Glueballs: Charmonium decay and $\overline{p}p$ annihilation

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The vector glueball O, made of 3 valence gluons, is expected to be "clean": it mixes less with quarkonia, but mediates OZI violations. The recent 0^{++} glueball candidate and the persistence of the $J/\psi,\psi' \to \rho\pi$ puzzle suggest $m_O \simeq m_{J/\psi}$, with mixing angle $\sim 2°-4°$; hence, $\Gamma(O\to\rho\pi,\, K^+K^-,\, e^+e^-) \sim$ MeV, few keV, few eV. Lower and upper bounds on Γ_O can be argued from $e^+e^- \to \rho\pi$ energy scan data and the condition $B(O\to\rho\pi) > B(J/\psi\to\rho\pi)$. O dominance may explain the "large" OZI violation in ${}^1S_0(\overline{p}p)\to\phi\gamma$ vs $\omega\gamma$. [S0556-2821(97)04811-X]

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Glueballs are fundamental objects in the sense that, if all quarks were as heavy as charm or bottom quarks, we would still have neutral, quarkless mesons starting around 1–2 GeV in mass, and the lowest-lying ones would be stable. Our world is complicated, however, by the existence of an approximate flavor SU(3) symmetry at the QCD scale. The abundance of $q\overline{q}$ meson states in the 1–2 GeV region and glueball-quarkonium mixings makes the identification of the would-be lightest neutral hadrons extremely difficult. To date, we have not yet established any glueball state beyond doubt.

There has been, however, some recent progress [1] in the 0^{++} scalar glueball sector, where experiment and lattice results are converging. On the one hand, in part due to high statistics studies of $\overline{p}p\to\pi^0 M\overline{M}$ modes [1], there is now an excess of isoscalar 0^{++} mesons, namely, $f_0(1370)$, $f_0(1500)$, and $f_0(1720)$ [2]. Together with the $I=1/2$ and 1 mesons $K_0^*(1430)$ and $a_0(1450)$, they do not all fit into a $q\overline{q}$ nonet [1]. On the other hand, recent lattice calculations predict [1] the 0^{++} glueball mass to be 1600 ± 100 MeV. Although two groups [3,4] claim opposite ends of the above range, their close agreement is in fact quite remarkable. There are thus competing claims that either [5] $f_0(1500)$ or [6] $f_0(1720)$ is the 0^{++} scalar glueball G while the other is dominantly $s\overline{s}$. It is likely, however, that both states have large glueball admixtures [1]. As to the lattice expectation [1] of 2200–2500 MeV for a 2^{++} glueball, further evidence for the $\xi(2230)$ state has been reported [1,7] recently. All these states are seen in $J/\psi\to\gamma+X$ transitions [1], where the "glue content" [8] appears to be high.

The 0^{++} and 2^{++} are $2g$ glueballs in the constituent picture. They are clearly difficult to disentangle from nearby quarkonia. In this paper we are mainly concerned with the lowest-lying 1^{--} glueball state called O, which can only be made of three constituent gluons. Because of its composition, and because it should be heavier [9], asymptotic freedom implies that it would mix less with $q\overline{q}$ mesons. It should therefore retain more of its glueball character, and hence cleaner and easier to interpret [10] once it is seen. Unfortunately, such glueballs are harder to produce since they require three gluons to construct. This brings us naturally, however, to vector charmonium decay, which, according to perturbative QCD, proceeds via three gluons. Interestingly, there has long been [11] some "anomaly" in J/ψ vs ψ' decays that seems to call for the existence of O. Assuming that J/ψ, $\psi'\to 3g\to X$ differ only in the $c\overline{c}$ wave function at the origin, the ratio of branching ratios is expected to follow the so-called 15% rule:

$$R_{\psi'\psi} \equiv B(\psi'\to X)/B(J/\psi\to X)$$
$$\simeq B(\psi'\to e^+e^-)/B(J/\psi\to e^+e^-) \cong 0.15, \qquad (1)$$

which holds for $p\overline{p}$, $p\overline{p}+n\pi$, 5π, 7π, and the recently reported $b_1\pi$ [12] and ϕf_0 [13] modes. However, as originally reported by Mark II [11], and reconfirmed by BES, although quite abundant in J/ψ decay ($\sim 1\%$), the VP modes $\rho\pi$ and $K^*\overline{K}$ are not yet seen for ψ' [13]:

$$B(\psi'\to\rho\pi) < 2.9\times 10^{-5},$$
$$B(\psi'\to K^{*+}K^-) < 3.2\times 10^{-5}. \qquad (2)$$

