Effective action of (2+1)-dimensional QED: The effect of a finite fermion density

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The effective action of (2+1)-dimensional QED with a finite fermion density is calculated in a uniform electromagnetic field. It is shown that the integer quantum Hall effect and de Haas–van Alphen-like phenomenon in condensed matter physics are derived directly from the effective action. [S0556-2821(97)03608-4]

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I. INTRODUCTION

In (2+1)-dimensional spacetime, Fermi systems interacting through Maxwell field may have a dynamically induced Chern-Simons (CS) term in the effective Lagrangian [1,2]. That is, the low-energy effective action for the electromagnetic fields in the system, obtained by integrating out the fermionic degrees of freedom, has the induced CS term as a parity-odd part of the effective action. This CS term could describe the quantum Hall effect (QHE) [3]. In particular, an effective action at zero fermion density in uniform electromagnetic field was obtained by Redlich *et al.* [4,5], where the coefficient of the induced CS term represents the Hall conductivity of the quantum Hall effect [3]. However, real systems in the experiment of QHE [6] consist of a finite density of electrons, therefore, it is necessary to evaluate an effective action at finite fermion density and then to examine the behavior of the CS term as the external electromagnetic field is varied.

Recently, it was argued in Refs. [7,8] that the induced CS term in the presence of nonzero fermion density may describe the integer QHE; here, we use a different method from theirs, and calculate an effective action in a uniform electromagnetic field at finite fermion density. Then we show directly that the coeffecient of the induced CS term represents the QHE. Additionally, the free energy of the system is obtained as a parity-even part of the effective action. This exhibits a certain periodic dependence on the external field, which is similar to the de Haas–van Alphen effect [9].

The technique used here for deriving the effective action is the proper-time method, which was established by Schwinger first [10], and generalized to the case of finite fermion density in (3+1)-dimensional spacetime by Chodos *et al.* several years ago [11].

In Sec. II, we calculate the effective action of (2+1)-dimensional QED with finite fermion density in a constant uniform electromagnetic field. From this effective action, the integer quantum Hall effect and de Haas–van Alphen-like phenomenon in condensed matter physics are derived directly. In the last section, a summary of our results is given.

II. EFFECTIVE ACTION OF (2+1)-DIMENSIONAL QED WITH A FINITE FERMION DENSITY

We consider two-component Dirac fermions in a constant uniform electromagnetic field in (2+1)-dimensional spacetime. The Lagrangian of the Fermi systems at a finite fermion density [7,12] is given by

$$\mathcal{L}=\overline{\psi}(i\partial\!\!\!/-eA\!\!\!/)\psi-m\overline{\psi}\psi-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}-\mu\overline{\psi}\gamma^0\psi, \qquad (1)$$

where μ is a chemical potential, $A\!\!\!/=A_\mu\gamma^\mu$, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, and $\gamma^0=\sigma^3$, $\gamma^{1,2}=i\sigma^{1,2}$ with σ^i Pauli matrices. Here, the last term in $\mathcal{L}$, $\mu\rho$ with $\rho=\overline{\psi}\gamma_0\psi$, indicates that the system under consideration is composed of nonzero density of fermions. The fermion-mass term in the Lagrangian violates parity P and time-reversal T symmetries, and so a P- and T-odd term, which is the Chern-Simons term, is generated in the effective theory for the gauge field $F_{\mu\nu}$ [1].

The one-loop contribution to the effective action $S^1_{\rm eff}$ has the expression

$$S^1_{\rm eff}=-i\,\mathrm{Trln}(i\partial\!\!\!/-eA\!\!\!/-m-\mu\gamma_0), \qquad (2)$$

where Tr denotes the summation over spacetime coordinates as well as spinor indices. This can be immediately rewritten as [13]

$$S^1_{\rm eff}=-\frac{i}{2}[\mathrm{Trln}(\widetilde{D\!\!\!/}-m)-\mathrm{Trln}(\widetilde{D\!\!\!/}+m)]-\frac{i}{2}[\mathrm{Trln}(\widetilde{D\!\!\!/}-m)+\mathrm{Trln}(\widetilde{D\!\!\!/}+m)], \qquad (3)$$

where $\widetilde{D}^\mu=(p^0-eA^0-\mu,p^i-eA^i)$. Note that the first term in the right-hand side of Eq. (3), to be labeled as $S^{\rm odd}_{\rm eff}$, is explicitly odd under $m\to-m$ and corresponds to the parity-odd CS action, while, the second term, $S^{\rm even}_{\rm eff}$, is the ordinary effective action of even parity, which is negative value of the free energy of the system. From the analysis of eigenvalues of Dirac Hamiltonian for the above Lagrangian Eq. (1), the transformation $m\to-m$ results in the corresponding transformation of the chemical potential μ; $\mu\to-\mu$. This property comes from the fact that in 2+1 dimensions, two types of mass parameter in the Dirac equation are allowed, which holds only in 2+1 dimensions. So the induced CS term must involve the terms of the form $m/|m|$ or $\mu/|\mu|$ [see Eq. (17)].

