Lorentz-invariant actions for chiral p-forms

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We demonstrate how a Lorentz-covariant formulation of the chiral p-form model in $D=2(p+1)$ containing infinitely many auxiliary fields is related to a Lorentz-covariant formulation with only one auxiliary scalar field entering a chiral p-form action in a nonpolynomial way. The latter can be regarded as a consistent Lorentz-covariant truncation of the former. We make the Hamiltonian analysis of the model based on the nonpolynomial action and show that the Dirac constraints have a simple form and are all first class. In contrast with the Siegel model the constraints are not the square of second-class constraints. The canonical Hamiltonian is quadratic and determines the energy of a single chiral p-form. In the case of $D=2$ chiral scalars the constraint can be improved by use of a ''twisting'' procedure (without the loss of the property to be first class) in such a way that the central charge of the quantum constraint algebra is zero. This points to the possible absence of an anomaly in an appropriate quantum version of the model. [S0556-2821(97)03708-9]

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I. INTRODUCTION

Chiral p-forms, i.e., antisymmetric boson fields with self-dual $(p+1)$-form field strengths, form integral part and play an important role in many theoretical models such as $D=6$ and type IIB $D=10$ supergravity, heterotic strings [1], and M-theory five-branes ([2–5] and references therein). A particular feature of these fields is that, since the self-duality condition implies the fulfillment of first-order equations of motion, which puts the theory on the mass shell, there is a problem of construction manifestly Lorentz-invariant actions for the chiral p-forms [6] and, as a consequence, a problem of quantizing such fields. The analogous problems exist in manifestly electric-magnetic duality formulation of $D=4$ Maxwell theory [7], where the Maxwell field can be considered as a complex chiral two-form.

Nonmanifestly covariant actions were proposed for $D=2$ chiral scalars in [8], for $D=4$ duality symmetric Maxwell fields in [9,10], for $D=2(p+1)$ chiral p-forms in [11], and for duality symmetric fields in space-time of any dimension in [12]. All of these actions lead to second-class constraints on the chiral boson phase space, which complicates the quantization procedure.

In [13] a $D=2(p+1)$ Lorentz-invariant action for chiral p-forms was constructed by squaring the second-class constraints and introducing first-class constraints thus obtained into the action with Lagrange multipliers. However, though the Lagrange multipliers do not contribute to the equations of motion of this model, it is not clear whether in $D>2$ $(p>1)$ there is enough local symmetry to completely gauge them away [13]. At the same time, even in $D=2$ the Siegel action for chiral scalars is not easy to quantize (in particular because of an anomaly problem) and an extensive literature has been devoted to studying this point (see, for example, [14]).

Another covariant (Hamiltonian) formulation was proposed for $D=2$ chiral scalars by McClain, Wu, and Yu [15] (see also [16]) and generalized to the case of higher order chiral p-forms in [17,18]. The construction is based on a procedure of converting the second-class constraints into first-class ones by introducing auxiliary fields [19]. In the case at hand, this required an infinite set of auxiliary $(p+1)$-forms. By use of a Legendre transformation it is possible to write down a manifestly Lorentz-invariant form of the chiral boson actions [20]. The chiral scalar and free Maxwell theory were consistently quantized in such a formulation, respectively, in [15] and [17].

It is of interest and somehow indicative that for a chiral four-form in ten dimensions the Lorentz-covariant formulation of [15–18,20] was, actually independently, derived from type IIB closed superstring field theory in [21,22].

The infinite set of auxiliary fields in the chiral boson models requires caution to deal with when one studies equations of motion, makes Hamiltonian analysis, imposes admissible gauge-fixing conditions, and quantizes the models [15–22], since, in particular, this infinite set corresponds to the infinite number of local symmetries and first-class constraints which cause problems with choosing the right regularization procedure. For instance, in [15] a strong group-theoretical argument based on the existence of a symmetry of the quantum theory was used to justify the regularization which leads to the correct partition function of the chiral scalar.