A similar situation now seems [13] to be emerging for VT modes such as ωf_2, ρa_2 and $K^*\overline{K}_2$. The simplest and most attractive explanation is [10,14] to invoke a nearby $3g$ resonance O that (see Fig. 1) enhances greatly the J/ψ decay into these anomalous channels. However, the BES experiment has recently reported [15] an energy scan of $J/\psi\to\rho\pi$, which appears to rule out the vector glueball O in the so-called Brodsky-Lepage-Tuan (BLT) domain [16]

$$|m_O - m_{J/\psi}| < 80 \text{ MeV}, \quad \Gamma_O < 160 \text{ MeV}. \qquad (3)$$

In this paper, we make a careful assessment of these recent data. We find that 0^{++} data and ψ' results support $m_O \simeq m_{J/\psi}$, while the conclusion drawn from the BES scan

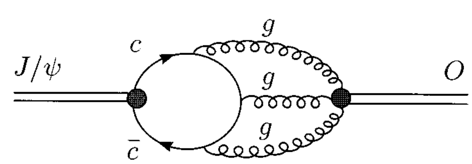

FIG. 1. Mechanism for J/ψ-O mixing. Likewise for other V-O mixings.

is questionable. A consistent decay scenario for O emerges. Further evidence for O is argued from the so-called $\overline{p}p\to\phi+X$ vs $\omega+X$ anomaly.

Shortly after the J/ψ discovery, Freund and Nambu (FN) postulated [17] the existence of a state O which mediates the Okubo-Zweig-Iizuka- (OZI-) [18] violating $\phi\to\rho\pi$ decay (assuming ideal ϕ-ω mixing). This is a ''Pomeron daughter,'' a ''closed string without quarks,'' and hence a (1^{--}) glueball in present terms. Its mass was argued from dual dynamics to be $\sim$1.4–1.8 GeV, and $J/\psi\to\rho\pi$ was predicted to be a dominant decay mode. From a constituent gluon picture, the low-lying $3g$ glueball spectrum was studied [10] by Hou and Soni (HS), assuming two-body forces only. Taking the constituent mass $m_g\sim 500$ MeV [19], it was found that $m_O\cong 4.8m_g\simeq 2.4$ GeV, which is considerably heavier than the estimate of FN. As $\Gamma(J/\psi\to\rho\pi)\cong 1.1$ keV turned out to be much smaller than predicted, the O-V mixings (Fig. 1) were allowed to have the QCD-motivated scale dependence [10]

$$f_{O\omega}:f_{O\phi}:f_{O\psi}=(\sqrt{2}:-1:1)f(q^2). \tag{4}$$

To explain the freshly reported [11] $\rho\pi, K^*\overline{K}$ anomaly, HS invoked [10] a pole dominance or resonance enhancement model: (i) $J/\psi\to O\to\rho\pi \gg J/\psi\to ggg|_{\rm cont}\to\rho\pi$, (ii) $J/\psi\to O\to$other$\ll J/\psi\to ggg|_{\rm cont}\to$other, and (iii) $\psi'\to O\to$any$\ll\psi'\to ggg|_{\rm cont}\to$any, where ''cont'' stands for continuum, and likewise for $K^*\overline{K}$. This leads to the ratio

$$\frac{\Gamma(\psi'\to O\to\rho\pi)}{\Gamma(J/\psi\to O\to\rho\pi)}\cong\left(\frac{m_\psi^2-m_O^2}{m_{\psi'}^2-m_O^2}\right)^2\frac{f_{O\psi'}^2}{f_{O\psi}^2}. \tag{5}$$

As the anomaly deepened, implying [20] that O has to be rather degenerate with J/ψ, BLT [16] included the O width $(m_V^2-m_O^2)^2\to(m_V^2-m_O^2)^2+m_O^2\Gamma_O^2$ and argued that the range of Eq. (3) was implied. Fortuitous as it may seem, recent BES data [13] on $\psi'\to VP$ modes, Eq. (2), continue to support this. We now wish to argue from 0^{++} data that the range of Eq. (3) is also motivated from outside of charmonium physics.

