Gravitational theory without the cosmological constant problem

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We develop the principle of nongravitating vacuum energy, which is implemented by changing the measure of integration in the action from $\sqrt{-g}d^Dx$ to an integration in an internal space of D scalar fields φ_a. As a consequence of such a choice of the measure, the matter Lagrangian L_m can be changed by adding a constant while no cosmological term is induced. Here we develop this idea to build a new theory which is formulated through the first order formalism, for example, when using vielbein e^μ_a and spin connection ω^{ab}_μ $(a,b=1,2,\ldots,D)$ as independent variables. The equations obtained from the variation of e^μ_a and the fields φ_a imply the existence of a nontrivial constraint. This approach can be made consistent with invariance under arbitrary diffeomorphisms in the internal space of scalar fields φ_a (as well as in ordinary space-time), provided that the matter model is chosen so as to satisfy the above-mentioned constraint. If the matter model is not chosen so as to satisfy automatically this constraint, the diffeomorphism invariance in the internal space is broken. In this case the constraint is dynamically implemented by the degrees of freedom that become physical because of the breaking of the internal diffeomorphism invariance. However, this constraint always dictates the vanishing of the cosmological constant term and the gravitational equations in the vacuum coincide with vacuum Einstein's equations with zero cosmological constant. The requirement that the internal diffeomorphisms be a symmetry of the theory points towards the unification of forces in nature such as in the Kaluza-Klein scheme. [S0556-2821(97)02010-9]

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I. INTRODUCTION

In 1917, Einstein realized [1] that his field equations can be modified by introducing the ''cosmological constant term.'' This ''Λ term'' appears in the Einstein's equations in the form

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\Lambda g_{\mu\nu}=\frac{\kappa}{2}T_{\mu\nu}, \tag{1}$$

where $\kappa\equiv 16\pi G$. Although Einstein considered the introduction of such a term a mistake, the fact is that such a term does not violate any symmetry. Furthermore, quantum field theory (QFT) predicts the existence a vacuum energy because of the zero point fluctuations, which gives an infinite contribution to $T_{\mu\nu}$ which is of the form $g_{\mu\nu}\rho_0$, ($\rho_0=$const), that is, indistinguishable from the Λ term. Even if the infinity problem could be avoided, QFT naturally predicts a very large Λ term, since on purely dimensional ground, QCD would give a vacuum energy of order 1 GeV^4 and in quantum gravity one expects 10^{76} GeV^4, while observations require the vacuum energy to be less then 10^{-46} GeV^4. For a historic overview see [2] and for reviews of the modern attempts to solve this puzzle see [3].

In this paper we will develop an approach where, as a consequence of a nontrivial constraint imposed by the variational principle, which has a highly geometrical motivation, any Λ term is forbidden. When this constraint is satisfied in an automatic form by the matter models, we have an additional local symmetry in the model. The triviality of the constraint or the associated local gauge symmetry implies the physical irrelevance of certain degrees of freedom. It should be pointed out that the vanishing of the cosmological term is achieved even if the constraint is nontrivially implemented. In this case the degrees of freedom mentioned above become nontrivial and are dynamically active in the mechanism that eliminates the cosmological constant.

This approach is based on a paper by us [4], where the ''principle of nongravitating vacuum energy (NGVE)'' was formulated. There the usual measure of integration that is $\sqrt{-g}$, was changed by another scalar density Φ which is also a total derivative, built from D scalar fields (if D is the dimension of space-time). In an explicit form

$$\Phi\equiv\varepsilon_{a_1a_2\cdots a_D}\varepsilon^{\alpha_1\alpha_2\cdots\alpha_D}(\partial_{\alpha_1}\varphi_{a_1})(\partial_{\alpha_2}\varphi_{a_2})\cdots(\partial_{\alpha_D}\varphi_{a_D}), \tag{2}$$

