rho_t=m_t^2/m_0^2$, and m_0 is *the mass of the lightest neutral Higgs boson*. The functions f and g are defined as

$$f(r)\equiv\frac{r}{2}\int_0^1 dx\frac{1-2x(1-x)}{x(1-x)-r}\ln\left[\frac{x(1-x)}{r}\right],$$

$$g(r)\equiv\frac{r}{2}\int_0^1 dx\frac{1}{x(1-x)-r}\ln\left[\frac{x(1-x)}{r}\right]. \tag{17}$$

We take $m_t=175$ GeV, $m_b=4.8$ GeV, $m_\tau=1.777$ GeV, $m_W=80.0$ GeV, and the fine structure constant $\alpha=1/137$, neglecting the running up to the scale of m_0.

The EDM generated from the b and the τ loops is [30]

$$d_\mu^{b,\tau\text{ loop}} = -(4N_cQ^2\tan^2\beta)\frac{em\alpha\sqrt{2}G_F}{(4\pi)^3}\times[f(\rho_f)+g(\rho_f)](\mathrm{Im}Z_0+\mathrm{Im}\widetilde{Z}_0), \tag{18}$$

where N_c is the color factor and Q is the charge. For the b and the τ, $4N_cQ^2$ is equal to 4/3 and 4, respectively.

The leading contribution from two-loop diagrams with the W boson [17–19] is

$$d_1^{W\text{ loop}} = (\sin^2\beta)\frac{em\alpha\sqrt{2}G_F}{(4\pi)^3}[4I_1(\rho_W)+2I_2(\rho_W)]\mathrm{Im}Z_0, \tag{19}$$

$$I_1(\rho_W)=3f(\rho_W)+\frac{23}{4}g(\rho_W)+\frac{3}{4}h(\rho_W), \tag{20}$$

$$I_2(\rho_W)=\frac{f(\rho_W)-g(\rho_W)}{\rho_W}, \tag{21}$$

$$h(r)\equiv\frac{r}{2}\int_0^1 dx\frac{1}{x(1-x)-r}\left[\frac{r}{x(1-x)-r}\ln\left(\frac{x(1-x)}{r}\right)-1\right], \tag{22}$$

where $\rho_W=m_W^2/m_0^2$, and the functions f and g are defined in Eq. (17). There are another two sets of diagrams with the W boson [17] contributing to the muon EDM. The contribution from these additional diagrams has an opposite sign to that of $d_1^{W\text{ loop}}$ and therefore reduces the magnitude of the full contribution from the W boson. In our analysis, we have employed the formulas in Ref. [17] to evaluate the complete contributions from the W loops.

B. One-loop diagrams

The one-loop diagrams contributing to the muon EDM are illustrated in Figs. 1(a) and 1(b). The one-loop contribution to the muon EDM (d_μ) is

$$d_\mu^{\text{one loop}} = \frac{em\sqrt{2}G_F\tan^2\beta}{(4\pi)^2}I(\rho)(\mathrm{Im}Z_0+\mathrm{Im}\widetilde{Z}_0), \tag{23}$$

where $\rho=m^2/m_0^2$. The function $I(\rho)$ is defined as

$$I(\rho)\equiv\rho\int_0^1 dx\frac{x^2}{\rho x^2-x+1} = 1+\frac{1}{\rho(z_1-z_2)}\times\left[(z_1-1)\ln\left(\frac{z_1-1}{z_1}\right)-(z_2-1)\ln\left(\frac{z_2-1}{z_2}\right)\right], \tag{24}$$

where z_1 and z_2 are roots of $\rho x^2-x+1=0$. For $\rho\ll 1$, $I(\rho)$ approaches $-\rho[\ln(\rho)+3/2]$. In this limit, the one-loop contribution to the the muon or the electron EDM has a simple form:

$$d_{(e,\mu)}^{\text{one loop}} = -\frac{e\sqrt{2}G_F\tan^2\beta}{(4\pi)^2}(m^3/m_0^2)[\ln(m^2/m_0^2)+3/2]\times(\mathrm{Im}Z_0+\mathrm{Im}\widetilde{Z}_0). \tag{25}$$

where $m=m_e, m_\mu$ and ($m_0\gg m$) is assumed. Therefore, the one-loop contribution to the muon or the electron EDM is proportional to $(m^3/m_0^2)\ln(m^2/m_0^2)$.