Note also that a direct cutting of the infinite series of fields at a number of N results in an action which does not describe a single chiral p-form [20].

An alternative Lorentz-invariant action for chiral p-forms was proposed in [23–25]. This formulation involves finite number of auxiliary fields and, as a consequence, a finite number of local symmetries being sufficient to gauge these fields away. Upon an appropriate gauge fixing one gets nonmanifestly covariant models of Refs. [8,11,12,5]. The ad-

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vantage of the covariant approach is that one should not bother about proving Lorentz invariance which may be rather cumbersome [11,12,5].

A minimal version of this covariant formulation contains (in space-time of any even dimension[1]) only one scalar auxiliary field entering the action in a nonpolynomial way. In the case of $D=4$ Maxwell theory this scalar field was assumed to be of an axion nature [24].

The purpose of the present paper is, on the one hand, to show how the McClain-Wu-Yu approach and the approach of Refs. [23–25] relate to each other, and, on the other hand, to make the Hamiltonian analysis of the nonpolynomial version and to demonstrate that in spite of the nonpolynomiality the structure of the constraints (which all belong to the first class) is rather simple, the canonical Hamiltonian is quadratic and describes the energy of a single chiral p-form boson. In the case of a chiral scalar in $D=2$ the form of the first-class constraint allows improvement by ''twisting'' its auxiliary field term, which at the quantum level allows the central charge of the constraint algebra to be zero. This points to possible absence of anomaly in a quantum version of the model.

The paper is organized as follows. In Sec. II we review the Lorentz-invariant chiral form actions of Refs. [15–22] and [23–25] and demonstrate a relationship between them by either trying to get rid of the nonpolynomiality and eliminate the scalar auxiliary field at the expense of introducing auxiliary $(p+1)$-forms or, vice versa, by a consistent truncation of the McClain-Wu-Yu infinite tail with putting on its end the auxiliary scalar field. In Sec. III we analyze the classical Hamiltonian structure of the chiral form model with the single auxiliary scalar and, in the $D=2$ case, discuss the problem of quantum anomaly of local symmetry of the model.

In the Conclusion (Sec. IV) open problems and prospectives are discussed.

To simplify notation and convention we consider $D=2$ chiral scalars and $D=6$ chiral two-forms. However, upon fitting numerical coefficients one can straightforwardly generalize all the expressions obtained to the generic case of chiral p-forms. We use almost positive signature of space-time, i.e., $(-,+,\ldots,+)$. Latin letters stand for space-time indices $(l,m,n,\ldots=0,1,\ldots,D-1)$ and Greek letters are spacial indices running from 1 to $D-1$.

II. RELATIONSHIP BETWEEN LORENTZ-INVARIANT CHIRAL FORM ACTIONS

A. The infinite series action

In a reduced form considered in [16,22] [where part of an infinite number of auxiliary fields was eliminated by gauge fixing an infinite number of local symmetries (see the first paper of Ref. [22] for details)], the chiral boson action of [15–20] in $D=6$ looks as

$$S=\int d^6x\left[-\frac{1}{6}F_{pqr}F^{pqr}+\Lambda^{pqr}_{(1)}F_{pqr}-\sum_{n=0}^{\infty}(-1)^n\Lambda^{pqr}_{(n+1)}\Lambda_{(n+2)pqr}\right],\tag{1}$$

where $F_{pqr}=\partial_{[p}A_{qr]}$, and $\Lambda_{(n+1)pqr}=[(-1)^n/3!]\varepsilon_{pqrlmn}\Lambda^{lmn}_{(n+1)}$ form an infinite set of (anti-)self-dual auxiliary three-form fields. The action (1) describes a single physical chiral two-form A_{mn} satisfying the self-duality condition

$$\mathcal{F}^{lmn}\equiv F_{lmn}-\frac{1}{3!}\varepsilon_{lmnpqr}F^{pqr}=0.\tag{2}$$

To arrive at Eq. (2), one should make an assumption that allowables are only those solutions to the equations of motion derived from Eq. (1) which contain only a finite number of nonzero fields $\Lambda_{(n+1)}$. This restriction, though it looks somewhat artificial, ensures the energy of the model to be well defined. Note that one cannot make such a truncation and eliminate all fields with n greater than a given number N directly in the action since this results in a model which does not describe a single chiral field, but an ordinary (chiral plus antichiral) antisymmetric gauge field, or a pair of chiral forms depending on the parity of N. The reader may find a detailed analysis of the model in [18,20,22].

