Decay constants of B, B^* and D, D^* mesons in the relativistic mock meson model

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We derive formulas for the decay constants f_P and f_V of pseudoscalar and vector mesons in the relativistic mock meson model. Using these formulas, we obtain f_P, f_V, and f_V/f_P of B_s, B_d, D_s, and D_d mesons as functions of the mock meson parameter β. Then by using the values of β which are obtained by the variational calculation in the relativistic quark model, we obtain the numerical values of the decay constants f_P and f_V of the B and D mesons and their ratios f_V/f_P. The results are compared with other calculations and existing experimental results. [S0556-2821(97)01611-1]

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B meson decays are expected to provide the CP violation phenomena [1], for which we have had only the $K_L \to \pi\pi$ decay [2] and the charge asymmetry in the decay $K_L \to \pi^{\pm} l^{\mp} \nu$ [3] for more than 30 years. The B factories at KEK and SLAC are under construction with this anticipation. The mechanism of CP violation through the complex phase of the Kobayashi-Maskawa (KM) three family mixing matrix [4] in the Weinberg-Salam model is presently the standard model for the CP violation. In order to understand and precisely probe this model, it is crucial to know the values of the KM matrix elements accurately and confirm the unitarity triangle [5]. However, the uncertainty in the knowledge of the decay constants of B and D mesons hinders seriously the precise extraction of the KM matrix elements from experimental data. For example, the magnitudes of the B_d-$\overline{B}_d$ and B_s-$\overline{B}_s$ mixings are proportional to $f_{B_d}^2 |V_{td}|^2$ and $f_{B_s}^2 |V_{ts}|^2$, respectively [6], so it is essential to know f_{B_d} and f_{B_s} reliably in order to obtain the values of $|V_{td}|$ and $|V_{ts}|$ from the results of their mixing experiments. However, the theoretical calculation of the B and D meson decay constants is difficult because it is in the realm of nonperturbative QCD and the motion of the light quark in B and D mesons is relativistic. On the other hand, understanding the decay constant better is also invaluable because its information reveals the inside structure of the hadron. Especially, a clear understanding of the difference between the decay constant of vector (B^*, D^*) and pseudoscalar (B, D) mesons will provide sound insight for their structures.

The Van Royen–Weisskopf formula [7] is used in many occasions for the meson decay constants. This formula is obtained in the two-component spinor limit where the spinors of the quarks inside meson are approximated to two-component Pauli spinors [7,8], and in this limit the decay constants f_P and f_V of the peudoscalar and vector mesons become the same when we approximate their masses the same [9]: $f_P = f_V = \sqrt{12/m_M}\ |\psi(0)|$, where m_M is the meson mass and $\psi(0)$ is the spatial wave function at origin. The relation $f_P = f_V$ is commonly used for the B and D mesons. However, in the B or D meson the light quark moves with large velocity and describing the quarks inside meson by two-component spinors is not legitimate. They should be described fully by four-component Dirac spinors. Then the different spin structures of pseudoscalar and vector mesons make f_P and f_V have different values. We will show this fact clearly by deriving the formulas for both f_P and f_V simultaneously. Of course, the Van Royen–Weisskopf formula is also modified by this four-component spinor consideration of the quarks inside the meson. We will obtain the numerical results of f_P, f_V and their ratios f_V/f_P of the B and D mesons.

We work in the relativistic mock meson model of Godfrey and co-workers [10–12], in which the heavy meson state composed of a light quark q and a heavy antiquark $\overline{Q}$ is represented as

$$|M(\mathbf{K})\rangle = \sqrt{2m_M} \int d^3p\ \Phi(\mathbf{p})\ \chi_{s\bar{s}}\,\phi_{c\bar{c}} \times \left| q\left(\frac{m_q}{m}\mathbf{K}+\mathbf{p},s\right)\ \overline{Q}\left(\frac{m_{\overline{Q}}}{m}\mathbf{K}-\mathbf{p},\bar{s}\right)\right\rangle, \tag{1}$$

where $\mathbf{K}$ is the mock meson momentum, $m \equiv m_q + m_{\overline{Q}}$, and $\Phi(\mathbf{p})$, $\chi_{s\bar{s}}$, and $\phi_{c\bar{c}}$ are momentum, spin, and color wave functions respectively. We take the momentum wave function $\Phi(\mathbf{p})$ as a Gaussian wave function

$$\Phi(\mathbf{p}) = \frac{1}{(\sqrt{\pi}\beta)^{3/2}} e^{-\mathbf{p}^2/2\beta^2}. \tag{2}$$

The decay constants of the pseudoscalar and vector mesons, f_P and f_V, respectively, are defined by

$$\langle 0|\overline{Q}\gamma^{\mu}\gamma_5 q\ |M_P(\mathbf{K})\rangle = f_P K^{\mu},$$

$$\langle 0|\overline{Q}\gamma^{\mu} q\ |M_V(\mathbf{K},\varepsilon)\rangle = f_V m_V \varepsilon^{\mu}. \tag{3}$$