We first calculate the parity-odd part of the action $S^{\rm odd}_{\rm eff}$. To evaluate the $S^{\rm odd}_{\rm eff}$ term, we may proceed in two steps [2,4]: first, the ground-state currrent density of the system, $\langle J_\mu\rangle$, is calculated, and then by integrating with respect to the vector potential A_μ, using the relation between an effective action and current density:

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$$\frac{\delta S_{\rm eff}}{\delta A^{\mu}}=\langle J_{\mu}\rangle, \tag{4}$$

the parity-odd part of the effective action may be obtained. The ground-state current $\langle J_{\mu}\rangle$ may be expressed in terms of the Green's function G:

$$\langle J_{\mu}\rangle=ie{\rm Tr}(\gamma_{\mu}G). \tag{5}$$

Here, the Green's function $G(x,y)$ is defined by

$$(i\partial\!\!\!/-eA\!\!\!/-m-\mu\gamma_0)G(x,y)=\delta(x-y). \tag{6}$$

We confine ourselves to the case where only the magnetic field B is turned on and the electric field $\vec{E}$ is zero. At the end of calculation, we can express the evaluated effective action in a Lorentz-covariant form. Under the Lorentz transformation, $F_{\mu\nu}\rightarrow F'_{\mu\nu}$; therefore, the nonzero electric- as well as magnetic-field effects may be deduced.

As in Ref. [11], the solution of Eq. (6) for G in the momentum space is given by

$$G=(D\!\!\!\!/+m)\left\{-i\int_0^{\infty}ds\exp[is(\tilde{D\!\!\!\!/}^2-m^2)]\,\theta[(p_0-\mu){\rm sgn}p_0]\right.$$
$$\left.+i\int_0^{\infty}ds\exp[-is(\tilde{D\!\!\!\!/}^2-m^2)]\,\theta[(\mu-p_0){\rm sgn}p_0]\right\}, \tag{7}$$

where $\tilde{D}^{\mu}=(p^0-\mu,\vec{D})$ with $\vec{D}\equiv\vec{p}-e\vec{A}$ and $F_{0i}=0$, $F_{12}=-B$. Now we use the identity

$$\tilde{D\!\!\!\!/}^2=(p^0-\mu)^2-\vec{D}^2$$
$$+\frac{e}{2}\sigma_{\mu\nu}F^{\mu\nu},\quad \text{with } \sigma_{\mu\nu}\equiv-\frac{i}{2}(\gamma_{\mu},\gamma_{\nu}). \tag{8}$$

Inserting the expression for G, Eq. (7), into Eq. (5) and introducing the proper-time parameter s [10,11] leads to

$$\langle J_{\mu}\rangle=ie{\rm tr}\gamma_{\mu}\left\langle x\left|-i\int_0^{\infty}ds\exp\left(is\left[-m^2+(p^0-\mu)^2+\frac{e}{2}\sigma\cdot F-\vec{D}^2\right]\right)(D\!\!\!\!/+m)\,\theta[(p^0-\mu){\rm sgn}p^0]\right.\right.$$
$$\left.\left.+i\int_0^{\infty}ds\exp\left[-is\left(-m^2+(p^0-\mu)^2+\frac{e}{2}\sigma\cdot F-\vec{D}^2\right)\right]\cdot(D\!\!\!\!/+m)\,\theta[(\mu-p^0){\rm sgn}p^0]\right|x\right\rangle, \tag{9}$$

where $\langle x|$ denotes the eigenket of the spacetime coordinate operator x^{μ} in Tr operation in Eq. (5). Note that we are led to a kind of dynamical problem; that is, $\exp(-is\vec{D}^2)$ plays the role of an evolution operator in the time variable s, governed by the Hamiltonian $\vec{D}^2$. Setting $|x\rangle\equiv|x^0\rangle|\vec{x}\rangle$ and $\langle\vec{x},s|\equiv\langle\vec{x}|\exp(-is\vec{D}^2)$, $\langle J_{\mu}\rangle$ is rewritten as