B. Chiral form action with a finite number of auxiliary fields

The Lorentz-invariant self-dual action of Refs. [23–25] have the following form in $D=6$:

$$S=\int d^6x\left[-\frac{1}{6}F_{lmn}F^{lmn}+\frac{1}{2(u_qu^q)}u^m\mathcal{F}_{mnl}\mathcal{F}^{nlr}u_r-\varepsilon^{mnpqrs}u_m\partial_n\Lambda_{pqrs}\right].\tag{3}$$

Equation (3) contains the anti-self-dual three-form $\mathcal{F}_{mnl}$ defined in Eq. (2) (whose turning to zero on the mass shell results in the self-duality of F_{lmn}); an auxiliary vector field $u_m(x)$ and a four-form field Λ_{lmnp}.

The action (3) is invariant under the local transformations

$$\delta A_{mn}=\partial_{[m}\phi_{n]}(x),\quad \delta\Lambda_{lmnp}=\partial_{[l}\phi_{mnp]}(x)\tag{4}$$

(which are the ordinary gauge symmetries of massless antisymmetric fields),

$$\delta A_{mn}=\frac{1}{2}u_{[m}\varphi_{n]}(x),\quad \delta\Lambda_{lmnp}=\frac{1}{u^2}\varphi_{[l}u_m\mathcal{F}_{np]q}u^q,\tag{5}$$

$$\delta u_m=0$$

(where $u^2=u_\alpha u_\alpha-u_0u_0$), and

$$\delta u_m=\partial_m\varphi(x),\quad \delta A_{mn}=\frac{\varphi(x)}{2u^2}\mathcal{F}_{mnp}u^p,\tag{6}$$

$$\delta\Lambda_{mnpq}=-\frac{\varphi(x)}{(u^2)^2}u^r\mathcal{F}_{r[mn}\mathcal{F}_{pq]s}u^s.$$

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[1]Remember that if p is odd the chiral form is complex in $D=2(p+1)$.

Note that the transformations (4) and (5) are finite-step reducible, which is harmless. For instance, if in Eqs. (5), $\varphi_m = \phi(x) u_m$ then the variations of A_{mn} and Λ_{mnpq} are zero.

The equation of motion of Λ_{mnpq} reads

$$\frac{\delta S}{\delta \Lambda} \Rightarrow \partial_{[m} u_{n]} = 0. \tag{7}$$

Its general solution is

$$u_m = \partial_m a(x), \tag{8}$$

with $a(x)$ being a scalar field. Under Eqs. (6), $a(x)$ transforms as a Goldstone field $[\delta a(x) = \varphi(x)]$ and can be completely gauge fixed. Thus, u_m is an auxiliary field. When one takes u_m to be a unit timelike vector (for instance, $u_m = \delta_m^0$), the model loses manifest Lorentz invariance and reduces to the noncovariant model of Refs. [8,11,12]. If, instead, one chooses a spacelike gauge $u_m = \delta_m^5$, the action (3) reproduces the free chiral field formulation of Ref. [5].