C. Numerical values

In this section, we discuss the numerical value of the muon EDM with several choices of m_0 and $\tan\beta$. The contributions from one-loop diagrams (one loop) and two-loop diagrams involving the W boson (W loop), the t quark (t loop), the b quark (b loop) and the τ lepton (τ loop), are presented in Table I for $\tan\beta=$ 2 and 20 with $m_0=100$, 200, and 400 GeV, in units of (a) Im Z_0 and (b) Im $\widetilde{Z}_0$.

To study the muon EDM dependence on the lightest Higgs boson mass (m_0), we present the muon EDM generated from one-loop and two-loop diagrams in units of (a) Im Z_0 and (b) Im $\widetilde{Z}_0$, as a function of m_0, with $\tan\beta=$ 1, 7, and

TABLE I. The muon EDM from one-loop diagrams (one-loop) and two-loop diagrams with the W boson (W loop), the top quark (t loop), the bottom quark (b loop), the τ lepton (τ loop), and their total (Total) for $\tan\beta=2$ ($\tan\beta=20$) with the lightest Higgs boson mass $m_0=100$, 200, and 400 GeV, in units of (a) $\mathrm{Im}Z_0\times10^{-24}e$ cm and (b) $\mathrm{Im}\widetilde{Z}_0\times10^{-24}e$ cm.

	m_0 (GeV)		
Diagrams	100	200	400
(a) Im $Z_0\times10^{-24}e$ cm			
One loop	$+0.01$ $(+1.19)$	$+0.003$ $(+0.33)$	$+0.001$ $(+0.09)$
W loop	$+2.47$ $(+3.07)$	$+1.16$ $(+1.45)$	$+0.20$ $(+0.25)$
t loop	-1.88 (-1.88)	-1.23 (-1.23)	-0.72 (-0.72)
b loop	-0.06 (-5.61)	-0.021 (-2.08)	-0.007 (-0.73)
τ loop	-0.04 (-3.97)	-0.014 (-1.36)	-0.004 (-0.44)
Total	$+0.50$ (-7.2)	-0.10 (-2.8)	-0.53 (-1.5)
(b) Im $\widetilde{Z}_0\times10^{-24}e$ cm			
One loop	$+0.01$ $(+1.19)$	$+0.003$ $(+0.33)$	$+0.001$ $(+0.09)$
W loop	$+0.0$	$+0.0$	$+0.0$
t loop	-0.33 (-0.33)	-0.21 (-0.21)	-0.11 (-0.11)
b loop	-0.06 (-5.61)	-0.021 (-2.08)	-0.007 (-0.73)
τ loop	-0.04 (-3.97)	-0.014 (-1.36)	-0.004 (-0.44)
Total	-0.42 (-8.7)	-0.24 (-3.3)	-0.13 (-1.2)

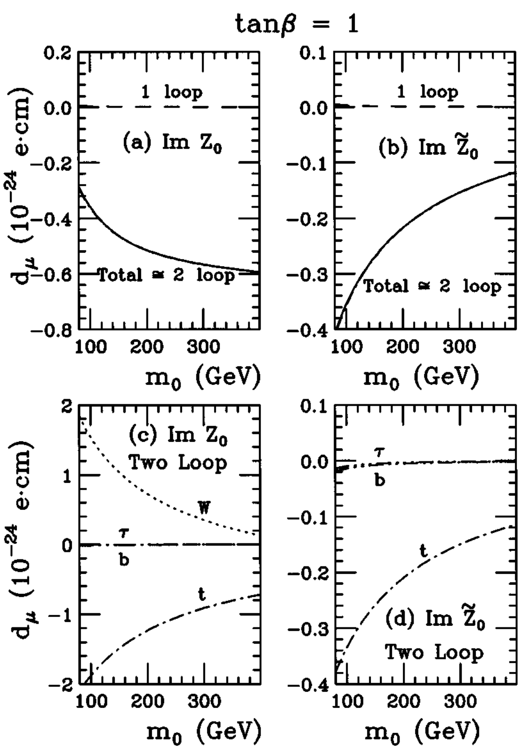

FIG. 2. The muon EDM generated from one-loop diagrams (dashed), two-loop diagrams (dash-dotted), and their total (solid) in units of (a) Im Z_0 and (b) Im $\widetilde{Z}_0$, as a function of m_0, with $\tan\beta=1$, where m_0 is the mass of the lightest neutral Higgs boson. Also shown are the contributions from two-loop diagrams involving the W boson (dot), the t quark (dash-dotted), the b quark (dash-dot-dotted), and the τ lepton (dash-dot-dot-dotted), in units of (c) Im Z_0 and (d) Im $\widetilde{Z}_0$.

