Correspondence principle for black holes and strings

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For most black holes in string theory, the Schwarzschild radius in string units decreases as the string coupling is reduced. We formulate a correspondence principle, which states that (i) when the size of the horizon drops below the size of a string, the typical black hole state becomes a typical state of strings and D-branes with the same charges, and (ii) the mass does not change abruptly during the transition. This provides a statistical interpretation of black hole entropy. This approach does not yield the numerical coefficient, but gives the correct dependence on mass and charge in a wide range of cases, including neutral black holes. [S0556-2821(97)00110-0]

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I. INTRODUCTION

A few years ago, Susskind proposed [1] that there was a one-to-one correspondence between Schwarzschild black holes and fundamental string states. This is based on the fact that as one increases the string coupling, the size of a highly excited string state becomes less than its Schwarzschild radius, so it must become a black hole. Conversely, as one decreases the coupling, the size of a black hole eventually becomes less than the string scale. At this point, the metric is no longer well defined near the horizon, so it can no longer be interpreted as a black hole. Susskind suggested that the configuration should be described in terms of some string state. At large values of the mass, the typical state consists of a small number of highly excited strings, so the black hole should reduce to such a state at weak coupling. In fact, the single-string entropy approximates the total entropy up to subleading terms, and so one can focus on states of a single highly excited string. Further evidence for this correspondence between black holes and excited string states has recently been given [2,3].

It is widely believed that there is a discrepancy between the entropy of a free fundamental string (which is proportional to the mass of the string state) and the Bekenstein-Hawking entropy (which is proportional to the *square* of the mass of the black hole). This apparent discrepancy must clearly be resolved in order for the proposed correspondence to be valid. Susskind has suggested that a large gravitational redshift might account for the difference.

We will show that the standard formulas for the string and black hole entropies can be related directly to one another. More generally, when the black hole carries Ramond-Ramond charge, the weak-coupling limit involves D-branes [4]. The correspondence between black holes at strong coupling and strings and D-branes at weak coupling can be stated in terms of the following principle: (i) When the curvature at the horizon of a black hole (in the string metric) becomes greater than the string scale, the typical black hole state becomes a typical state of strings and D-branes with the same charges and angular momentum; (ii) the mass changes by at most a factor of order unity during the transition.

The condition on the curvature in (i) is just the condition for α' corrections to become important near the horizon, so this is a natural point for the transition to occur. An immediate consequence of this principle is that the black hole entropy must be comparable to the string entropy. One may wonder how this is possible, since we explicitly assume that the mass does not change significantly during the transition to a string state. The point is that as the string coupling g is varied, the mass of a black hole is constant in Planck units, while the mass of a string (ignoring gravitational corrections) is constant in string units. So they can agree for only one value of g. It is natural to equate the masses at the value of g when the black hole becomes a string. By the above principle, this occurs when the size of the horizon is of order the string scale. We will show that when the black hole mass and string mass are set equal at this scale, their respective entropies are also equal, up to a factor of order unity that depends on exactly when the black hole forms. Turning the argument around, if we follow a given state (that is, fixing the entropy[1]) adiabatically through the transition between a black hole and a string, its mass changes by a factor which is of order one, rather than being parametrically large.

For the Schwarzschild black hole in four dimensions, the equality of the single-string and black hole entropies for string-sized black holes was pointed out by Susskind [1]. We extend this to all dimensions and to black holes carrying a variety of charges. (Adding angular momentum typically changes the entropy by at most a factor of two, so its effect is difficult to see just from the correspondence principle.) In some cases the typical state on the weak-coupling side is a single long string, but in others it is gas of massless strings

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[1]For a system with rapidly growing density of states, a narrow band of states can be labeled by its entropy.

on D-branes; two distinctly different kinds of gas (free and interacting) arise. Thus the correspondence principle unifies the known results on black hole entropies and enables us to understand many new cases.

Section II treats black holes without Ramond-Ramond (RR) charges, starting with Schwarzschild in any dimension, and then including electric Neveu-Schwarz (NS) charges.[2] Section III treats black p-branes with a single Ramond-Ramond charge. The near extremal entropy of these solutions have been discussed previously [7] where it was argued that in most cases it could not be understood simply in terms of the known light degrees of freedom on the brane. We will see that when the correspondence principle is applied, the near extremal entropy in all cases agrees with the D-brane counting[3] (up to factors of order unity). We will also discuss two ways of compactifying the black p-brane space time and show that their entropies are reproduced by two different configurations of D-branes. Section IV presents a discussion of the relation to other work and some concluding remarks.

