Null strings in Schwarzschild spacetime

Mariusz P. Dąbrowski[*]

Institute of Physics, University of Szczecin, Wielkopolska 15, 70-451 Szczecin, Poland

Arne L. Larsen[†]

Department of Physics, University of Alberta, Faculty of Science, 412 Avadh Bhatia, Physics Laboratory, Edmonton, Canada T6G 2J1

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The null string equations of motion and constraints in Schwarzschild spacetime are given. The solutions are those of the null geodesics of general relativity appended by a null string constraint in which the ''constants of motion'' depend on the world-sheet spatial coordinate. Because of the extended nature of a string, the physical interpretation of the solutions is completely different from the point particle case. In particular, a null string is generally not propagating in a plane through the origin, although each of its individual points is. Some special solutions are obtained and their physical interpretation is given. Especially, the solution for a null string with a constant radial coordinate r moving vertically from the south pole to the north pole around the photon sphere is presented. A general discussion of classical null or tensile strings as compared to massless or massive particles is given. For instance, tensile circular solutions with a constant radial coordinate r do not exist at all. The results are discussed in relation to the previous literature on the subject. [S0556-2821(97)03010-5]

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I. INTRODUCTION

It is well known that the classical evolution of strings even in the simplest curved backgrounds, such as the Schwarzschild spacetime, is described by a complicated system of second-order nonlinear coupled partial differential equations. In the Schwarzschild spacetime the system is actually nonintegrable and it subjects chaotic behavior [1], so one may just try to find the exact evolution for some special configurations [2–7] or perform some numerical calculations [1,3,8]. This means that there is no hope for making the full classification of the possible classical trajectories of strings in the Schwarzschild spacetime similar to the one for the point particles; see, for instance, the standard textbook by Chandrasekhar [9].

In the case of the null strings (tensionless strings), as first introduced by Schild [10] and later developed, among others, by Karlhede and Lindström [11], this situation is simplified since the null strings, similarly to the massless point particles, essentially sweep out the light cone, and their equations of motion are essentially just geodesic equations of general relativity appended by an additional constraint. General relativistic first integrals for point particles are known for most of the symmetric spacetimes, and we can apply them to null strings with almost no hesitation. Then, depending on the assumed shape of a null string, in principle, one can solve the null string equations of motion in many cases. Such calculations have been performed recently, though not completely, by Kar [12] for Minkowski, Rindler, Schwarzschild, and Robertson-Walker spacetimes following an earlier idea originally suggested by Roshchupkin and Zheltukhin [13] for Robertson-Walker spacetimes.

The task of this paper is to discuss the null string evolu-

tion in the Schwarzschild spacetime in more detail, and thereby also to shed light on the solutions obtained by Kar [12]. We present the general equations of motion for strings in the Schwarzschild spacetime and give the general solutions in quadratures in the case of null strings in Sec. II. In Sec. III we solve the equations of motion completely in closed form for circular null strings and we discuss their physical interpretation. In Sec. IV we present a very interesting exact solution which describes a string moving vertically up and down around the photon sphere. In Sec. V we briefly discuss the relation between the tensile and the null strings in the context of our solutions. Finally in Sec. VI, we summarize our results and give some concluding remarks.

II. STRINGS IN THE SCHWARZSCHILD SPACETIME

In any curved spacetime the spacetime coordinates describing a string configuration, in general, depend on both of the string coordinates τ and σ, so we may use the notation

$$X^0 = t(\tau, \sigma), \quad X^1 = r(\tau, \sigma),$$

$$X^2 = \theta(\tau, \sigma), \quad X^3 = \varphi(\tau, \sigma). \tag{2.1}$$

Let us consider the tensile string (finite tension) and the null string (zero tension) equations of motion in a compact formula:

$$\ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = \lambda (X''^\mu + \Gamma^\mu_{\nu\rho} X'^\nu X'^\rho). \tag{2.2}$$

The constraints read as

$$g_{\mu\nu} \dot{X}^\mu \dot{X}^\nu = -\lambda g_{\mu\nu} X'^\mu X'^\nu, \tag{2.3}$$

$$g_{\mu\nu} \dot{X}^\mu X'^\nu = 0. \tag{2.4}$$

Here $(\cdots)^\cdot \equiv \partial/\partial\tau$ and $(\cdots)' \equiv \partial/\partial\sigma$. For $\lambda = 1$ we have the tensile strings while $\lambda = 0$ applies for the null strings. In

*Electronic address: mpdabfz@uoo.univ.szczecin.pl

†Electronic address: alarsen@phys.ualberta.ca

Refs. [14–17], expansion schemes were considered, essentially using λ as a continuous expansion parameter; here we simply use λ as a discrete parameter discriminating between tensile and null strings.

