String propagation in an exact four-dimensional black hole background

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We study string propagation in an exact, stringy, four-dimensional dyonic black hole background. The exact solutions in terms of elliptic functions describing string configurations in the $J=0$ limit are obtained by solving the string equations of motion and constraints. By using the covariant formalism, we also investigate the propagation of physical perturbations along the string in the given curved background. [S0556-2821(97)03710-7]

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I. INTRODUCTION

The study of string dynamics in curved space-time started in [1] has become an interesting area of research in recent times. The classical string equations of motion and the constraints in curved space-time become highly nonlinear, coupled partial differential equations and hence it is often difficult to find exact and complete solutions of these equations. However, exact solutions to these equations of motion and constraints have been obtained in some specific curved space-times, such as, cosmic string backgrounds [2], black hole space-times [3,4], de Sitter space-times [5], gravitational wave backgrounds [6], the cosmological background [7], and the wormhole background [8]. The string equations of motion and constraints are exactly integrable in D-dimensional de Sitter space-times [5]. The novel feature of strings in the de Sitter space-time is the appearance of multistring solutions [9], where there is only one single world sheet, but infinitely many different and independent strings. Also, by using certain appropriate *Ansätze*, such as a circular string *Ansatz* [10] or a stationary string *Ansatz* [11], one can obtain a general family of exact solutions. In stationary, axially symmetric space-times, the circular string *Ansatz* corresponds to decoupling of the dependence of x^μ on the world sheet parameter σ and consequently reduces the equations of motion and the constraints to simpler coupled ordinary differential equations. On the other hand, the stationary string *Ansatz* decouples the dependence of x^μ on the world sheet coordinate τ and again the string equations of motion and constraints reduce to separated first-order ordinary differential equations, which are easier to handle. It has also been noticed that consideration of stationary strings in static or time-independent backgrounds simplifies the problem a lot, where the stationary string configuration in static backgrounds can be described by geodesic equations in a certain three-dimensional ''unphysical'' space [3]. After knowing the extremal string configuration, one can also consider small perturbations around the stationary strings. Larsen and Frolov [12] have developed a covariant formalism for small perturbations propagating along a string in curved space-time (see also [13]) and have subsequently analyzed stationary strings in quasi-Newtonian, Rindler, black hole, and de-Sitter space-times. The second variation of the Polyakov action essentially gives the equations describing the propagation of perturbations along a stationary string in a static background. By doing the analysis in the properly chosen three-dimensional ''unphysical'' space-time, one obtains simple wave equations such as the Pöschl-Teller equation in quantum mechanics for strings in the Rindler and de-Sitter space-times. For the Schwarzschild black hole background, the wave equation has been explicitly analyzed by using the weak-field expansion and a scattering formalism has also been set-up [14].

In this paper, we analyze the equilibrium string configuration in the background of the exact, four-dimensional dyonic black hole of Lowe and Strominger [15], which has been obtained by tensoring the two-dimensional electrically charged black holes [these are related to the $(2+1)$-dimensional rotating black hole of Banados, Teitelboim, and Zanelli [16] by Kaluza-Klein reduction] with two-dimensional $SU(2)/Z(m)$ coset models. We make use of the stationary string *Ansatz* and in the $J=0$ limit, the solutions of the string equations of motion are obtained in terms of elliptic functions. However, we do not find any multistring solutions in the above case. We also investigate the nature of perturbations as well as their propagation around the string configuration.