II. NEUTRAL AND NS-NS BLACK HOLES

A. Schwarzschild black holes

We start with the familiar four-dimensional Schwarzschild black hole

$$ds^2=-\left(1-\frac{r_0}{r}\right)dt^2+\left(1-\frac{r_0}{r}\right)^{-1}dr^2+r^2d\Omega. \quad (2.1)$$

The mass of the black hole is $M_{\rm BH}=r_0/2G$. We want to equate this with the mass of a string state at excitation level N, which is $M_s^2\sim N/\alpha'$ at zero string coupling g. In four dimensions Newton's constant is related to the string coupling g and α' by $G\sim g^2\alpha'$. So it is clear that the mass of the black hole cannot equal the string mass for all values of g. If we want to equate them, we have to decide at what value of the string coupling they should be equal. Clearly, the natural choice is to let g be the value at which the string forms a black hole, which, by our correspondence principle is when the horizon is of order the string scale. Setting the masses equal when $r_0^2\sim\alpha'$ yields

$$M_{\rm BH}^2\sim\frac{\alpha'}{G^2}\sim\frac{N}{\alpha'}. \quad (2.2)$$

The black hole entropy is then

$$S_{\rm BH}\sim\frac{r_0^2}{G}\sim\frac{\alpha'}{G}\sim\sqrt{N}. \quad (2.3)$$

So the Bekenstein-Hawking entropy is comparable to the string entropy [1]. They have the same dependence on the mass and only differ by a factor of order unity which depends on exactly when the string state forms a black hole.

Put differently, consider following a particular state as the coupling is varied, which means holding the entropy fixed. The success of the above matching means that the mass changes only by a factor of order 1 during the transition from the black hole description to the string description. There are various large and small dimensionless numbers in the problem. One is the excitation level N. Another is the string coupling g at the transition; from the matching condition (2.2) and $G\sim g^2\alpha'$, one finds $g\sim N^{-1/4}$. It could have turned out that the mass changes during the transition by a factor which is parametrically large, such as a power of N, but this is not the case here or in any of the later examples.

Since the string forms a black hole at the string scale, which is not large compared to the compactification scale, it is important to see whether this agreement continues to hold for black holes in higher dimensions. The Schwarzschild metric in d spatial dimensions is similar to Eq. (2.1) except that r_0/r is replaced by $(r_0/r)^{d-2}$. The mass is now $M_{\rm BH}\sim r_0^{d-2}/G$. We again equate this with the string mass when the black hole is of order the string size

$$M_{\rm BH}^2\sim\frac{(\alpha')^{d-2}}{G^2}\sim\frac{N}{\alpha'}. \quad (2.4)$$

The black hole entropy is thus

$$S_{\rm BH}\sim\frac{r_0^{d-1}}{G}\sim\frac{(\alpha')^{(d-1)/2}}{G}\sim\sqrt{N}. \quad (2.5)$$

So once again the black hole entropy is comparable to the string entropy.

One might have been concerned that the typical string state is much larger than the string scale and so does not sensibly match onto the black hole [1]. This is a somewhat involved question, about which we will have more to say in a future paper [11], but in fact it is not really relevant here. A highly excited string is like a random walk, with an entropy proportional to its length. Even if we restrict attention to highly excited string states that are small, constrained to lie in just a few string volumes, the entropy is still proportional to the length, just with a numerically smaller coefficient to which we are not sensitive. In fact there is an offsetting effect due to the gravitational self-interaction [11].

B. Charged black holes

We now consider charges that can be carried by fundamental strings, i.e., electric Neveu-Schwarz charges associated with momentum and winding modes. For a black hole with these charges, the dilaton is not constant, so the string metric differs from the Einstein metric. The correspondence between strings and black holes occurs when the curvature of the string metric at the horizon is of order the string scale. We will see that this implies that the size of the horizon in the string metric is of order the string scale. We will equate the mass of the black hole to the mass of the string at this point and show that the black hole entropy (which is proportional to the horizon area in the Einstein metric) is then comparable to the usual string entropy.

For a string propagating on a circle with radius R, the left- and right-moving momenta are defined to be

[2]Black holes with magnetic Neveu-Schwarz charge are the one example in which the horizon does not become smaller than a string at weak coupling. An approach for understanding these entropies has been discussed in [5,6].

[3]For other discussions of the near extremal entropy, see [8–10].

$$p_L=\frac{n}{R}-\frac{mR}{\alpha'},\quad p_R=\frac{n}{R}+\frac{mR}{\alpha'},\tag{2.6}$$

where n,m are the integer momentum and winding numbers. The string entropy is

$$S_s\sim\sqrt{N_L}+\sqrt{N_R},\tag{2.7}$$

where

$$\frac{N_L}{\alpha'}\sim M_s^2-p_L^2,\quad \frac{N_R}{\alpha'}\sim M_s^2-p_R^2.\tag{2.8}$$

To obtain the four-dimensional black hole solution with these charges we start with the five-dimensional black string solution [12,13] (in the string metric)

$$ds^2=F\left[-\left(1-\frac{r_0}{r}\right)dt^2+dz^2\right]+\left(1-\frac{r_0}{r}\right)^{-1}dr^2+r^2d\Omega,\tag{2.9}$$

where

$$F^{-1}=1+\frac{r_0\sinh^2\gamma_1}{r}\tag{2.10}$$

and the dilaton is $e^{2\phi_5}=F$. One can now boost along the z direction (to add momentum) and reduce to four dimensions to obtain

$$ds^2=-\Delta^{-1}\left(1-\frac{r_0}{r}\right)dt^2+\left(1-\frac{r_0}{r}\right)^{-1}dr^2+r^2d\Omega,\tag{2.11}$$

$$\Delta=\left(1+\frac{r_0\sinh^2\gamma_1}{r}\right)\left(1+\frac{r_0\sinh^2\gamma_p}{r}\right).$$