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In this paper we work in the meson rest frame (where $\mathbf{p}_q=-\mathbf{p}_{\bar{Q}}$), then the meson state in Eq. (1) is written as

$$|M_P(\mathbf{0})\rangle=\sqrt{2m_P}\int\frac{d^3p_q}{(2\pi)^{3/2}\sqrt{2E_q2E_{\bar{Q}}}}\times\Phi(\mathbf{p}_q)\frac{1}{\sqrt{N_c}}\frac{1}{\sqrt{2}}[a_\uparrow^\dagger(\mathbf{p}_q,c)b_\downarrow^\dagger(\mathbf{p}_{\bar{Q}},\bar{c})-a_\downarrow^\dagger(\mathbf{p}_q,c)b_\uparrow^\dagger(\mathbf{p}_{\bar{Q}},\bar{c})]\,|0\rangle, \tag{4}$$

where the arrow indicates a state with spin up (down) along a fixed axis and c is the color index which is summed. Whereas we wrote the pseudoscalar meson state in Eq. (4), we can also write the vector meson state in the same way with the spin combinations for the vector states, which are given by $(\uparrow\uparrow)$, $1/\sqrt{2}\,(\uparrow\downarrow+\downarrow\uparrow)$, and $(\downarrow\downarrow)$. In Eq. (4) we adopted the normalization of the creation and annihilation operators given by $\{a(\mathbf{p},s),a^\dagger(\mathbf{p}',s')\}=(2\pi)^3 2E\delta_{ss'}\delta^3(\mathbf{p}-\mathbf{p}')$, and then the meson state in Eq. (4) is normalized by $\langle M_P(\mathbf{0})|M_P(\mathbf{0})\rangle=2m_P\delta^3(\mathbf{0})$, and also in the same way for the vector meson states. We note that there is an ambiguity of relativistic covariance in extracting the meson decay constant from the relativistic mock meson model [12]. We avoid this ambiguity by working in the meson rest frame.

Since we are concerned with the matrix elements in the left-hand side of Eq. (3) with the meson states in Eq. (4), it is convenient to represent the meson states by

$$\Psi_P\equiv-\langle 0|\,q\bar{Q}\,|M_P(\mathbf{0})\rangle,\quad \Psi_V\equiv-\langle 0|\,q\bar{Q}\,|M_V(\mathbf{0})\rangle, \tag{5}$$

with which the formulas in Eq. (3) are written as

$$\mathrm{Tr}(\gamma^0\gamma_5\Psi_P)=f_Pm_P,\quad \mathrm{Tr}(\gamma^\mu\Psi_V)=f_Vm_V\varepsilon^\mu. \tag{6}$$

If both quarks inside the meson are static, the spinor combinations of $u(\mathbf{0})\overline{v(\mathbf{0})}$ for the pseudoscalar and vector meson states are given, respectively, as [13,14]

$$P(\mathbf{0},\mathbf{0})=-\frac{1}{\sqrt{2}}\frac{1+\gamma^0}{2}\gamma^5,\quad V(\mathbf{0},\mathbf{0},\varepsilon)=\frac{1}{\sqrt{2}}\frac{1+\gamma^0}{2}\not{\varepsilon}, \tag{7}$$

where the polarization vectors of the vector meson are given by $\varepsilon_\pm^\mu=(1/\sqrt{2})(0,1,\pm i,0)$ and $\varepsilon_3^\mu=(0,0,0,1)$. However, since the quarks inside the mock meson represented by Eq. (4) are not static, we boost the spinors by using the formulas

$$u^{(\alpha)}(k)=\frac{\not{k}+m}{\sqrt{2m(m+E)}}u^{(\alpha)}(m,\mathbf{0}),$$

$$\overline{v^{(\alpha)}}(k)=\overline{v^{(\alpha)}}(m,\mathbf{0})\frac{-\not{k}+m}{\sqrt{2m(m+E)}}. \tag{8}$$

Then, through this procedure we obtain Ψ_P and Ψ_V in Eq. (5) as

$$\Psi_I=\sqrt{2m_I}\int\frac{d^3p_q}{(2\pi)^{3/2}}\Phi(\mathbf{p}_q)\frac{\sqrt{N_c}}{\sqrt{2E_q\,2E_{\bar{Q}}}}\times\frac{\not{p}_q+m_q}{\sqrt{2m_q(m_q+E_q)}}S_I\frac{-\not{p}_{\bar{Q}}+m_{\bar{Q}}}{\sqrt{2m_{\bar{Q}}(m_{\bar{Q}}+E_{\bar{Q}})}}, \tag{9}$$

where $I=P$ or V, and S_P and S_V are, respectively, $P(\mathbf{0},\mathbf{0})$ and $V(\mathbf{0},\mathbf{0},\varepsilon)$ in Eq. (7). By incorporating Eq. (9) into Eq. (6), we obtain the following formulas for the decay constants of pseudoscalar and vector mesons in the relativistic mock meson model:

$$f_I=\frac{2\sqrt{3}}{\sqrt{m_I}}\int\frac{d^3p}{(2\pi)^{3/2}}\Phi(\mathbf{p})\left(\frac{E_q+m_q}{2E_q}\frac{E_{\bar{Q}}+m_{\bar{Q}}}{2E_{\bar{Q}}}\right)^{1/2}\times\left(1+a_I\frac{\mathbf{p}^2}{(E_q+m_q)(E_{\bar{Q}}+m_{\bar{Q}})}\right), \tag{10}$$

where $I=P$ or V and

$$a_P=-1,\quad a_V=+\tfrac{1}{3}. \tag{11}$$

We note that the above formulas for the decay constants become the Van Royen–Weisskopf formula $f_I=\sqrt{12/m_I}|\psi(0)|$ in the two-component spinor limit which corresponds to taking the $\mathbf{p}\to\mathbf{0}$ limit in the last two factors of Eq. (10). The formula for f_P in Eqs. (10) and (11) was already obtained by Godfrey in Ref. [11], however, we derived the formulas for both f_P and f_V simultaneously, from which the similarity and difference between f_P and f_V can be seen clearly.

