# Bosonic thermal masses in supersymmetry

D. Comelli
*Instituto de Física Corpuscular–IFIC/CSIC, Departmento de Física Teòrica, Universidad de València, 46100 Burjassot, València, Spain*

J. R. Espinosa
*Deutsches Elektronen Synchrotron DESY, Notkestrasse 85, 22603 Hamburg, Germany*
(Received 26 June 1996)

Effective thermal masses of bosonic particles in a plasma play an important role in many different phenomena. We compute them in general supersymmetric models at leading order. The origin of the different corrections is explicitly shown for the formulas to be applicable when some particles decouple. The correct treatment of Boltzmann decoupling in the presence of trilinear couplings and mass mixing is also discussed. As a relevant example, we present results for the minimal supersymmetric standard model. [S0556-2821(97)02510-1]

PACS number(s): 11.10.Wx, 11.30.Pb

## I. INTRODUCTION

If supersymmetry is realized in nature, it would have manifold and very important implications on the history of the early Universe. In fact, much effort has been devoted to the study of supersymmetric cosmology and supersymmetric solutions to old cosmological problems have been proposed (while new problems have also arisen; see [1] for a review and references). However, the (weak scale[1]) supersymmetric generalization of the standard model (SM) is not uniquely defined. First, the introduction of arbitrary soft supersymmetry-breaking parameters, to prevent the mass degeneracy between ordinary and supersymmetric particles, generates a lot of freedom; second, there are various options related to the particle content and the gauge group definition. This limits the generality of the predictions that can be made although it still permits us to confront different classes of models and theoretical assumptions by examining their cosmological implications.

At the high temperatures of the early Universe, supersymmetric particles would be thermally pair created and would populate the plasma. One of the simplest consequences of this fact is that the effective thermal mass of a generic particle present in that plasma would change due to interactions with supersymmetric ambient particles. It is obvious that knowledge of these effective thermal masses is fundamental to describe the behavior and properties of the plasma. Moreover, it is well known that these quantities play a crucial role in many interesting aspects of the evolution of the early Universe.

In the case of gauge vector bosons (see, e.g., [2,3]) the effective thermal mass for longitudinal components corresponds to the usual Debye mass, i.e., the inverse screening length of electric potentials in the plasma. At leading order (one loop in perturbation theory) the Debye mass is $m_D \sim gT$, where $g$ is the corresponding gauge coupling constant. Transverse components have instead zero thermal mass at leading order. For Abelian gauge bosons this is true also to all orders, corresponding to the nonscreening of magnetic fields, but for non-Abelian gauge bosons a magnetic mass of order $g^2T$ is expected to appear nonperturbatively. Supersymmetric particles in the plasma will have an influence on Debye masses (see [4]). In this paper we will consider only thermal masses at leading order so that magnetic masses will be taken to be zero.

Let us turn now to scalar thermal masses. As pointed out by Kirzhnits and Linde [5], spontaneously broken symmetries are generally restored at high temperatures (see also [6]). This can be understood in terms of the effective thermal mass of the (Higgs) scalars driving the symmetry breaking. Consider as a particularly relevant example the case of electroweak gauge symmetry. Call $\phi$ the Higgs field responsible for the breaking. The one-loop approximation for the effective potential of $\phi$, including the effects of finite temperature, is of the form

$$V(\phi,T)=\frac{1}{2}(\kappa T^2-m^2)\phi^2-ET(\phi^2)^{3/2}+\frac{1}{4}\lambda(T)\phi^4, \quad (1)$$

where $E$, $\kappa$, and $\lambda(T)$ are some functions of the masses and couplings, easily calculable in a given model. For low temperatures the negative $T=0$ mass squared dominates, favoring the formation of a condensate, while at sufficiently high $T$, the (leading order) Higgs boson thermal mass $\sqrt{\kappa}T \sim [\text{coupling}]\times T$ dominates over the negative $T=0$ mass, disfavoring a nonzero condensate. Furthermore, it is clear that knowledge of the thermal mass for the Higgs bosons allows an estimate of the critical temperature [2] of the transition ($T_c^2 \sim m^2/\kappa$). When the scalar potential allows for

[1]We concentrate here on temperatures of that order, relevant, for example, in studies of the electroweak phase transition.

[2]Of course, a precise determination of the critical temperature must take into account also the rest of the terms in Eq. (1) and higher order effects (for example, $m^2$ also depends on $T$, etc.). In many cases, however, the simple analytical estimate $T_c^2 \sim m^2/\kappa$ is useful.

 

different patterns of symmetry breaking (as is the case if, e.g., $m^2<0$ along different field directions) knowledge of the critical temperatures associated with the different directions is crucial to determine the thermal history of the Universe.

The order of these transitions is also related to the value of thermal masses in a more indirect way. From Eq. (1) it is clear that the presence of the nonanalytic cubic term causes the transition to be first order. In fact, the jump in the order parameter is

$$\frac{\phi(T_c)}{T_c}=\frac{2E}{\lambda(T_c)} \tag{2}$$

[here $T_c$ is defined by the coexistence of two degenerate vacua in Eq. (1)]. The quantity (2) is of the utmost importance for the viability of electroweak baryogenesis (for review and references, see, e.g., [7]). Now, the cubic term in Eq. (1) is a purely finite temperature effect and comes from the interaction of the Higgs field with the static modes of different species of bosons in the plasma (fermions do not contribute to this term because they do not have static modes). In fact, each bosonic degree of freedom, with $(T=0)$ field-dependent mass $M_i(\phi)$, contributes to the potential $V(\phi,T)$ a term

$$\Delta_i V=-\frac{T}{12\pi}[M_i^2(\phi)]^{3/2}. \tag{3}$$

Beyond the one-loop approximation for the potential, every mass $M_i(\phi)$ in Eq. (3) should be substituted by the corresponding effective thermal mass, obtained from

$$M_{ij}^2(\phi)\rightarrow M_{ij}^2(\phi)+\kappa_{ij}T^2, \tag{4}$$

where the last piece comes from the interaction of the particles with the surrounding plasma. Substituting Eq. (4) into Eq. (3) resums an infinite series of higher order diagrams, the so-called daisies. The net effect of this resummation is to screen the cubic term in Eq. (1), effectively reducing the $E$ parameter and thus weakening the strength of the phase transition. In the standard model, where the dominant contribution to the cubic term in the potential comes from gauge bosons, the screening of the longitudinal modes is very effective while it is zero at leading order for the transverse modes. Then, daisy improvement of the effective potential leads to a reduction of the strength of the transition [8] roughly by a factor of 2/3. In the minimal supersymmetric standard model (MSSM), if stops are light, they give the dominant contribution to the cubic term in the Higgs potential (see [9] for the effect of Debye screening on the electroweak phase transition in the MSSM) and the final strength of the transition will be sensitive to the value of the top squark thermal masses.

In addition to the effects explained, (bosonic) thermal mass corrections are very important because they represent the starting point of a resummation of perturbation theory [6,10]. This resummation is necessary to take care of the infrared problems that plague theories at finite temperature if they contain massless bosons in the symmetric phase, e.g., Yang-Mills theories [11]. The problem appears when we probe our system at low scales[3] $O(gT)$ compared to the temperature $T$. At this scale, an infinite number of diagrams can give contributions of the same order and, to improve the usual perturbative series, they need to be resummed. The effective thermal masses provide then an IR cutoff taming the perturbative expansion.[4] One example is provided by the cubic term in the potential discussed previously. Its nonanalytic behavior signals its infrared singular origin: It comes from (bosonic) zero Matsubara frequency modes. Note that fermions do not cause infrared problems because they do not have zero Matsubara modes. In fact, at sufficiently high temperatures (or for distances much larger than $1/T$), fermions decouple from the effective three-dimensional (3D) theory at finite $T$. For that reason we concentrate here on bosons only.

Other examples where effective thermal masses play a role (in supersymmetric contexts) are studies on the 3D reduced effective theory in the MSSM [12], analysis of charge- and color-breaking minima [13] at finite temperature [14], nonrestoration of symmetries at very high temperature in general supersymmetric models [15], inverse symmetry breaking at some range of temperatures [16,17], different details of the spontaneous mechanism for electroweak baryogenesis [18], etc.

The aim of this paper is then to compute thermal masses for bosons (scalars or gauge vectors) in general softly broken supersymmetric models (Sec. II). In Sec. II A these masses are presented for temperatures much larger than all particle masses. In that case all particles in the theory are thermally produced and form part of the plasma. In Sec. II B we present the more complicated case in which the temperature is lower than the mass of some particles which decouple from the thermal plasma and then do not contribute to the effective masses of other particles. Section III applies these results to the particular case of the MSSM (some of the results presented have already appeared in the literature [4,9,19–21]).

## II. GENERAL SOFTLY BROKEN SUPERSYMMETRIC MODEL

Since Bose-Einstein and Fermi-Dirac distributions are different, the thermal bath is populated by different amounts of on-shell fermions and bosons. In that sense, in a supersymmetry (SUSY) theory, temperature effects can invalidate various cancellations implied by the symmetry between fermions and bosons [22]. This observation is relevant in particular for the computation of effective thermal masses.