From the above we can see that for the null strings we have the null geodesic equations supplemented by the constraint (2.4), which ensures that each point of the string propagates in the direction perpendicular to the string. In the Schwarzschild spacetime we have, from Eq. (2.2),

$$\ddot{t} - \lambda t'' + 2\frac{M/r^2}{1-2M/r}(\dot{t}\dot{r} - \lambda t'r') = 0, \qquad (2.5)$$

$$\ddot{r} - \lambda r'' - \frac{M/r^2}{1-2M/r}(\dot{r}^2 - \lambda r'^2) + \frac{M}{r^2}\left(1 - \frac{2M}{r}\right)(\dot{t}^2 - \lambda t'^2)$$

$$- r\sin^2\theta\left(1 - \frac{2M}{r}\right)(\dot{\varphi}^2 - \lambda \varphi'^2)$$

$$- r\left(1 - \frac{2M}{r}\right)(\dot{\theta}^2 - \lambda \theta'^2) = 0, \qquad (2.6)$$

$$\ddot{\varphi} - \lambda \varphi'' + \frac{2}{r}(\dot{r}\dot{\varphi} - \lambda r'\varphi') + 2\frac{\cos\theta}{\sin\theta}(\dot{\theta}\dot{\varphi} - \lambda \theta'\varphi') = 0, \qquad (2.7)$$

$$\ddot{\theta} - \lambda \theta'' + \frac{2}{r}(\dot{r}\dot{\theta} - \lambda r'\theta') - \sin\theta\cos\theta(\dot{\varphi}^2 - \lambda \varphi'^2) = 0. \qquad (2.8)$$

In the case of the null strings ($\lambda = 0$), Eqs. (2.5) and (2.7) easily integrate. The only difference from that of the general relativistic point particle case is that now the ''constants of motion'' must depend on the string coordinate σ: i.e.,

$$\dot{t} = \frac{E(\sigma)}{1-2M/r}, \qquad (2.9)$$

$$\dot{\varphi} = \frac{L(\sigma)}{r^2\sin^2\theta}. \qquad (2.10)$$

Combining Eq. (2.8) with Eqs. (2.9) and (2.10) we obtain, for $\lambda = 0$,

$$r^4\sin^2\theta\,\ddot{\theta} + 2r^3\dot{r}\sin^2\theta\,\dot{\theta} - L^2(\sigma)\frac{\cos\theta}{\sin\theta} = 0, \qquad (2.11)$$

which integrates in a standard way

$$r^4\sin^2\theta\,\dot{\theta}^2 = -L^2(\sigma)\cos^2\theta + K(\sigma)\sin^2\theta, \qquad (2.12)$$

and the non-negative function $K(\sigma)$ generalizes Carter's ''fourth constant'' of motion (see, for instance, [18]). The standard potential equation for the radial coordinate is then obtained by integrating Eq. (2.6) in the case $\lambda = 0$:

$$\dot{r}^2 + V(r) = 0, \qquad (2.13)$$

where

$$V(r) = -E^2(\sigma) + \frac{1}{r^2}\left(1 - \frac{2M}{r}\right)[L^2(\sigma) + K(\sigma)], \qquad (2.14)$$

and the constraint (2.3) has been taken into account. As we can easily see $L(\sigma)$ refers directly to the coordinate φ while $K(\sigma)$ refers directly to the coordinate θ. One can also define the generalized impact parameter as [9]

$$D(\sigma) \equiv \frac{\sqrt{L^2(\sigma) + K(\sigma)}}{E(\sigma)}. \qquad (2.15)$$

It should be noticed that if there was not the ''constant of motion'' $K(\sigma)$, the only solution of Eq. (2.12) with $L \neq 0$ would be given by $\theta = \text{const} = \pi/2$. As is very well known in the case of point particles, one can of course put $K = 0$ without loss of generality. However, because of the fact that the string is an extended object this is not the case here, and the explicit dependence on the string coordinate σ must be given. In particular, the null string does *not* in general move on a plane through the origin, although each individual point of the string actually does.