20 in Figs. 2, 3, and 4. Figure 5 shows the effect of varying $\tan\beta$ on the muon EDM, for the case of $m_0=100$ GeV. From these figures, we conclude that (i) for $\tan\beta$ close to one, contributions from two-loop diagrams involving the top quark and the W boson dominate; (ii) for $\tan\beta\gtrsim10$ contributions from two-loop diagrams involving the b quark and the τ lepton are dominant [30]; (iii) for $8\gtrsim\tan\beta\gtrsim4$, significant cancellation occurs among the contributions from two-loop diagrams and one-loop diagrams dominate for $\tan\sim7$.

To compare the muon EDM with the electron EDM, we present the ratio of (d_μ/d_e) to (m_μ/m_e) in Fig. 6. For the electron EDM, the two-loop contribution is about 10^5 (for $\tan\beta\gtrsim10$) to 10^6 (for $\tan\beta\sim1$) larger than that from the one-loop diagrams. For the muon EDM, the one-loop diagrams make important contributions which dominate for $\tan\beta\sim7$. For $\tan\beta\gtrsim10$, the one-loop contribution to the muon EDM is about 10% of that from the two-loop diagrams. We find that (i) for $\tan\beta\sim1$, $d_\mu\approx(m_\mu/m_e)$; (ii) for $\tan\beta\gtrsim10$, $d_\mu\approx0.9(m_\mu/m_e)$, since there is a cancellation for the muon EDM between the one-loop contribution and the two-loop contribution involving the b quark and the τ lepton; (iii) for $8\gtrsim\tan\beta\gtrsim4$, $|d_\mu|$ can be two to three times $|(m_\mu/m_e)d_e|$.

The muon EDM can be expressed as

$$d_\mu=d_A\mathrm{Im}Z_0+d_B\mathrm{Im}\widetilde{Z}_0=\frac{d_A+d_B}{2}(\mathrm{Im}Z_0+\mathrm{Im}\widetilde{Z}_0)+\frac{d_A-d_B}{2}(\mathrm{Im}Z_0-\mathrm{Im}\widetilde{Z}_0), \quad (26)$$

where d_A and d_B are the total coefficients of $\mathrm{Im}Z_0$ and $\mathrm{Im}\widetilde{Z}_0$ from one-loop diagrams, the W loop, the t loop, the b loop, and the τ loop. Applying unitarity constraints in Eq. (15), we can define the maximal $|d_\mu|$ as

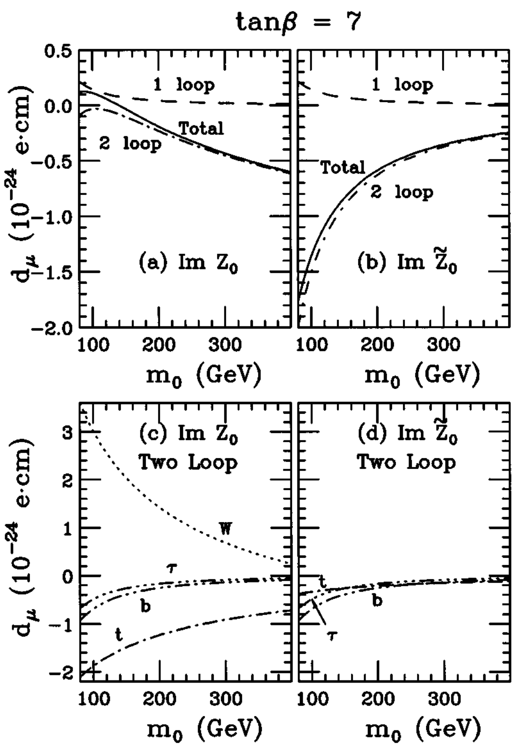

FIG. 3. The same as in Fig. 2, except that $\tan\beta = 7$.