We should note that because of the presence of the norm of u_m in the denominator in the action (3), the gauge-fixing condition $u_m = 0$ $[a(x) = \text{const}]$ (or, more generally, $u_m u^m = 0$) cannot be applied directly and in this sense is inadmissible. So, in what follows we shall require $u_m u^m \neq 0$. This situation is analogous to that in gravity, where one requires the existence of the inverse space-time metric. However, in principle, one can arrange a consistent limit of $u_m \to 0$ with an appropriate simultaneous limit of other fields in such a way that the physical contents of the model are the same as at other gauge points.[2]

The equation of motion of A_{mn} is

$$\epsilon^{lmnpqr} \partial_n \left(\frac{1}{u^2} u_p \mathcal{F}_{qrs} u^s \right) = 0. \tag{9}$$

Its general solution has the form [when Eq. (7) is taken into account]

$$\mathcal{F}_{lmn} u^n = u^2 \partial_{[l} \Phi_{m]} + u^n (\partial_n \Phi_{[l}) u_{m]} + u_{[l} (\partial_{m]} \Phi_n) u^n, \tag{10}$$

where $\Phi_m(x)$ is an arbitrary vector function. One can check that the right-hand side (RHS) of Eq. (10) has the same form as the transformation of $\mathcal{F}_{qrs} u^s$ under Eqs. (5), thus one can use this symmetry to gauge fix the RHS of Eq. (10) to zero. As a result, because of the anti-self-duality, the whole $\mathcal{F}_{lmn}$ becomes equal to zero and we get the self-duality of F_{lmn}, Eq. (2) [12,23–25]. In this gauge the equation of motion of u_m reduces to

$$\frac{\delta S}{\delta u} \Rightarrow \varepsilon^{mnpqrs} \partial_n \Lambda_{pqrs} = 0, \tag{11}$$

from which, in view of the local symmetry (4), it follows that Λ_{mnpq} has only pure gauge degrees of freedom.

[2]The problem of an admissible gauge choice also exists for the infinitely many-field actions [22]. There it is caused by a requirement of convergency of infinite series. It might happen that such ''critical'' gauge points in both approaches have a unique nature.

Thus, the model based on action (3) indeed describes the classical dynamics of a single chiral two-form field A_{mn}.

We can simplify this action by substituting u_m with its expression in terms of $a(x)$, Eq. (8). Then Eq. (3) takes the form which contains only one auxiliary scalar field $a(x)$:

$$S = \int d^6x \left[-\frac{1}{6} F_{lmn} F^{lmn} + \frac{1}{2(\partial_q a \partial^q a)} \times \partial^m a(x) \mathcal{F}_{mnl} \mathcal{F}^{nlr} \partial_r a(x) \right]. \tag{12}$$

This action possesses the same symmetries as Eq. (3) with only difference that now $u_m = \partial_m a$, and Λ_{lmnp} is absent from Eqs. (4)–(6). Notice that the variation of the action (12) over $a(x)$ is identically zero on the solutions (10) of Eq. (9). It is simply Eq. (9) multiplied by $(1/u^2) \mathcal{F}_{lmf} u^f$ and, hence, does not produce new field equations. This reflects the presence of the local symmetry (6).

C. Passing from one action to another

Now, let us try to relate action (12) to the action (1) containing infinite number of auxiliary fields. For this we should first get rid of the nonpolynomiality of Eq. (12) or Eq. (3) by introducing new auxiliary three-form fields in an appropriate way.[3] We write

$$S = \int d^6x \left[-\tfrac{1}{6} F_{lmn} F^{lmn} + \hat{\Lambda}_{(1)mnl} \mathcal{F}^{mnl} - \tfrac{1}{2} \hat{\Lambda}_{(1)lmn} \hat{\Lambda}^{lmn}_{(1)} + \hat{\Lambda}^{lmn}_{(2)} (\hat{\Lambda}_{(1)lmn} - \hat{\Lambda}_{(0)lmp} \partial^p a \partial_n a) \right]. \tag{13}$$

One can directly check that upon eliminating the auxiliary fields $\hat{\Lambda}$ by solving their algebraic equations of motion one returns back to the action (12).