As is well known, only self-energy diagrams which are quadratically divergent at $T=0$ contribute to the leading thermal masses. Typical diagrams that enter such calculation are depicted in Fig. 1. Although the second diagram is not quadratically divergent, it can give a contribution in the presence of Boltzmann decoupling and should be kept. Note that for our purpose the external momentum can be set to zero. If fermion-boson cancellations were still operative at nonzero

[3]Here $g$ stands for a typical gauge coupling or a Yukawa coupling. For power counting quartic scalar couplings are $\lambda\sim g^2$.

[4]Of course, there remains an infrared problem for transverse gauge bosons, associated with physics at the scale $g^2T$.

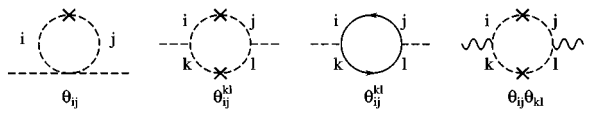


FIG. 1. Different types of diagrams contributing to thermal masses and responsible for the indicated $\theta$ symbols.

temperature in the supersymmetric case, we would obtain zero thermal masses. However, it can be shown that fermionic contributions come with an extra factor of $(-1/2)$. More explicitly, if a bosonic integral gives

$$I_b=\kappa(\Lambda^2+T^2)+\cdots,$$

where $\Lambda^2$ is the $T=0$ quadratic divergence and $\kappa T^2$ the associated finite temperature contribution to the thermal mass, the fermionic counterpart will be

$$I_f=-\kappa\left(\Lambda^2-\frac{1}{2}T^2\right)+\cdots.$$

Then, instead of a cancellation of thermal masses there is a reinforcement:

$$I_b+I_f=\frac{3}{2}\kappa T^2+\cdots.$$

Explicit examples of this effect can be found in the next sections.

### A. Thermal masses in the limit $T\gg M$

As we will see, in the limit when the temperature is much larger than any mass in the theory the contributions to the various self-energies depend only on the gauge structure of the theory and the dimensionless parameters of the superpotential $W$, which reads

$$W=\frac{1}{2}\mu_{ij}\phi_i\phi_j+\frac{1}{3!}W_{ijk}\phi_i\phi_j\phi_k.$$

Latin indices $i,j,k,\ldots$ will be used for scalar fields. The corresponding fermionic partners carry a tilde: $\tilde{k},\tilde{l},\ldots$ . Latin indices $a,b,c,\ldots$ are reserved for gauge bosons and the tilded version for gauginos. Unless stated otherwise, a sum over repeated indices is always implied.

The scalar potential is then

$$V_0(\phi)=\sum_i\left|\frac{\partial W}{\partial\phi_i}\right|^2+\frac{1}{2}g_a^2|\phi_i^*T_{ij}^a\phi_j|^2+m_i^2|\phi_i|^2+\left(\frac{1}{2}B_{ij}\phi_i\phi_j+\frac{1}{3!}A_{ijk}\phi_i\phi_j\phi_k+\text{H.c.}\right).$$

The remaining soft breaking terms[5] are gaugino masses:

$$\tilde{V}_aM_{\tilde{a}}\tilde{V}_a+\text{H.c.}$$

Leading order thermal masses for scalars can be obtained simply taking derivatives from the one-loop finite $T$ effective potential which reads

$$V(\phi)=V_0(\phi)+V_1(\phi)+V_T(\phi),$$

where $V_1(\phi)$ is the $T=0$ one-loop correction and

$$V_T(\phi)=\frac{T^4}{2\pi^2}\sum_s n_sJ_s[m_s^2(\phi)/T^2],$$

with $J_s=J_+(J_-)$ if the $s$th particle is a boson (fermion) with $n_s$ degrees of freedom (defined negative for fermions) and

$$J_\pm(y^2)=\int_0^\infty dx\,x^2\ln[1-(\pm)e^{-\sqrt{x^2+y^2}}].$$

The behavior of $J_\pm(m^2/T^2)$ is very simple in two limiting cases. First of all, the expansion of $J_\pm(m^2/T^2)$ for large values of $m/T$ gives contributions that are exponentially suppressed $\sim e^{-m/T}$, while the expansion for small values of $m/T$ gives the leading contributions $O(T^2)$:

$$V_T\sim\frac{T^2}{24}\sum_{\text{boson}}n_bm_b^2-\frac{T^2}{48}\sum_{\text{fermion}}n_fm_f^2+\cdots.$$

Then, we will simply use a step approximation for the effective potential to compute the thermal mass corrections:

$$V_T=\frac{T^2}{24}\left[\sum_{\text{boson}}n_bm_b^2\theta_b-\frac{1}{2}\sum_{\text{fermion}}n_fm_f^2\theta_f\right],\tag{5}$$

where the sum runs over all mass eigenstates calculated in the theory at zero temperature and[6] $\theta_{b,f}=1$ if $m_{b,f}\ll T$ and 0 if $m_{b,f}\gg T$. Of course this is a crude approximation but gives the correct results in the two limiting cases of interest. Now we study the limit in which the temperature is the largest mass scale. In that case $\theta_{b,f}=1$ for all bosons and fermions and we can forget the complications induced by soft breaking mass terms and supersymmetric massive parameters present in the superpotential.

Setting then all $\theta$'s to 1 in Eq. (5) and using the fact that

$$\text{Str}M^2(\phi)\equiv3\text{Tr}M_V^2+\text{Tr}M_S^2-2\text{Tr}M_F^2=\sum_s n_sm_s^2(\phi)$$

$$=K-2g_aD^a\text{Tr}T^a,$$

with $K$ a field-independent constant and $D^a=-g_a\phi_i^*T_{ij}^a\phi_j$, we obtain

$$V_T\sim-\frac{T^2}{16}\sum_f n_fm_f^2-\frac{T^2}{12}g_aD^a\text{Tr}T^a+\frac{T^2}{24}K.\tag{6}$$

[5]If the model contains matter fermions in the adjoint representation of some group, soft masses that mix them with the corresponding gauginos can be written. However, these soft terms are generically absent in supergravity scenarios.

[6]To be precise one should define $\theta_b=\theta(2\pi T-m_b)$ and $\theta_f=\theta(\pi T-m_f)$; however, for masses of the same order of the temperature the step approximation is too rough.

The constant term is irrelevant for our purposes and the $D$ term vanishes if we further assume $\mathrm{Tr}Y=0$ for any U(1) gauge group present.

Then, up to a field-independent constant, we can write

$$V_T(\phi_i,T)=-\frac{T^2}{16}\sum_f n_f m_f^2 = \frac{T^2}{8}\left[\sum_{i,k}|W_{ik}|^2+4\sum_a g_a^2\sum_{i,k}\phi_i^*(T^aT^a)_{ik}\phi_k\right]. \tag{7}$$

And from this,

$$\Pi_{ij}=\frac{\partial^2 V_T(\phi,T)}{\partial\phi_i\partial\phi_j^*}=\frac{T^2}{8}\left[\sum_{k,l}W_{ikl}W_{jkl}^*+4\sum_a g_a^2(T^aT^a)_{ij}\right]. \tag{8}$$

Writing $(T^aT^a)_{ij}=C_a(R)\delta_{ij}$ and using a convenient basis for the fields $\phi_i$ we get

$$\Pi_{ij}=\delta_{ij}\frac{T^2}{8}\left[\sum_{k,l}|W_{ikl}|^2+4\sum_a g_a^2 C_a(R_i)\right], \tag{9}$$

which gives the thermal mass corrections for scalars. This diagonal correction should be added to the $T=0$ mass matrix. The eigenvalues of this thermally corrected matrix are the end point of our calculation.

The leading order thermal masses for longitudinal gauge bosons, $\Pi_V$, get contributions from scalar, fermion, and gauge boson loops plus their supersymmetric partners (note that we can describe the chiral supermultiplet contributions either as $S+\tilde{S}$ or $\tilde{F}+F$; we use both below in the understanding that no $F$ corresponds to any $\tilde{S}$):

$$\Pi_V=\Pi_V^{(S)}+\Pi_V^{(\tilde{S})}+\Pi_V^{(F)}+\Pi_V^{(\tilde{F})}+\Pi_V^{(V)}+\Pi_V^{(\tilde{V})} = \frac{3}{2}\Pi_V^{(S)}+3\Pi_V^{(F)}+\frac{3}{2}\Pi_V^{(V)},$$

where the last equality follows from supersymmetry as explained above. Now, the vector contribution is

$$\mathrm{U}(1):\ \Pi_V^{(V)}=0, \qquad \mathrm{SU}(N):\ \Pi_V^{(V)}=\frac{N}{3}g_N^2T^2, \tag{10}$$

and, according to our rule, gaugino loops contribute half this result. The contributions from scalars and (chiral) fermions are

$$\Pi_V^{(S)}=\frac{1}{3}\sum_S g^2 t_2(R_S)T^2, \quad \Pi_V^{(F)}=\frac{1}{6}\sum_F g^2 t_2(R_F)T^2, \tag{11}$$

with $\mathrm{Tr}(T^aT^b)=t_2(R)\delta^{ab}$. The contribution from sfermion loops is twice that from fermion loops. Higgsino loops, on the other hand, give only a half of the Higgs contribution. Also note that the contribution from nonchiral fermions would be twice larger than that from chiral ones.