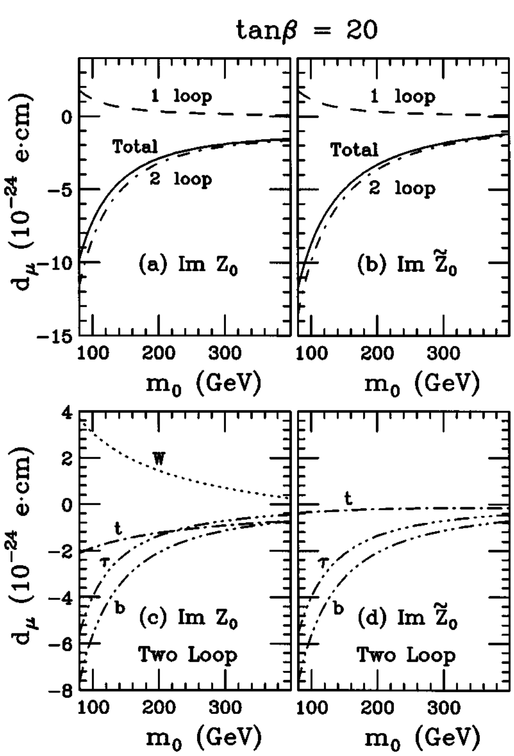

FIG. 4. The same as in Fig. 4, except that $\tan\beta = 20$.

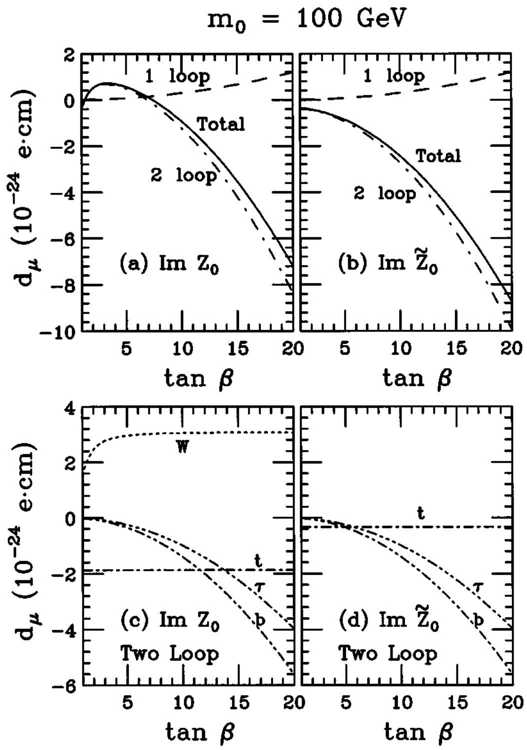

FIG. 5. The muon EDM generated from one-loop diagrams (dashed), two-loop diagrams (dash-dotted) and their total (solid) in units of (a) Im Z_0 and (b) Im $\widetilde{Z}_0$, as a function of $\tan\beta$, with $m_0 = 100$ GeV. Also shown are the contributions from two-loop diagrams involving the W boson (dotted), the t quark (dash-dotted), the b quark (dash-dot-dotted), and the τ lepton (dash-dot-dot-dotted), in units of (c) Im Z_0 and (d) Im $\widetilde{Z}_0$.

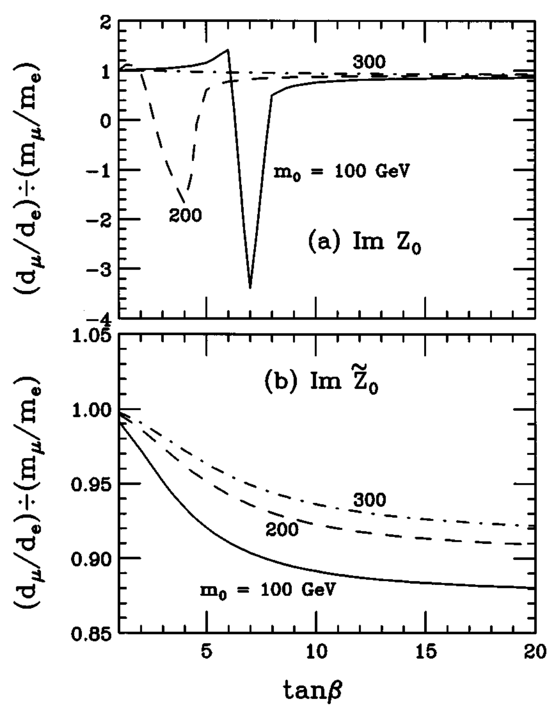

FIG. 6. The ratio of (d_μ/d_e) to (m_μ/m_e) in units of (a) Im Z_0 and (b) Im $\widetilde{Z}_0$, as a function of $\tan\beta$, for $m_0 =$ 100 GeV (solid), 200 GeV (dashed), and 300 GeV (dash-dotted).