We can make another step and replace the term $\hat{\Lambda}_{(0)lmp} \partial^p a \partial_n a$ in Eq. (13) with an arbitrary three-form $\Lambda_{(3)mnl}$ and, for not spoiling the model, add to the action one more term of the form

$$\int dx^6 \hat{\Lambda}^{lmn}_{(4)} (\hat{\Lambda}_{(3)lmn} - \hat{\Lambda}_{(0)lmp} \partial^p a \partial_n a).$$

Introducing more and more auxiliary three-forms we can make any number N of steps of this kind and push the term containing $a(x)$ as far from the beginning of the series under construction as we like:

$$S = \int d^6x \left[-\frac{1}{6} F_{lmn} F^{lmn} + \hat{\Lambda}_{(1)mnl} \mathcal{F}^{mnl} - \frac{1}{2} \hat{\Lambda}_{(1)lmn} \hat{\Lambda}^{lmn}_{(1)} + \sum_{n=0}^{2N-1} (-1)^n \hat{\Lambda}^{pqr}_{(n+1)} \hat{\Lambda}_{(n+2)pqr} + \hat{\Lambda}^{lmn}_{(2N+2)} (\hat{\Lambda}_{(2N+1)lmn} - \hat{\Lambda}_{(0)lmp} \partial^p a \partial_n a) \right]. \tag{14}$$

[3]Another way to eliminate nonpolynomiality is to consider u_m to be a unit-norm harmoniclike variable, i.e., to impose the constraint $u^2 = -1$. Such a version of the model was discussed in [23,24].

At $N\to\infty$, we get exactly the action (1) upon splitting $\hat{\Lambda}_{(n+1)pqr}$ $(n=0,\ldots,2N+1)$ into self-dual and anti-self-dual parts and redefining them in an appropriate way.

On the other hand, if we start from the action (1) with the infinite number of fields, the procedure considered above prompts how one can consistently truncate the infinite series without spoiling the physical contents of the model at least at the classical level. The prescription is as follows: if in Eq. (1) one wants to put to zero all $\Lambda_{(n+1)}$ with $n>N'$, then one should replace the sum of the self-dual and anti-self-dual form $\Lambda_{(N')lmn}+\Lambda_{(N'+1)lmn}$ with a term of the form $\partial^p a \hat{\Lambda}_{(0)p[lm}\partial_{n]}a$ (where $\hat{\Lambda}_0$ is an arbitrary three-form).

Thus, the chiral form action with infinite number of auxiliary fields is related to the action (12) through the consistent truncation of the infinite tail of the former. The truncation leads to a reconstruction of symmetries in the model which become of the type written in Eqs. (4)–(6).

III. HAMILTONIAN ANALYSIS OF THE NONPOLYNOMIAL ACTION

The Hamiltonian structure which follows from the chiral form action with infinite number of fields was discussed in detail in [15,18,20] and we refer the reader to these papers.

Below, we shall make the Hamiltonian analysis of models based on action (12). As an instructive example, we start with the action for a chiral boson in $D=2$ and compare its Hamiltonian structure with other versions [13,15,16] of the chiral boson model.

A. $D=2$ chiral bosons

Action (12) takes the form [25]

$$S=\int d^2x\frac{1}{2}\left[\partial_+\phi\partial_-\phi-\frac{\partial_+a}{\partial_-a}(\partial_-\phi)^2\right], \tag{15}$$

where $\partial_\pm\equiv\partial_0\pm\partial_1$.

And the action-invariance transformations (4)–(6) reduce to

$$\delta a=\varphi, \quad \delta\phi=\frac{\varphi}{\partial_-a}\partial_-\phi. \tag{16}$$

The essential difference of the action (15) from the Siegel model [13] is that the second term in Eq. (15) contains derivatives of the scalar field $a(x)$ and not an arbitrary Lagrange multiplier $\lambda_{++}(x)$ as in the Siegel case.