The four-dimensional dilaton (which differs from ϕ_5 by a factor of the length of the fifth dimension) is $e^{-4\phi}=\Delta$. The horizon is at $r=r_0$, and the Arnowitt-Deser-Misner (ADM) mass is

$$M_{\rm BH}=\frac{r_0}{8G}(2+\cosh2\gamma_1+\cosh2\gamma_p),\tag{2.12}$$

where G is the four-dimensional Newton's constant. The integer normalized charges corresponding to the momentum and winding numbers are[4]

$$n=\frac{r_0R}{8G}\sinh2\gamma_p,\quad m=\frac{r_0\alpha'}{8GR}\sinh2\gamma_1.\tag{2.13}$$

The left- and right-moving momenta are thus

$$p_L=\frac{r_0}{8G}(\sinh2\gamma_p-\sinh2\gamma_1),$$

$$p_R=\frac{r_0}{8G}(\sinh2\gamma_p+\sinh2\gamma_1).\tag{2.14}$$

The horizon area in the Einstein metric $ds_E^2=e^{-2\phi}ds^2$ is

$$A=4\pi r_0^2\cosh\gamma_1\cosh\gamma_p.\tag{2.15}$$

The curvature of the full ten dimensional string metric [which is the product of Eq. (2.9) with a five torus] has two independent components near the horizon. One is proportional to $1/r_0^2$ and the other is proportional to $(\tanh^2\gamma_1)/r_0^2$. So the first is always larger, and the curvature is of order the string scale when $r_0^2\sim\alpha'$. Setting the mass and charges of the black hole equal to those of the string at this scale yields

$$\frac{N_L}{\alpha'}\sim M_{\rm BH}^2-p_L^2\sim\frac{\alpha'}{G^2}[3+2(\cosh2\gamma_1+\cosh2\gamma_p)+\cosh2(\gamma_1+\gamma_p)],$$

$$\frac{N_R}{\alpha'}\sim M_{\rm BH}^2-p_R^2\sim\frac{\alpha'}{G^2}[3+2(\cosh2\gamma_1+\cosh2\gamma_p)+\cosh2(\gamma_1-\gamma_p)].\tag{2.16}$$

The largest term or terms in N_L or N_R is always of order $\alpha'^2G^{-2}\cosh2(|\gamma_1|+|\gamma_p|)$, which for all γ_1,γ_p is the same as $\alpha'^2G^{-2}\cosh^2\gamma_1\cosh^2\gamma_p$ up to a factor of order one. The string entropy is then

$$S_s\sim\sqrt{N_L}+\sqrt{N_R}\sim\frac{\alpha'}{G}\cosh\gamma_1\cosh\gamma_p.\tag{2.17}$$

This is the same as the black hole entropy $S_{\rm BH}\sim A/G$, where the area is given in Eq. (2.15), at the point $r_0^2\sim\alpha'$ where the matching is done. Thus we find that the black hole entropy always agrees with the string entropy up to factors of order unity. In particular, it has the same dependence on the mass and charge. We have also checked that this agreement extends to charged black holes in higher dimensions.

It should be noted that even after the transition to the weakly coupled regime, the gravitational dressing remains large. The stringy behavior at $r\sim r_0$ smears out the zero in $(1-r_0/r)$ and so this does not cause a large correction (essentially, this is part of the correspondence principle). But the factor Δ differs significantly from unity over a much greater scale when γ_1 or γ_p is large, and its value near the horizon is of order $\cosh^2\gamma_1\cosh^2\gamma_p$. We must use the corrected local metric in calculating the string entropy. This does not, however, affect the result. Consider first the case $\gamma_p=0$, so there is only winding charge; the dressing (2.9) is then a uniform rescaling of the zt plane. Near the extremal limit, the mass relation becomes

$$M_{\rm BH}-\frac{mR}{\alpha'}\sim\frac{N}{mR}.\tag{2.18}$$

The left-hand side is the free energy, the excess energy above the rest mass of the winding strings. Its value near the string is greater than its asymptotic value by the redshift $\cosh\gamma_1$. But also the radius R is contracted by the same factor, so the value of N and hence the entropy are the same as

[4]We follow the conventions of [14,15]. The left and right boost parameters of [16] are related to γ_1, γ_p by $\alpha=\gamma_p-\gamma_1$ and $\beta=\gamma_p+\gamma_1$.

would follow from the asymptotic values. For the second charge, the compact momentum, we do not need a detailed analysis: it is simply a result of applying a boost to both the black hole and string configurations.

The redshift does have one notable effect. The asymptotic temperature at the matching point is $T\sim(\alpha')^{1/2}\cosh\gamma_1\cosh\gamma_p)^{-1}$. With the redshift included, the local temperature at the string is $T\sim(\alpha')^{-1/2}$, the string scale, just as in the Schwarzschild case [1].