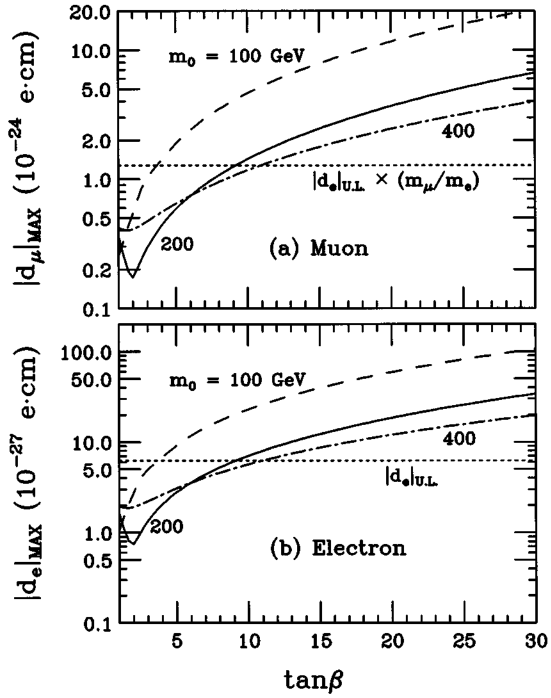

FIG. 7. The maximal EDM allowed by unitarity [Eq. (27)] for (a) the muon EDM and (b) electron EDM, as a function of tanβ for $m_0=100$ (dashed), 200 (solid), and 400 GeV (dash-dotted). Also shown is the experimental upper limit (U.L.) on the electron EDM at 95% C.L. (dotted).

$$|d_\mu|_{\max}=\left|\frac{d_A+d_B}{2}\right||\mathrm{Im}Z_0+\mathrm{Im}\widetilde{Z}_0|+\left|\frac{d_A-d_B}{2}\right||\mathrm{Im}Z_0-\mathrm{Im}\widetilde{Z}_0|$$
$$=\frac{|d_A+d_B|}{4}\cot\beta(1+\tan^2\beta)^{1/2}$$
$$+\frac{|d_A-d_B|}{4}\tan\beta(1+\cot^2\beta)^{1/2}. \tag{27}$$

In Fig. 7, we present the maximal value allowed by unitarity for the muon and the electron EDM's, as a function of tanβ with $m_0=100$, 200, and 400 GeV. Also shown is the experimental upper limit for the electron EDM (denoted by $|d_e|_{\mathrm{U.L.}}$). In this model, for tan$\beta\gtrsim 10$, the simple scaling $(m_\mu/m_e)|d_e|_{\mathrm{U.L.}}$ could be treated as an upper limit for the muon EDM since $d_\mu\approx(m_\mu/m_e)d_e$. A measured value of the muon EDM above this bound could arise if the muon EDM and the electron EDM are generated from different sources, i.e., if this model applies only to the muon EDM.

There are several interesting aspects to note from the different contributions: (i) The contributions from one-loop diagrams and two-loop diagrams with the b and the τ are proportional to $(\mathrm{Im}Z_0+\mathrm{Im}\widetilde{Z}_0)$. (ii) For the the t loop, the coefficient of the Im $\widetilde{Z}_0$ is much smaller than that of the Im Z_0. (iii) The W loop does not contribute to the Im $\widetilde{Z}_0$ term. (iv) The contributions from one-loop diagrams and the W loop have a positive sign while the contributions from heavy fermion loops have a negative sign. Therefore, the total muon EDM is slightly reduced by cancellation. (v) For large tanβ, two-loop diagrams involving the b and the τ dominate and the electron EDM and the muon EDM are almost proportional to their masses. Therefore, $d_\mu\approx(m_\mu/m_e)d_e$ for tan$\beta\gtrsim 10$.

IV. CONCLUSIONS

Our results may be summarized as follows.

For tanβ close to one, contributions from two-loop diagrams involving the t quark and the W boson dominate.

For tan$\beta\gtrsim 10$, contributions from two-loop diagrams involving the b quark and the τ lepton are dominant.

For tan$\beta\sim 1$ or tan$\beta\gtrsim 10$, $d_\mu\approx(m_\mu/m_e)d_e$.

For $8\gtrsim$ tan$\beta\gtrsim 4$, significant cancellations occur among the contributions from two-loop diagrams and the one-loop contribution dominates for tan$\beta\sim 7$.