$$\langle J_{\mu}\rangle=ie{\rm tr}\gamma_{\mu}\left\{-i\int_0^{\infty}ds\exp\left[-is\left(m^2-\frac{e}{2}\sigma\cdot F\right)\right]\right.$$
$$\times\int_{-\infty}^{\infty}\frac{dp^0}{2\pi}\exp[is(p^0-\mu)^2]\langle\vec{x},s|D^i\gamma_i+(p^0-\mu)\gamma_0+m|\vec{x}\rangle\;\theta[(p^0-\mu){\rm sgn}p^0]-i\int_0^{\infty}ds\exp\left[is\left(m^2-\frac{e}{2}\sigma\cdot F\right)\right]$$
$$\left.\times\int_{-\infty}^{\infty}\frac{dp^0}{2\pi}\exp[-is(p^0-\mu)^2]\langle\vec{x},s|D^i\gamma_i+(p^0-\mu)\gamma_0+m|\vec{x}\rangle\;\theta[(\mu-p^0){\rm sgn}p^0]\right\}. \tag{10}$$

To evaluate the rather formal expression for $\langle J_{\mu}\rangle$, one may use the quantities

$${\rm tr}\;\exp\left(-\frac{i}{2}es\;\sigma_{\mu\nu}F^{\mu\nu}\right)=\cos(es|B|)+i\;\frac{\gamma^{\alpha}{}^*F_{\alpha}}{|B|}+\sin(es|B|), \tag{11}$$

and

$$\langle\vec{x},s|D^i|\vec{x}\rangle=0,\quad \langle\vec{x},s|\vec{x}\rangle=\frac{-i}{4\pi}\,\frac{e|B|}{\sin(es|B|)}, \tag{12}$$

where ${}^*F_{\alpha}=\frac{1}{2}\epsilon_{\alpha\beta\gamma}F^{\beta\gamma}$ with ${}^*F_0=-B$, ${}^*F_i=0$.

Note that only the time component of $\langle J_{\mu}\rangle$, $\langle J_0\rangle$, is sufficient in determining the coefficient of CS term [2,4]. Substituting Eqs. (11) and (12) into Eq. (10) and using $\int_{-\infty}^{\infty}dx\exp(isx^2)=\sqrt{\pi/s}\exp(i\pi/4)$, one may get that

$$\langle J_0\rangle=\frac{me^2}{4\pi^{3/2}}\exp\left(\frac{i\pi}{4}\right){}^*F_0\int_0^{\infty}\frac{ds}{\sqrt{s}}\exp(-ism^2)-\frac{me^2}{2\pi^2}{}^*F_0{\rm Re}\int_0^{\mu}dx\int_0^{\infty}ds\exp[is(x^2-m^2)]$$
$$-\frac{e^2}{2\pi^2}{}^*F_0{\rm Im}\int_0^{\mu}dx\;x\int_0^{\infty}ds\exp[is(x^2-m^2)]\cot(es|B|), \tag{13}$$

where Re (Im) denote the real (imaginary) part of the corresponding quantity.

To obtain the final expression for $\langle J_0\rangle$ in Eq. (13), one has to take the integration over both s and x variables. The integration over s in the first term is taken by letting $s\to -i\,s$. As may be checked, in the calculation of the second term in the right-hand side (RHS) of Eq. (13), one had better integrate over s variable first and then over x variable. In this way, one gets the $\theta(\mu^2-m^2)$ function. On the other hand, some care is needed in the calculation of the third term in the above equation; in this case, an integration over x may first be performed. And then in the integration over s, the integration contour in the real axis $(0,\infty)$ does have the meaning of $(0-i\epsilon,\infty-i\epsilon)$ [11,14]. Thus, the third term in $\langle J_0\rangle$ in Eq. (13) leads to the expression

$$\varepsilon(\mu)\theta(\mu^2-m^2)\pi^2\left[\frac{\mu^2-m^2}{2e|B|}+\sum_{n=1}^{\infty}\frac{1}{\pi n}\sin\left(\pi n\,\frac{(\mu^2-m^2)}{e|B|}\right)\right], \tag{14}$$

where $\varepsilon(\mu)$ is defined by $\varepsilon(\mu)=\mu/|\mu|$ and $\theta(x)$ is the step function such that $\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$. Adding up three terms in Eq. (13), we have a more familiar expression for $\langle J_0\rangle$:

$$\begin{aligned}\langle J_0\rangle=&\frac{e^2}{4\pi}\frac{m}{|m|}[1-\theta(\mu^2-m^2)]{}^*F_0\\&-\frac{e^2}{2\pi}\varepsilon(\mu)\theta(\mu^2-m^2)\left[\frac{\mu^2-m^2}{2e|B|}+\sum_{n=1}^{\infty}\frac{1}{\pi n}\sin\left(\pi n\,\frac{(\mu^2-m^2)}{e|B|}\right)\right]{}^*F_0.\end{aligned} \tag{15}$$