When the meson and quark masses are given, f_P and f_V can be calculated from Eqs. (10) and (11) for a given value of the parameter β in Eq. (2). We obtained numerically f_P and f_V of B_s, B_d, D_s, and D_d mesons as functions of β by using the meson masses given by [15]

$$m_{B_s}=5.375\text{ GeV},\quad m_{B_d}=5.279\text{ GeV},$$

$$m_{D_s}=1.969\text{ GeV},\quad m_{D_d}=1.869\text{ GeV},$$

$$m_{B_s^*}=5.422\text{ GeV},\quad m_{B_d^*}=5.325\text{ GeV},$$

$$m_{D_s^*}=2.110\text{ GeV},\quad m_{D_d^*}=2.010\text{ GeV}, \tag{12}$$

and the constituent quark masses given by [16]

$$m_u=m_d=0.33\text{ GeV},\quad m_s=0.53\text{ GeV},$$

$$m_c=1.78\text{ GeV},\quad m_b=5.17\text{ GeV}. \tag{13}$$

The reason why we use the constituent quark masses for the quark masses in Eq. (10) in the present work is because we will deal with the potential $V(r)$ in Eq. (20) of the potential model which is the effective potential from the contribution of gluon, and the constituent quark masses are effective quark masses while the current ones are bare ones. In Figs. 1 and 2 we present the obtained f_P and f_V as functions of β for

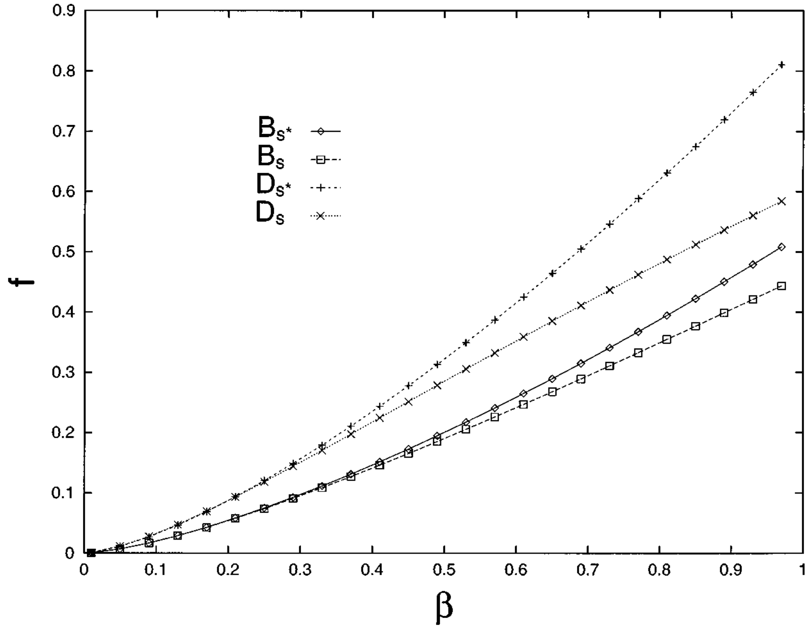

FIG. 1. $f_{B_s^*}$, f_{B_s}, $f_{D_s^*}$, and f_{D_s} (GeV) as functions of the parameter β (GeV).

the B and D mesons having s and d quarks as their light quark, and in Fig. 3 the ratios f_V/f_P also as a function of β.

From Figs. 1–3 we can see the dependence of the decay constants and their ratios on the parameter β, and we find that the value of β is very important in the calculation of the decay constants. Then, the problem is determining which values of β are the right values for the physical B and D mesons. Capstick and Godfrey calculated the pseudoscalar meson decay constants f_P by using the values of β obtained from the effective-harmonic oscillator potential [11,12]. They also gave the results obtained by using the β's from the variational calculation of Schrödinger wave functions by Isgur, Scora, Grinstein, and Wise [17] (β_B=0.41 GeV, β_D=0.39 GeV), and the β's obtained by fitting harmonic-oscillator wave functions to the rms radii of the wave functions from the variational calculation of the relativistic Hamiltonian [10,12] (β_{B_s}=0.636 GeV, β_{B_d}=0.580 GeV, β_{D_s}=0.651 GeV, β_{D_d}=0.601 GeV).

Since it is essential to obtain accurate values of β in order to get reliable results for f_P, f_V, and f_V/f_P of the B and D mesons, we calculate in this paper the β's by using the following six different potentials [18–23], which we also display in Fig. 4 [24]. We note in Fig. 4 the tendency that the potential which has higher values of potential energy in the short range has lower values in the long range, and vice versa.