The constraint (2.4) now takes the form

$$E(\sigma)t' - \frac{\dot{r}}{1-2M/r}r' - L(\sigma)\varphi' - r^2\dot{\theta}\theta'\sin^2\theta = 0, \qquad (2.16)$$

with $\dot{r}$ and $\dot{\theta}$ given by Eqs. (2.12) and (2.13). Equation (2.16) means that we have a constraint on the functions $E(\sigma)$, $L(\sigma)$, and $K(\sigma)$.

Finally, we notice that the invariant string size (the length of the string) $S(\tau)$ for the null string is given by

$$S(\tau) = \int_0^{2\pi} S(\tau,\sigma)d\sigma, \qquad (2.17)$$

where

$$S(\tau,\sigma) = \sqrt{g_{\mu\nu}X'^{\nu}X'^{\nu}}$$

$$= \left[-\left(1 - \frac{1}{2M}\right)t'^2 + \left(1 - \frac{1}{2M}\right)^{-1}\right.$$

$$\left.\times r'^2 + r^2\theta'^2 + r^2\sin^2\theta\,\varphi'^2\right]^{1/2}. \qquad (2.18)$$

III. CIRCULAR NULL STRINGS

As a first example of exact solutions, we consider the circular ansatz for a null string in the Schwarzschild spacetime:

$$t = t(\tau), \quad r = r(\tau), \quad \theta = \theta(\tau), \quad \varphi = \sigma. \qquad (3.1)$$

Inserting Eq. (3.1) into Eqs. (2.9)–(2.13) we have

$$\dot{t} = \frac{E(\sigma)}{1-2M/r}, \qquad (3.2)$$

$$\dot{r}^2 = E^2(\sigma) - \frac{K(\sigma)}{r^2}\left(1 - \frac{2M}{r}\right), \qquad (3.3)$$

$$r^4 \dot{\theta}^2 = K(\sigma), \tag{3.4}$$

and we have also used the constraint (2.16) which gives the condition for L to be equal to zero. It is also clear from Eqs. (3.2)–(3.4) that E and K must be *constants* (independent of σ) in this case. This is not the case in general, of course.

The simplest solutions come if we put $K=0$, i.e.,

$$\theta = \text{const}, \tag{3.5}$$

$$r - r_0 + 2M \ln\frac{r-2M}{r_0-2M} = \pm(t-t_0), \tag{3.6}$$

and they describe the "cone strings" which start with finite size and then sweep out the cones $\theta = \text{const}$. These strings play a similar role as the radial null geodesics in general relativity. Notice that the extended nature of the string means that configurations corresponding to different values of the constant polar angle θ are physically different: for $\theta = \pi/2$, the string is in a plane through the origin while for any other value of θ it is not. In fact, for $\theta = \pi/2$, the string winds around the black hole in the equatorial plane while for $\theta \neq \pi/2$, it is on a parallel plane moving perpendicular to the equatorial plane. In contrast, a point particle is always in a plane through the origin. This illustrates the fact that although the null string equations are similar to the massless geodesic equations, in particular when E and K are constants, the physical interpretation of the solutions is completely different.

For the sake of comparison we mention that in Minkowski spacetime ($M=0$) the logarithmic term vanishes giving simply

$$\theta = \text{const}, \tag{3.7}$$

$$r - r_0 = \pm(t-t_0). \tag{3.8}$$

The "cone strings" also appear in the anti–de Sitter spacetime since there

$$\theta = \text{const}, \tag{3.9}$$

$$r - r_0 = \pm\frac{1}{H}\tan H(t-t_0), \tag{3.10}$$

with $H = \text{const} = \sqrt{-\Lambda/3}$, where Λ is the cosmological constant. Such "cone strings" can also easily be constructed in other static spherically symmetric spacetimes, but we shall not go into further details here.