II. STATIONARY STRING ANSATZ AND FOUR-DIMENSIONAL DYONIC BLACK HOLE BACKGROUND

The family of exact, four-dimensional dyonic black holes in string theory [15] are constructed as a tensor product of electrically charged two-dimensional black holes with the angular magnetic monopole conformal field theory (CFT) obtained by quotienting a SU(2) Wess-Zumino-Witten (WZW) model by the discrete subgroup $Z(m)$ [17], where m is an integer. The level of the corresponding WZW model is denoted as $k_{\rm SU}$. The two-dimensional, electrically charged black hole part is obtained by a Kaluza-Klein reduction [18] of the string analogue of the $(2+1)$-dimensional, rotating black hole solution [16]. This tensor product leads to a solution describing the throat limit of a four-dimensional black

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hole with electric and magnetic charge. The corresponding metric is given by [15]

$$ds^2=-\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)dt^2+\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right)^{-1}dr^2 + \frac{1}{4}k_{SU}d\theta^2+\frac{1}{4}k_{SU}\sin^2\theta d\phi^2, \quad (1)$$

where k_{SU} is the level of the SU(2) WZW model as discussed before, M is the mass of the black hole, J is the angular momentum, and the cosmological constant is proportional to l^2.

Next, we analyze the string configuration in the background of the above four-dimensional dyonic blackhole by using the stationary string *Ansatz*. We consider strings in the equatorial plane for which $\theta=(\pi/2)$. The stationary string *Ansatz* is given by

$$t=\tau, \quad r=r(\sigma), \quad \phi=\phi(\sigma). \quad (2)$$

The string equations of motion and the constraints are given by

$$\ddot{x}^\mu-x''^\mu+\Gamma^\mu_{\rho\sigma}(\dot{x}^\rho\dot{x}^\sigma-x'^\rho x'^\sigma)=0, \quad (3)$$

$$g_{\mu\nu}\dot{x}^\mu x'^\nu=g_{\mu\nu}(\dot{x}^\mu\dot{x}^\nu+x'^\mu x'^\nu)=0, \quad (4)$$

where dots and primes denote derivatives with respect to τ and σ, respectively, and μ, ν run over t, r, θ, and ϕ. Since we consider strings in the equatorial plane, we only have to determine the functions $r(\sigma)$ and $\phi(\sigma)$ by solving the string equations of motion and the constraints. The metric of a static space-time can, in general, be written as

$$g_{\mu\nu}=\begin{pmatrix}-F & 0\\ 0 & H_{ij}/F\end{pmatrix}, \quad (5)$$

where $\partial_t F=0$; $\partial_t H_{ij}=0$ and $i,j=1,2,3$. H_{ij} is the metric of the three-dimensional unphysical space. For the stationary string *Ansatz*, the equations of motion for the Nambu-Goto action reduce to

$$x''^i+\tilde{\Gamma}^i_{jk}x'^j x'^k=0, \quad (6)$$

where $\tilde{\Gamma}^i_{jk}$ is the Christoffel symbol for the metric H_{ij}. Equation (6) is nothing but the geodesic equation in the three-dimensional unphysical space, with the line element

$$d\tilde{s}^2=H_{ij}dx^i dx^j, \quad (7)$$

and

$$H_{ij}x'^i x'^j=1\,. \quad (8)$$

For the four-dimensional dyonic black hole background,

$$F=-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}, \qquad H_{ij}=\mathrm{diag}(1,\Delta,\Delta\ \sin^2\theta), \quad (9)$$

where

$$\Delta=\frac{1}{4}k_{SU}\left(-M+\frac{r^2}{l^2}+\frac{J^2}{4r^2}\right). \quad (10)$$

The nonzero components of the Christoffel symbol $\tilde{\Gamma}^i_{jk}$ for the metric in the "unphysical" space are given by

$$\tilde{\Gamma}^r_{\phi\phi}=-\frac{k_{SU}}{4}\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right)\sin^2\theta = \tilde{\Gamma}^r_{\theta\theta}\sin^2\theta,$$
$$\tilde{\Gamma}^\theta_{r\theta}=\tilde{\Gamma}^\phi_{r\phi} = \frac{1}{F}\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right), \quad (11)$$
$$\tilde{\Gamma}^\theta_{\phi\phi}=-\cos\theta\,\sin\theta,$$
$$\tilde{\Gamma}^\phi_{\phi\theta}=\cot\theta.$$