(A) Coulomb-plus-linear potential of Eichten *et al.* [18]:

$$V(r) = -\frac{\alpha_c}{r} + Kr, \tag{14}$$

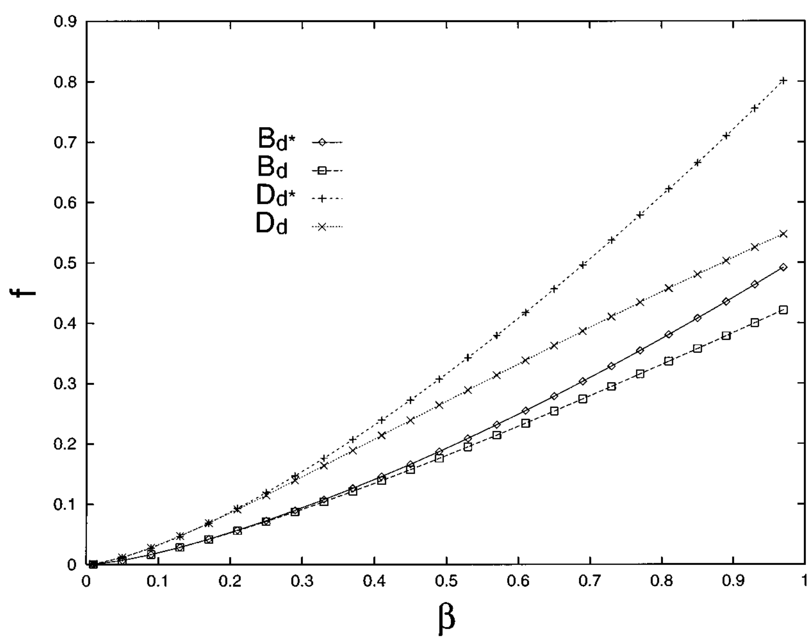

FIG. 2. $f_{B_d^*}$, f_{B_d}, $f_{D_d^*}$, and f_{D_d} (GeV) as functions of the parameter β (GeV).

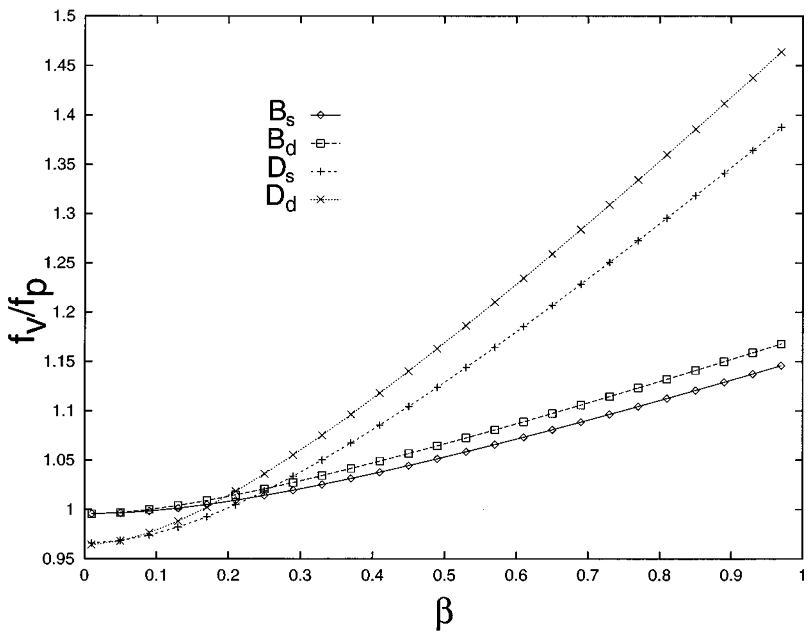

FIG. 3. $f_{B_s^*}/f_{B_s}$, $f_{B_d^*}/f_{B_d}$, $f_{D_s^*}/f_{D_s}$, and $f_{D_d^*}/f_{D_d}$ as functions of the parameter β (GeV).

with $\alpha_c = 0.52$, $K = 1/(2.34)^2$ GeV^2, $m_c = 1.84$ GeV, $m_b = 5.18$ GeV.

(B) Coulomb-plus-linear potential (14) of Hagiwara *et al.* [19], with $\alpha_c = 0.47$, $K = 0.19$ GeV^2, $m_c = 1.32$ GeV, $m_b = 4.75$ GeV.

(C) Power-law potential of Martin [20]:

$$V(r) = -8.064\ \text{GeV} + (6.898\ \text{GeV})(r\times 1\ \text{GeV})^{0.1}, \tag{15}$$

with $m_c = 1.8$ GeV, $m_b = 5.174$ GeV.

(D) Power-law potential of Rosner *et al.* [21]:

$$V(r) = -0.772\ \text{GeV} + 0.801[(r\cdot 1\ \text{GeV})^{\alpha} - 1]/\alpha, \tag{16}$$

with $\alpha = -0.12$, $m_c = 1.56$ GeV, $m_b = 4.96$ GeV.

(E) Logarithmic potential of Quigg and Rosner [22]:

$$V(r) = -0.6635\ \text{GeV} + (0.733\ \text{GeV})\ln(r\cdot 1\ \text{GeV}), \tag{17}$$

with $m_c = 1.5$ GeV, $m_b = 4.906$ GeV.

