Color-octet mechanism and J/ψ polarization at CERN LEP

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Polarized heavy quarkonium productions in Z^0 decays are considered. We find that polarizations of the produced quarkonia are independent of that of the parent Z^0 provided that one considers the energy distribution or the total production rate. Produced J/ψ's via the color-octet and the color-singlet mechanisms are expected to be 19% and 29% longitudinally polarized, respectively. The energy dependence of $\eta_{1,8}(x) \equiv d\Gamma_{1,8}^L/dx/d\Gamma_{1,8}/dx$ is very sensitive to the production mechanism, and, therefore, the measurement of $\eta(x)_{\rm expt}$ will be an independent probe of the color-octet mechanism. [S0556-2821(97)04511-6]

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Since Braaten and Fleming put forward the idea of the color-octet mechanism [1] as a possible solution to the so-called ψ' puzzle at the Fermilab Tevatron [2], there have been many activities applying this idea to other processes: heavy quarkonium (both S- and P-wave charmonium and bottomium) production at the Tevatron [3], in B decays [4], fixed target experiments [5], γp collisions [6], e^+e^- annihilations at CLEO [7], and Z^0 decays at the CERN e^+e^- collider LEP [8–10]. Polarized heavy quarkonium production was also considered as an independent check of the color-octet mechanism [11,12]. It is adopted in the calculations of the ψ' polarizations at the Tevatron [13]. A new way to regularize the ultraviolet or infrared divergences in heavy quarkonium calculations was proposed in Ref. [14]. Also, some nonrelativistic QCD (NRQCD) matrix elements relevant to S- and P-wave heavy quarkonium decays were calculated on the lattice [15]. Some reviews of earlier literatures can be found in Ref. [16].

In the color-singlet model, the prompt J/ψ production rate in Z^0 decays is dominated by charm quark fragmentation [17]. However, recent reports by the OPAL Collaboration [18] claim that they have observed an excess of events for $Z^0 \to \Upsilon(nS)+X$ (for $n=1,2,3$), larger than the theoretical expectation by a factor of ~ 10, compared to the b-quark fragmentation contribution [17]. A similar excess was also observed in the prompt J/ψ and ψ' production in Z^0 decays, although the experimental errors are quite large [19]. It turns out that the color-octet gluon fragmentation suggested by Braaten and Fleming could fix this discrepancy through $Z^0 \to q\overline{q}+g$ followed by color-octet gluon fragmentation into J/ψ with emission of soft gluons [8,9]. In Refs. [8, 9], the energy distribution of the produced J/ψ via the color-octet $c\overline{c}(^3S_1^{(8)})$ intermediate state was shown to be dramatically different from that of the J/ψ produced via the color-singlet mechanism. Therefore, the J/ψ energy distribution in the Z^0 decays could be another good test of the idea of the color-octet mechanism. In Ref. [10], the present authors have considered the angular distribution of J/ψ's in Z^0 decays, and one can find whether the color-octet mechanism is working or not.

In this work, we suggest another observable, the polarization of J/ψ at LEP produced via the color-singlet and the color-octet mechanisms. In short, J/ψ's produced via the two channels, $c\overline{c}(^3S_1^{(1)}) \to J/\psi$ and $c\overline{c}(^3S_1^{(8)}) \to J/\psi$, have distinctively different polarizations. The J/ψ produced by the color-octet mechanism is about 19% longitudinally polarized, whereas J/ψ by the color-singlet mechanism is about 29% longitudinally polarized.