The nonvanishing components of the Riemann tensor are

$$\tilde{R}_{r\phi r\phi}=\sin^2\theta\tilde{R}_{r\theta r\theta} = \sin^2\theta\,\frac{k_{SU}}{4F}\left(\frac{M}{l^2}+\frac{3J^2M}{4r^4}-\frac{3J^2}{2r^2l^2}-\frac{J^4}{8r^6}\right),$$
$$\tilde{R}_{\theta\phi\theta\phi}=\frac{k_{SU}}{4}\sin^2\theta\left[F-\frac{k_{SU}}{4}\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right)^2\right]. \quad (12)$$

For strings in the equatorial plane ($\theta=\pi/2$), the geodesic equations are given by

$$\phi''+2\tilde{\Gamma}^\phi_{r\phi}r'\phi'=0, \quad (13)$$

$$r''+\tilde{\Gamma}^r_{\phi\phi}\phi'^2=0. \quad (14)$$

The constraint equation (8) is given by

$$r'^2(\sigma)+\Delta\phi'^2(\sigma)=1. \quad (15)$$

The string configuration is known by solving Eqs. (13), (14), and (15). Integrating the ϕ equation (13), one obtains

$$\phi'(\sigma)=\frac{b}{\Delta}. \quad (16)$$

Integrating Eq. (14) using Eq. (16), one obtains

$$r'(\sigma)=\pm\sqrt{1-\frac{b^2}{\Delta}}, \quad (17)$$

where b is an integration constant. These are the two first-order differential equations, whose solutions will give the string configuration. The r equation of motion can also be written as

$$r'^2+V(r)=0, \quad (18)$$

where, $V(r)$ is the effective potential given by

$$V(r)=-\left(1-\frac{b^2}{\Delta}\right). \tag{19}$$

Qualitatively the possible string configurations can also be known by studying the zeros of the potential. The solution of Eqs. (16) and (17) are obtained in terms of elliptic functions. The world sheet coordinate σ can be replaced in terms of conformal string parameter σ_c, so that $d\sigma_c=d\sigma/F$. Hence we obtain

$$r'(\sigma_c)=\pm\frac{4\Delta}{k_{SU}}\sqrt{1-\frac{b^2}{\Delta}}, \tag{20}$$

$$\phi'(\sigma_c)=\frac{4b}{k_{SU}}, \tag{21}$$

where, $4\Delta/k_{SU}=-M+r^2/l^2+J^2/4r^2$. These two equations are difficult to solve for nonzero J. Hence, we restrict ourselves to the case when $J=0$ and also take $M=1$. In this case, the solution for $r^2(\sigma_c)$ is obtained in terms of elliptic functions,

$$r^2(\sigma_c)=l^4\wp(\sigma_c+z_0;g_2,g_3)+\frac{2l^2}{3}\left(1+\frac{2b^2}{k_{SU}}\right), \tag{22}$$

where $\wp(z)$ is the Weierstrass elliptic $\wp$ function [19] with the variants g_2 and g_3 taking the value

$$g_2=\frac{4}{l^6}\left(\frac{l^2}{3}+\frac{4b^2l^2}{3k_{SU}}+\frac{16}{3}\frac{b^4l^2}{k_{SU}^2}\right),$$
$$g_3=-\frac{4}{3l^8}\left(\frac{2l^2}{9}+\frac{4}{3}\frac{b^2l^2}{k_{SU}}-\frac{16}{3}\frac{b^4l^2}{k_{SU}^2}-\frac{128}{9}\frac{b^6l^2}{k_{SU}^3}\right). \tag{23}$$