where φ_a, $(a=1,2,\ldots,D)$ are scalar fields. In this case $\int L_m\Phi d^Dx$ is invariant under the change $L_m\rightarrow L_m+$const, since then we just add to the integrand $L_m\Phi$ a total derivative term. We should then remember that usually the cosmological constant piece in Eq. (1) is generated from a term of the form $\Lambda\int\sqrt{-g}d^Dx$, which with the change of measure becomes an irrelevant total divergence. In spite of this, in such a model an integration constant, that plays a role which resembles that of a cosmological term, appears in the equations (although for nonvanishing values of this integration constant, maximally symmetric spaces are not available [4]). In addition, the equations deviate from those of general relativity and a new physical massless ''dilaton'' appears, with the corresponding phenomenological problems.

In contrast, here we will find that when formulating the theory in a way which is invariant under diffeomorphisms in the manifold of fields φ_a, then no term that plays the role of

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a cosmological constant term can appear. Together with this, no propagating dilaton appears. This is achieved in a simple way by formulating the model in terms of vielbeins and allowing the possibility of torsion. This is quite a natural approach since vielbeins and torsion appear in any case if fermions are introduced.

In the context of this formulation of the theory, the local symmetry mentioned before, is actually the group of diffeomorphisms in the space of the scalar fields φ_a. If this group is indeed a symmetry, the vanishing of the cosmological constant is a trivial consequence. If this group is not a symmetry, still the variation with respect to these degrees of freedom leads to the constraint that makes the cosmological constant vanishing.

As was mentioned above, the key idea of the theory is replacing the measure $\sqrt{-g}d^Dx$ by Φd^Dx, Φ being given by Eq. (2) and we are then led to the following action for gravity plus matter:

$$S=\int L\Phi d^Dx\equiv\int\left(-\frac{1}{\kappa}R+L_m\right)\Phi d^Dx, \tag{3}$$

where R is the scalar curvature. To define R, we can use the standard Riemannian definition in terms of $g_{\mu\nu}$. This leads to the theory studied in Ref. [4].

A different approach, which will be shown in this paper to be *physically inequivalent*, is to allow a more general form for R which allows for the possibility of torsion. In this case we define [5]

$$R(\omega,e)=e^{a\mu}e^{b\nu}R_{\mu\nu ab}(\omega), \tag{4}$$

$$R_{\mu\nu ab}(\omega)=\partial_\mu\omega_{\nu ab}-\partial_\nu\omega_{\mu ab}+(\omega^c_{\mu a}\omega_{\nu cb}-\omega^c_{\nu a}\omega_{\mu cb}), \tag{5}$$

where $e^{a\mu}=\eta^{ab}e^\mu_b$, η^{ab} is the diagonal $D\times D$ matrix with elements $+1,-1,\ldots,-1$ on the diagonal, e^μ_a are the vielbeins, and ω^{ab}_μ $(a,b=1,2,\ldots,D)$ is the spin connection. The matter Lagrangian L_m that appears in Eq. (3) does not depend on the scalar fields φ_a and it is now a function of matter fields, vielbeins, and spin connection, considered as independent fields. We assume for simplicity that L_m does not depend on the derivatives of vielbeins and spin connection.

It should be noted that the same results can be obtained in the first order formalism where we use the metric $g_{\mu\nu}$ (or vielbeins $e_{a\mu}$) and the connection $\Gamma^\lambda_{\mu\nu}$ as independent variables.

As it is well known [6], if fermions contribute to L_m, the vielbein formalism becomes unavoidable anyway. This could be regarded as an argument to view the use of Eqs. (4) and (5) as a more fundamental starting point than that of using the Riemannian definition for R. As we will see, in the NGVE theory studied here, $R(\omega,e)\neq$ Riemann scalar, even in the case $L_m=0$.