When treating polarized quarkonium productions, one should take care of the soft process $Q\overline{Q}(^{2S+1}L_J) \to H$. Recently, Braaten and Chen developed a method for treating the polarized heavy quarkonium production [11] which we use here. Beneke and Rothstein also pointed out that the interference among different ^3P_J states occurs in a polarized heavy quarkonium production [12]. In general, one expresses the free particle amplitude, where $Q\overline{Q}$ makes a transition into the physical quarkonium state, in a power series of the relative momentum $\mathbf{q}$ of Q and $\overline{Q}$ in the $Q\overline{Q}$ rest frame. Then, one can find out what specific spectroscopic state of the $Q\overline{Q}$ pair is initially produced in the hard process amplitude. Finally, one may consider the soft transition in which the initially produced $Q\overline{Q}$ system transforms into the physical heavy quarkonium state in which one is interested. In the case of heavy quarkonium production by the color-singlet mechanism, the soft process does not change any spectroscopic quantum numbers such as color and angular momentum up to a v^2 order correction (which contains relativistic correction and double $E1$ transitions):

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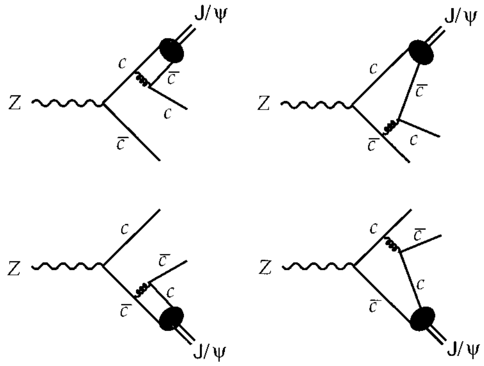

FIG. 1. Feynman diagrams for the color-singlet mechanism for $Z^0 \to (c\overline{c})(^3S_1^{(1)}) + c\overline{c}$.

$$Q\overline{Q}(^3S_1^{(1)}) \to Q\overline{Q}(^3P_J^{(8)}) \to H(^3S_1^{(1)}). \tag{1}$$

In Z^0 decay, the color-singlet production process mainly comes from the Feynman diagram shown in Fig. 1, whereas the color-octet production process mainly comes from the soft process $Q\overline{Q}(^3S_1^{(8)}) \to J/\psi$ as shown in Fig. 2. If we consider J/ψ polarization, we should take into account the relation of the J/ψ polarization and the angular momentum of the initial $Q\overline{Q}$ pair. If J/ψ is produced via the color-singlet mechanism

$$Q\overline{Q}(^3S_1^{(1)}) \to J/\psi(^3S_1^{(1)}), \tag{2}$$

the polarization vector of J/ψ is identical to spin wave function of the initially produced $Q\overline{Q}$ pair. The polarization vector of the J/ψ produced via the color-octet mechanism through double $E1$ transitions,

$$Q\overline{Q}(^3S_1^{(8)}) \to Q\overline{Q}(^3P_J^{(8)}) \to J/\psi(^3S_1^{(1)}), \tag{3}$$

is the same as the spin polarization vector of the initially produced $Q\overline{Q}(^3S_1^{(8)})$, since the $E1$ transition conserves spin and the total angular momenta of the $Q\overline{Q}(^3S_1^{(8)})$ and J/ψ are the same. Therefore, in these two channels, there is no problem, even though we treat the polarization vector of the produced J/ψ and the spin wave function of the $Q\overline{Q}$ pair to be the same. The only factor involving the polarization of the produced J/ψ is the hard processes shown in Figs. 1 and 2 which produce a $Q\overline{Q}$ pair at a short distance. Since the $Q\overline{Q}(^3S_1^{(8)})$ produced via the color-octet mechanism comes from the gluon propagator, it seems to be strongly transversely polarized. The quantitative number for the color-octet-produced J/ψ polarization can be obtained only after the full calculations, which will be presented below along with the numerical results.