Here z_0 is an integration constant. The discriminant Δ is given by

$$\Delta=\frac{16\beta^2}{l^{12}}(1+\beta)^2, \tag{24}$$

where $\beta=4b^2/k_{SU}$. Since $\Delta>0$, all the roots e_1, e_2, e_3 are real. The roots are given by

$$e_1=\frac{1}{3l^2}(1+2\beta),$$
$$e_2=\frac{1}{3l^2}(1-\beta), \tag{25}$$
$$e_3=-\frac{2}{3l^2}\left(1+\frac{\beta}{2}\right),$$

where $e_1>e_2>e_3$. We can also express the solution (22) in terms of Jacobi's elliptic function, which is given by

$$r^2(\sigma_c)=l^2\mu^2 ns^2\left[\frac{\mu}{l}(\sigma_c+z_0),k\right], \tag{26}$$

where, $\mu^2=1+\beta$ and k is the elliptic modulus. This solution can be compared with strings in the background of a (2 +1)-dimensional black hole anti-de-Sitter space-time (for $J=0$ and $M=1$) given by [10]

$$r^2(\sigma)=l^4\wp(\sigma-\sigma_0;g_2,g_3)+\frac{2}{3}l^2 \tag{27}$$

with variants

$$g_2=\frac{4}{l^6}\left(\frac{l^2}{3}+L^2\right),$$
$$g_3=-\frac{4}{3l^8}\left(\frac{2l^2}{9}+L^2\right), \tag{28}$$

where L is an integration constant. The $\phi'(\sigma_c)$ equation (21) is simple and the solution is given by

$$\phi(\sigma_c)=\left(\frac{4b}{k_{SU}}\right)\sigma_c+\text{const.} \tag{29}$$

which means that σ_c is periodic with period $\pi k_{SU}/2b$. In this case, there is no multistring solution possible, whereas for strings in the background of a (2+1)-dimensional black hole anti-de-Sitter space-time, one does obtain multistring solutions.

III. STRAIGHT STRING AND CIRCULAR STRING CONFIGURATION

Here we obtain a simple string configuration by choosing the parameter $b=0$. In this limit, for the case of $J=0$ $M=1$, the two ordinary differential equations reduce to

$$r'^2(\sigma_c)=\left(\frac{r^2}{l^2}-1\right)^2,$$
$$\phi'(\sigma_c)=0. \tag{30}$$

Now the solution for the radial coordinate is obtained in terms of the hyperbolic function given by

$$r^2(\sigma_c)=l^2\tanh^2\left(\frac{\sigma_c}{l}\right) \tag{31}$$

and

$$\phi(\sigma_c)=\text{const.} \tag{32}$$

This is nothing but a straight string configuration.

The circular string configuration is given by $r=$const, ϕ as a periodic function of σ and $\theta=\pi/2$. Let us consider the case when $4b^2/k_{SU}=1$. In this case, the solutions of Eqs. (20) and (21) reduce to

$$r^2(\sigma_c)=l^2,2l^2,$$
$$\phi(\sigma_c)=\frac{\sigma_c}{b}, \tag{33}$$

Though they satisfy the two first-order differential equations, the analysis of the zeros of the effective potential shows that they will not lead to a circular string configuration. The effective potential is given by

$$V(r(\sigma_c)) = -\left(\frac{r^2}{l^2}-1\right)\left(\frac{r^2}{l^2}-1-\frac{4b^2}{k_{\rm SU}}\right). \tag{34}$$

This implies that the potential vanishes at $r=l$ and at $r=r_0=l[1+4b^2/k_{\rm SU}]^{1/2}$. For $4b^2/k_{\rm SU}=1$, r takes the value $r=r_0=\sqrt{2}l$. But they do not lead to a circular string configuration as they do not satisfy the condition $(\partial V/\partial r)|_{r=l,\sqrt{2}l}=0$, which is necessary for the original second-order string equations of motion (3) and the constraints (4) [or equivalently Eqs. (13), (14), and (15)] to be satisfied. On the other hand, a circular string configuration is possible for nonzero J. Consider a circular string *Ansatz*

$$t=\tau, \quad r=C_1, \quad \phi=\frac{\sigma}{C_2}, \quad \theta=\frac{\pi}{2}. \tag{35}$$