Coming back to Eqs. (3.2)–(3.4) for arbitrary K, we first notice that Eq. (3.3) is exactly equivalent to the equation for photons moving in the equatorial plane with nonvanishing angular momentum ($L \neq 0, \theta = \pi/2 \Rightarrow K=0$). This, of course, just reflects the fact that point particles always move in a plane through the origin. However, since a string (even a null string) is an extended object, the physical interpretation of the solutions to Eq. (3.3) is completely different from that of point particle solutions. In particular, none of the string solutions we will obtain here are propagating in a plane through the origin. But the qualitative and quantitative picture for the string solutions can still be extracted from the well-known results for point particles (see, for instance,

[9,18]). It is possible because in the assumed ansatz (3.1), all the spacetime coordinates are just functions of one of the two string coordinates, and we can derive the equation which relates the coordinates r and θ from Eqs. (3.2) and (3.3) in a similar way as one usually does for the massless particles [9], although now, instead of φ we have θ, and instead of L we have $\sqrt{K}$: i.e.,

$$\left(\frac{du}{d\theta}\right)^2 = 2M\left(u+\frac{1}{6M}\right)\left(u-\frac{1}{3M}\right)^2 + \frac{1}{M^2}\left(\frac{M^2}{D^2}-\frac{1}{27}\right)$$

$$= 2Mu^3 - u^2 + \frac{1}{D^2}, \tag{3.11}$$

where $u=1/r$ and D is defined by Eq. (2.15). From this equation we can immediately derive the trajectories of the null strings similarly as in [9]. In our discussion we just use directly the Eq. (3.3) looking for the turning points ($\dot{r}=0$) of it, satisfying

$$\frac{K}{4M^2E^2} = \frac{(r/2M)^2}{1-(r/2M)^{-1}}. \tag{3.12}$$

Equation (3.12) has two solutions outside the horizon provided

$$\frac{K}{M^2E^2} = \frac{D^2}{M^2} > 27, \tag{3.13}$$

one solution in case of equality and otherwise no solutions. Consider first the case where $K < 27M^2E^2$ ($D < 3\sqrt{3}M$) and a circular string incoming from spatial infinity (say) $\theta = 0, r = \infty$. The plane of the string is always parallel to the equatorial plane and the string approaches the south pole of the black hole, and since there is no turning point in this case, the string will eventually fall into the black hole. If $K > 27M^2E^2$ ($D > 3\sqrt{3}M$), the string again approaches the south pole of the black hole, but in this case it will scatter off and escape towards infinity again. Notice that in both cases the string can make a number of turns, moving vertically from the south pole to the north pole and back again and so on, around the black hole [during which it actually collapses ($r\sin\theta = 0$) several times], but always with its plane parallel to the equatorial plane, before its fate is determined. Similarly one can consider strings starting very close to (but outside) the horizon with increasing $r(\tau)$. If $K < 27M^2E^2$, the string escapes to infinity while if $K > 27M^2E^2$, it will hit the barrier and fall back into the black hole. In the next section we will consider a limiting case of this kind of dynamics.

IV. NULL STRINGS ON THE PHOTON SPHERE

In the special case when the impact parameter $D = 3\sqrt{3}M$, Eq. (3.11) [with the circular ansatz (3.1) valid] factorizes and the simplest solution for the constant radial coordinate $r = 1/u = 3M$ comes immediately. If $r = 3M$, then we conclude from Eqs. (3.11) and (3.2) that

$$t(\tau) = 3E\tau, \quad D^2 = 27M^2. \tag{4.1}$$

Then, one is able to integrate Eq. (3.4) to give

$$\theta = \pm \frac{E\tau}{\sqrt{3}M} + \theta_0, \qquad (4.2)$$

and $\theta_0 =$ const. Finally, we conclude that a circular string solution is described explicitly by

$$t = 3E\tau, \quad r = \text{const} = 3M,$$

$$\theta = \pm \frac{E\tau}{\sqrt{3}M} + \theta_0, \quad \varphi = \sigma, \qquad (4.3)$$

which means that the string may move vertically from the south pole to the north pole and back again and so on around the photon sphere ($r = 3M$). Notice that the factor $3E$ in Eq. (4.1) can be scaled away. The invariant string size (2.17) of the string solution (4.3) is simply

$$S = 6\pi M \sin\left(\pm \frac{E\tau}{\sqrt{3}M} + \theta_0 \right). \qquad (4.4)$$