String states with $N_R=0$ are supersymmetric. On the black hole side this corresponds to an extremal limit $r_0/G\to 0$, $\gamma_p,\gamma_1\to\infty$ keeping $(r_0/G)\sinh 2\gamma_1$ and $\gamma_1-\gamma_p$ fixed. In Planck units (constant G), the horizon area (2.15) vanishes in this limit, which led Sen [17] to compare the number of string states with the area of a ''stretched horizon'' where the curvature of the extremal solution was of order the string scale. Since we start with nonextremal black holes and set the size of the horizon in the string metric to be of order the string scale,[5] the entropy remains finite in the limit and agrees with the string expression.

III. BLACK p-BRANES

A. Ten dimensions

In this section we consider black p-branes with a single Ramond-Ramond charge. The string metric is given by [12]

$$ds^2=f^{-1/2}\left[-\left(1-\frac{r_0^n}{r^n}\right)dt^2+dy^i dy_i\right]+f^{1/2}\left[\left(1-\frac{r_0^n}{r^n}\right)^{-1}dr^2+r^2 d\Omega_{n+1}\right], \tag{3.1}$$

where

$$f=1+\frac{r_0^n\sinh^2\alpha}{r^n}. \tag{3.2}$$

The y_i are $p=7-n$ spatial coordinates along the brane which we assume are compactified on a large torus of volume V. The dilaton is $e^{2\phi}=f^{(n-4)/2}$. The energy, RR charge, and entropy of the p-brane are

$$E\sim\frac{r_0^n V}{g^2\alpha'^4}\left(\frac{n+2}{n}+\cosh 2\alpha\right),$$

$$Q\sim\frac{r_0^n}{g\alpha'^{n/2}}\sinh 2\alpha, \tag{3.3}$$

$$S_{\rm BH}\sim\frac{r_0^{n+1}V}{g^2\alpha'^4}\cosh\alpha.$$

We have dropped overall constants of order unity since they will not be needed for testing the correspondence principle. However, it should be noted that the constants in front of E and Q are the same, so that in the extremal limit ($r_0\to 0$, $\alpha\to\infty$, with Q fixed) we have $E=QV/g\alpha'^{(8-n)/2}$.

In the limit of weak string coupling, this extremal limit corresponds to Q Dirichlet p-branes [4]. The nonextremal solution should correspond to an excited state of these D-branes and strings. To determine when this weak-coupling description is applicable, we consider the curvature of Eq. (3.1) at the horizon $r=r_0$. The largest contribution comes from the angular part of the metric and is of order $(r_0^2\cosh\alpha)^{-1}$. By the correspondence principle, the matching between the black p-brane and the strings and D-branes occurs when this is of order $1/\alpha'$ or

$$r_0=\frac{\alpha'^{1/2}}{(\cosh\alpha)^{1/2}}. \tag{3.4}$$

That is, for given Q and S, Eq. (3.4) determines the value of g at which the description changes. At this point we wish to compare the Bekenstein-Hawking entropy to that of an assembly of strings and D-branes with the same charge and mass. Note that since $e^{\phi}=(\cosh\alpha)^{(n-4)/2}$ on the horizon, Eq. (3.4) implies that $ge^{\phi}<1/Q$. Thus the local string coupling remains small.

There are two qualitatively different kinds of excited states of D-branes. The first consists of adding a small number of long strings.[6] In this state the entropy is $S_1\sim\alpha'^{1/2}\Delta E$, where

$$\Delta E=E-\frac{QV}{g\alpha'^{(p+1)/2}} \tag{3.5}$$

is the excess energy above the D-brane rest mass. The second class of states consists of exciting a large number of massless open strings on the D-branes. There are Q^2 species of open string, as will be explained below, so the excess energy and the entropy (dropping numerical constants) are

$$\Delta E\sim Q^2T^{p+1}V,$$

$$S_2\sim Q^2T^pV \tag{3.6}$$

which implies $S_2\propto(\Delta E)^{p/(p+1)}$. Not surprisingly, for large excess energy, long string states are more numerous, while the string gas has higher entropy when ΔE is small. The transition occurs when $\Delta E\sim Q^2V/\alpha'^{(p+1)/2}$, i.e., when T is of order one in string units.

We now wish to compare these weak-coupling entropies with $S_{\rm BH}$. The procedure is to match the energy and the charge of the strings and D-branes with that of the black p-brane when Eq. (3.4) is satisfied. At this point,

$$\Delta E\sim\frac{V}{g^2\alpha'^{(p+1)/2}}(\cosh\alpha)^{-n/2},$$

[5]Since $r_0\sim(\alpha')^{1/2}$, the string coupling g must become large as r_0/G vanishes.