Before presenting the results for the J/ψ polarization at LEP, we first argue that the Z^0 polarization at LEP does not affect the J/ψ polarization in Z^0 decays. The Z^0 produced at LEP is polarized as a result of unequal vector and axial vector couplings between electron and Z^0 boson. Therefore, the density matrix $\rho_Z^{\mu\nu}$ of Z^0 is given by [20]

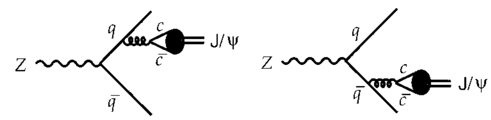

FIG. 2. Feynman diagrams for the color-octet mechanism for $Z^0 \to q\overline{q} + J/\psi$ with $q = u,d,c,s,b$.

$$\rho_Z^{\mu\nu} = \frac{1}{3} I^{\mu\nu} - \frac{i}{2M_Z} \varepsilon^{\mu\nu\lambda\tau} Z_\lambda \mathcal{P}_\tau - \frac{1}{2} \mathcal{Q}^{\mu\nu}, \tag{4}$$

where $I^{\mu\nu} \equiv -g^{\mu\nu} + Z^\mu Z^\nu / M_Z^2$, and Z^μ is four-momentum of Z^0. $\mathcal{P}^\mu$ and $\mathcal{Q}^{\mu\nu}$ are vector and tensor polarization of a Z^0 boson:

$$\mathcal{P}^\mu = \frac{\Delta^\mu}{M_Z} \frac{g_V^2 - g_A^2}{g_V^2 + g_A^2}, \tag{5}$$

$$\mathcal{Q}^{\mu\nu} = -\frac{1}{3} I^{\mu\nu} + \frac{\Delta^\mu \Delta^\nu}{M_Z^2}, \tag{6}$$

where $\Delta^\mu \equiv (k_1 - k_2)^\mu$ with k_1 and k_2 being the four-momenta of e^- and e^+ at LEP, and $g_{V,A}$ are the vector and the axial vector couplings between e and Z^0 boson. We can write the decay rate of Z^0 as

$$d\Gamma = \frac{1}{2M_Z} \rho_Z^{\mu\nu} \int [dp] \int d_2(PS) H_{\mu\nu}, \tag{7}$$

where $[dp] \equiv d^3p/(2\pi)^3 2p^0$ is the invariant phase space of the produced heavy quarkonium with four-momentum p_μ. By the Lorentz covariance, the integration $\int d_2(PS) H_{\mu\nu}$ gives terms proportional to $g_{\mu\nu}$, $Z_\mu Z_\nu$, $Z_\mu p_\nu$, $Z_\nu p_\mu$, $p_\mu p_\nu$, and $\epsilon_{\mu\nu\alpha\beta} Z^\alpha p^\beta$. Here, $\epsilon_{\mu\nu\alpha\beta} Z^\alpha p^\beta$ is the only nonvanishing term after being contracted with the vector polarization term in $\rho_Z^{\mu\nu}$, and the result is proportional to $\cos\theta^*$, where θ^* is the angle between the initial electron beam and the produced quarkonium directions. When they are contracted with the tensor polarization contribution in $\rho_Z^{\mu\nu}$, only $p_\mu p_\nu$ gives a nonzero quantity, proportional to $3\cos^2\theta^* - 1$. In calculating the energy distribution or the total decay rate, we integrate over the angle θ^* by which all of the contributions from the polarization dependence vanish. Therefore $\rho_Z^{\mu\nu}$ can be safely replaced by $\frac{1}{3} I^{\mu\nu}$ in our calculations, effectively.

When one considers the J/ψ polarization, it is convenient to define η to be the ratio of the production rate (Γ_L) of the longitudinal J/ψ to the total production rate ($\Gamma_{\text{tot}} \equiv \Gamma_L + \Gamma_T$) as follows:

$$\eta \equiv \frac{\Gamma_L}{\Gamma_{\text{tot}}} = \frac{\Gamma_L}{\Gamma_L + \Gamma_T}. \tag{8}$$

This ratio η can be determined experimentally from the measurement of the angular distribution of the leptons in the subsequent decay $J/\psi \to l^+ l^-$ [21]. Defining θ_l^* to be the angle between the three momentum of J/ψ in the Z^0 rest frame and the three-momentum of the daughter lepton (say l^-) in the rest frame of J/ψ, the angular distribution of a lepton in the decaying J/ψ rest frame has the form

TABLE I. Longitudinal production fraction η of quarkonium produced in the Z^0 decay and the asymmetry α of the angular distribution of the quarkonium decay in its rest frame.