The final result is then

$$\Pi_{\mathrm{U}(1)}=\frac{1}{2}g_1^2T^2\left[\sum_S Y_S^2+\sum_F Y_F^2\right]=\frac{1}{2}g_1^2T^2\left[\sum_A Y_A^2\right], \tag{12}$$

$$\Pi_{\mathrm{SU}(N)}=\frac{1}{2}g_N^2T^2\left[N+\sum_S t_2(R_S)+\sum_F t_2(R_F)\right] = \frac{1}{2}g_N^2T^2\left[N+\sum_A t_2(R_A)\right], \tag{13}$$

where the index $A$ runs over chiral supermultiplets.

We see explicitly that all possible self-energies depend only on the gauge quantum numbers of the spectrum and on the Yukawa couplings $W_{ijk}$ that appear in the superpotential. In practice, at very high temperature the masses of the underlying $T=0$ are irrelevant. In that case, the computation of the leading thermal corrections is simplified and they can be derived directly from an exactly conformal supersymmetric theory.

### B. Thermal masses for general $T$

The study of the case in which the scale of the temperature is not the dominant one is a bit more involved. The mass scales present in the theory, aside from possible nonzero background fields, are the soft SUSY-breaking terms and the massive coefficients in the bilinear terms of the superpotential. These scales can have very different values and there always exists some range of temperatures in which decoupling and mixing effects have to be taken into account.

The obvious new effect is the Boltzmann decoupling of particles with mass $m\gg T$. Then, their contribution to the thermal masses of other particles should be dropped. This effect will be taken into account by writing every contribution with the corresponding $\theta(2\pi T-m_b)$ or $\theta(\pi T-m_f)$ that will take care of the decoupling in a step approximation.

As long as field-background effects can be neglected, i.e., as long as no particle is decoupled because of a large background-dependent mass, the thermal self-energies will only mix particles with the same quantum numbers. The reason for this is that leading thermal masses arise from quadratically divergent diagrams that can already be drawn in the $T=0$ unbroken theory. As an example consider $W_3$-$B$ mixing. One can certainly draw diagrams at $T=0$ that mix those particles at one loop. However, when summing such diagrams over complete SU(2) multiplets these contributions cancel. At finite $T$ this is reflected in the nondiagonal self-energy

$$\Pi_{W_{3L}B_L}=\frac{1}{6}g_1g_2T^2[\mathrm{Tr}_S(\theta Y T_3)+2\mathrm{Tr}_F(\theta Y T_3)],$$

which gives zero when all $\theta$'s are 1. In a background that breaks SU(2) $\times$ U(1) if some particle acquires a mass larger than $T$ its contribution to $\Pi_{W_{3L}B_L}$ will drop and only then will a nonzero contribution result. Here we will assume that background masses are always smaller than the temperature (which is usually the case in most applications of interest) so that we will not encounter this complication. In that case one can compute thermal masses at zero background (corrections from nonzero background effects will be suppressed by pow-

ers of $T$). The prescription to obtain thermally corrected masses is then to write the $T=0$ mass matrices in whatever field background one is interested (provided it is smaller than $T$), add the thermal corrections, and, afterwards, rotate or diagonalize the mass matrix.

Setting then a zero background we can in principle compute thermal self-energies using either an interaction basis or a mass eigenstate basis. The first option is more convenient and it is simple to rotate to the mass basis in particular cases (note that $\theta$'s are naturally defined in the mass basis, so that, to decouple some particle, the rotation should be made). We will express our general results in terms of some convenient $\theta$ symbols which vary with the origin of the contributions as shown in Fig 1. The rules to rotate these symbols from one basis to another are explained below.

The fields in the interaction basis, $\phi_i$, can be written as a linear combination of the mass eigenstates $\varphi_\alpha$:

$$\phi_i = U_i^\alpha \varphi_\alpha ,$$

where we stress that the unitary matrices $U$ diagonalize the $M^2$ mass matrix calculated at zero background and zero temperature. The symbol $\theta_{ij}$ comes from the contraction of $\phi_i^* - \phi_j$ to close the loop as shown in Fig. 1. It is defined by rotating to the mass basis as

$$\theta_{ij} = \theta_{U_i^\alpha \varphi_\alpha , U_j^\beta \varphi_\beta} = U_i^{\alpha *} \;\; \theta_{\alpha\beta} U_j^\beta \;\; = U_i^{\alpha *} \;\; \theta_{\alpha\alpha} \;\; U_j^\alpha , \tag{14}$$

with

$$\theta_{\alpha\beta} = \theta_{\alpha\alpha}\delta_{\alpha\beta} = \begin{cases} 1 & \text{if} \quad m_\alpha \ll T, \\ 0 & \text{if} \quad m_\alpha \gg T. \end{cases} \tag{15}$$

The $\theta_{ij}$ symbol defined applies both to fermion or boson contractions.

We define also the four-index symbol $\theta_{ij}^{kl}$ for the second and third diagrams shown in Fig. 1. For these objects the rotation from the interaction basis to the mass basis is

$$\theta_{ij}^{kl} = U_i^{\beta *} \;\; U_j^\beta \;\; \theta_{\beta\beta}^{\alpha\alpha} \;\; U_k^{\alpha *} \;\; U_l^\alpha . \tag{16}$$

Now there is a difference between the fermionic and bosonic cases. For fermions we have simply (tildes omitted)

$$\theta_{\beta\beta}^{\alpha\alpha} = \theta_{\alpha\alpha}\theta_{\beta\beta} , \tag{17}$$

while for bosons,

$$\theta_{\beta\beta}^{\alpha\alpha} = \frac{\theta_{\alpha\alpha} - \theta_{\beta\beta}}{m_\alpha^2 - m_\beta^2} . \tag{18}$$

The reason for this is the following. Note that the bosonic diagram is not quadratically divergent (in particular $\theta_{\beta\beta}^{\alpha\alpha} = 0$ if $\theta_{\alpha\alpha} = \theta_{\beta\beta} = 1$). However, it contributes to the thermal masses if one of the particles running in the loop, say, $\alpha$, decouples. Then, the diagram behaves effectively as the first one, with the heavy line in the loop collapsed to a point. In other words, in the effective theory that results after integrating out the heavy particle there are new quartic couplings proportional to $1/m_\alpha^2$. The symbol (18) takes this into account.

There is another effect we have to mention before presenting the results. Suppose that the scalar fields $\phi_i$ and $\phi_j$ have the same quantum numbers but opposite Abelian charges. The mixing $\phi_i$-$\phi_j^*$ by thermal mass effects is not possible in the nondecoupling limit analyzed in the previous subsection [Eqs. (7) and (8)] but becomes possible if the thermal contributions from some particles are Boltzmann suppressed. We allow for such a possibility in our general formulas. The corresponding thermal self-energy will be denoted by $\Pi_{\phi_i , \phi_j^*}$.

Also note that we give our results in terms of thermal polarizations and $\theta$'s for complex scalar fields. This assumes that real and imaginary components behave in the same way; e.g., they decouple together when some mass parameter is made heavy. That is no longer true in the presence of large backgrounds (which we assume not to be the case) or for singlet fields. The real and imaginary components of a singlet can have different masses and should be treated separately. Our formalism can be trivially generalized to take this possibility into account by relations such as

$$S = \frac{1}{\sqrt{2}}(S^r + iS^i) \Rightarrow \begin{cases} \theta_{SS} = \frac{1}{2}[\,\theta_{S^rS^r} + \theta_{S^iS^i}], \\ \theta_{SS^*} = \frac{1}{2}[\,\theta_{S^rS^r} - \theta_{S^iS^i}]. \end{cases}$$

The general results are the following.