The canonical momenta of the fields $\phi(x)$ and $a(x)$ are

$$P_\phi=\frac{\delta L}{\delta\dot{\phi}}=\dot{\phi}-\frac{\partial_+a}{\partial_-a}\partial_-\phi=\phi'-2a'\frac{\partial_-\phi}{\partial_-a}, \tag{17}$$

$$P_a=\frac{\delta L}{\delta\dot{a}}=a'\left(\frac{\partial_-\phi}{\partial_-a}\right)^2, \tag{18}$$

where the ''dot'' and ''prime'' denote time and spatial derivative, respectively.

From Eqs. (17) and (18) we get the primary constraint

$$C\equiv\frac{1}{4}(P_\phi-\phi')^2-P_aa'\equiv\frac{1}{4}(P_\phi-\phi')^2-\frac{1}{4}(P_a+a')^2+\frac{1}{4}(P_a-a')^2=0. \tag{19}$$

The canonical Hamiltonian of the model has the form

$$H_0=\frac{1}{2}\int dx^1(P_\phi+\phi')^2. \tag{20}$$

It does not contain the field $a(x)$ and describes the energy of a single chiral boson mode.

The constraint (19) strongly commutes with H_0 under the equal-time Poisson brackets

$$\{P_\phi(x),\phi(y)\}=\delta(x-y), \quad \{P_a(x),a(y)\}=\delta(x-y). \tag{21}$$

Hence, there are no secondary constraints in the model, and one can check that Eq. (19) is the first-class constraint associated with the symmetry transformations (16). The Poisson brackets of $C(x)$ have the properties of a classical Virasoro stress tensor:

$$\{C(x),C(y)\}=-\delta(x-y)C'(y)+2\partial_x\delta(x-y)C(y). \tag{22}$$

In contrast with the Siegel model [13] where the constraint is

$$(P_\phi-\phi')^2=0, \tag{23}$$

the first-class constraint (19) is not the square of a second-class constraint.

If we partially fix the gauge under the transformations (6) (i.e., under $\delta a=\varphi$) by imposing the gauge condition

$$C_2=P_a-a'=0, \tag{24}$$

we again find a relation of the present model with the McClain-Wu-Yu approach.

Indeed, the constraint C_2 is of the second class $[\{C_2,C_2\}=2\delta'(x-y)]$. And with taking it into account in Eq. (19) we can split the latter into the product of two multipliers

$$C=\frac{1}{4}(P_\phi-\phi'-P_a-a')(P_\phi-\phi'+P_a+a')=0, \tag{25}$$

either of which can be taken as independent constraint (since constraints are always defined up to a field-dependent factor). For instance, let us take

$$C_1=\frac{1}{2}(P_\phi-\phi'+P_a+a')=0. \tag{26}$$

This constraint is still first class and strongly commutes with itself and Eq. (24) under the classical Poisson brackets (21).

If now one would like to convert the second-class constraint into a first-class one by use of the standard conversion procedure [19], which implies introducing new auxiliary fields, one arrives at the model with an infinite set of first-class constraints for an infinite set of fields considered in detail in [15].

Let us discuss prospectives for a consistent quantization of the model based on action (15). One of the problems one should address is the problem of gauge symmetry anomalies. The indication that an anomaly might exist is the appearance of a nonzero central charge in the quantum commutator of constraints which are classically first class.

In our case the quantum commutator acquires the central charge $c=3$ because of the sum of three Virasoro-like terms in Eq. (19).

Remember that in the Siegel model [13] the central charge is equal to 1, and to cancel the anomaly the authors of [14] proposed to improve Eq. (23) by adding to it the total derivative term $\partial_1^2\phi(x)$ with an appropriate coefficient. Though this way one can cancel the quantum anomaly, the model looses the gauge symmetry at the classical level since classically the new constraint is not first class anymore.