	$\eta^{J/\psi}/\eta^{\Upsilon}$	$\alpha^{J/\psi}/\alpha^{\Upsilon}$
Fragmentation (color singlet) [21]	0.31/0.31	0.053/0.053
Color singlet (this work)	0.29/0.24	0.10/0.23
Color octet (this work)	0.19/0.22	0.36/0.28
Octet+singlet (this work)	0.21/0.22	0.31/0.28

$$\frac{d\Gamma(J/\psi\to l^+l^-)}{d\cos\theta_l^*}\propto 1+\alpha\cos^2\theta_l^*, \tag{9}$$

where

$$\alpha=\frac{1-3\eta}{1+\eta}. \tag{10}$$

The unpolarized J/ψ corresponds to $\eta=1/3$, and $\alpha=0$.

The polarized J/ψ production in Z^0 decays in the color-singlet model was calculated in Ref. [21] using the fragmentation approximation. In that paper, the authors showed that the asymmetry α is rather small, i.e., $\sim 5\%$. Also, α is independent of the produced quarkonium mass so that α's are the same both for J/ψ and Υ production in their fragmentation approach [21].

In our work, we calculated all the Feynman diagrams without any fragmentation approximation in the color-singlet and color-octet contributions. We recover their results in the limit of $\lambda\equiv m^2_{J/\psi(\Upsilon)}/M_Z^2\to 0$. Our results shown in the Appendix depend explicitly on the parameter λ, and numerical values are shown in Table I. Note that α's are considerably different for $Z^0\to J/\psi+X$ and $Z^0\to\Upsilon+X$. Also, the fragmentation approximation is not that accurate at calculating the Υ polarization in the Z^0 decays because of a rather large mass of Υ. α's are enhanced compared to those calculated in the fragmentation approximation.

When we compare the polarization of J/ψ produced via the color-singlet and the color-octet mechanisms, we observe that there is a considerable difference between $\alpha_1=0.10$ for the singlet and $\alpha_8=0.36$ for the octet $c\bar{c}$ contribution to J/ψ production. Adding the singlet and the octet contributions, we get $\alpha_{\rm tot}^{J/\psi}=0.31$, which is appreciably different from $\alpha_1^{J/\psi}=0.10$ or $\alpha_{\rm frag}^{J/\psi}=0.053$. These numerical values are consistent with those mentioned in Ref. [22]. We have used the following numerical values for the matrix elements of NRQCD appearing in the J/ψ production rates from the Z^0 decays:

$$\langle 0|\mathcal{O}_1^{J/\psi}(^3S_1)|0\rangle=0.73\ \text{GeV}^3, \tag{11}$$

$$\langle 0|\mathcal{O}_8^{J/\psi}(^3S_1)|0\rangle=0.015\ \text{GeV}^3. \tag{12}$$

We remark that both $\alpha_1^{J/\psi}$ and $\alpha_8^{J/\psi}$ are independent of these NRQCD matrix elements, since they cancel in the numerator and the denominator when we take the ratio in η. On the other hand, $\alpha_{\rm tot}^{J/\psi}$ does depend on the numerical values of NRQCD matrix elements in Eqs. (11) and (12), each of which is known only within a factor of ~ 2. Therefore, the definite test of the color-octet mechanism in $Z^0\to J/\psi+X$ will be a deviation of the measured $\alpha_{\rm expt}^{J/\psi}$ from the singlet prediction, $\alpha_1^{J/\psi}$ or $\alpha_8^{J/\psi}$, in the direction of a larger value of $\alpha_{\rm expt}^{J/\psi}$. Deviation of the J/ψ polarizations (or α) from the color-singlet prediction ($\alpha_1^{J/\psi}=0.10$) may be used as a probe to check the color-octet mechanism in heavy quarkonium productions, once a few thousand decays of $J/\psi\to l^+l^-$ are observed in Z^0 decays. In the case of $Z^0\to\Upsilon+X$, α^{Υ} is not so sensitive to the singlet and octet mechanisms: $\alpha_8^{\Upsilon}=0.28$ and $\alpha_1^{\Upsilon}=0.23$, which are considerably larger than the prediction $\alpha_{\rm frag}^{\Upsilon}=0.053$ based on the fragmentation approach.