### 1. Scalars

*a. Yukawa contribution from fermion loops:*

$$\Pi_{\phi_i , \phi_j} = \frac{T^2}{24} W_{ikl} W_{jrs}^* \theta_{\tilde{r}\tilde{k}}^{\tilde{s}\tilde{l}} . \tag{19}$$

*b. Yukawa contributions from scalar loops:*

$$\Pi_{\phi_i , \phi_j} = \frac{T^2}{12} W_{rik} W_{rjl}^* \theta_{lk} , \qquad \Pi_{\phi_i , \phi_j^*} = \frac{T^2}{24} W_{rij} W_{rkl}^* \theta_{kl*} . \tag{20}$$

*c. Trilinear contributions*: Note that these terms are proportional to $\theta_{kl}^{ij}$ and thus give zero in the limit $T \gg M$:

$$\begin{aligned} \Pi_{\phi_i , \phi_j} = \frac{T^2}{24}\{ & A_{ikl} A_{jrs}^* \theta_{ls}^{kr} + 2 W_{ikl} \mu_{lm}^* W_{jrs}^* \mu_{rn} ( \theta_{mn}^{sk} + \theta_{ms*}^{nk*} ) \\ & + W_{mkl}^* \mu_{mi} W_{rsn} \mu_{rj}^* \theta_{nl}^{sk} + [\, 2 W_{ikl} \mu_{km}^* W_{nrs} \mu_{jr}^* \theta_{l*s}^{mn} \\ & + 2 A_{ikl} W_{jrs}^* \mu_{mr} \theta_{m*l}^{sk} + A_{ikl} W_{rsm} \mu_{jr}^* \theta_{m*l}^{s*k} \\ & + (\text{H.c.}, i \leftrightarrow j)]\}, \end{aligned} \tag{21}$$

$$\begin{aligned} \Pi_{\phi_i , \phi_j^*} = \frac{T^2}{24}\{ & A_{ikl} A_{jrs} \theta_{l*s}^{k*r} + 2 W_{ikl} \mu_{km}^* W_{jrs} \mu_{rn}^* ( \theta_{ms}^{ln} \\ & + \theta_{mn*}^{ls*} ) + W_{rkl}^* \mu_{ir} W_{mns}^* \mu_{jm} \theta_{s*l}^{n*k} \\ & + [\, 2 A_{ikl} W_{jmn} \mu_{mr}^* \theta_{lr}^{kn*} + A_{ikl} W_{mnr}^* \mu_{jm} \theta_{lr}^{kn} \\ & + 2 W_{rkl}^* \mu_{ir} W_{jmn} \mu_{ms}^* \theta_{s*l}^{nk} + (i \leftrightarrow j)]\}. \end{aligned} \tag{22}$$

*d. Gauge contributions from fermion loops*: We change momentarily our notation from $\phi_i$ to $N_\alpha$ where $N$ refers to a

given representation of the group and $\alpha$ to the group index (not to be confused with mass eigenstate indices). Of course no change is needed for U(1)'s. The results are

$$\Pi_{N_\alpha,P_\beta}=\frac{T^2}{6}g_a g_b T^a_{\alpha\gamma}T^b_{\delta\beta}\theta^{\tilde{\gamma}\tilde{\delta}}_{\tilde{b}\tilde{a}},$$

$$\Pi_{N_\alpha,P^*_\beta}=\frac{T^2}{6}g_a g_b T^a_{\alpha\gamma}T^b_{\beta\delta}\theta^{\tilde{b}\tilde{\gamma}}_{\tilde{a}\tilde{\delta}}. \tag{23}$$

The second contribution can be nonzero only if the model contains matter fermions in the adjoint representation as discussed in footnote 5.

*e. Gauge contribution from scalar loops*: The general result is

$$\Pi_{N_\alpha,P_\beta}=\frac{T^2}{12}g^2[\,\delta_{NP}T^a_{\beta\alpha}\mathrm{Tr}_S(T^a_{\gamma\delta}\theta_{\gamma\delta})+T^a_{\gamma\alpha}T^a_{\beta\delta}\theta_{\gamma\delta}], \tag{24}$$

$$\Pi_{N_\alpha,P^*_\beta}=\frac{T^2}{12}g^2[T^a_{\gamma\alpha}T^a_{\delta\beta}\theta_{\delta^*\gamma}]. \tag{25}$$

For SU($N$) with all nonsinglet fields in the fundamental representation,

$$\Pi_{N_\alpha,P_\beta}=\frac{T^2}{24}g^2\left[\,2\,\delta_{NP}T^a_{\beta\alpha}\mathrm{Tr}_S(T^a\theta)+\delta_{\alpha\beta}\theta_{N_\gamma P_\gamma}-\frac{1}{N}\theta_{\alpha\beta}\right],$$

$$\Pi_{N_\alpha,P^*_\beta}=\frac{T^2}{24}g^2\left[\,\theta_{P^*_\alpha N_\beta}-\frac{1}{N}\theta_{P^*_\beta N_\alpha}\right].$$

For $\mathrm{U}(1)_Y$,

$$\Pi_{\phi_i,\phi_j}=\frac{T^2}{12}g_1^2[\,\delta_{ij}Y_i\mathrm{Tr}_S(Y\theta)+Y_iY_j\theta_{ij}],$$

$$\Pi_{\phi_i,\phi^*_j}=\frac{T^2}{12}g_1^2Y_iY_j\theta_{ij*}.$$

*f. Gauge contribution from gauge boson loops:* The general result is

$$\Pi_{P_\alpha,P_\beta}=\frac{T^2}{4}g_A g_B T^a_{\beta\gamma}T^b_{\gamma\alpha}\theta_{ab}\,. \tag{26}$$

For SU($N$), when $\theta_{ab}=\delta_{ab}\theta_{aa}$,

$$\Pi_{P_\alpha,P_\beta}=\frac{T^2}{4}g_N^2C_N(R_P)\,\delta_{\alpha\beta}\theta_G\,.$$

For U(1),

$$\Pi_{\phi_i,\phi_j}=\frac{T^2}{4}g_1^2Y_i^2\,\delta_{ij}\theta_B\,.$$

#### 2. Gauge bosons

As already mentioned, only longitudinal gauge bosons get a nonzero thermal mass at leading order. The following thermal polarizations should then be understood as polarizations for the temporal components $\Pi_{00}$ of the gauge fields $V^a_0, V^b_0$.

*a. Scalar contribution:*

$$\Pi_{ab}=\frac{T^2}{12}g_A g_B(\{T^a,T^b\}_{\beta\gamma}\theta_{P_\beta P_\gamma}+[\,T^a_{\beta\alpha}T^b_{\delta\gamma}(\,\theta_{M_\beta N_\gamma}\theta_{N_\delta M_\alpha}-\theta_{M_\beta N^*_\delta}\theta_{N_\gamma M^*_\alpha})+\mathrm{H.c.}]), \tag{27}$$

where in principle two different groups, with coupling constants $g_A$, $g_B$, are considered.

For SU($N$) and fields in the fundamental representation $M,P$, etc., the general result is

$$\Pi_N\equiv\frac{1}{N^2-1}\sum_a\Pi_{aa}=\frac{T^2}{12}g_N^2\left\{\sum_M\theta_M+\frac{1}{N^2-1}\left[\,\theta_{M_\alpha P_\alpha}\theta_{P_\beta M_\beta}-\theta_{M_\beta P^*_\alpha}\theta_{M^*_\alpha P_\beta}-\frac{1}{N}(\,\theta_{M_\alpha P_\beta}\theta_{P_\beta M_\alpha}-\theta_{M^*_\beta P_\alpha}\theta_{M_\alpha P^*_\beta})\right]\right\},$$

where $\theta_M\equiv(1/N)\Sigma_\alpha\theta_{M_\alpha M_\alpha}$.

For U(1),

$$\Pi_{B_L}=\frac{T^2}{6}g_1^2\sum_{ij}Y_iY_j[\,\theta_{ij}+\theta^2_{ij}-\theta^2_{ij*}].$$

*b. Contributions from matter fermion loops:*

$$\Pi_{ab}=\frac{T^2}{6}g_A g_B T^a_{\alpha\beta}T^b_{\gamma\delta}\theta_{\tilde{P}_\beta\tilde{N}_\gamma}\theta_{\tilde{P}_\alpha\tilde{N}_\delta}. \tag{28}$$

*c. Contributions from gaugino loops:*

$$\Pi_{ab}=\frac{T^2}{6}g_A g_B f^A_{dea}f^B_{ghb}\theta^{\tilde{d}\tilde{h}}_{\tilde{e}\tilde{g}}, \tag{29}$$

where $f^A_{abc}$ are the structure constants of the group $A$ ($[T^a,T^b]=if_{abc}T^c$).

## III. MINIMAL SUPERSYMMETRIC STANDARD MODEL

In this section we apply our general results to a particularly relevant example, the MSSM. It is the simplest supersymmetric extension of the SM and is described by the superpotential

$$W=\mu H_1\cdot H_2+h_{U_i}Q_i\cdot H_2U_i+h_{D_i}H_1\cdot Q_iD_i+h_{E_i}H_1\cdot L_iE_i$$

embedded into the SU(3)×SU(2)×U(1) gauge group. We will consider general soft SUSY-breaking terms although with negligible intergenerational mixing. Then, the only fields that mix at zero background are $H_1$ and $H_2$.