[6]The black p-brane should match onto long strings lying in approximately the volume of the p-brane, but this constraint will only affect the coefficient in the entropy. Also, long open strings ending on the D-branes are more numerous (and hence more likely) than long closed strings, but the effect on the entropy is of subleading order.

$$Q \sim \frac{1}{g} \sinh 2\alpha (\cosh\alpha)^{-n/2}, \tag{3.7}$$

and so

$$S_1 \sim \frac{V}{g^2 \alpha'^{p/2}} (\cosh\alpha)^{(p-7)/2},$$

$$S_2 \sim \frac{V}{g^2 \alpha'^{p/2}} (\tanh\alpha)^{2/(p+1)} (\cosh\alpha)^{(p-6)/2}. \tag{3.8}$$

Large α corresponds to near extremal configurations (with $S_2 > S_1$), while small α corresponds to configurations far from extremality (with $S_1 > S_2$). In either case, the larger of the two entropies agrees with the black hole entropy, which from Eqs. (3.3) and (3.4) is

$$S_{\rm BH} \sim \frac{V}{g^2 \alpha'^{p/2}} (\cosh\alpha)^{(p-6)/2}. \tag{3.9}$$

Thus the correspondence principle correctly reproduces the entropy of all black p-branes with a RR charge.

As in the case of near extreme black holes with NS charges, there is a large redshift near the D-branes, but it does not affect the result. From the metric (3.1) f is a uniform rescaling of the D-brane world volume, to which the massless string gas is insensitive. In other words, in the ideal gas relations (3.6) we need to include a redshift factor $\gamma = (\cosh\alpha)^{1/2}$ in ΔE and T and γ^{-p} in the volume, so these equations continue to hold. The redshift again raises the asymptotic temperature at the matching point $T \sim (\alpha' \cosh\alpha)^{-1/2}$ to the string scale $(\alpha')^{-1/2}$.

Now we must justify the assumption of Q^2 degrees of freedom in the gas regime. The massless fields on the D-branes are the non-Abelian gauge fields and collective coordinates [18]. These are Q^2 in number, but the collective coordinates have a potential proportional to $\mathrm{Tr}([X^i, X^j]^2)$ so that the moduli space is of dimension Q; similarly the gauge fields have a self-interaction. These interaction terms will restrict the number of effective degrees of freedom only if they are large compared to the kinetic term, which turns out not to be the case. Treating the interaction as a perturbation on a free gas of Q^2 species, we can estimate its ratio to the kinetic term as follows. By the usual large-N counting, the ratio will have a factor of $g e^{\phi} Q$. The quartic coupling g in $p+1$ space time dimensions has units of $(\mathrm{mass})^{3-p}$, so the dimensionless expansion parameter is $g e^{\phi} Q T^{p-3}$. We have seen that at the horizon $T \sim (\alpha')^{-1/2}$ and $g e^{\phi} Q \sim (\alpha')^{(p-3)/2}$, so the expansion parameter is of order 1.[7] That is, the two energies are of the same order. The potential is positive so we have underestimated the energy of the gas, but only by a numerical factor. This is within the accuracy of the correspondence principle.[8]

The correspondence principle is thus confirmed for the large class of solutions (3.1), but we should note that it is working even better than one might expect in some cases. Specifically, for $n > 4$ ($p = 0,1,2$) and α large, spheres outside the horizon $r = r_0$ grow *smaller* as r increases, reaching a minimum size at $r \sim r_0 (\sinh\alpha)^{2/n}$. Correspondingly, the maximum curvature outside the horizon occurs at a finite distance away from the horizon. Thus as one decreases the string coupling, the α' expansion first breaks down away from the horizon, and there is a range of couplings where the horizon and asymptotic region are both described by low-energy gravity but an intermediate region has string corrections. In applying the correspondence principle we have implicitly assumed that the expressions for the black p-brane (3.3) remain valid until the curvature at the horizon itself is of order the string scale. It is not clear why this is justified. Incidentally, if one attempts to match the black hole to the D-branes when the curvature away from the horizon first becomes of order the string scale, one finds that the entropies do not agree—the D-brane entropy is too small.

B. Compactification

We now consider compactification of the black p-branes below ten dimensions. We begin with the simplest case of zero charge ($\alpha = 0$), when the solutions are just the product of a torus T^p and the $(10-p)$-dimensional Schwarzschild metric. This is one form of compactification, and was considered in Sec. II. However there is another possibility: One can consider a p-dimensional array of ten-dimensional Schwarzschild black holes. An array of finite size would not be static, but an infinite array does lead to a static solution [20]. Identifying after one period, the array has the same mass and entropy as a single ten-dimensional black hole $M \sim r_0^7 / g^2 \alpha'^4$ and $S \sim r_0 M$, while the product solution has $M' \sim \rho_0^n V / g^2 \alpha'^4$, and $S' \sim \rho_0 M'$ where ρ_0 is the Schwarzschild radius of the lower-dimensional black hole, V is the volume of the internal space, and $n = 7 - p$. Setting the masses equal $r_0^7 \sim \rho_0^n V$, yields

$$\frac{S}{S'} \sim \frac{r_0}{\rho_0} \sim \left(\frac{V}{r_0^p} \right)^{1/n}. \tag{3.10}$$

So the array has greater entropy as long as $V > r_0^p$, below which point the images start to merge. This suggests that the product solution is unstable in this regime, a fact which has been confirmed by studying the linearized perturbations [21].

Both the entropy of the product solution and the array can be understood by our correspondence principle, since we saw in Sec. II that it reproduces the entropy of Schwarzschild black holes in any dimension. The difference between the two cases is the following. As we decrease the string coupling, the mass of the black hole in string units increases. For the product solution, the curvature at the horizon reaches the

[7] Curiously the position dependence of ϕ and of the effective T cancel, so one comes to the same conclusion by erroneously using the asymptotic values.