The energy dependence of $\eta(x)$ (with $x\equiv 2E_{J/\psi}/M_Z$) differs greatly depending on the J/ψ production mechanisms, as we can see in Fig. 3 for the case of J/ψ and Fig. 4 for the case of Υ. The J/ψ's produced via the color-singlet mechanism are almost unpolarized in almost the entire energy range, while the J/ψ's produced via the color-octet mechanism are highly transversal (especially at high energy). Therefore, if we observe quarkonium of a particular energy range, we can greatly increase the polarization sensitivity if there are enough data. For example, if we observe the J/ψ only in the range of $0.7\leqslant x\leqslant 0.9$, where most of the color-singlet J/ψ is produced [8], $\eta_8^{J/\psi}=0.076$ and $\eta_1^{J/\psi}=0.30$. These values correspond to $\alpha_8^{J/\psi}=0.72$ and $\alpha_1^{J/\psi}=0.077$. In the same energy range, $\eta_8^{\Upsilon}=0.12$ and $\eta_1^{\Upsilon}=0.28$ for the case of Υ, which correspond to $\alpha_8^{\Upsilon}=0.57$ and $\alpha_1^{\Upsilon}=0.13$, respectively. We can also observe Υ's in the energy range $0.3\leqslant x\leqslant 0.4$, where most of color-octet Υ's are produced [8]. In this energy range, a color-singlet Υ is more transversal

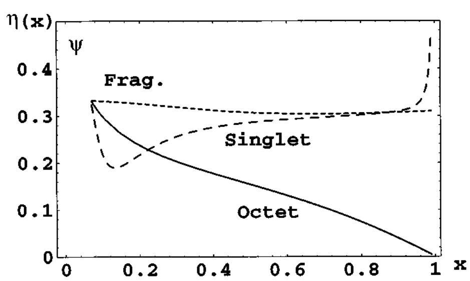

FIG. 3. Energy dependence of $\eta(x)$ in case of $Z^0\to J/\psi+X$: $\eta_8^{J/\psi}(x)$ in the solid curve, $\eta_1^{J/\psi}(x)$ in the dashed curve, and $\eta_{\rm frag}^{J/\psi}(x)$ in the dotted curve.

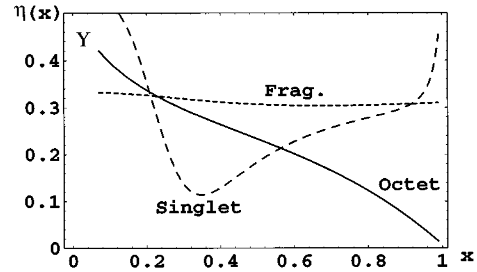

FIG. 4. Energy dependence of $\eta(x)$ in case of $Z^0\to\Upsilon+X$: $\eta_8^{\Upsilon}(x)$ in the solid curve, $\eta_1^{\Upsilon}(x)$ in the dashed curve, and $\eta_{\rm frag}^{\Upsilon}(x)$ in the dotted curve.

than a color-octet Υ, where $\eta_8^{\Upsilon}=0.28$ and $\eta_1^{\Upsilon}=0.12$, corresponding to $\alpha_8^{\Upsilon}=0.13$ and $\alpha_1^{\Upsilon}=0.57$, respectively.

In conclusion, we have calculated polarization in heavy quarkonium (J/ψ and Υ) production in Z^0 decay. The polarization of J/ψ's produced via the color-octet mechanism is more transversal compared to those produced via the color-singlet mechanism (Table I). Therefore, the measurement of polarizations provides another independent test of the idea of the color-octet mechanism.