II. GENERAL FEATURES OF THE NGVE THEORY

In this section we study the general features of the NGVE theory, which are consequences only of the fact that the scalar fields φ_a enter just in the measure of integration and not in the total Lagrangian density $L\equiv-(1/\kappa)R+L_m$. First, notice that Φ is the Jacobian of the mapping $\varphi_a=\varphi_a(x^\alpha)$, $a=1,2,\ldots,D$. If this mapping is nonsingular ($\Phi\neq 0$), then (at least locally) there is the inverse mapping $x^\alpha=x^\alpha(\varphi_a)$, $\alpha=0,1\ldots,D-1$. Since $\Phi d^Dx=D!d\varphi_1\wedge d\varphi_2\wedge\cdots\wedge d\varphi_D$, we can think Φd^Dx as integrating in the internal space variables φ_a. In addition, if $\Phi\neq 0$ then there is a coordinate frame where the coordinates are the scalar fields themselves.

The field Φ is invariant under the volume-preserving diffeomorphisms in internal space: $\varphi'_a=\varphi'_a(\varphi_b)$ where

$$\varepsilon_{a_1a_2\cdots a_D}\frac{\partial\varphi'_{b_1}}{\partial\varphi_{a_1}}\frac{\partial\varphi'_{b_2}}{\partial\varphi_{a_2}}\cdots\frac{\partial\varphi'_{b_D}}{\partial\varphi_{a_D}}=\varepsilon_{b_1b_2\cdots b_D}. \tag{6}$$

Such infinite-dimensional symmetry leads to an infinite number of conservation laws. To see this, notice that from the volume-preserving symmetries $\varphi'_a=\varphi'_a(\varphi_b)$, defined by Eq. (6), which for the infinitesimal case imply

$$\varphi'_a=\varphi_a+\lambda\varepsilon^{aa_1\cdots a_D}\frac{\partial F_{a_1a_2\cdots a_{D-1}}(\varphi_b)}{\partial\varphi_{a_D}} \tag{7}$$

$(\lambda\ll 1)$, we obtain through Noether's theorem the conserved quantities

$$j^\mu_V=A^\mu_a\left(-\frac{1}{\kappa}R+L\right)\varepsilon_{aa_1\cdots a_D}\frac{\partial F_{a_1a_2\cdots a_{D-1}}(\varphi_b)}{\partial\varphi_{a_D}}. \tag{8}$$

We now want to notice that the form of the action (3) implies the existence of a very special set of equations. These are the equations of motion obtained by variation of the action (3) with respect to the scalar fields φ_b and they are

$$A^\mu_b\partial_\mu\left(-\frac{1}{\kappa}R+L_m\right)=0, \tag{9}$$

where

$$A^\mu_b\equiv\varepsilon_{a_1a_2\cdots a_{D-1}b}\varepsilon^{\alpha_1\alpha_2\cdots\alpha_{D-1}\mu}(\partial_{\alpha_1}\varphi_{a_1})\times(\partial_{\alpha_2}\varphi_{a_2})\cdots(\partial_{\alpha_{D-1}}\varphi_{a_{D-1}}). \tag{10}$$

It follows from Eq. (2) that $A^\mu_b\partial_\mu\varphi_{b'}=D^{-1}\delta_{bb'}\Phi$ and taking the determinant of both sides, we get $\det(A^\mu_b)=(D^{-D}/D!)\Phi^{D-1}$. Therefore, if $\Phi\neq 0$, which we will assume in what follows, the only solution for Eq. (9) is

$$L\equiv-\frac{1}{\kappa}R+L_m=\text{const}\equiv M. \tag{11}$$

Finally, we show that the same structure of this action, which leads to the very special set of equations displayed above, is associated with another, even more puzzling set of symmetries than the volume-preserving diffeomorphisms. In fact, let us consider the following infinitesimal shift of the fields φ_a by an arbitrary infinitesimal function of the total Lagrangian density $L\equiv-(1/\kappa)R+L_m$, that is

$$\varphi'_a=\varphi_a+\epsilon g_a(L),\quad \epsilon\ll 1. \tag{12}$$

In this case the action is transformed according to

$$\delta S=\epsilon D\int A^{\mu}_{a}L\partial_{\mu}g_{a}(L)d^{D}x=\epsilon\int \partial_{\mu}\Omega^{\mu}d^{D}x, \tag{13}$$