This string solution with constant radial coordinate $r = 3M$ is, besides the solution

$$t = \tau, \quad r = \text{const} = 2M, \quad \theta = \text{const}, \quad \varphi = \sigma, \quad (4.5)$$

considered by Kar [12], the only string solution with constant r. Notice that the solution $r = 2M$ is stationary while the solution $r = 3M$ is highly dynamical. It must be stressed, however, that the solution $r = 3M$ is unstable. The situation is similar to the case of a photon on a circular orbit at $r = 3M$ [9,18] and it means that there exist two asymptotic solutions in addition to the one given by Eq. (4.3), one of which describes an incoming string approaching the photon sphere from infinity ($r = \infty$), then spiraling around it infinitely many times, and the other one, an outgoing string starting from somewhere close to (but outside) the horizon approaching the photon sphere by also spiraling around it infinitely many times. The exact expressions for such trajectories have been already given in [9]. For the string approaching the photon sphere from infinity (the orbit of ''the first kind'' as called in [9]) we have, from Eqs. (3.2)–(3.4),

$$\frac{1}{r} = -\frac{1}{6M} + \frac{1}{2M}\tanh^2\left(\frac{1}{2}(\theta + \theta_0) \right), \qquad (4.6)$$

$$\frac{dt}{d\tau} = \frac{E}{4/3 - \tanh^2[1/2(\theta + \theta_0)]}, \qquad (4.7)$$

$$\frac{d\theta}{d\tau} = \pm \frac{E}{4\sqrt{3}M}\left[-1 + 3\tanh^2\left(\frac{1}{2}(\theta + \theta_0) \right) \right]^2, \quad (4.8)$$

and, for the outgoing string approaching the photon sphere (the orbit of ''the second kind''),

$$\frac{1}{r} = \frac{1}{3M} + \frac{2z}{M(z-1)^2}, \qquad (4.9)$$

$$\frac{dt}{d\tau} = \frac{E}{1/3 - \tan^2[2\arctan(\sqrt{z})]}, \qquad (4.10)$$

$$\frac{dz}{d\tau} = \pm \frac{E}{\sqrt{3}M}\frac{z[z^2 + 4z + 1]^2}{(z-1)^4}, \qquad (4.11)$$

where $z = \exp\theta$ and $\theta_0 =$ const. The exact solutions of these equations are given in terms of elliptic functions and we will not discuss them further on since their physical meaning is clear from the qualitative considerations given above and at the end of Sec. III. One can easily see from Eqs. (4.6)–(4.8) that in the limit $\theta \to \infty$, $r = 3M$, $t = 3E\tau$, and $d\theta/d\tau = \pm E/\sqrt{3}M$. Similarly in Eqs. (4.9)–(4.11) for $\theta \to \infty$, $r = 3M$, $t = 3E\tau$, and $d\theta/d\tau = \pm E/\sqrt{3}M$, thus in both cases we obtain the limiting case (4.3) as we should.

V. TENSILE CIRCULAR STRINGS IN THE SCHWARZSCHILD BACKGROUND

In this section we briefly consider the case of tensile strings and start with the circular ansatz of Sec. III given by Eq. (3.1). The equations of motion (2.5)–(2.8) are given by

$$\ddot{t} - \frac{E}{1 - 2M/r} = 0, \qquad (5.1)$$

$$\ddot{r} - \frac{M/r^2}{1 - 2M/r}\dot{r}^2 + \frac{M}{r^2}\left(1 - \frac{2M}{r} \right)\dot{t}^2 + r\sin^2\theta\left(1 - \frac{2M}{r} \right)$$

$$- r\left(1 - \frac{2M}{r} \right)\dot{\theta}^2 = 0, \qquad (5.2)$$

$$\ddot{\theta} + 2\frac{\dot{r}}{r}\dot{\theta} + \sin\theta\cos\theta = 0, \qquad (5.3)$$

and the constraint (2.4) is automatically fulfilled, while the constraint (2.3) gives

$$\frac{E^2 - \dot{r}^2}{(1 - 2M/r)} - r^2\dot{\theta}^2 - r^2\sin^2\theta = 0. \qquad (5.4)$$

These equations can be easily integrated in the equatorial plane and the solutions have been discussed in Refs. [3–5] so we shall not repeat them here. The first special solutions of Eqs. (5.1)–(5.3) outside the equatorial plane one might look for are the ones with constant radial coordinate $r =$ const (cf. the discussion of the null strings in the previous section). However, as one can easily check by simple substitution $\dot{r} = 0$ in Eqs. (5.1)–(5.3) and then differentiation of Eq. (5.2) with respect to τ, this results in contradiction with Eq. (5.3). This means that tensile strings with a constant value of the radial coordinate r do not exist at all. This is a very big difference from the point particles since for massive particles there are circular orbits which are stable for $r > 6M$ and unstable for $r < 6M$. It seems that it is impossible to keep such a symmetric and stationary configuration because of the self-interaction of the tensile strings.