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APPENDIX

In this appendix, we show the analytic forms of $\Gamma_{1(8),L}$ and $\Gamma_{1(8),\mathrm{tot}}$, defined in Eq. (8);

$$\Gamma_{8,\mathrm{tot}}=\frac{\alpha_s^2(2m_c)}{18}\,\Gamma(Z\to q\overline{q})\,\frac{\langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle}{m_c^3}\int_{2\sqrt{\lambda}}^{1+\lambda}dx\left\{\ln\left(\frac{x+\sqrt{x^2-4\lambda}}{x-\sqrt{x^2-4\lambda}}\right)\frac{[x^2-2x+2+2\lambda(2-x)+2\lambda^2]}{x}-2\sqrt{x^2-4\lambda}\right\}, \tag{A1}$$

$$\Gamma_{8,L}=\frac{2\alpha_s^2(2m_c)}{9}\,\Gamma(Z\to q\overline{q})\,\frac{\langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle}{m_c^3}\int_{2\sqrt{\lambda}}^{1+\lambda}dx\,\frac{\lambda}{x^2-4\lambda}\left\{\ln\left(\frac{x+\sqrt{x^2-4\lambda}}{x-\sqrt{x^2-4\lambda}}\right)\frac{[x-1+\lambda(x-2)-\lambda^2]}{x}\right.$$
$$\left.+\frac{1}{2\lambda}(1+\lambda)(1+\lambda-x)\sqrt{x^2-4\lambda}\right\}, \tag{A2}$$

$$\Gamma_{1,\mathrm{tot}}=\frac{\alpha_s^2(2m_c)}{243}\,\Gamma(Z\to c\overline{c})\,\frac{\langle\mathcal{O}_1^{J/\psi}(^3S_1)\rangle}{m_c^3}\int_{2\sqrt{\lambda}}^{1}dx\left[4\lambda\,\ln\left(\frac{x\sqrt{1-x+\lambda}+\sqrt{(x^2-4\lambda)(1-x)}}{x\sqrt{1-x+\lambda}-\sqrt{(x^2-4\lambda)(1-x)}}\right)\Big(10\lambda^3(x^2+4)+\lambda^2(-5x^4+20x^3\right.$$
$$+8x^2-80x+80)+\lambda(9x^5-59x^4-8x^3+68x^2-128x+64)+4x^2(5x^2-4)+\frac{g_V^2-g_A^2}{g_V^2+g_A^2}\,[2\lambda^3(x^2+4)+\lambda^2(5x^4-60x^3$$
$$+24x^2-48x+48)+\lambda x^2(-9x^3+73x^2-76)+32x^4(-x+1)]\Big)\Big/[x^3(2-x)^2]-8\sqrt{\frac{(x^2-4\lambda)(1-x)}{1-x+\lambda}}$$
$$\times\Big\{2\lambda^4(x+2)(5x^3-38x^2+60x-40)+\lambda^3(-5x^6+66x^5-286x^4+888x^3-992x^2+960x-480)$$
$$+6\lambda^2(2x^7-25x^6+118x^5-324x^4+384x^3-360x^2+288x-96)+\lambda(-5x^8+76x^7-411x^6+1168x^5-1384x^4$$
$$+1248x^3-1456x^2+1024x-256)-4x^2(x-1)^2(5x^4-32x^3+72x^2-32x+16)+\frac{g_V^2-g_A^2}{g_V^2+g_A^2}\,[2\lambda^4(x+2)(x^3+18x^2+12x$$
$$-8)+\lambda^3(5x^6-78x^5+274x^4-840x^3+384x^2+448x-224)+4\lambda^2(-3x^7+45x^6-211x^5+528x^4-504x^3$$
$$\left.+44x^2+144x-48)+\lambda x^2(x-1)(5x^5-73x^4+328x^3-712x^2+560x-80)]\Big\}\Big/[x^2(2-x)^6]\right], \tag{A3}$$