For tan$\beta>15$ and the lightest Higgs boson mass $m_0<300$ GeV, CP violation mediated by Higgs boson exchange in a two Higgs doublet model could produce a muon EDM which is close to the reach of the proposed BNL experiment [7].

High precision experimental measurements of the muon EDM could provide interesting information about m_0 and tanβ as well as CP violation parameters, ImZ_0, and Im$\widetilde{Z}_i$, $i=0,1,2$. A positive result for the muon EDM measurement could shed light on non-standard CP violation and help pin down the value for tanβ in two Higgs doublet models. A negative result, however, could mean (i) that tanβ is small than 10; or, (ii) the CP violation parameters ImZ_0, and Im$\widetilde{Z}_i$ are smaller than their unitarity bounds; or, (iii) the masses of the neutral Higgs scalars and the Higgs pseudoscalar are very close to one another [10].

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[1] F. Hoogeveen, Nucl. Phys. **B341**, 322 (1990).
[2] I. B. Khriplovich and M. Pospelov, Sov. J. Nucl. Phys. **53**, 638 (1991).
[3] M. Booth, University of Chicago Report No. EFI-93-01, 1993 (unpublished).
[4] I. S. Altarev *et al.*, Phys. Lett. B **276**, 242 (1992); K. F. Smith *et al.*, *ibid.* **234**, 191 (1990).
[5] E. Commins *et al.*, Phys. Rev. A **50**, 2960 (1994); K. Abdullah *et al.*, Phys. Rev. Lett. **65**, 2347 (1990).
[6] J. Bailey *et al.*, J. Phys. G **4**, 345 (1978); J. Bailey *et al.*, Nucl. Phys. **B150**, 1 (1979).
[7] E821 Collaboration at BNL, Y. Semertzidis *et al.* (unpublished).
[8] E821 Collaboration at BNL, ''A New Precision Measurement

of the Muon (g-2) Value at the level of 0.35 ppm,'' Design Report No. BNL AGS E821, 1995 (unpublished).
[9] S. Weinberg, Phys. Rev. Lett. **63**, 2333 (1989).
[10] S. Weinberg, Phys. Rev. D **42**, 860 (1990).
[11] S. Barr and A. Zee, Phys. Rev. Lett. **65**, 21 (1990); **65**, 2920(E) (1990).
[12] S. L. Glashow and S. Weinberg, Phys. Rev. D **15**, 1958 (1977).
[13] G. C. Branco and M. N. Rebelo, Phys. Lett. **160B**, 117 (1985).
[14] J. Liu and L. Wolfenstein, Nucl. Phys. **B289**, 1 (1987).
[15] J. F. Donoghue and L.-F. Li, Phys. Rev. D **19**, 945 (1979).
[16] L. Hall and M. Wise, Nucl. Phys. **B187**, 397 (1981).
[17] R. G. Leigh, S. Paban, and R.-M. Xu, Nucl. Phys. **B352**, 45 (1991).
[18] J. Gunion and R. Vega, Phys. Lett. B **251**, 157 (1990).
[19] D.-W. Chang, W.-Y. Keung, and T.-C. Yuan, Phys. Rev. D **43**, 14 (1991).
[20] C. Kao and R.-M. Xu, Phys. Lett. B **296**, 435 (1992).
[21] CLEO Collaboration, M. S. Alam *et al.*, Phys. Rev. Lett. **74**, 2885 (1995).
[22] R. G. Ellis, G. C. Joshi, and M. Matsuda, Phys. Lett. **179B**, 119 (1986).
[23] T. Rizzo, Phys. Rev. D **38**, 820 (1988).
[24] B. Grinstein and M. Wise, Phys. Lett. B **201**, 274 (1988).
[25] W.-S. Hou and R. Wiley, Phys. Lett. B **202**, 591 (1988).
[26] C.-Q. Geng and J. Ng, Phys. Rev. D **38**, 2857 (1988).
[27] V. Barger, J. L. Hewett, and R. J. N. Phillips, Phys. Rev. D **41**, 3421 (1990).
[28] W. Bernreuther, Z. Phys. C **56**, S97 (1992).
[29] W. Bernreuther and M. Suzuki, Rev. Mod. Phys. **63**, 313 (1991).
[30] A. Das and C. Kao, Phys. Lett. B **372**, 106 (1996).