Here, the charge density vector $\langle J_0\rangle$ is one component of the covariant current vector $\langle J_\mu\rangle$. Note that under a Lorentz transformation, $|B|=|{}^*F|$ is an invariant quantity; that is, $|{}^*F|=|B|\to|{}^*F'|=\sqrt{B'^2-E'^2}$. The present situation is the zero temperature limit of relativistic thermodynamics. Thermodynamics can be formulated in the relativistically covariant way [15]. The chemical potential μ is a Lorentz scalar and defined by the value in the rest frame of Fermi gases; therefore, $\mu^2\to\mu^2$ under the Lorentz transformation. One may write the parity-odd part of the effective action in the covariant form

$$S_{\rm eff}^{\rm odd}=\frac{1}{2}\int\langle J'_\mu\rangle A'^{\mu}d^3x'. \tag{16}$$

The corresponding effective Lagrangian is

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\rm odd}=\frac{e^2}{16\pi}\Bigg\{&\frac{m}{|m|}[1-\theta(\mu^2-m^2)]-2\,\varepsilon(\mu)\theta(\mu^2-m^2)\\&\times\left[\frac{\mu^2-m^2}{2e|{}^*F|}+\sum_{n=1}^{\infty}\frac{1}{\pi n}\sin\left(\pi n\,\frac{(\mu^2-m^2)}{e|{}^*F|}\right)\right]\Bigg\}\\&\times\epsilon^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma},\end{aligned} \tag{17}$$

where the expression has the covariant form and the prime was suppressed for the simplicity of notation. Note carefully that taking the derivative of Eq. (17) with respect to A^0 leads to Eq. (15). The first μ-independent term was the obtained one in Refs. [4,5]. It is interesting to notice that the CS term for a vacuum fermion system (without the μ-dependent term in the Lagrangian) vanishes by the presence of arbitrary small fermion density. One may see that the $\varepsilon(\mu)$ term is odd under $m\to -m$; that is, this transformation leads to the corresponding transformation in μ such as $\mu\to-\mu$.

The coefficient of CS term corresponds to the Hall conductivity [3]. This may be easily calculated in the frame where the electric field $\vec{E}$ vanishes. Then, the chemical potential μ satisfies $\mu^2=m^2+2|eB|n$, when the fermions in the system occupy Landau levels up to certain integer n. And the infinite series term [14] in $\mathcal{L}_{\rm eff}^{\rm odd}$ vanishes. Then, Eq. (17) is reduced as follows:

$$\begin{aligned}\mathcal{L}_{\rm eff}^{\rm odd}&=-\frac{e^2}{8\pi}\varepsilon(\mu)\,\frac{\mu^2-m^2}{2e|{}^*F|}\,\epsilon^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma}\\&=-\frac{e^2}{8\pi}\varepsilon(\mu)n\;\epsilon^{\alpha\beta\gamma}A_\alpha F_{\beta\gamma}.\end{aligned} \tag{18}$$

Thus we get the Hall conductivity σ_{xy}:

$$\sigma_{xy}=\frac{e^2}{2\pi}\varepsilon(\mu)n, \tag{19}$$

where n is some integer number corresponding to the filled Landau levels in the system. In this case of a finite fermion density [16], the conductivity is 2 times the one for the vacuum system for each state n. This is the well-known integer Quantum Hall effect.

We now calculate the parity-even part of the effective action, $S_{\rm eff}^{\rm even}$, in Eq. (3). This may be calculated as in the $(3+1)$-dimensional case,

$$\begin{aligned}S_{\rm eff}^{\rm even}&=-\frac{i}{2}[\mathrm{Trln}(\tilde{\not{D}}-m)+\mathrm{Trln}(\tilde{\not{D}}+m)]\\&=-\frac{i}{2}\mathrm{Trln}(\tilde{\not{D}}^2-m^2).\end{aligned} \tag{20}$$

As for the odd part, we first consider pure magnetic case and then extend to the general case at the end of the calculation. Following the procedure in Refs. [10,11] and using the Green's function in Eq. (7), one may obtain $S_{\rm eff}^{\rm even}$:

$$S_{\rm eff}^{\rm even}=\frac{i}{2}{\rm tr}\int_0^\infty\frac{ds}{s}\langle x|\exp[is(\tilde{D\!\!\!/}^2-m^2)]\,\theta[(p_0-\mu){\rm sgn}p_0]|x\rangle+\frac{i}{2}\,{\rm tr}\int_0^\infty\frac{ds}{s}\langle x|\exp[-is(\tilde{D\!\!\!/}^2-m^2)]\,\theta[(\mu-p_0){\rm sgn}p_0]|x\rangle. \tag{21}$$