With this *Ansatz*, Eq. (13) is trivially satisfied. Equation (14) determines r as

$$r=C_1=\sqrt{\frac{Jl}{2}}. \tag{36}$$

Then Eq. (15) determines the constant C_2 as

$$C_2=\frac{1}{2}\sqrt{k_{\rm SU}\left(\frac{J}{l}-M\right)}. \tag{37}$$

Hence, $r=\sqrt{Jl/2}$ and $\phi=2\sigma/\sqrt{k_{\rm SU}(J/l-M)}$, $\theta=\pi/2$, $t=\tau$ is an acceptable circular string solution provided $J>Ml$. However, for the horizons to exist, we need the condition $\Delta=0$ and this implies that the solutions for r^2 are given by

$$r^2=\frac{1}{2}[Ml^2\pm l\sqrt{M^2l^2-J^2}].$$

As one can see from the above expression there is no real root if $J>Ml$. So this limit essentially gives us a geometry with no horizon. In fact, for stationary black hole solutions, which are not spherically symmetric, there cannot be any circular string configuration. This can be seen in the following way. Consider the general nonspherically symmetric metric of the form

$$ds^2=-a(r)dt^2+\frac{dr^2}{a(r)}+B(d\theta^2+\sin^2\theta d\phi^2), \tag{38}$$

where B is a constant. For the dyonic black hole metric $a(r)=[-M+r^2/l^2+J^2/4r^2]$ and $B=\frac{1}{4}k_{\rm SU}$. Let us consider the circular string *Ansatz* as discussed before. Now Eq. (13) is trivially satisfied. From Eq. (14), we obtain the condition $a'(r)=0$ (this will determine r for specific solutions). Now, from the constraint equation (15), one obtains $C_2=\sqrt{Ba(r)}$. The existence of horizon implies the condition $a(r)=0$, which means that the constraint equation is no longer satisfied. Hence, there cannot be any circular string configuration if the geometry in the above case has a horizon. The possibility of a circular string is also ruled out in the case of the Schwarzschild black hole [11]. However, one obtains closed string configurations for the wormhole geometry [8] and de-Sitter space-time [12].

IV. THE PROPAGATION OF PERTURBATION ALONG THE STRING

The stability properties of the above obtained solutions can be investigated by studying the propagation of small perturbations along the strings in curved space-time. The propagation of small waves on a membrane and cosmic strings have been considered before [20]. The equations of motion for small perturbations along the strings in arbitrary curved space-time can be obtained by the second variation of the Polyakov action. The first variation of the action gives the equations of motion for the strings in curved space-time itself. The variations of equations of motion and constraints for strings also gives the propagation of perturbations for the corresponding string configurations. A covariant formalism for the analysis of perturbation has been developed by Larsen and Frolov [12] and in Ref. [13].

Let x^μ be a solution of Eqs. (3) and (4). Introducing two vectors n_R^μ $(R=2,3)$ normal to the string world sheet, the perturbation can be decomposed as

$$\delta x^\mu=\delta x^R n_R^\mu+\delta x^A x^\mu_{,A}, \tag{39}$$

where $A=0$, 1 denote the string world sheet coordinates τ and σ, respectively, and the comma denotes partial differentiation. The normal vectors satisfy the conditions

$$g_{\mu\nu}n_R^\mu n_S^\nu=\delta_{RS}; \quad g_{\mu\nu}x^\mu_{,A}n_R^\nu=0. \tag{40}$$

Since the second term in the variation equation (39) leaves the action invariant due to reparametrization invariance, we will consider the first term in Eq. (39), i.e., $\delta x^\mu=\delta x^R n_R^\mu$. The effective action for the physical perturbation is obtained in terms of the second fundamental form and the normal fundamental form [12]. In the stationary string *Ansatz*, where $t=x^0=\tau$ and $x^i=x^i(\sigma)$, we have

$$x^\mu_{,0}=(1,0,0,0); \quad x^\mu_{,1}=(0,x'^i). \tag{41}$$

Therefore, the components of the normal vectors can be written as $n_R^\mu=(0,n_R^i)$.