(F) Richardson potential [23]:

$$V(r) = \frac{8\pi}{33-2n_f}\Lambda\left(\Lambda r - \frac{f(\Lambda r)}{\Lambda r}\right),$$

$$f(t) = 1 - 4\int_1^{\infty}\frac{dq}{q}\frac{e^{-qt}}{[\ln(q^2-1)]^2+\pi^2}, \tag{18}$$

with $n_f = 3$, $\Lambda = 0.398$ GeV, $m_c = 1.491$ GeV, $m_b = 4.884$ GeV.

We note that in this paper we used the variational method with the Gaussian trial wave function in Eqs. (2) or (21) in the same spirit as the approaches of Refs. [10–12,17,25]. In

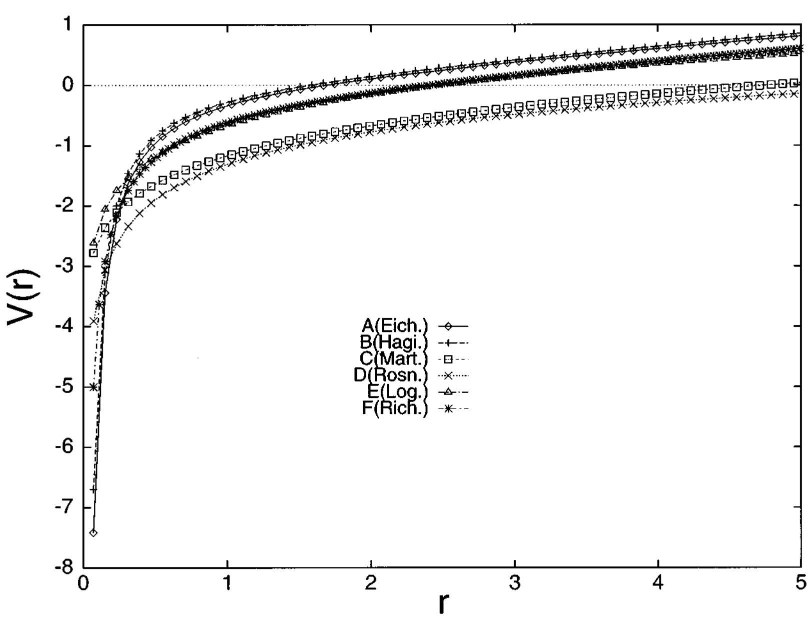

FIG. 4. The interquark potentials of the potential models in [18–23]. The radial distance of the horizontal axis is in the unit of GeV^{-1} (1 GeV^{-1} = 0.197 fm), and the potential energy of the vertical axis is in the unit of GeV.

the recent work Veseli and Dunietz [26] calculated the decay constants of P and D wave heavy-light mesons as well as of S wave ones. They considered the relativistic Hamiltonian given in Eq. (20) with the Coulomb-plus-linear potential

$$V(r)=-\frac{4}{3}\frac{\alpha_s}{r}+br+c. \tag{19}$$

In their calculations they used the method which diagonalizes the Hamiltonian matrix in a particular (truncated) basis, with the basis states depending on some variational parameter. They displayed the results for various decay constants as functions of the number of basis states included in the calculations and pointed out that there is a pronounced lack of convergence for the S wave. They also concluded that the calculation of the decay constants of S wave mesons using the relativistic meson model is not reliable, because the resulting values of the decay constants are quite sensitive to the particular choice of parameters of the model. They noted that the above problems are rooted in the fact that the wave function at the origin of the S wave is not well defined and in fact divergent for realistic short-distance potentials [26,27]. In the same work, they also noted that one possible solution to the above problems would be to replace the $1/r$ potential with the one-loop single gluon exchange potential, i.e., $\alpha_s \to \alpha_s(r)$. With this replacement the divergence of the wave function is only logarithmic [27], and this should lead to much more stable results and the results should also be much less dependent on the specific choice of the model parameters. In the present paper, we used six different potentials (A)–(F). The potentials (A) and (B) are the Coulomb-plus-linear potentials which are the same type as that in Eq. (19), and our results from these two potentials are subject to the above problems which Veseli and Dunietz pointed out. In this context, we would like to consider our calculations as being based on the assumption that the ground-state wave function of the S wave meson can be effectively represented by a Gaussian function with an appropriate variational parameter β, which is a kind of effective harmonic-oscillator approach. On the other hand, the other four potentials (C)–(F) are much less singular than the Coulomb-plus-linear potential (19). In fact, the potential (F) is the same type potential as that given by the replacement of $\alpha_s \to \alpha_s(r)$, and the potentials (C)–(E) are even less singular than the potential (F) as can be seen from Fig. 4 and Eqs. (15)–(18).