The scalar-fermionic soft Lagrangian reads

$$L_{\mathrm{soft}}=\left[\,A_{E_i}H_1\cdot\tilde{L}_i\tilde{E}_i+A_{D_i}H_1\cdot\tilde{Q}_i\tilde{D}_i+A_{U_i}\tilde{Q}_i\cdot H_2\tilde{U}_i-m_3^2H_1\cdot H_2+\sum_{\tilde{g}}\tilde{g}M\tilde{g}+\mathrm{H.c.}\right]+\sum_i m_i^2|\phi_i|^2.$$

### A. Limit of very large $T$

The bosonic self-energies in the case in which the temperature is the larger mass scale, i.e.,

*$T \gg \mu,\ A_\phi,\ m_\phi,\ m_3,\ M$, are obtained from Sec. II A directly as*

$$\Pi_{\tilde{U}_{L_i}} = \Pi_{\tilde{D}_{L_i}} = \frac{2}{3}g_3^2T^2 + \frac{3}{8}g_2^2T^2 + \frac{1}{72}g_1^2T^2 + \frac{1}{4}(h_{U_i}^2 + h_{D_i}^2)T^2,$$

$$\Pi_{\tilde{U}_{R_i}} = \frac{2}{3}g_3^2T^2 + \frac{2}{9}g_1^2T^2 + \frac{1}{2}h_{U_i}^2T^2,$$

$$\Pi_{\tilde{D}_{R_i}} = \frac{2}{3}g_3^2T^2 + \frac{1}{18}g_1^2T^2 + \frac{1}{2}h_{D_i}^2T^2,$$

$$\Pi_{\tilde{e}_{L_i}} = \Pi_{\tilde{\nu}_{L_i}} = \frac{3}{8}g_2^2T^2 + \frac{1}{8}g_1^2T^2,$$

$$\Pi_{\tilde{e}_{R_i}} = \frac{1}{2}g_1^2T^2,$$

$$\Pi_{H_1^0} = \Pi_{H_1^\pm} = \frac{3}{8}g_2^2T^2 + \frac{1}{8}g_1^2T^2 + \frac{3}{4}h_b^2T^2,$$

$$\Pi_{H_2^0} = \Pi_{H_2^\pm} = \frac{3}{8}g_2^2T^2 + \frac{1}{8}g_1^2T^2 + \frac{3}{4}h_t^2T^2,$$

$$\Pi_{g_L} = \frac{9}{2}g_3^2T^2,$$

$$\Pi_{W_L} = \frac{9}{2}g_2^2T^2,$$

$$\Pi_{B_L} = \frac{11}{2}g_1^2T^2. \tag{30}$$

For $H_1$ and $H_2$ we only keep third-generation Yukawa couplings. Also, note that particles in the same gauge multiplet receive the same thermal mass correction.

### B. Explicit formulas in the general case

If there is not a defined hierarchy between the scales $T,\ \mu,\ A_\phi,\ m_i,\ m_3,\ M$, we must apply the formulas of Sec. II B which have as asymptotic limit, for high $T$, the equations presented in the previous subsection.

In the formulas that follow we write most of the self-energies in the interaction basis. Also, we use $\theta_i = \theta_{ii}$, $\theta_i^j = \theta_{ii}^{jj}$, etc., to simplify the notation. For all fields besides $H_1$ and $H_2$ the $\theta_{ij}$ functions in the gauge basis are diagonal and coincide with the definition in the mass eigenstate basis (at zero background). The treatment of $H_{1,2}$ is as follows: As is well known, there are three mass parameters in the tree-level Higgs potential of the MSSM:

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_3^2(H_1 \cdot H_2 + \text{H.c.}) + \text{quartic terms.}$$

Two of these mass parameters $(m_1, m_2)$ can be traded by the $T=0$ vacuum expectation values $v_1$ and $v_2$ [with $v_1^2 + v_2^2 = v^2 = (174\ \text{GeV})^2$ and $\tan\beta = v_2/v_1$], leaving only one free mass parameter, conventionally taken to be the mass of the pseudoscalar Higgs boson, $m_A$. Then we have two scales in the Higgs sector, $v$ (or $M_Z$) and $m_A$, and the only nontrivial case at finite temperature corresponds to $M_Z \ll T \ll m_A$. When $m_A \gg M_Z$ one (linear combination) of the two Higgs doublets is heavy ($\sim m_A$) and one light ($\sim M_Z$).

In order to obtain the mass eigenstates at zero background, we can work with the full doublets. Diagonalization of the $2\times 2$ mass matrix defines the mixing angle $\beta_0$. In the only nontrivial case with $m_A \gg M_Z$, it is straightforward to see that $\beta_0 \to \beta$ so that, in this limit, we can define the doublets $H$ (light) and $\Phi$ (heavy) by the rotation

$$\overline{H_1} = H\cos\beta - \Phi\sin\beta,$$

$$H_2 = H\sin\beta + \Phi\cos\beta, \tag{31}$$

where $\overline{H_1} = (-H_1^+, H_1^{0*})^T$. The doublets $H$ and $\Phi$ are the mass eigenstates so that Eq. (31) is our equation $\phi_i = U_i^\alpha \varphi_\alpha$ in this case. From Eq. (31) and using rules (14) and (16) we can express all $\theta$ symbols for Higgs bosons in terms of $\theta_H$ and $\theta_\Phi$ or, equivalently, $\theta(2\pi T - M_Z)$ and $\theta(2\pi T - m_A)$.

From Eq. (31) it follows that

$$\theta_{H_1^0 H_1^0} = \theta_{H^0H^0}\cos^2\beta + \theta_{\Phi^0\Phi^0}\sin^2\beta,$$

$$\theta_{H_1^{0*} H_2^0} = (\theta_{H^0H^0} - \theta_{\Phi^0\Phi^0})\cos\beta\sin\beta,$$

$$\theta_{H_1^- * H_2^+} = -(\theta_{H^+H^+} - \theta_{\Phi^+\Phi^+})\cos\beta\sin\beta,$$

and so on. The rest of $\theta$ symbols are trivial to handle. For squarks remember that gauge invariance requires equal soft mass $m_{\tilde{Q}}$ for $\tilde{U}_L$ and $\tilde{D}_L$ so that $\theta_{\tilde{U}_L} = \theta_{\tilde{D}_L} \equiv \theta_{\tilde{Q}}$.

Also note that although $\theta$'s for gauge bosons will always take the value 1 (because they are massless at zero background), we write them explicitly.

We also use

$$6\,\mathrm{Tr}_S(\theta Y) = -3\sum_j (\theta_{\tilde{\nu}_{L_j}} + \theta_{\tilde{e}_{L_j}} - 2\theta_{\tilde{e}_{R_j}}) + 3(\theta_{H_2^\pm} + \theta_{H_2^0}) - 3(\theta_{H_1^\pm} + \theta_{H_1^0}) + N_c\sum_j (\theta_{\tilde{U}_{L_j}} + \theta_{\tilde{D}_{L_j}} - 4\theta_{\tilde{U}_{R_j}} + 2\theta_{\tilde{D}_{R_j}}),$$

$$2\mathrm{Tr}_S(\theta T_3)=\sum_j\ (\theta_{\tilde{\nu}_{L_j}}-\theta_{\tilde{e}_{L_j}})+N_c\sum_j\ (\theta_{\tilde{U}_{L_j}}-\theta_{\tilde{D}_{L_j}})+\theta_{H_1^0}-\theta_{H_1^\pm}+\theta_{H_2^\pm}-\theta_{H_2^0}.$$

Note, however, that this last trace would be nonzero only in a $SU(2)_L$-breaking background.

In the formulas that follow the reader can easily check sector by sector that fermionic contributions are always half of the corresponding bosonic ones.

### 1. Squarks

Thermal self-energies are diagonal in color space unless a color-breaking background that decouples some contribution is present. As we assume this is not the case the color index structure is trivial and is suppressed:

$$\Pi_{\tilde{U}_{L_i}}=\frac{1}{6}g_3^2T^2\frac{N_c^2-1}{4N_c}[3\theta_g+\theta_{\tilde{U}_{L_i}}+2\theta_{\tilde{g}}\theta_{U_{L_i}}]+\frac{1}{48}g_2^2T^2[6\theta_{W^\pm}+3\theta_{W_3}+\theta_{\tilde{U}_{L_i}}+2\theta_{\tilde{D}_{L_i}}+2\mathrm{Tr}_S(\theta T_3)+2\theta_{U_{L_i}}\theta_{\tilde{W}_3}+4\theta_{D_{L_i}}\theta_{\tilde{W}^\pm}]$$
$$+\frac{1}{432}g_1^2T^2[3\theta_B+\theta_{\tilde{U}_{L_i}}+6\mathrm{Tr}_S(\theta Y)+2\theta_{\tilde{B}}\theta_{U_{L_i}}]+\Delta^0_{\tilde{U}_{R_i}}+\Delta^{\pm}_{\tilde{D}_{R_i}}+\frac{1}{12}h_{U_i}^2T^2[\theta_{H_2^0}+\theta_{\tilde{U}_{R_i}}+\theta_{\tilde{H}_2^0}\theta_{U_{R_i}}]$$
$$+\frac{1}{12}h_{D_i}^2T^2[\theta_{H_1^\pm}+\theta_{\tilde{D}_{R_i}}+\theta_{\tilde{H}_1^\pm}\theta_{D_{R_i}}],$$