One of the main uncertainties in the potential model is the constituent or dynamical mass $m_g=500\pm 200$ MeV [19]. The 0^{++} glueball G is predicted [21] to have mass $m_G\simeq 2.3m_g$ while the 1^{--} glueball O has mass [10] $m_O\simeq 4.8m_g$. Perhaps the ratio $m_O/m_G\simeq 2.1$ is more trustworthy. Taking $m_G=1400$, 1500, and 1600 MeV, we find $m_g\simeq 610$, 650, and 700 MeV, respectively, which is remarkably close to twice the constituent quark mass $2m_q$. The predicted O mass then is 2920, 3130, and 3340 MeV, respectively, which is near that [22] of Eq. (3). The consequences of this upward shift from the original HS paper to Eq. (3) turns out to be more self-consistent and in better agreement with the data, but were not explored in detail by BLT. First, HS advocated [10] a direct search via $J/\psi,\psi'\to G+O$. Now that $m_G+m_O>m_{\psi'}$, these modes are clearly forbidden, although a $\psi'\to(\pi\pi)_{I=0}+O(\to\rho\pi,K^*\overline{K})$ search should continue. Perhaps one can search for $\Upsilon(1S)\to G+O(\to\rho\pi,K^*\overline{K})$ at CLEO and at future B Factories. Second, assuming O saturation, the known $J/\psi\to\rho\pi$ width provides an important constraint $f(m_{J/\psi}^2)\cong 0.025$ GeV $\times[(m_{J/\psi}^2-m_O^2)^2+m_O^2\Gamma_O^2]^{1/4}$, which leads to the mixing angle

$$\sin\theta_{O\psi}\simeq f(m_{J/\psi}^2)/\sqrt{(m_{J/\psi}^2-m_O^2)^2+m_O^2\Gamma_O^2}$$
$$\simeq 0.025\ \text{GeV}/[(m_{J/\psi}^2-m_O^2)^2+m_O^2\Gamma_O^2]^{1/4}. \tag{6}$$

Refining the mass range of Eq. (3) to (the bounds on Γ_O would be explained later)

$$20\ \text{MeV}<|m_O-m_{J/\psi}|<80\ \text{MeV},$$
$$4\ \text{MeV}\lesssim\Gamma_O\lesssim 30\text{–}50\ \text{MeV}, \tag{7}$$

since the degeneracy of m_O to within 20 MeV (an arbitrarily chosen *ad hoc* value) of J/ψ would be too fortuitous, we find

$$0.035<\sin\theta_{O\psi}<0.071, \tag{8}$$

which is reasonably small. In what follows, we shall use $m_O=3180$ MeV for numerical illustration, where $f(m_{J/\psi}^2)\simeq 0.018$ GeV^2 and $\sin\theta_{O\psi}\simeq 0.034$.

One can now see that, because of the paucity of isocalar 1^{--} mesons, J/ψ-O mixing introduces the chief $q\overline{q}$ content to the state O, while ψ', ϕ, ω mixings with O are suppressed by propagator factors. The two physical states O and J/ψ can be written as [23,24]

$$|J/\psi\rangle\cong+\cos\theta_{O\psi}|c\overline{c}(1S)\rangle+\sin\theta_{O\psi}|ggg\rangle,$$
$$|O\rangle\cong-\sin\theta_{O\psi}|c\overline{c}(1S)\rangle+\cos\theta_{O\psi}|ggg\rangle, \tag{9}$$