$$\Gamma_{1,L}=\frac{\alpha_s^2(2m_c)}{243}\,\Gamma(Z\to c\overline{c})\,\frac{\langle\mathcal{O}_1^{J/\psi}(^3S_1)\rangle}{m_c^3}\int_{2\sqrt{\lambda}}^{1}dx\left[-4\lambda\,\ln\left(\frac{x\sqrt{1-x+\lambda}+\sqrt{(x^2-4\lambda)(1-x)}}{x\sqrt{1-x+\lambda}-\sqrt{(x^2-4\lambda)(1-x)}}\right)\Big(24\lambda^4(x^2+4)+8\lambda^3(x^4-2x^3\right.$$
$$-x^2-24x+28)+\lambda^2(-3x^6+24x^5-128x^4+64x^3-112x^2-128x+128)+\lambda x^2(-x^5+3x^4+56x^3+60x^2-64)+4x^4$$
$$\times(-5x^2+4)+\frac{g_V^2-g_A^2}{g_V^2+g_A^2}\,\lambda[8\lambda^3(-x^2-4)+8\lambda^2(4x^4-12x^3+7x^2+12)+\lambda x^2(3x^4-52x^3+176x^2-208x+16)$$
$$+x^4(x+2)(x^2-3x+6)]\Big)\Big/[x^3(x^2-4\lambda)(2-x)^2]+8\sqrt{\frac{(x^2-4\lambda)(1-x)}{1-x+\lambda}}\Big\{8\lambda^5(x+2)(9x^3-62x^2+108x-72)$$

$$+8\lambda^4(3x^6-25x^5+76x^4+176x^3-584x^2+912x-480)+\lambda^3(-9x^8+108x^7-756x^6+2936x^5-8744x^4+11424x^3$$
$$-12096x^2+10752x-4224)+2\lambda^2(-15x^8+216x^7-1114x^6+3520x^5-3864x^4+1984x^3-2176x^2+2304x-768)$$
$$+\lambda x^2(3x^8-36x^7+149x^6-128x^5-1264x^4+2048x^3-16x^2-1280x+512)+4x^4(x-1)^2(3x^4-24x^3+64x^2$$
$$-32x+16)+\frac{g_V^2-g_A^2}{g_V^2+g_A^2}3\lambda[-8\lambda^4(x+2)(x^3-14x^2+12x-8)+8\lambda^3(4x^6-37x^5+108x^4-256x^3+184x^2+16x-32)$$
$$+\lambda^2(3x^8-88x^7+740x^6-2600x^5+5672x^4-5728x^3+1600x^2+768x-384)+4\lambda x^2(4x^6-53x^5+231x^4-624x^3$$
$$+844x^2-496x+96)+x^4(-x+1)(x^5-13x^4+56x^3-128x^2+80x+16)]\Bigg\} \Bigg/ [3x^2(x^2-4\lambda)(2-x)^6]\Bigg], \qquad \text{(A4)}$$

with

$$\lambda\equiv\frac{4m_c^2}{M_Z^2}. \qquad \text{(A5)}$$

The same formulas apply to the Υ case, with the substitution of m_b for m_c, the corresponding change of couplings g_V and g_A, and the corresponding long-range matrix elements.

[1] E. Braaten and S. Fleming, Phys. Rev. Lett. **74**, 3327 (1995).

[2] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **69**, 3704 (1992); **71**, 2537 (1993); CDF Collaboration, M. Mangano, in *Proceedings of the 27th International Conference on High Energy Physics*, Glasgow, Scotland, 1994, edited by P. J. Bussey and I. G. Knowles (IOP, London, 1995).