[8] The gas picture does not apply for $p = 0$, but the result is the same: the potential determines the magnitude of the fluctuations of the Q^2 degrees of freedom.

string scale at a larger value of the coupling than for the array. Hence the energy of the resulting string state and associated entropy is smaller.

Now we consider the near extremal solutions $\alpha \gg 1$. (Solutions far from extremality are qualitatively similar to the case of zero charge.) We first consider a periodic coordinate transverse to the p-brane, say, $x^9 \sim x^9 + 2\pi R$. Again there are two kinds of black hole, the array and the translationally invariant solution. For the extremal black p-brane, the array is given by replacing f in Eq. (3.2) with

$$f_1 = 1 + r_0^n \sinh^2\alpha_1 \sum_{k=-\infty}^{\infty} \frac{1}{|x - x_k|^n}, \tag{3.11}$$

where $x_k^9 = 2\pi R k$ are the image positions, and taking $r_0 \to 0$ with $r_0^n \sinh^2\alpha$ fixed. The array of nonextremal solutions $r_0 > 0$ is more complicated [22], but for $r_0 \ll R$ it is easy to construct an approximate solution. The metric for a single nonextreme p-brane is indistinguishable from the extreme solution when $r \gg r_0$. So one can approximate the nonextreme array by keeping f_1 as above and inserting factors of $(1 - r_0^n/r^n)$ just as in the ten-dimensional solution (3.1) (where r is a radius from each p-brane).

The homogeneous solution, which is translationally invariant in the x^9 direction, is

$$ds^2 = f_2^{-1/2}\left[-\left(1 - \frac{\rho_0^{n-1}}{\rho^{n-1}}\right) dt^2 + dy^i dy_i \right] + f_2^{1/2}\left[\left(1 - \frac{\rho_0^{n-1}}{\rho^{n-1}}\right)^{-1} d\rho^2 + \rho^2 d\Omega_n + dx_9^2 \right], \tag{3.12}$$

where

$$f_2 = 1 + \frac{\rho_0^{n-1}\sinh^2\alpha_2}{\rho^{n-1}}. \tag{3.13}$$

The array solution (3.11) has the same energy, charge and entropy as the ten-dimensional solution, Eq. (3.3), while for the homogeneous solution (3.12) these are

$$E' \sim \frac{\rho_0^{n-1} R V}{g^2 \alpha'^4}\left(\frac{n+1}{n-1} + \cosh 2\alpha_2\right),$$

$$Q' \sim \frac{\rho_0^{n-1} R}{g \alpha'^{n/2}} \sinh 2\alpha_2, \tag{3.14}$$

$$S'_{\rm BH} \sim \frac{\rho_0^n R V}{g^2 \alpha'^4} \cosh\alpha_2.$$

For large R, one would expect the compactification to have little effect and so the array solution (3.11) would appear to be more physical. As before, we can determine which solution is stable by seeing which has more entropy for given mass and charge. For equal masses and charges, Eqs. (3.3) and (3.14) imply that $\alpha_1 \sim \alpha_2$ and $\rho_0^{n-1} R \sim r_0^n$. It follows that

$$\frac{S_{\rm BH}}{S'_{\rm BH}} \sim \frac{r_0}{\rho_0} \sim \left(\frac{R}{r_0}\right)^{1/(n-1)}. \tag{3.15}$$

Once again, the array has greater entropy as long as $R > r_0$, below which point the array solution approaches the translationally invariant one. Notice that in the near extremal limit, r_0 is very small, so the homogeneous solution is almost always unstable.

Let us now consider the weak-coupling description in terms of D-branes. The array corresponds to Q coincident D-branes. The translationally invariant solution corresponds to Q D-branes evenly distributed in x^9. To count the number of excited states, it is convenient to apply T duality. The D p-branes then become $(p+1)$-branes, extended in the x^9 direction. The x^9 coordinate is T dual to the D-brane Wilson line [18], so for the coincident D-branes the Wilson line is the identity while for the distributed D-branes its eigenvalues are uniformly distributed. In the latter case, one can go to a basis in which the Wilson line is the shift matrix,

$$W_{ij} = \delta_{i,j+1} \quad (i \equiv i + Q). \tag{3.16}$$

With this Wilson line the D-brane fields $\phi_{i,j}$ (both the collective coordinates and the gauge fields) are periodically identified with $\phi_{i+1,j+1}$ and one essentially has Q species (distinguished by $|i-j|$) on a D-brane of length $2\pi Q R' = 2\pi Q \alpha'/R$. We refer to this as the wrapped system since it describes one D-brane wrapped Q times around the circle [19]. For $W = 1$, the unwrapped system, there are Q^2 species on a D-brane of length $2\pi R'$.