where $|c\overline{c}\rangle$ and $|ggg\rangle$ are pure $c\overline{c}$ and ggg states. The pole dominance model with near degeneracy of O and J/ψ then implies that $\Gamma(O\to\rho\pi)\simeq\Gamma(J/\psi\to\rho\pi)/\sin^2\theta_{O\psi}\simeq 1$ MeV, and similarly $\Gamma(O\to K^*\overline{K})\simeq 0.7$ MeV. In contrast, $O\to e^+e^-$ proceeds via its $c\overline{c}$ content, and hence $\Gamma(O\to e^+e^-)\simeq\Gamma(J/\psi\to e^+e^-)\times\sin^2\theta_{O\psi}\simeq 6$ eV, which is extremely small. This is in strong contrast to usual neutral $q\overline{q}$ mesons [10,17]. Assuming ω [10] and ϕ [17] dominance, respectively, one finds that $\Gamma(O\to p\overline{p})\simeq 10$ keV and $\Gamma(O\to K\overline{K})\sim 6$ keV, which is much smaller than $\rho\pi$ and $K^*\overline{K}$ modes. These numbers fit the resonance enhancement model prescription for the $\rho\pi$ anomaly fairly well.

A generic lower bound on $B(O\to\rho\pi)$, and hence an upper bound on Γ_O, can be argued from J/ψ and ψ' data. If $B(O\to\rho\pi)\lesssim B(J/\psi\to\rho\pi)\sim 1\%$, then the $|ggg\rangle$ component of J/ψ would saturate the J/ψ width, and many more modes would violate the 15% rule of Eq. (1). Since this is not the case, we expect $B(O\to\rho\pi)\gtrsim$few %, and $\Gamma_O\lesssim 30$ MeV [hence the upper bound on Γ_O in Eq. (7)] for $m_O=3180$ MeV, which is relatively narrow for such a heavy flavorless hadron. The bound on Γ_O decreases as $m_O\to m_{J/\psi}$.

The immediate question to address is the absence of evidence for an O state in the vicinity of J/ψ. Scanning the $J/\psi\to\rho\pi$ mode over a 40 MeV energy interval, the BES experiment has recently reported [15] the bound

$$\sigma_{O+I}/\sigma_{J/\psi}<0.098, \tag{10}$$

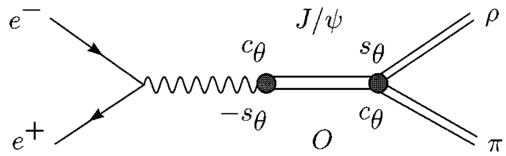

FIG. 2. $e^+e^-\to\rho\pi$ via J/ψ and O intermediate states.

at the 90% confidence level, where σ_{O+I} is the extra cross section due to O and its interference with J/ψ in the energy window. After some analysis, BES claims [15] that a broad and nondegenerate O (with J/ψ) is ruled out. This is quite puzzling, since intuitively a broad state not too close to J/ψ should have been harder to discern. Note that, according to Eq. (9), $e^+e^-\to J/\psi\to\rho\pi$ and $e^+e^-\to O\to\rho\pi$ should have equal total cross sections (see Fig. 2), but the peak cross section for the latter is far less than the former, weighed down by the factor $\Gamma^2_{J/\psi}/\Gamma^2_O$. This is borne out by our numerical example. On closer inspection, one finds that the assumption stated in Eq. (8) of Ref. [15], viz., $B[\psi(2S)\to\rho\pi]/(\sigma_{J/\psi}/\sigma_{\rm tot})\simeq 0.15$, is self-contradictory, since it ignores the $|ggg\rangle$ content of the physical state $|J/\psi\rangle$ which is responsible for $J/\psi\to\rho\pi$ enhancement. Taking Eq. (9) into due account, the scan result of Eq. (10) cannot rule out a glueball state O with $|m_O-m_{J/\psi}|$ and Γ_O greater than a few times the BES energy resolution $\Delta E\simeq 2$ MeV [25,26]. Preliminary analysis along similar lines in the search for O in the $\rho\pi$ invariant mass spectrum of $\psi'\to\pi^+\pi^-+\rho\pi$ decay leads to the bound [27] $\Gamma_O>4$ MeV for $m_O\simeq 3180$ MeV, implying that $B(O\to\rho\pi)<25\%$.