(x'^i,n_2^i,n_3^i) form an orthogonal system in the three-dimensional ''unphysical'' space with the metric H_{ij}. By defining $n_R^i/\sqrt{F}=\tilde{n}_R^i$ we have the obvious relations

$$H_{ij}\tilde{n}_R^i\tilde{n}_S^j=\delta_{RS}, \qquad H_{ij}x'^i\tilde{n}_R^j=0. \tag{42}$$

The equations of motion for the physical perturbations are given by

$$(\partial^2_{\sigma_c}-\partial^2_{\sigma_c})\delta x_R=U_{RS}\delta x_S, \tag{43}$$

where U_{RS} is the matrix potential given by

$$U_{RS}=V\delta_{RS}+F^{-1}V_{RS}. \tag{44}$$

and,

$$V=\frac{3}{4F^2}\left(\frac{dF}{d\sigma_c}\right)^2-\frac{1}{2F}\frac{d^2F}{d\sigma_c^2}, \tag{45}$$

$$V_{RS}=x'^i(\sigma_c)x'^j(\sigma_c)\widetilde{R}_{ijkl}n_R^k n_s^l, \tag{46}$$

where $\widetilde{R}_{ijkl}$ is the Riemann tensor for the metric H_{ij} and $d\sigma_c=d\sigma/F$. The first term in Eq. (44) is connected with the time delay effect in a static gravitational field. The second nondiagonal term is connected with the curvature of the three-dimensional unphysical space. We need to calculate the matrix potential to determine the time-dependent propagation of the perturbation. The normal vectors perpendicular and parallel to the strings are chosen as

$$n_\perp^i=\frac{2}{\sqrt{k_{\rm SU}}}(0,1,0),$$

$$n_\parallel^i=\frac{2}{\sqrt{k_{\rm SU}}}\left(-b,0,(r^2/l^2-1)^{-1}\frac{dr}{d\sigma_c}\right). \tag{47}$$

Using Eqs. (44), (45), and (46) we obtain

$$V=P-\frac{b^2}{\Delta}\left[P+\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right)^2\right],$$

$$V_{\perp\perp}=-P\left(\frac{r^2}{l^2}-1\right)+\frac{4b^2}{k_{\rm SU}}\left[P+\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right)^2-\frac{1}{\Delta}\left(\frac{r^2}{l^2}-1\right)^2\right],$$

$$V_{\parallel\parallel}=-P\left(\frac{r^2}{l^2}-1\right), \tag{48}$$

where

$$P=\left(\frac{M}{l^2}+\frac{3J^2M}{4r^4}-\frac{3J^2}{2r^2l^2}-\frac{J^4}{8r^6}\right). \tag{49}$$

The components of the matrix potential are given by

$$U_{\perp\perp}=-\frac{16b^2}{k_{\rm SU}^2},$$

$$U_{\parallel\parallel}=-\frac{b^2}{\Delta}\left[P+\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right)^2\right], \tag{50}$$

$$U_{\perp\parallel}=0,$$

where $\Delta=(k_{\rm SU}/4)[-M+r^2/l^2+J^2/4r^2]$. Now the equations of motion for the time-dependent perturbations in perpendicular and parallel directions are respectively given by

$$(\partial_{\sigma_c}^2-\partial_r^2)\delta x_\perp+\frac{16b^2}{k_{\rm SU}^2}\delta x_\perp=0 \tag{51}$$

and

$$(\partial_{\sigma_c}^2-\partial_\tau^2)\delta x_\parallel+\frac{4b^2}{k_{\rm SU}[r^2/l^2-M+J^2/4r^2]}\times\left[P+\left(\frac{r}{l^2}-\frac{J^2}{4r^3}\right)^2\right]\delta x_\parallel=0. \tag{52}$$