$$\Pi_{\tilde{D}_{L_i}}=\frac{1}{6}g_3^2T^2\frac{N_c^2-1}{4N_c}[3\theta_g+\theta_{\tilde{D}_{L_i}}+2\theta_{\tilde{g}}\theta_{D_{L_i}}]+\frac{1}{48}g_2^2T^2[6\theta_{W^\pm}+3\theta_{W_3}+\theta_{\tilde{D}_{L_i}}+2\theta_{\tilde{U}_{L_i}}-2\mathrm{Tr}_S(\theta T_3)+2\theta_{D_{L_i}}\theta_{\tilde{W}_3}+4\theta_{U_{L_i}}\theta_{\tilde{W}^\pm}]$$
$$+\frac{1}{432}g_1^2T^2[3\theta_B+\theta_{\tilde{D}_{L_i}}+6\mathrm{Tr}_S(\theta Y)+2\theta_{\tilde{B}}\theta_{D_{L_i}}]+\Delta^0_{\tilde{D}_{R_i}}+\Delta^{\pm}_{\tilde{U}_{R_i}}+\frac{1}{12}h_{U_i}^2T^2[\theta_{H_2^\pm}+\theta_{\tilde{U}_{R_i}}+\theta_{\tilde{H}_2^\pm}\theta_{U_{R_i}}]$$
$$+\frac{1}{12}h_{D_i}^2T^2[\theta_{H_1^0}+\theta_{\tilde{D}_{R_i}}+\theta_{\tilde{H}_1^0}\theta_{D_{R_i}}],$$

$$\Pi_{\tilde{U}_{R_i}}=\frac{1}{6}g_3^2T^2\frac{N_c^2-1}{4N_c}[3\theta_g+\theta_{\tilde{U}_{R_i}}+2\theta_{\tilde{g}}\theta_{U_{R_i}}]+\frac{1}{108}g_1^2T^2[12\theta_B+4\theta_{\tilde{U}_{R_i}}-6\mathrm{Tr}_S(\theta Y)+8\theta_{\tilde{B}}\theta_{U_{R_i}}]+\Delta^0_{\tilde{U}_{L_i}}+\Delta^{\pm}_{\tilde{D}_{L_i}}$$
$$+\frac{1}{12}h_{U_i}^2T^2[\theta_{H_2^0}+\theta_{H_2^\pm}+\theta_{\tilde{U}_{L_i}}+\theta_{\tilde{D}_{L_i}}+\theta_{\tilde{H}_2^0}\theta_{U_{L_i}}+\theta_{\tilde{H}_2^\pm}\theta_{D_{L_i}}],$$

$$\Pi_{\tilde{D}_{R_i}}=\frac{1}{6}g_3^2T^2\frac{N_c^2-1}{4N_c}[3\theta_g+\theta_{\tilde{D}_{R_i}}+2\theta_{\tilde{g}}\theta_{D_{R_i}}]+\frac{1}{216}g_1^2T^2[6\theta_B+2\theta_{\tilde{D}_{R_i}}+6\mathrm{Tr}_S(\theta Y)+4\theta_{\tilde{B}}\theta_{D_{R_i}}]+\Delta^0_{\tilde{D}_{L_i}}+\Delta^{\pm}_{\tilde{U}_{L_i}}$$
$$+\frac{1}{12}h_{D_i}^2T^2[\theta_{H_1^0}+\theta_{H_1^\pm}+\theta_{\tilde{U}_{L_i}}+\theta_{\tilde{D}_{L_i}}+\theta_{\tilde{H}_1^0}\theta_{D_{L_i}}+\theta_{\tilde{H}_1^\pm}\theta_{U_{L_i}}],$$

where

$$\Delta^c_{\tilde{U}_P}=\frac{T^2}{12}h_U^2[|A_{U_P}|^2\theta^{\tilde{U}_P}_{H_2^c}+|\mu|^2\theta^{\tilde{U}_P}_{H_1^c}+(A_{U_P}\mu+A^*_{U_P}\mu^*)\theta^{\tilde{U}_P}_{H_1^cH_2^{c*}}],$$

$$\Delta^c_{\tilde{D}_P}=\frac{T^2}{12}h_D^2[|A_{D_P}|^2\theta^{\tilde{D}_P}_{H_1^c}+|\mu|^2\theta^{\tilde{D}_P}_{H_2^c}-(A_{D_P}\mu+A^*_{D_P}\mu^*)\theta^{\tilde{D}_P}_{H_1^cH_2^{c*}}].$$

Rotating from $H_1,H_2$ to $H,\Phi$ as explained, the previous $\Delta$'s can be written as

$$\Delta^c_P=\frac{T^2}{12}h_P^2\left[|\tilde{A}_P^+|^2\frac{\theta_P-\theta_{Hc}}{m_P^2-m_Z^2}+|\tilde{A}_P^-|^2\frac{\theta_P-\theta_{\Phi c}}{m_P^2-m_A^2}\right].$$

Here $P=\tilde{U}_{L_i},\tilde{D}_{L_i},\tilde{U}_{R_i},\tilde{D}_{R_i}$, $c=0,\pm$, and

$$\tilde{A}^+_{U_i}=A_{U_i}\sin\beta+\mu^*\cos\beta,\quad \tilde{A}^-_{U_i}=A_{U_i}\cos\beta-\mu^*\sin\beta,$$

$$\tilde{A}^+_{D_i}=A_{D_i}\cos\beta+\mu^*\sin\beta,\quad \tilde{A}^-_{D_i}=A_{D_i}\sin\beta-\mu^*\cos\beta.$$

### 2. Sleptons

$$\Pi_{\tilde{\nu}_{L_i}} = \frac{1}{48} g_2^2 T^2 [6\theta_{W^\pm} + 3\theta_{W_3} + \theta_{\tilde{\nu}_{L_i}} + 2\theta_{\tilde{e}_{L_i}} + 2\mathrm{Tr}_S(\theta T_3) + 2\theta_{\nu_{L_i}}\theta_{\tilde{W}_3} + 4\theta_{e_{L_i}}\theta_{\tilde{W}^\pm}]$$
$$+ \frac{1}{144} g_1^2 T^2 [9\theta_B + 3\theta_{\tilde{\nu}_{L_i}} - 6\mathrm{Tr}_S(\theta Y) + 6\theta_{\tilde{B}}\theta_{\nu_{L_i}}] + \Delta^{\pm}_{\tilde{E}_{R_i}} + \frac{1}{12} h_{E_i}^2 T^2 [\theta_{H_1^\pm} + \theta_{\tilde{E}_{R_i}} + \theta_{\tilde{H}_1^\pm}\theta_{E_{R_i}}],$$

$$\Pi_{\tilde{e}_{L_i}} = \frac{1}{48} g_2^2 T^2 [6\theta_{W^\pm} + 3\theta_{W_3} + \theta_{\tilde{e}_{L_i}} + 2\theta_{\tilde{\nu}_{L_i}} - 2\mathrm{Tr}_S(\theta T_3) + 2\theta_{e_{L_i}}\theta_{\tilde{W}_3} + 4\theta_{\nu_{L_i}}\theta_{\tilde{W}^\pm}]$$
$$+ \frac{1}{144} g_1^2 T^2 [9\theta_B + 3\theta_{\tilde{e}_{L_i}} - 6\mathrm{Tr}_S(\theta Y) + 6\theta_{\tilde{B}}\theta_{e_{L_i}}] + \Delta^0_{\tilde{E}_{R_i}} + \frac{1}{12} h_{E_i}^2 T^2 [\theta_{H_1^0} + \theta_{\tilde{E}_{R_i}} + \theta_{\tilde{H}_1^0}\theta_{E_{R_i}}],$$

$$\Pi_{\tilde{E}_{R_i}} = \frac{1}{72} g_1^2 T^2 [18\theta_B + 6\theta_{\tilde{E}_{R_i}} + 6\mathrm{Tr}_S(\theta Y) + 12\theta_{\tilde{B}}\theta_{E_{R_i}}] + \Delta^0_{\tilde{E}_{L_i}} + \Delta^{\pm}_{\tilde{\nu}_{L_i}} + \frac{1}{12} h_{E_i}^2 T^2 [\theta_{H_1^0} + \theta_{H_1^\pm} + \theta_{\tilde{\nu}_{L_i}} + \theta_{\tilde{e}_{L_i}} + \theta_{\tilde{H}_1^0}\theta_{e_{L_i}} + \theta_{\tilde{H}_1^\pm}\theta_{\nu_{L_i}}],$$

where the $\Delta$'s follow the same notation used for squarks and now

$$\tilde{A}^+_{E_i} = A_{E_i}\cos\beta + \mu^* \sin\beta, \quad \tilde{A}^-_{E_i} = A_{E_i}\sin\beta - \mu^*\cos\beta.$$