Collecting results, we find 4 MeV$\lesssim\Gamma_O<30$–50 MeV, and few %$\lesssim B(O\to\rho\pi)\lesssim 25\%$ for $m_O\simeq 3180$ MeV. Unlike the old [10] result of HS, where $\Gamma(O\to\rho\pi)$ was estimated to be ~ 50 MeV (since m_O was far away from $m_{J/\psi}$), and hence must be a predominant decay mode, our present result of a dominant but not predominant $O\to\rho\pi$ mode is more plausible. The smallness of Γ_O is in part because $O\to GG$ is phase space and P wave suppressed.

But what about the emerging VT anomaly [13], where the ωf_2, ρa_2, and $K^*\overline{K_2}$ modes are also seen to be suppressed in ψ' decays? Note that the observed $J/\psi\to VP,VT$ and $\eta_c\to VV$ modes are all rather prominent, each of order 1%. As suggested by Anselmino, Genovese, and Kharzeev [23], the η_c could also mix with a 0^{-+} $3g$ glueball (containing sizable $2g$ content [10], which explains the large $\eta_c\to VV$ width compared to $J/\psi\to VP$), which mediates the VV modes. Interestingly, the potential model predicts [10] altogether *four* lowest-lying $3g$ glueballs: one pseudoscalar $0^{-+}(1)$, *two* vectors $1^{--}(0)$ and $1^{--}(2)$, and a spin-3 state $3^{--}(2)$, where the number in parentheses is the total spin of any $(gg)_8$ pair. They are all roughly *degenerate* at $4.8m_g\simeq m_{J/\psi}\simeq m_{\eta_c}$. It appears then that the two final state mesons tend to ''remember'' the original spin configurations. One might picture the glueball as decaying via $g_8(gg)_8\to(q\overline{q})_8(q'\overline{q}')_8$, and the q and q' undergoes some Fierz-like rearrangement before hadronizing. With the near degeneracy of the 1^{--} and 0^{-+} $3g$ states to J/ψ and η_c, one has a crude but common scenario for the observed prominence of $J/\psi\to VP,VT$ and $\eta_c\to VV$ modes. The VP and VV modes are probably highly suppressed in ψ' and

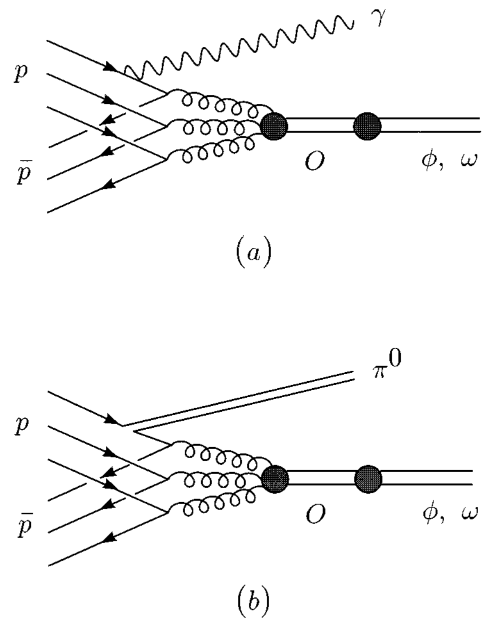

FIG. 3. Scenario for S-state $\overline{p}p\to V\gamma,V\pi^0$ via O dominance.

η_c' decays by total hadron helicity conservation (HHC) [28]. The observation [13] of a suppressed but nonvanishing $\psi'\to\rho a_2$ mode is consistent with the $\psi'\to VT$ modes being allowed by HHC, and the absence of a nearby O pole [20].

Recall that O was originally introduced [17] to explain OZI [18] dynamics. Indeed, using Eq. (4) one obtains from $\phi\to\rho\pi$ decay $f(m^2_\phi)\sim 0.5$ GeV^2, which is of typical hadronic scale, but $f(m^2_\phi)/(m^2_O-m^2_\phi)\sim 0.05$–0.06. The latter is roughly the O-ϕ mixing angle $\sin\theta_{O\phi}$. We note that $\theta_{O\phi}\simeq 3°$–3.4° is very close to the deviation from ideal ω-ϕ mixing, $\delta\cong 3.7°$. Thus, OZI violation in a 1^- nonet is probably rooted in the heaviness of O.