In our case things differ because of the presence in Eq. (19) of a b-c ghostlike term containing the auxiliary field $a(x)$. Without spoiling the property of the constraint (19) to be first class, we can add to it the total derivative term $\lambda\partial_1(P_a a)$ (where λ is an arbitrary constant) and to get an improved constraint in the form

$$C_{(\lambda)}=\frac{1}{4}(P_\phi-\phi')^2-P_a a'+\lambda\partial_1(P_a a)$$
$$=\frac{1}{4}(P_\phi-\phi')^2-(1-\lambda)P_a a'+\lambda P_a' a=0. \quad (27)$$

This procedure is akin to ghost ''twisting'' commonly used in conformal field and string theory. The contribution of the terms containing $a(x)$ to the quantum central charge is $2(6\lambda^2-6\lambda+1)$ [26]. So, the central charge appearing in the RHS of the quantum commutator of Eq. (27) is

$$c=1+2(6\lambda^2-6\lambda+1). \quad (28)$$

It vanishes at $\lambda=\frac{1}{2}$.

Thus, we can assume that, due to operator ordering, the quantum theory can be reconstructed in such a way that the central charge of the quantum constraint [containing a contribution from ghosts (if any)] is equal to zero, and the anomaly associated with the local symmetry of the model does not arise. We hope to carry out detailed study of this point in future work.

B. Chiral two-forms in $D=6$

Let us analyze from the Hamiltonian point of view the model based on action (3). (To simplify a bit, the form of expressions we shall denote $\partial_m a(x)\equiv u_m$.) The canonical momenta of A_{mn} and $a(x)$ are

$$P_{A_{0\alpha}}=0,$$
$$P_{A\alpha\beta}=\frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta\rho}F_{\gamma\delta\rho}+2\frac{(u_\gamma)^2}{u^2}\mathcal{F}_{0\alpha\beta}-\frac{u_0}{u^2}u_\gamma\varepsilon_{\alpha\beta\gamma\delta\rho}\mathcal{F}_{0\delta\rho}-\frac{2}{u^2}u_{[\beta}\mathcal{F}_{0\alpha]\gamma}u_\gamma, \quad (29)$$

$$P_{a(x)}=2\frac{u_0}{(u^2)^2}(u_\gamma)^2(\mathcal{F}_{0\alpha\beta})^2-\frac{(u_0)^2+(u_\gamma)^2}{2(u^2)^2}\times u_\gamma\varepsilon_{\alpha\beta\gamma\delta\rho}\mathcal{F}_{0\beta\gamma}\mathcal{F}_{0\delta\rho}-4\frac{u_0}{(u^2)^2}(\mathcal{F}_{0\alpha\beta}u_\beta)^2. \quad (30)$$

Remember that $\alpha,\beta,\ldots=1,\ldots,5$, $\varepsilon_{\alpha\beta\gamma\delta\rho}\equiv\varepsilon_{0\alpha\beta\gamma\delta\rho}$, $u^2=u_\gamma u_\gamma-u_0u_0$, and $\mathcal{F}_{\alpha\beta\gamma}=\frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta\rho}\mathcal{F}_{0\delta\rho}$ are the components of the anti-self-dual tensor $\mathcal{F}_{lmn}$ defined in Eq. (2).

The canonical Poisson brackets are

$$\{P_{A0\alpha},A_{0\beta}\}=\delta_{\alpha\beta}\delta^{(5)}(x-y),$$
$$\{P_{A\alpha\beta},A_{\gamma\delta}\}=\delta_{\alpha[\gamma}\delta_{\delta]\beta}\delta^{(5)}(x-y), \quad (31)$$
$$\{P_a(x),a(y)\}=\delta^{(5)}(x-y).$$

The parts of the momenta corresponding to the self-dual and anti-self-dual part of $A_{\alpha\beta}$ are

$$\Pi^+_{\alpha\beta}=P_{A\alpha\beta}+\frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta\rho}F_{\gamma\delta\rho},$$
$$\Pi_{\alpha\beta}=P_{A\alpha\beta}-\frac{1}{6}\varepsilon_{\alpha\beta\gamma\delta\rho}F_{\gamma\delta\rho}. \quad (32)$$

Note that $\{\Pi^+,\Pi\}=0$.