### 3. Higgs bosons

$$\Pi_{H_1^0} = \frac{1}{48} g_2^2 T^2 [6\theta_{W^\pm} + 3\theta_{W_3} + \theta_{H_1^0} + 2\theta_{H_1^\pm} + 2\mathrm{Tr}_S(\theta T_3) + 2\theta_{\tilde{H}_1^0}\theta_{\tilde{W}_3} + 4\theta_{\tilde{H}_1^\pm}\theta_{\tilde{W}^\pm}] + \frac{1}{144} g_1^2 T^2 [9\theta_B + 3\theta_{H_1^0} - 6\mathrm{Tr}_S(\theta Y)$$
$$+ 6\theta_{\tilde{B}}\theta_{\tilde{H}_1^0}] + \Delta_1 + \frac{1}{12} T^2 \sum_i [N_c h_{D_i}^2 (\theta_{\tilde{D}_{L_i}} + \theta_{\tilde{D}_{R_i}} + \theta_{D_{L_i}}\theta_{D_{R_i}}) + h_{E_i}^2 (\theta_{\tilde{e}_{L_i}} + \theta_{\tilde{E}_{R_i}} + \theta_{e_{L_i}}\theta_{E_{R_i}})],$$

$$\Pi_{H_1^\pm} = \frac{1}{48} g_2^2 T^2 [6\theta_{W^\pm} + 3\theta_{W_3} + \theta_{H_1^\pm} + 2\theta_{H_1^0} - 2\mathrm{Tr}_S(\theta T_3) + 2\theta_{\tilde{H}_1^\pm}\theta_{\tilde{W}_3} + 4\theta_{\tilde{H}_1^0}\theta_{\tilde{W}^\pm}] + \frac{1}{144} g_1^2 T^2 [9\theta_B + 3\theta_{H_1^\pm} - 6\mathrm{Tr}_S(\theta Y)$$
$$+ 6\theta_{\tilde{B}}\theta_{\tilde{H}_1^\pm}] + \Delta_1 + \frac{1}{12} T^2 \sum_i [N_c h_{D_i}^2 (\theta_{\tilde{U}_{L_i}} + \theta_{\tilde{D}_{R_i}} + \theta_{U_{L_i}}\theta_{D_{R_i}}) + h_{E_i}^2 (\theta_{\tilde{\nu}_{L_i}} + \theta_{\tilde{E}_{R_i}} + \theta_{\nu_{L_i}}\theta_{E_{R_i}})],$$

$$\Pi_{H_2^\pm} = \frac{1}{48} g_2^2 T^2 [6\theta_{W^\pm} + 3\theta_{W_3} + \theta_{H_2^\pm} + 2\theta_{H_2^0} + 2\mathrm{Tr}_S(\theta T_3) + 2\theta_{\tilde{H}_2^\pm}\theta_{\tilde{W}_3} + 4\theta_{\tilde{H}_2^0}\theta_{\tilde{W}^\pm}]$$
$$+ \frac{1}{144} g_1^2 T^2 [9\theta_B + 3\theta_{H_2^\pm} + 6\mathrm{Tr}_S(\theta Y) + 6\theta_{\tilde{B}}\theta_{\tilde{H}_2^\pm}] + \Delta_2 + \frac{1}{12} T^2 \sum_i N_c h_{U_i}^2 (\theta_{\tilde{D}_{L_i}} + \theta_{\tilde{U}_{R_i}} + \theta_{D_{L_i}}\theta_{U_{R_i}}),$$

$$\Pi_{H_2^0} = \frac{1}{48} g_2^2 T^2 [6\theta_{W^\pm} + 3\theta_{W_3} + \theta_{H_2^0} + 2\theta_{H_2^\pm} - 2\mathrm{Tr}_S(\theta T_3) + 2\theta_{\tilde{H}_2^0}\theta_{\tilde{W}_3} + 4\theta_{\tilde{H}_2^\pm}\theta_{\tilde{W}^\pm}]$$
$$+ \frac{1}{144} g_1^2 T^2 [9\theta_B + 3\theta_{H_2^0} + 6\mathrm{Tr}_S(\theta Y) + 6\theta_{\tilde{B}}\theta_{\tilde{H}_2^0}] + \Delta_2 + \frac{1}{12} T^2 \sum_i N_c h_{U_i}^2 (\theta_{\tilde{U}_{L_i}} + \theta_{\tilde{U}_{R_i}} + \theta_{U_{L_i}}\theta_{U_{R_i}}),$$

$$\Pi_{H_2^0 H_1^{0*}} = -\frac{1}{48} T^2 [(g_2^2 + g_1^2)\theta_{H_2^0 H_1^{0*}} - 2g_2^2 \theta_{H_2^\pm H_1^{\mp *}}] + \Delta_{12}, \tag{32}$$

$$\Pi_{H_2^\pm H_1^{\mp}} = -\frac{1}{48} T^2 [(g_2^2 + g_1^2)\theta_{H_2^\pm H_1^{\mp *}} - 2g_2^2 \theta_{H_2^0 H_1^{0*}}] - \Delta_{12}, \tag{33}$$

with

$$\Delta_1 = \frac{T^2}{12} \sum_i \{N_c [h_{U_i}^2 |\mu|^2 \theta^{\tilde{Q}_{L_i}}_{\tilde{U}_{R_i}} + h_{D_i}^2 |A_{D_i}|^2 \theta^{\tilde{Q}_{L_i}}_{\tilde{D}_{R_i}}] + h_{E_i}^2 |A_{E_i}|^2 \theta^{\tilde{L}_i}_{\tilde{E}_{R_i}}\},$$

$$\Delta_2=\frac{T^2}{12}\sum_i \{N_c[h_{U_i}^2|A_{U_i}|^2\theta^{\tilde{Q}_{L_i}}_{\tilde{U}_{R_i}}+h_{D_i}^2|\mu|^2\theta^{\tilde{Q}_{L_i}}_{\tilde{D}_{R_i}}]+h_{E_i}^2|\mu|^2\theta^{\tilde{L}_i}_{\tilde{E}_{R_i}}\},$$

$$\Delta_{12}=\mu\frac{T^2}{12}\sum_i \{N_c[h_{U_i}^2A_{U_i}\theta^{\tilde{Q}_{L_i}}_{\tilde{U}_{R_i}}+h_{D_i}^2A_{D_i}\theta^{\tilde{Q}_{L_i}}_{\tilde{D}_{R_i}}]+h_{E_i}^2A_{E_i}\theta^{\tilde{L}_i}_{\tilde{E}_{R_i}}\}.$$

As an example of how to rotate $\theta$'s and $\Pi$'s consider the case in which only one (combination) of the Higgs doublets is light compared to the temperature, while the other is heavy and Boltzmann suppressed (this limit is realized for a large pseudoscalar mass and has been considered at finite temperature in studies of the electroweak phase transition). We will concentrate in the Higgs loop contribution to Higgs thermal self-energies only. The rest of the terms are trivial to deal with. In terms of $\theta_H,\theta_\Phi$, the off-diagonal thermal mixing between $H_1$ and $H_2$ [Eqs. (32) and (33)] has the form

$$\Pi_{H_2^0H_1^{0*}}=-\frac{1}{48}T^2[(g_2^2+g_1^2)(\theta_{H^0}-\theta_{\Phi^0})+2g_2^2(\theta_{H^\pm}-\theta_{\Phi^\pm})]\sin\beta\cos\beta,$$

$$\Pi_{H_2^\pm H_1^\pm}=\frac{1}{48}T^2[(g_2^2+g_1^2)(\theta_{H^\pm}-\theta_{\Phi^\pm})+2g_2^2(\theta_{H^0}-\theta_{\Phi^0})]\sin\beta\cos\beta.$$

In the neutral sector then, setting $\theta_{\Phi^0}=0$, it is easy to obtain

$$\Pi_{H^0}=\Pi_{H_1^0}\cos^2\beta+\Pi_{H_2^0}\sin^2\beta+2\Pi_{H_2^0H_1^{0*}}\cos\beta\sin\beta=\frac{1}{48}(g_1^2+g_2^2)(2\theta_{H^0}+\theta_{H^\pm})\cos^2 2\beta.$$

It can be checked that this is the correct result by noting that the standard model result is

$$\Pi_H^{\text{scalar}}=\frac{1}{4}\lambda T^2$$

(with the quartic Higgs coupling in the potential normalized to $V=\frac{1}{2}\lambda|H|^4$), while in the MSSM, the quartic self coupling of $H$, defined by Eq. (31), is $\lambda=\frac{1}{4}(g_1^2+g_2^2)\cos^2 2\beta$.

#### *4. Gauge bosons*

*a. SU(3)$_C$*:

$$\Pi_{g_L}=\frac{1}{12}g_3^2T^2\left[4N_c\theta_g+2\sum_j(\theta_{\tilde{U}_{L_j}}+\theta_{\tilde{D}_{L_j}}+\theta_{\tilde{U}_{R_j}}+\theta_{\tilde{D}_{R_j}})+\sum_j(\theta_{U_{L_j}}+\theta_{D_{L_j}}+\theta_{U_{R_j}}+\theta_{D_{R_j}})+2N_c\theta_{\tilde{g}}\right].$$

To simplify the contribution coming from squark loops we have used

$$\theta_{\tilde{q}}+\theta_{\tilde{\bar{q}}}^2=2\theta_{\tilde{q}}. \tag{34}$$

Note, however, that if we were to rotate the squark basis, the expression on the left-hand side should be used.