Originally, the potentials (A)–(F) were obtained by fitting the data of the J/ψ and Υ families (mainly their spectra) which are composed of two heavy quarks, however, we use them in the study of heavy-light mesons in the present work. We expect this usage is reasonable for the following reason. We obtain $\bar{\beta}=0.61\pm0.02$ GeV and 0.51 ± 0.01 GeV for the B_d and D_d meson, respectively, which give the root mean square radius $\langle r^2\rangle^{1/2}=3/(2\bar{\beta})=2.46\ \mathrm{GeV}^{-1}$ and $2.94\ \mathrm{GeV}^{-1}$ ($1\ \mathrm{GeV}^{-1}=0.197$ fm), respectively. So, the dominant interquark distance in the B and D mesons is the confining long-distant linear potential part, as we can see in Fig. 4. When we perform the same calculations for the J/ψ and Υ mesons, we get $\bar{\beta}=0.67\pm0.04$ GeV and 1.19 ± 0.07 GeV [28], which give $\langle r^2\rangle^{1/2}=2.24\ \mathrm{GeV}^{-1}$ and $1.26\ \mathrm{GeV}^{-1}$, respectively. Therefore, as we see in Fig. 4, even though the asymptotic-free short-distant Coulombic potential part is more influential for the J/ψ and Υ mesons than for the B and D mesons, the linear potential part is more important than the Coulombic potential part even for the J/ψ and Υ mesons, especially for the J/ψ meson. Moreover, since the string tension K which is the coefficient of the linear potential is widely believed to be universally given as $K=0.18\sim0.19\ \mathrm{GeV}^2$, we think it is reasonable to use the potentials in Fig. 4 for the study of the B and D mesons. The relative vertical shifts among the potentials in Fig. 4 are not relevant in our study, because we calculate the values of $\bar{\beta}$ which minimize the expectation value of the Hamiltonian.

In order to obtain the values of $\bar{\beta}$, we apply the variational method to the following relativistic Hamiltonian since the motion of the light quark inside the B or D meson is relativistic [24,29]:

$$H=\sqrt{\mathbf{p}^2+m_q{}^2}+\sqrt{\mathbf{p}^2+m_Q{}^2}+V(r), \tag{20}$$

where $\mathbf{r}$ and $\mathbf{p}$ are the relative coordinate and its conjugate momentum. The Hamiltonian in Eq. (20) represents the energy of the meson in the meson rest frame, since in this reference frame the magnitude of the momentum of each quark is the same as that of the conjugate momentum of the relative coordinate. We should note that we have not attained the covariant Hamiltonian of the relativistic bound state; instead, we have incorporated the relativistic kinetic energy term in the Hamiltonian (20). The usual potential is a nonrelativistic construct, and it is not clear how to obtain corrections to it in a consistent expansion. Also, the confining part of the potential only works out to a certain distance r before it becomes energetically favorable to create real quark-antiquark pairs out of the vacuum.

We take the Gaussian wave function in Eq. (2) as the trial wave function with the variational parameter β. The Fourier transform of $\Phi(\mathbf{p})$ in Eq. (2) gives the spatial wave function

$$\psi(\mathbf{r})=\left(\frac{\beta}{\sqrt{\pi}}\right)^{3/2}e^{-\beta^2r^2/2}, \tag{21}$$

which is also Gaussian. The ground state is given by minimizing $\langle H\rangle=\langle\psi|H|\psi\rangle=E(\beta)$, that is, $dE(\beta)/d\beta=0$ at $\beta=\bar{\beta}$, where $\bar{\beta}$ represents the inverse size of the meson $(\langle r^2\rangle^{1/2}=3/(2\bar{\beta}))$, and $\bar{E}\equiv E(\bar{\beta})$ the meson mass m_M. In this variational calculation, we took the potential $V(r)$ in Eq. (20) from six different potentials of Refs. [18–23]. For the light quark mass m_q in Eq. (20) we use the constituent quark masses m_s and m_d in Eq. (13), however, for the heavy quark masses m_b and m_c we use the values given in each potential model as the fitted parameter values. We present the results of the variational calculation in Table I, which gives the following average values of $\bar{\beta}$:

$$\bar{\beta}(B_s)=0.65\pm0.02\ \mathrm{GeV},\quad \bar{\beta}(B_d)=0.61\pm0.02\ \mathrm{GeV},$$
$$\bar{\beta}(D_s)=0.55\pm0.02\ \mathrm{GeV},\quad \bar{\beta}(D_d)=0.51\pm0.01\ \mathrm{GeV}, \tag{22}$$

where the specified errors are values calculated from six different results from six potential models. Equation (22) shows $\bar{\beta}(B_s)>\bar{\beta}(B_d)>\bar{\beta}(D_s)>\bar{\beta}(D_d)$, that is, $\bar{\beta}$ becomes bigger

TABLE I. The values of the variational parameter β of the Gaussian wave function which minimize $\langle H\rangle$, and the corresponding values of the minimum energy.