For $TR' > 1$, meaning large R' or large energy density, these two systems behave essentially the same, with

$$\Delta E \sim Q^2 V R' T^{p+2},$$

$$S \sim Q^2 V R' T^{p+1}. \tag{3.17}$$

When $TR' < 1$, the modes of the unwrapped system can no longer propagate in the compact direction and the system behaves like a p-dimensional gas, while the wrapped system continues to behave like a $(p+1)$-dimensional gas. We now show that when the correspondence principle is applied, these two systems reproduce the entropy of the array and homogeneous black p-branes respectively. The entropy of the array is the same as a single black p-brane in ten dimensions, since the compactification has little effect. We have already seen that its near extremal entropy is reproduced by a gas of Q^2 degrees of freedom in p dimensions which is just the unwrapped system. If we apply T duality to the homogeneous solution, the effect on the metric (3.12) is $g_{99} \to 1/g_{99}$ and we obtain the solution for a black $(p+1)$-brane. The energy and entropy are duality invariant. Thus the near extremal entropy is reproduced by a gas of Q^2 degrees of freedom in $(p+1)$ dimensions which is the wrapped D-brane. Notice that the wrapped D-brane has *less* entropy than the unwrapped one.

What is the role of the transition point $TR' \sim 1$ in the black p-brane picture? As we have seen, the asymptotic temperature satisfies $T \sim (\alpha' \cosh\alpha)^{-1/2}$, so the radius above which the wrapped and unwrapped systems become indistinguishable is $R' \sim (\alpha' \cosh\alpha)^{1/2}$. The T-dual radius is $R = (\alpha'/\cosh\alpha)^{1/2} = r_0$, which is just the radius below which the array overlaps into a translationally invariant system.

Thus we see a detailed correspondence between wrapping/unwrapping for D-branes and the two kinds of compactified black holes.

Now let us consider compactification of one of the directions parallel to the p-brane, i.e., we suppose one direction is much smaller than the rest. To make use of the previous discussion (to which it is T dual), we start with a black $(p+1)$-brane, compactify with periodicity $2\pi R'$, and denote the volume of the $(p+1)$-brane by $2\pi R'V$. We saw in Sec. III A that the near extremal entropy is reproduced by a $(p+1)$-dimensional gas with Q^2 species. For $R'T>1$ this can be represented in weak coupling either by wrapped or unwrapped D-branes. However, for $R'T<1$, only the wrapped system yields the black $(p+1)$-brane entropy; the unwrapped branes have higher entropy. It may seem puzzling that the black $(p+1)$-brane ceases to be the lowest-entropy configuration at a rather large radius $R'\sim(\alpha'\cosh\alpha)^{1/2}$. Moreover, the higher-entropy configuration is hard to describe. It is T dual to the array. But the array is not invariant under x^9 translation—that is, the background has modes of nonzero p_9—so the dual background must have fields associated with nonzero winding number. We can see a signature of this in the black p-brane metric (3.1). The metric along the D-brane, at the horizon, is $dy^2(\cosh\alpha)^{-1/2}$, so that when $R'\sim(\alpha'\cosh\alpha)^{1/2}$ the size of the compactified direction *at the horizon* is only $\alpha'^{1/2}$ and so it is possible for stringy effects to arise.

It is interesting to note if we continue to reduce the radius until $QR'T<1$, the entropy of the wrapped D-brane changes from a $(p+1)$ dimensional gas to a p-dimensional gas with Q degrees of freedom. This is to be expected since the T-dual configuration now consists of Q widely separated D-branes. The strong-coupling limit of this would be Q near extremal black p-branes each with unit charge. Thus when $QR'T<1$, the entropy of the homogeneous p-brane is not only smaller than the array of charge Q p-branes, but also smaller than an array of charge one p-branes with spacing $1/Q$ of the previous period.

To summarize, we have seen that the correspondence principle works in great detail: We have considered two different kinds of compactified black p-branes and two configurations of D-branes, and the entropies match in detail both for the higher-entropy and the lower-entropy system.

C. Black holes with two or more RR charges

Recently, a precise agreement (including the numerical coefficient) has been found [23,24] between the entropy of certain extreme and near extreme black holes and states of D-branes. These cases differ from the ones we have discussed in that there are at least two Ramond-Ramond charges. An example is the five-dimensional black hole carrying one-brane charge, five-brane charge, and compact momentum. Applying the correspondence principle to this black hole, there are eight cases to consider according to which of the three charges are large. These separate into three categories. When neither of the Ramond charges is large the results of Sec. II apply: The typical weak-coupling state is a long string, whose entropy matches that of the black hole. When one of the Ramond charges is large, the discussion from earlier in this section applies and the black hole entropy matches that of an interacting string gas with Q_1^2 or Q_5^2 degrees of freedom. When both Ramond charges are large, the entropy is reproduced precisely by a gas of Q_1Q_5 moduli [23–25]. This is now a free gas, in that the moduli have no potential.[9]

Note that in most of the cases we have discussed, if one sets the energy and charge of the weak-coupling state equal to that of the black hole at an arbitrary value of the Schwarzschild radius r_0, the two entropies have different dependence on r_0. For example, in the Schwarzschild black hole $S_{\rm BH}\propto r_0^2$ while $S_{\rm str}\propto r_0$ at fixed g. The matching of entropies then depends on a special value of r_0 given by the correspondence principle. For the cases where exact calculations of the entropy have been done, the r_0 dependence is the same on both sides and so the matching scale drops out.