A second first integral [besides Eq. (5.4)] of the system (5.1)–(5.3), which would guarantee its full integrability, is not known. In fact, numerical investigations strongly suggest that no such integral exists, i.e., the system is chaotic. The numerical investigations showed that essentially three possible evolution schemes are possible for the axisymmetric

tensile string in the Schwarzschild spacetime [1,3]: (1) the string simply passes the horizon and falls onto the black hole; (2) the string passes by the black hole, but part of its translational energy is transformed into oscillatory energy; (3) the string is ''trapped'' jumping chaotically around the black hole for a certain amount of time before it either falls into the black hole or escapes towards infinity. Notice that this is qualitatively the same kind of dynamics that we found for the null strings using exact analytical methods of Secs. III and IV. This suggests that by fine-tuning of the initial conditions, it should actually be possible to have (tensile) string solutions jumping around the black hole forever,[1] somewhat similar to the null string on the photon sphere as discussed in Sec. IV, but with the radial coordinate not exactly constant. In the present case of tensile strings it is however not known whether such solutions would be stable under small perturbations, and their existence might demand *infinite* fine-tuning of the initial conditions. We finally notice that in the case of electrically charged strings, such solutions actually do exist [1]; their stability being guaranteed by a Coulomb barrier for small radius and a tension barrier for large radius. For the ordinary uncharged tensile strings, such solutions have not yet been found, either by analytical or by numerical methods.

VI. SUMMARY

After our analysis of the evolution of strings in the above cases we now come to the following discussion which compares the behavior of classical null strings and tensile strings to the behavior of massless and massive point particles in curved spacetimes.

A classical massive point particle in a curved spacetime experiences only the interaction with the gravitational field. It means that taking the limit of zero mass changes the world line of the particle smoothly since photon subjects the same rule and they also interact with the gravitational field only. Thus from the point of view of world lines only, the limit of zero mass is relatively mild. For strings the situation is somewhat more complicated. What distinguishes the strings from the point particles in such situations is that the strings not only interact with the gravitational field, but also self-interact due to the tension. This makes an important physical difference with what we have for the point particles in the sense that now taking the limit of zero tension is quite *dramatic*. It is because in this limit the self-interaction of strings totally vanishes, while the interaction with the gravitational field changes smoothly (as for point particles).

It is thus obvious that the null particles (like photons)

have something to do with massive particles. The general question is then, whether the null strings actually have anything to do with the tensile strings. Referring to that, there have been suggestions recently [14–17] that the tensile string equations of motion may be expanded perturbatively in inverse string tension parameter α' (or a parameter related to it by rescaling) having the null string equations of motion as the zeroth-order approximation. However, although the equations of the null strings are mathematically much simpler than the equations of the tensile strings, one can question (cf. the above discussion) whether it is physically meaningful to consider a null string as a zeroth-order approximation of a tensile string. It seems also that there is some disagreement in the literature [14–17] about how to define such an expansion scheme correctly. Other physical aspects of tensionless strings, such like their classical interactions, have also been discussed recently (see [19] and references therein).

On the other hand, it is certainly meaningful and interesting to consider null strings by themselves (assuming that such objects actually exist). The dynamics of null strings in a curved spacetime is however very simply obtained if the dynamics of point particles is known. As discussed in this paper, it is essentially a question of interpreting the well-known point particle results in the framework of an extended object. Referring to our calculations one can distinguish two different physical situations. These are when the constraint (2.4) is or is not automatically fulfilled by the ansatz. If it is automatically fulfilled the evolution of the string is almost trivial, in the sense that each point of the string follows a null geodesic (a trajectory of a massless particle) without any correlations with the rest of the string. Then, the string motion reduces to the motion of a collection of massless point particles moving quite *independently*. On the other hand, if the constraint (2.4) is not automatically fulfilled by the ansatz then there are some nontrivial correlations between the different points of the string, the nature of which are purely ''stringy.'' It appears that the ''stringy'' nature is absent in any axially symmetric spacetime for the circular ansatz, thus to look for more complicated dynamics one should consider either some less symmetric background spacetimes or some different string shapes.

Similar problems as discussed here might appear for the null p-branes in curved spacetimes [20] which generalizes strings to higher-dimensional objects.

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[1] We thank D. Page for interesting discussions on this point.

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