Model	$\overline{\beta}(B_s)$	$\overline{E}(B_s)$	$\overline{\beta}(B_d)$	$\overline{E}(B_d)$	$\overline{\beta}(D_s)$	$\overline{E}(D_s)$	$\overline{\beta}(D_d)$	$\overline{E}(D_d)$
A (Eich.) [18]	0.636	6.100	0.591	5.988	0.547	2.843	0.515	2.722
B (Hagi.) [19]	0.616	5.723	0.574	5.609	0.512	2.406	0.484	2.280
C (Power 1) [20]	0.652	5.317	0.612	5.208	0.563	2.034	0.531	1.915
D (Power 2) [21]	0.675	5.514	0.628	5.406	0.555	2.223	0.519	2.102
E (Log.) [22]	0.655	5.559	0.612	5.450	0.545	2.263	0.512	2.141
F (Rich.) [23]	0.653	5.569	0.610	5.459	0.545	2.285	0.514	2.164
(Average)	0.648	5.630	0.605	5.520	0.545	2.342	0.513	2.221

for the bigger quark masses inside the meson. This aspect can be understood since the distance between quarks inside the meson becomes shorter when the quark masses are bigger and the meson size is inversely proportional to $\overline{\beta}$. In Table I the meson masses $\overline{E}$ are rather bigger than the experimental values of $(m_P+3m_V)/4$ from Eq. (12). However, if we incorporate a constant term V_0 in the potential $V(r)$ in Eq. (20) which is commonly used for the B and D meson systems (in Ref. [30] Fulcher assigned $V_0=0$ for Υ and ψ systems, -0.213 GeV for B mesons, and -0.244 GeV for D mesons), we get reasonable values of $\overline{E}$ for the B and D meson masses. We note that we did not include the chromomagnetic hyperfine interaction term in the above variational calculation by considering this term as a perturbation term. If we treat this spin-dependent Hamiltonian nonperturbatively, it would probably give spin-dependent corrections to the wave functions, then the values of $\overline{\beta}$ in Eq. (22) would receive corrections since $|\psi(0)|=(\overline{\beta}/\sqrt{\pi})^{3/2}$ from Eq. (21). This nonperturbative treatment is expected to make $|\psi_V(0)/\psi_P(0)|$ smaller than 1 [31], where V and P mean vector and pseudoscalar, respectively, then $\overline{\beta}_V$ will be smaller than $\overline{\beta}_P$. If this situation happens, the ratios f_V/f_P in Eq. (24) will become smaller than the values presented in Eq. (24), and the inequality in Eq. (25) will not be, in general, satisfied either. However, in this paper we do not take into account this correction.

With the values of $\overline{\beta}$ in Table I we calculated the decay constants by using the formulas in Eqs. (10) and (11). In this calculation we used the meson masses in Eq. (12), the constituent quark masses for the light quarks m_s and m_d in Eq. (13), and the heavy quark masses m_b and m_c given in each potential model. The results we obtained are as follows in MeV units [32,33]:

$$f_{B_s}=266\pm 10, \quad f_{B_d}=231\pm 9,$$

$$f_{D_s}=309\pm 15, \quad f_{D_d}=271\pm 14;$$

$$f_{B_s^*}=289\pm 11, \quad f_{B_d^*}=252\pm 10,$$

$$f_{D_s^*}=362\pm 15, \quad f_{D_d^*}=327\pm 13. \tag{23}$$

We also present the detailed results in Table II, from which we get the ratios of the vector and pseudoscalar meson decay constants:

$$\frac{f_{B_s^*}}{f_{B_s}}=1.09\pm 0.01, \quad \frac{f_{B_d^*}}{f_{B_d}}=1.09\pm 0.01,$$

$$\frac{f_{D_s^*}}{f_{D_s}}=1.17\pm 0.02, \quad \frac{f_{D_d^*}}{f_{D_d}}=1.21\pm 0.02. \tag{24}$$

In Table III we compare our results of the ratios in Eq. (24) with other works: Neubert's results by the heavy quark effective theory [14] and the lattice results of the ELC group [34]. In Eq. (24) we see that the ratios for D mesons are bigger than those for B mesons, which can be understood by the fact that the second term in the last factor of Eq. (10) contributes more for D mesons than for B mesons. We also find that Eq. (10) gives the inequality

TABLE II. The pseudoscalar and vector meson decay constants (MeV) in six different potential models.

Model	f_{B_s}	$f_{B_s^*}$	f_{B_d}	$f_{B_d^*}$	f_{D_s}	$f_{D_s^*}$	f_{D_d}	$f_{D_d^*}$
A (Eich.) [18]	261	281	224	243	319	365	281	329
B (Hagi.) [19]	248	269	215	234	279	331	245	301
C (Power 1) [20]	269	291	235	256	328	380	290	343
D (Power 2) [21]	280	305	242	265	315	372	275	332
E (Log.) [22]	270	293	234	256	306	362	268	326
F (Rich.) [23]	268	292	233	255	306	362	269	328
(Average)	266 ± 10	289 ± 11	231 ± 9	252 ± 10	309 ± 15	362 ± 15	271 ± 14	327 ± 13

TABLE III. The ratios of the decay constants of vector and pseudoscalar mesons.

	$f_{B_s^*}/f_{B_s}$	$f_{B_d^*}/f_{B_d}$	$f_{D_s^*}/f_{D_s}$	$f_{D_d^*}/f_{D_d}$
This Work	1.09 ± 0.01	1.09 ± 0.01	1.17 ± 0.02	1.21 ± 0.02
Neubert [14]		1.07 ± 0.02		1.35 ± 0.05
ELC [34]		1.12 ± 0.05		1.30 ± 0.06

$$\sqrt{m_V}\, f_V \geqslant \sqrt{m_P}\, f_P . \tag{25}$$

In Eq. (25) the equality holds in the two-component spinor limit in which the Van Royen–Weisskopf formula holds. Another point which we note from the results in Eq. (23) is that the ratios of f_B/f_D are enhanced compared with the nonrelativistic scaling law $f_B/f_D = \sqrt{m_D/m_B}$ [9] by the factors 1.42, 1.43, 1.28, and 1.25 for f_{B_s}/f_{D_s}, f_{B_d}/f_{D_d}, $f_{B_s^*}/f_{D_s^*}$, and $f_{B_d^*}/f_{D_d^*}$, respectively.