[3] M. Cacciari, M. Greco, M. L. Mangano, and A. Petrelli, Phys. Lett. B **356**, 560 (1995); M. Cacciari and M. Greco, Phys. Rev. Lett. **73**, 1586 (1994); E. Braaten, M. A. Doncheski, S. Fleming, and M. L. Mangano, Phys. Lett. B **333**, 548 (1994); D. P. Roy and K. Sridhar, *ibid.* **339**, 141 (1994); P. Cho and A. K. Leibovich, Phys. Rev. D **53**, 150 (1996); **53**, 6203 (1996); C. S. Kim, Jungil Lee, and H. S. Song, Phys. Rev. D (to be published).

[4] G. T. Bodwin, E. Braaten, T. C. Yuan, and P. Lepage, Phys. Rev. D **46**, R3703 (1992); P. Ko, Jungil Lee, and H. S. Song, *ibid.* **53**, 1409 (1996); S. Fleming, O. F. Hernandez, I. Maksymyk, and H. Nadeau, *ibid.* **55**, 4098 (1997).

[5] W.-K. Tang and M. Vänttinen, Phys. Rev. D **53**, 4851 (1996); **54**, 4349 (1996); S. Fleming and I. Maksymyk, *ibid.* **54**, 3608 (1996); S. Gupta and K. Sridhar, *ibid.* **54**, 5545 (1996).

[6] M. Cacciari and M. Krämer, Phys. Rev. Lett. **76**, 4128 (1996); P. Ko, Jungil Lee, and H. S. Song, Phys. Rev. D **54**, 4312 (1996); J. Amundson, S. Fleming, and I. Maksymyk, Report No. UTTG-10-95, hep-ph/9601298 (unpublished); R. Godbole, D. P. Roy, and K. Sridhar, Phys. Lett. B **373**, 328 (1996); J. P. Ma, Nucl. Phys. **B460**, 109 (1996); P. Ko, J. Korean Phys. Soc. **29**, S261 (1996).

[7] E. Braaten and Y.-Q. Chen, Phys. Rev. Lett. **76**, 730 (1996).

[8] K. Cheung, W.-Y. Keung, and T. C. Yuan, Phys. Rev. Lett. **76**, 877 (1996).

[9] P. Cho, Phys. Lett. B **368**, 171 (1996).

[10] Seungwon Baek, Pyungwon Ko, Jungil Lee, and H. S. Song, Phys. Lett. B **389**, 609 (1996).

[11] E. Braaten and Y-.Q. Chen, Phys. Rev. D **54**, 3216 (1996).

[12] M. Beneke and I. Z. Rothstein, Phys. Lett. B **372**, 157 (1996); Phys. Rev. D **54**, 2005 (1996).

[13] A. K. Leibovich, Report No. CALT-68-2082, hep-ph/9610381 (unpublished); M. Beneke and M. Krämer, Phys. Rev. D (to be published); P. Ko, Jungil Lee and H. S. Song (work in progress).

[14] E. Braaten and Y.-Q. Chen, Report No. OHSTPY-HEP-T-96-018, hep-ph/9610401 (unpublished).

[15] G. T. Bodwin, D. K. Sinclair, and S. Kim, Phys. Rev. Lett. **77**, 2376 (1996).

[16] See, for example, E. Braaten, S. Fleming, and T. C. Yuan, Annu. Rev. Nucl. Part. Sci. (to be published).

[17] E. Braaten, K. Cheung, and T. C. Yuan, Phys. Rev. D **48**, 4230 (1993).

[18] OPAL Collaboration, Phys. Lett. B **370**, 185 (1996).

[19] OPAL Collaboration, ‘‘Observation of J/ψ and ψ' productions in hadronic Z^0 decays,’’ OPAL Physics Note No. PN178, 1995 (unpublished).

[20] H. S. Song, Phys. Rev. D **33**, 1252 (1986).

[21] A. F. Falk, M. Luke, M. J. Savage, and M. B. Wise, Phys. Lett. B **312**, 486 (1993).

[22] K. Cheung, W.-Y. Keung, and T. C. Yuan, Report No. hep-ph/9609268, 1996 (unpublished).