Substituting Eq. (8) into Eq. (21) and using Eqs. (11) and (12), we have the following expression for the parity-even effective Lagrangian $\mathcal{L}_{\rm eff}^{\rm even}$:

$$\mathcal{L}_{\rm eff}^{\rm even}=-\frac{\exp(i\pi/4)}{8\pi^{2/3}}\int_0^\infty\frac{ds}{s^{5/2}}\exp(-ism^2)es|B|\cot(es|B|)-\frac{1}{4\pi^2}\,{\rm Re}\int_0^\mu dx\int_0^\infty\frac{ds}{s^2}\exp[is(x^2-m^2)]es|B|\cot(es|B|). \tag{22}$$

The first term in the right-hand side of Eq. (22) is calculated by deforming the path of integration $s\to -is$; for $m=0$ it can be analytically integrated [4]. The integration over the s variable in the second term is performed by choosing a proper contour in the complex s plane just as the third term in Eq. (13) [11]. Then, it leads to the two terms

$$\frac{e|B|}{2\pi^2}\int_0^\mu dx\,{\rm Im}\sum_{n=1}^\infty\frac{1}{n}\exp\left(i\pi n\frac{x^2-m^2}{e|B|}\right)\theta(x^2-m^2)+\frac{1}{12\pi}\theta(\mu^2-m^2)(|\mu|-m)^2(|\mu|+2m). \tag{23}$$

Thus, expressing this in the covariant form as in Eq. (16), we get the parity-even part of the one-loop effective action

$$\mathcal{L}_{\rm eff}^{\rm even}=\frac{1}{8\pi^{2/3}}\int_0^\infty\frac{ds}{s^{5/2}}\exp(-m^2s)[es|{}^*F|\coth(es|{}^*F|)-1]+\frac{1}{12\pi}\theta(\mu^2-m^2)(|\mu|-m)^2(|\mu|+2m)$$
$$+\frac{(e|{}^*F|)^{3/2}}{\pi^2}\sum_{n=1}^\infty\frac{1}{n^{3/2}}\sin^{3/2}\left(\pi n\frac{\mu^2-m^2}{2e|{}^*F|}\right)\cos^{1/2}\left(\pi n\frac{\mu^2-m^2}{2e|{}^*F|}\right), \tag{24}$$

where for renormalization one term has been added to the first term on the right-hand side. Notice that the first μ-independent term in this effective Lagrangian is the previously obtained result in Ref. [4]. The third term in the right-hand side of Eq. (24) shows a periodic behavior as $|{}^*F|$ is decreased; the analogous term also appears in the effective Lagrangian of $(3+1)$-dimensional QED [11,14]. In Ref. [14] it is argued that the infinite series term when $\vec{E}=0$ may describe the de Haas–van Alphen effect [9] in condensed matter physics.

III. DISCUSSION

In this article we obtained an exact one-loop effective action of (2+1)-dimensional QED with finite fermion density in a uniform electromagnetic field. The obtained total effective Lagrangian $\mathcal{L}_{\rm eff}^1=\mathcal{L}_{\rm eff}^{\rm odd}+\mathcal{L}_{\rm eff}^{\rm even}$ in Eq. (17) and Eq. (24) agrees with the previously calculated one in Refs. [4,5] where the chemical potential μ-dependent term is not introduced into the Lagrangian.

We showed that the coefficient of the induced CS term exhibits the step-function behavior, when the fermions fill some Landau levels completely, and that this corresponds to the phenomena of the integer quantum Hall effect [7,8]. In particular, the induced fermion density in the vacuum vanishes if a finite density of fermions is added to the system. Then, the Chern-Simons term corresponding to the induced fermion density in the vacuum disappears. Similar behavior has already been observed in Ref. [17]; that is, the fermion density in the vacuum corresponding to the same system under consideration in this paper evaporates for any nonzero temperature $T\neq 0$. This nonanalytic behavior under the effect of finite temperature or finite chemical potential seems to be a very general phenomenon.

In addition, the third term in $\mathcal{L}_{\rm eff}^{\rm even}$ in Eq. (24) for $\vec{E}=0$ has a periodic behavior as a function of $(\mu^2-m^2)/2e|B|$. This agrees with the frequency for the de Haas–van Alphen effect [14]. So the infinite series over n in Eq. (24) seems to be related to the de Haas–van Alphen effect [9].

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