Recently, there has been much development in the heavy quark effective theory. This theory relates the axial currents in Eq. (3) in different energy scales, and gives the relation [35]

$$\frac{f_B}{f_D} = \left(\frac{m_D}{m_B}\right)^{1/2} \left(\frac{\alpha_s(m_B)}{\alpha_s(m_D)}\right)^{-6/(33-2N_f)}, \tag{26}$$

where we take $N_f = 4$ which is the number of flavors appropriate to the mass scale in the interval between m_B and m_D. Using the formula for the running coupling constant $\alpha_s(q^2) = 12\pi/[(33-2N_f)\ \ln(-q^2/\Lambda^2)]$ with $\Lambda = \Lambda_4 = 0.28$ GeV [15], Eq. (26) gives $f_{B_d}/f_{D_d} = 0.66$, which is smaller than our result $f_{B_d}/f_{D_d} = 0.85$ which we obtain from Eq. (23).

We compare our results of the pseudoscalar meson decay constants with other theoretical calculations and existing experimental results in Table IV. The reason why we do not include the vector meson decay constants in this table is because the previous calculations did not obtain those values by distinguishing them from the pseudoscalar meson decay constants, except for the calculations by Neubert and the ELC lattice group presented in Table III. The second row is the results of Capstick and Godfrey in Ref. [12], in which they employed the spatial components of the first formula in Eq. (3). As Capstick and Godfrey pointed out in Ref. [12], there exists an ambiguity of relativistic covariance in extracting the pseudoscalar meson decay constant from the relativistic mock meson model and they used the spatial components, while Godfrey used the time component [11] which is equivalent to our calculation of f_P. We avoid this ambiguity by working in the meson rest frame. For resolving this ambiguity it is necessary to have a completely relativistic representation of the mock meson which is lacking at the present time because of the complicated structure of the meson. Dominguez performed the calculation by the QCD sum rules approach [36]. The fourth and fifth rows are from the lattice calculations of the UKQCD group [37] and Bernard *et al.* [38]. The existing experimental results of the WA75 [39], CLEO [40,41], BES [42], and E653 [43] groups are also presented in Table IV. The presently available experimental results for the heavy-light meson decay constants are only f_{D_s} from the measurements of the decay rate of the purely leptonic $D_s^+ \to \mu^+ \nu$ process. CLEO 1 and CLEO 2 are old and new results of the CLEO group, respectively. We note that our result of f_{D_s} is in the middle of the mean values of the CLEO's old and new results.

In conclusion, we derived the formulas for f_P and f_V simultaneously in the relativistic mock meson model. From them we could see the similarity and difference between f_P and f_V clearly. By using the formulas we obtained the numerical values of f_P, f_V, and their ratios, and then compared the results with other theoretical calculations and existing experimental measurements. We note that the spin-dependent interaction Hamiltonian would probably introduce corrections to the wave functions when it is treated nonperturbatively, whereas we did not take into account this correction in the present work by considering it as a perturbative term. In this paper, we have focused on how the different mock meson spin structures influence the meson decay constants, and investigated its results in detail.

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TABLE IV. The values (MeV) and ratios of the decay constants from different calculations and experimental results.

	f_{B_s}	f_{B_d}	f_{B_s}/f_{B_d}	f_{D_s}	f_{D_d}	f_{D_s}/f_{D_d}
This Work	266 ± 10	231 ± 9	1.15 ± 0.01	309 ± 15	271 ± 14	1.14 ± 0.01
Cap. God. [12]	210 ± 20	155 ± 15	1.35 ± 0.18	290 ± 20	240 ± 20	1.21 ± 0.13
Doming. [36]	193 ± 28	158 ± 25	1.22 ± 0.02	222 ± 48	187 ± 48	1.21 ± 0.06
UKQCD [37]	194^{+6+62}_{-5-9}	160^{+6+53}_{-6-19}	$1.22^{+0.04}_{-0.03}$	212^{+4+46}_{-4-7}	185^{+4+42}_{-3-7}	1.18 ± 0.02
BLS [38]	$207 \pm 9 \pm 40$	$187 \pm 10 \pm 37$	$1.11 \pm 0.02 \pm 0.05$	$230 \pm 7 \pm 35$	$208 \pm 9 \pm 37$	$1.11 \pm 0.02 \pm 0.05$
WA75 [39]				$232 \pm 45 \pm 52$		
CLEO 1 [40]				$344 \pm 37 \pm 67$		
CLEO 2 [41]				$284 \pm 30 \pm 30 \pm 16$		
BES [42]				$4.3^{+1.5+0.4}_{-1.3-0.4} \times 10^2$		
E653 [43]				$194 \pm 35 \pm 20 \pm 14$		

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