*b. SU(2)$_L$*:

$$\Pi_{W_{3L}}=\frac{1}{24}g_2^2T^2\Big[18\theta_{W^\pm}-2\theta_{gh}+8\theta_{\tilde{W}^\pm}+N_c\sum_j(2\theta_{\tilde{U}_{L_j}}+2\theta_{\tilde{D}_{L_j}}+\theta_{U_{L_j}}+\theta_{D_{L_j}})+\sum_j(2\theta_{\tilde{\nu}_{L_j}}+2\theta_{\tilde{e}_{L_j}}+\theta_{\nu_{L_j}}+\theta_{e_{L_j}})$$
$$+2(\theta_{H^0}+\theta_{H^\pm}+\theta_{\Phi^0}+\theta_{\Phi^\pm})+\theta_{\tilde{H}_1^0}+\theta_{\tilde{H}_1^\pm}+\theta_{\tilde{H}_2^\pm}+\theta_{\tilde{H}_2^0}\Big],$$

$$\Pi_{W_L^\pm}=\frac{1}{24}g_2^2T^2\Big[3(\theta_{W_3}+\theta_{W^\pm})+12\theta_{W_3}\theta_{W^\pm}-2\theta_{gh}+8\theta_{\tilde{W}^\pm}\theta_{\tilde{W}_3}+N_c\sum_j[(\theta_{\tilde{U}_{L_j}}+\theta_{\tilde{D}_{L_j}})^2+2\theta_{U_{L_j}}\theta_{D_{L_j}}]$$
$$+\sum_j[(\theta_{\tilde{\nu}_{L_j}}+\theta_{\tilde{e}_{L_j}})^2+2\theta_{\nu_{L_j}}\theta_{e_{L_j}}]+(\theta_{H^0}+\theta_{H^\pm})^2+(\theta_{\Phi^0}+\theta_{\Phi^\pm})^2+2\theta_{\tilde{H}_1^0}\theta_{\tilde{H}_1^\pm}+2\theta_{\tilde{H}_2^\pm}\theta_{\tilde{H}_2^0}\Big].$$

Here $\theta_{gh}$ gives the ghost piece and we have already rotated the Higgs contributions to the $H,\Phi$ basis using

$$\theta_{H_1^0}+\theta_{H_1^0}^2+\theta_{H_2^0}+\theta_{H_2^0}^2+\theta_{H_2^0H_1^{0*}}^2=\theta_{H^0}+\theta_{H^0}^2$$

and similarly for the charged $\theta$'s. We have further simplified our expression by using a relation similar to Eq. (34).

*c. U(1)$_Y$*:

$$\Pi_{B_L}=\frac{1}{216}g_1^2T^2\Bigg[18\sum_j\,(\theta_{\tilde{\nu}_{L_j}}+\theta_{\tilde{e}_{L_j}}+4\theta_{\tilde{e}_{R_j}})+9\sum_j\,(\theta_{\nu_{L_j}}+\theta_{e_{L_j}}+4\theta_{e_{R_j}})+18(\theta_{H^\pm}+\theta_{H^0}+\theta_{\Phi^\pm}+\theta_{\Phi^0})$$
$$+9(\theta_{\tilde{H}_1^\pm}+\theta_{\tilde{H}_1^0}+\theta_{\tilde{H}_2^\pm}+\theta_{\tilde{H}_2^0})+2N_c\sum_j\,(\theta_{\tilde{U}_{L_j}}+\theta_{\tilde{D}_{L_j}}+16\theta_{\tilde{U}_{R_j}}+4\theta_{\tilde{D}_{R_j}})+N_c\sum_j\,(\theta_{U_{L_j}}+\theta_{D_{L_j}}+16\theta_{U_{R_j}}+4\theta_{D_{R_j}})\Bigg].$$

Here, contributions from scalars, and in particular Higgs bosons, have been treated in the same way as explained for SU(2) and SU(3).

## ACKNOWLEDGMENTS

We would like to thank M. Carena, B. de Carlos, M. Pietroni, M. Quirós , and C.E.M. Wagner for help and discussions. The work of D.C. was supported by Ministerio de Educación y Ciencia (Spain). The work of J.R.E. was supported by the Alexander-von-Humboldt Stiftung.

[1] S. Sarkar, Rep. Prog. Phys. **59**, 1493 (1996).

[2] D. Gross, R. Pisarski, and L. Yaffe, Rev. Mod. Phys. **53**, 43 (1981).

[3] A. V. Smilga, Report No. ITEP-10/96-TH, hep-ph/9604367 (unpublished).

[4] J. Grundberg, T. H. Hansson, and U. Lindström, Report No. USITP-95-09, hep-th/9510045 (unpublished).

[5] D. A Kirzhnits, JETP Lett. **15**, 529 (1972); D. A Kirzhnits and A. D. Linde, Phys. Lett. **72B**, 471 (1972).

[6] S. Weinberg, Phys. Rev. D **9**, 3357 (1974); L. Dolan and R. Jackiw, *ibid.* **9**, 3320 (1974).

[7] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Annu. Rev. Nucl. Part. Sci. **43**, 27 (1993); V. A. Rubakov and M. E. Shaposhnikov, Sov. Phys. Usp. **39**, 461 (1996).

[8] M. Carrington, Phys. Rev. D **45**, 2933 (1992); M. E. Shaposhnikov, Phys. Lett. B **277**, 324 (1992); **282**, 483(E) (1992); M. Dine *et al.*, Phys. Rev. D **46**, 550 (1992); Phys. Lett. B **283**, 319 (1992); M. Shaposhnikov, Nucl. Phys. **B299**, 797 (1988); Phys. Lett. B **277**, 324 (1992); **282**, 483(E) (1992); J. R. Espinosa, M. Quirós, and F. Zwirner, *ibid.* **314**, 206 (1993); W. Buchmüller, Z. Fodor, T. Helbig, and D. Walliser, Ann. Phys. (N.Y.) **234**, 260 (1994).

[9] J. R. Espinosa, M. Quirós, and F. Zwirner, Phys. Lett. B **307**, 106 (1993); A. Brignole, J. R. Espinosa, M. Quirós, and F. Zwirner, *ibid.* **324**, 181 (1994).

[10] J. I. Kapusta, *Finite Temperature Field Theory* (Cambridge University Press, Cambridge, England); M. Quirós, Helv. Phys. Acta **67**, 451 (1994).

[11] A. D. Linde, Phys. Lett. **96B**, 289 (1980).

[12] J. M. Cline and K. Kainulainen, Nucl. Phys. **B482**, 73 (1996); M. Losada, Report No. RU-96-25, hep-ph/9605266 (unpublished); M. Laine, Nucl. Phys. **B481**, 43 (1996).

[13] J. M. Frere, D. R. T. Jones, and S. Raby, Nucl. Phys. **B222**, 11 (1983); C. Kounnas *et al.*, *ibid.* **B236**, 438 (1984); J. F. Gunion, H. E. Haber, and M. Sher, *ibid.* **B306**, 1 (1988); H. Komatsu, Phys. Lett. B **215**, 323 (1988); P. Langacker and N. Polonsky, Phys. Rev. D **51**, 379 (1995); J. A. Casas, A. Lleyda, and C. Muñoz, Nucl. Phys. **B471**, 3 (1996); Phys. Lett. B **389**, 305 (1996); T. Falk, K. A. Olive, L. Roszkowski, and M. Srednicki, *ibid.* **367**, 183 (1996).

[14] M. Claudson, L. J. Hall, and I. Hinchliffe, Nucl. Phys. **B228**, 501 (1983); A. Riotto and E. Roulet, Phys. Lett. B **377**, 60 (1996); A. Kusenko, P. Langacker, and G. Segre, Phys. Rev. D **54**, 5824 (1996).

[15] H. E. Haber, Phys. Rev. D **26**, 1317 (1982); M. Mangano, Phys. Lett. **147B**, 307 (1984); G. Dvali and K. Tamvakis, Phys. Lett. B **378**, 41 (1996).

[16] A. Masiero, D. V. Nanopoulos, and T. Yanagida, Phys. Lett. **138B**, 91 (1984).

[17] D. Comelli and M. Pietroni, Phys. Lett. B **306**, 67 (1993); J. R. Espinosa, J. M. Moreno, and M. Quirós, *ibid.* **319**, 505 (1993); D. Comelli, M. Pietroni, and A. Riotto, Nucl. Phys. **B412**, 441 (1994); Phys. Rev. D **50**, 7703 (1994); Phys. Lett. B **343**, 207 (1995).

[18] D. Comelli, M. Pietroni, and A. Riotto, Phys. Lett. B **354**, 91 (1995); P. Huet and A. Nelson, *ibid.* **355**, 229 (1995); Phys. Rev. D **53**, 4578 (1996); A. Riotto, *ibid.* **53**, 5834 (1996).

[19] M. Carena, M. Quirós, and C. E. M. Wagner, Phys. Lett. B **380**, 81 (1996).

[20] J. R. Espinosa, Nucl. Phys. **B475**, 273 (1996).

[21] J. M. Moreno, D. H. Oaknin, and M. Quirós, Nucl. Phys. **B483**, 267 (1997).

[22] A. Das and M. Kaku, Phys. Rev. D **18**, 4540 (1978); L. Girardello, M. Grisaru, and P. Salomonson, Nucl. Phys. **B178**, 331 (1981).