IV. DISCUSSION

We have proposed a correspondence principle which connects black holes to weakly coupled strings and D-branes, and shown that it leads to an agreement between the entropy of these two systems. Although we have not been able to compare the precise coefficients in the entropy formulas, they have the same dependence on the mass and charge in a wide variety of different contexts. This strengthens the idea that a black hole is an ordinary quantum mechanical system, and that string theory is a viable theory of quantum gravity. In all examples considered here, string theory provides the correct number of degrees of freedom to account for the black hole entropy.

We have seen that the typical string state depends in an essential way on the quantum numbers. With no large RR charges it is a single long string, with one it is an interacting string gas on D-branes, and for some examples with two or more RR charges it is a free string gas, a gas of moduli. The correspondence principle thus unifies various results in the literature.

The success of the correspondence principle does not mean that gravitational effects remain small whenever the string and D-brane picture is valid. For near extremal black holes, we have seen that the metric deviates from flat space over a region much larger than the horizon size. Thus there is a large gravitational dressing after the transition. This affects both the local energy and the size of the internal space, but suprisingly, the entropy is unaffected. In all cases, the local Hawking temperature at the matching point is of order the string scale.

One can trivially extend this agreement in certain ways, e.g., by adding momentum to the black p-branes discussed in Sec. III. The only change in both the black hole and weak-coupling descriptions is to apply a boost in some direction. Other extensions are probably possible, but require further investigation. For example consider magnetic NS charge. It

[9]There is a puzzle here, in that the local temperature at the transition is of order the string scale and so large enough that there will be some excitation of states other than the moduli. This does not affect the qualitative agreement required by the correspondence principle, but makes the precise agreement for near extremal entropies [25] somewhat puzzling.

is clear that when the charge is small, it has little effect on the entropy and the matching can be done as in Sec. II. In other cases, e.g., the near extremal black five-brane in IIB string theory with large charge, the correspondence principle cannot be applied directly since the horizon size never becomes small in string units. However, in this case the string coupling becomes large near the horizon. It thus seems appropriate to count states by going to the weakly coupled S-dual description. This is five-brane with RR charge, for which the correspondence principle can be applied and yields the correct entropy.

It appears difficult to extend our analysis to try to compare the precise coefficients in the formulas for the entropy. This would require a better understanding of the string state when it is of order the string scale.

Near extremal black p-branes with one RR charge have been discussed recently in the literature. It is perhaps useful if we comment on the relation between our discussion in Sec. III and some of this previous work. It has been noted that the black hole and D-brane entropies have different temperature (or equivalently r_0) dependence in general [7]. This is in accord with our point of view, since we expect them to match only at one point.[10]

The self-dual case $p=3$ is particularly interesting. If we consider the r_0 dependence of the entropy at fixed g and α, the black hole entropy is proportional to r_0^{8-p} while the string gas entropy is proportional to $r_0^{(7-p)(p+2)/(p+1)}$. Precisely for $p=3$ these are the same, so the matching scale drops out and one might hope to relate the entropies precisely. However, this is the case where the D-brane entropy seems to exceed that of the black hole by a factor of 4/3 [26]. We now have some understanding of the origin of this factor. The D-brane calculations were done with a gas of Q^2 species treated as free. We have seen, however, that the interactions are of order 1; neglecting them underestimates the energy of each state by a factor of order 2 and so overestimates the entropy (at fixed energy) by a similar factor. The interactions are quite complicated, however, and we do not see a way to obtain the precise factor.

In the case $p=5$, the black p-brane entropy has been shown to agree precisely with that of a gas of noncritical closed strings living on the five-brane [9]. We, on the other hand, have shown approximate agreement with the entropy of ordinary D-branes and open strings. Moreover, in the latter case the open string interactions are of order 1 and the gravitational dressing is large, while all such complications are blithely ignored in Ref. [9]. Is there some duality here?

Susskind has pointed out that our results give an approximate verification of string duality for non-BPS states. Many of our examples are related by duality. The simplest is just the Schwarzschild black hole, which might turn into a heterotic string at small g and a type-II string at large. The agreement of the nonextremal entropies of each string with that of the black hole implies agreement with each other. In other words, we can follow a given state from a long heterotic string, to a black hole, to a long type-II string. The new ingredient is that the black hole description gives a known g dependence of the mass in the intermediate regime.

Since the different weakly coupled string theories have different degeneracy of states, one might wonder whether they could all be consistent with the Bekenstein-Hawking entropy at a precise level (when the coefficients are better understood). There are at least two ways in which this could occur. The first is if the transition from the black hole to the string state takes place at slightly different values of the curvature in the different string theories. The second is that, as we have remarked several times, the black hole state may turn into only a subset of the available string states. This subset is large enough so that its entropy differs from the usual string entropy only by an overall coefficient. In this sense, there may not be a precise one-to-one correspondence between string states and black holes, and may explain the factor of two discrepancy in [3].

[10]In Ref. [8] it is shown that higher dimension operators in the action can in a rather general way correct the temperature dependence of the black hole entropy to that of the D-brane gas. Possibly there is some relation to our work, though the physical assumptions are rather different.

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