After some algebraic manipulations, one gets the primary constraints:

$$P_{A0\alpha}=0, \quad (33)$$

$$\Pi_{\alpha\beta}\partial_\beta a(x)=0, \quad (34)$$

$$\frac{1}{8}\varepsilon_{\alpha\beta\gamma\delta\rho}\Pi_{\alpha\beta}\Pi_{\gamma\delta}\partial_\rho a(x)+P_a[\partial_\gamma a(x)]^2=0. \quad (35)$$

The constraints are first class and correspond to the local symmetries (4)–(6), respectively.

The canonical Hamiltonian is

$$H_0=\int dx^5\left[\frac{1}{4}(\Pi^+_{\alpha\beta})^2-2P_{A\alpha\beta}\partial_\alpha A_{0\beta}\right]. \quad (36)$$

Commuting H_0 with Eq. (33), we get the secondary constraint, which is also first class and corresponds to the Gauss law for the gauge field A_{mn}

$$\partial_\alpha P_{A\alpha\beta}=0. \quad (37)$$

All other constraints strongly commute with the Hamiltonian. Thus, as in the $D=2$ case, there are no second-class constraints in the model, and the constraint (35) is not the square of a second-class constraint.

If we fix a Lorentz-noncovariant (timelike) gauge $u_\gamma = \partial_\gamma a = 0$, the definitions (29) and (30) of the momenta imply that the constraints (34) and (35) consistently reduce to

$$\Pi_{\alpha\beta} = 0, \quad P_a = 0, \tag{38}$$

where the first constraint belongs to the second class. More precisely, it is a mixture of first- and second-class constraints [11,20]. In this gauge we recover the noncovariant chiral form model of Refs. [11,12].

IV. CONCLUSION

We have demonstrated how the Lorentz-covariant formulation of the chiral p-form model containing infinitely many auxiliary fields is related to the Lorentz-covariant formulation with only one auxiliary field entering the chiral p-form action in a nonpolynomial way. The latter can be regarded as a consistent Lorentz-covariant truncation of the former.

The Hamiltonian analysis of the model based on the nonpolynomial action has shown that in spite of nonpolynomiality, the Dirac constraints have a simple form and are all first class. In contrast with the Siegel model, the constraints are not the square of second-class constraints. The canonical Hamiltonian is quadratic and describes a single chiral p-form.

We have seen that in the case of $D=2$ chiral scalars the constraint can be improved by use of ''twisting'' procedure (without the loss of the property to be first class) in such a way that the central charge of the quantum constraint algebra is zero. This points to the possible absence of anomaly associated with the local symmetry of the classical theory in an appropriate quantum version. To justify this conjecture one should carry out the quantization of the chiral form model in the formulation considered above, which is a goal still to be reached.

The chiral p-form action (12) allows coupling to gravity in the natural covariant way [23–25]. Thus, the long-standing problem of gravitational anomaly caused by chiral forms might also be studied in this formulation.

The nonpolynomial version can be supersymmetrized [23–25]. In $D=2$ case an $N=1/2$ superfield formulation of one scalar and one spinor chiral field exists [25], while in $D=4$ only a component $N=1$ supersymmetric version of duality symmetric Maxwell theory is known yet [12,24]. Recently, Berkovits [22] proposed superfield formulation for duality-symmetric super-Maxwell theory in the version with infinitely many fields. In view of the relationship considered above it would be of interest to truncate his supersymmetric model to a superfield version of the action (3) or (12).

Another interesting problem is to consider interaction of chiral forms with other fields and between themselves [9,7,22,5] with the aim, for instance, to construct complete actions for p-branes which have chiral form fields in their world volumes, such as the M-theory five-brane [3–5]. Our manifestly Lorentz-covariant approach might be useful in making progress in this direction.

We hope to address some of these problems in the future.

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