For the straight string configuration $b=0$. So the perturbation equations become identical and they reduce to plane wave equations:

$$(\partial_{\sigma_c}^2-\partial_r^2)\delta x_{\perp,\parallel}=0. \tag{53}$$

By a Fourier expansion of $\delta x_{\perp,\parallel}$, we get

$$\delta x_{\perp,\parallel}(\tau,\sigma_c)=\int e^{-i\omega\tau}D_\omega(\sigma_c)d\omega. \tag{54}$$

The solutions of Eq. (53) are given by

$$\delta x_{\perp,\parallel}(\tau,\sigma_c)=\int d\omega(A_\omega^{\perp,\parallel}e^{-i\omega(\tau-\sigma_c)}+B_\omega^{\perp,\parallel}e^{-i\omega(\tau+\sigma_c)}). \tag{55}$$

Now for the circular string configuration (which is possible when the geometry does not have a horizon because of the condition $J>Ml$), we have $4b^2/k_{\rm SU}=1$. The perturbation equations in this limit are given by

$$(\partial_\sigma^2-\partial_\tau^2)\delta x_\perp+\frac{1}{b^2}\delta x_\perp=0 \tag{56}$$

and

$$(\partial_\sigma^2-\partial_\tau^2)\delta x_\parallel-\frac{4}{l^2}\delta x_\parallel=0. \tag{57}$$

With the Fourier expansion of $\delta x_{\perp,\parallel}$ [Eq. (54)], the above two equations reduce to

$$\frac{d^2}{d\sigma_c^2}D_\omega+\left[\omega^2+\frac{1}{b^2}\right]D_\omega=0 \tag{58}$$

and

$$\frac{d^2}{d\sigma_c^2}D_\omega+\left[\omega^2-\frac{4}{l^2}\right]D_\omega=0. \tag{59}$$

For the circular string configuration $\phi(\sigma_c)=\sigma_c/b$, where σ_c is periodic with period $2\pi b$. Since the potentials for both the perturbations are constant, we can write down the solutions as

$$\delta x_\perp=\int d\omega[A_\perp e^{-(i/b)(r\sqrt{n^2-1}\pm n\sigma_c)}] \tag{60}$$

and

$$\delta x_\parallel=\int d\omega[A_\parallel e^{-(i/b)(r\sqrt{n^2+4b^2/l^2}\pm n\sigma_c)}], \tag{61}$$

where, n is an integer. We note that here there is no unstable mode in the solution for $\delta x_\parallel$, whereas, unstable modes can arise in the solution of $\delta x_\perp$ when $n=0$.

V. CONCLUSION

In this paper, we have used the stationary string *Ansatz* to study the exact solutions of string equations of motion and constraints in the background of an exact, stringy four-dimensional dyonic black hole obtained by tensoring the two-dimensional electrically charged black hole with $SU(2)/Z(m)$ coset models. Since solving the equations for nonzero J becomes quite complicated, we restrict ourselves to the $J=0$ and $M=1$ limits of the above background to solve the geodesic equations in the three-dimensional ''unphysical'' space with a metric H_{ij}. The straight string configuration is obtained in terms of Weierstrass elliptic functions. We have also analyzed the possibility of having circular string configuration. Unlike the case of the $(2+1)$-dimensional black hole anti–de Sitter background of Banados, Teitelboim, and Zanelli, there cannot be any circular string configuration in the above four-dimensional background if the geometry has to include a horizon. No multistring solution is found in this case. We have also studied the propagation of physical perturbations along the stationary string in the given curved space-time by using a covariant formalism. The equations of motion for the time-dependent perturbations are in the form of wave equations with a complicated matrix potential term. For the straight string configuration (i.e., when the parameter b goes to zero), the matrix potential becomes zero and the equations of motion for perturbations in both the perpendicular and parallel directions reduce to simple plane wave equations. The above method can also be applied to stringy cosmological backgrounds to obtain various string configurations and a systematic analysis of the propagation of perturbation along the string configuration will be an interesting problem. To conclude, this exercise is mainly an attempt to have a general understanding of the string dynamics in curved space-times.

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