It is fascinating to mention another recent OZI-violating ''anomaly.'' Several experiments have studied $\overline{p}p\to\phi+X$ vs $\omega+X$ with $\overline{p}p$ annihilating at rest [29]. One expects

$$R_X\equiv\frac{\sigma(\overline{p}p\to\phi+X)}{\sigma(\overline{p}p\to\omega+X)}\sim\tan^2\delta\lesssim 1\%, \tag{11}$$

which seems to be respected in most cases, but with two prominent exceptions [30]

$$R_\pi\simeq 0.1,\quad R_\gamma\simeq 0.24. \tag{12}$$

These two cases proceed via specific initial states [29,30] $^1S_0(\overline{p}p)\to V\gamma$ and $^3S_1(\overline{p}p)\to V\pi$. It is plausible that the common $I=0$, spin-1, excited $\overline{p}p$ system annihilates completely into three gluons without leaving behind some $q\overline{q}$'s. We conjecture that there is a substantial resonance contribution (see Fig. 3) from $^1S_0(\overline{p}p)\to(\overline{p}p)^*_{I=0}[\to O\to\phi,\omega]+\gamma$ and $^3S_1(\overline{p}p)\to(\overline{p}p)^*_{I=0}[\to O\to\phi,\omega]+\pi$, where O dominance gives the SU(3) prediction

$$R_X=(1/\sqrt{2})^2=1/2. \tag{13}$$

The experimental results for R_π and R_γ, Eq. (12), should necessarily be smaller since $f(m_\omega^2) > f(m_\phi^2)$, and since there should be more channels (e.g., ''incomplete'' $\overline{p}p$ annihilation) for ω final states, especially for $\omega\pi$. As for other modes $X = \eta$, ρ, ω, $\pi\pi$, etc., they typically involve more partial waves and there is considerably more cross section for final states without $s\overline{s}$. Our explanation of the anomalously large cross section for S-state $\overline{p}p \to \phi\gamma$ and $\phi\pi$, though qualitative, is cogent and simple compared to most other [29] model explanations: O mediates OZI violation. Since the $O \to p\overline{p}$ width is rather small, the strategy may be to search for its 0^{-+} partner as a bump [23] in $p\overline{p}$ cross section around $\sqrt{s} \simeq 3$ GeV.

Let us summarize the main points of this paper. The vector glueball state O, postulated 21 years ago, is alive and well. As the 0^{++} glueball G seems to have emerged with $m_G \sim 1500$–1700 GeV, we argue that m_O is plausibly above 3 GeV. The persistent absence of $\psi' \to \rho\pi, K^*\overline{K}$ modes compared with $B(J/\psi \to \rho\pi, K^*\overline{K}) \sim 1\%$ (the ''$\rho\pi$ anomaly'') then strongly suggest the mass range for O as given in Eq. (7). The absence of distortion in the recent BES energy scan of $J/\psi \to \rho\pi$ does *not* rule out $m_O \simeq m_{J/\psi}$, but serves to put a lower bound on Γ_O, while consistency requires $B(O \to \rho\pi) \gtrsim \text{few} \times B(J/\psi \to \rho\pi)$. The range for Γ_O is also summarized in Eq. (7). A consistent decay picture for O emerges, where few $\% \lesssim B(O \to \rho\pi) \lesssim 25\%$ is a dominant but not predominant mode. The $O \to e^+e^-$ mode is very suppressed, at the eV level, while K^+K^- and $p\overline{p}$ modes are quite suppressed compared to VP modes. The prominence of $J/\psi \to VP, VT$ and $\eta_c \to VV$ modes is explained by resonance enhancement due to nearby $1^{--}(0)$, $1^{--}(2)$, and $0^{-+}(1)$ $3g$ glueball states (the number in parentheses is gg pair spin) predicted by potential models. The OZI suppression of $\phi \to \rho\pi$ is traced to the heaviness of O. The S-state $\overline{p}p \to \phi\gamma$ and $\phi\pi$ annihilation anomaly may be due to O dominance, which is facilitated by the limited number of channels. The search for O should continue in $\psi' \to \pi^+\pi^- + (\rho\pi, K^*\overline{K})$, perhaps via $\Upsilon(1S) \to G + O$, and in $\overline{p}p$ annihilation in flight. Once clearly seen, the glueball nature of O should be unequivocal.

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