where $\Omega^{\mu}\equiv DA^{\mu}_{a}f_{a}(L)$ and $f_{a}(L)$ being defined from $g_{a}(L)$ through the equation $Ldg_{a}/dL=df_{a}/dL$. To obtain the last expression in the Eq. (13), it is necessary to note that $\partial_{\mu}A^{\mu}_{a}\equiv 0$. By means of the Noether's theorem, this symmetry leads to the conserved current

$$j^{\mu}_{L}=A^{\mu}_{a}(Lg_{a}-f_{a})\equiv A^{\mu}_{a}\int_{L_0}^{L}g_{a}(L')dL'. \tag{14}$$

The existence of this symmetry depends crucially on the independence of the Lagrangian density L on the scalar fields that define the measure. In fact, the existence of this symmetry could be used to justify the expectation that quantum corrections would keep that basic structure, provided it is present at the tree level.

III. THE NGVE THEORY—RIEMANNIAN APPROACH

Before studying the case when the definitions (4) and (5) are used, we will review the model studied in [4], where R in the action (3) is the Riemannian one and $L_m=L_m(g_{\mu\nu},\text{matter fields})$.

Variation of $S_g\equiv-(1/\kappa)\int R\Phi d^{D}x$ with respect to $g^{\mu\nu}$ leads to the result

$$\delta S_g=-\frac{1}{\kappa}\int \Phi[R_{\mu\nu}+(g_{\mu\nu}\Box-\nabla_{\mu}\nabla_{\nu})]\delta g^{\mu\nu}d^{D}x. \tag{15}$$

In order to perform the correct integration by parts we have to make use of the scalar field $\chi\equiv\Phi/\sqrt{-g}$, which is invariant under continuous general coordinate transformations, instead of the scalar density Φ. Then, integrating by parts and ignoring a total derivative term which has the form $\partial_{\alpha}(\sqrt{-g}P^{\alpha})$, where P^{α} is a vector field, we get

$$\frac{\delta S_g}{\delta g^{\mu\nu}}=-\frac{1}{\kappa}\sqrt{-g}[\chi R_{\mu\nu}+g_{\mu\nu}\Box\chi-\chi_{,\mu;\nu}]. \tag{16}$$

In a similar way, varying the matter part of the action (3) with respect to $g^{\mu\nu}$ and making use of the scalar field χ we can express a result in terms of the standard matter energy-momentum tensor $T_{\mu\nu}\equiv(2/\sqrt{-g})\partial(\sqrt{-g}L_m)/\partial g^{\mu\nu}$. Then, after some algebraic manipulations we get, instead of Einstein's equations,

$$G_{\mu\nu}=\frac{\kappa}{2}\Big\{T_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}[T^{\alpha}_{\alpha}+(D-2)L_m]\Big\}+\frac{1}{\chi}\left(\frac{D-3}{2}g_{\mu\nu}\Box\chi+\chi_{,\mu;\nu}\right), \tag{17}$$

where $G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}$.

By contracting Eq. (17) and using Eq. (11), we get

$$\Box\chi-\frac{\kappa}{D-1}\Big\{M+\tfrac{1}{2}[T^{\alpha}_{\alpha}+(D-2)L_m]\Big\}\chi=0. \tag{18}$$

By using Eq. (18) we can now exclude $T^{\alpha}_{\alpha}+(D-2)L_m$ from Eq. (17):

$$G_{\mu\nu}=\frac{\kappa}{2}[T_{\mu\nu}+Mg_{\mu\nu}]+\frac{1}{\chi}[\chi_{,\mu;\nu}-g_{\mu\nu}\Box\chi]. \tag{19}$$

Notice that Eqs. (9) and (17) are invariant under the addition to L_m a constant piece, since the combination $T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}[T^{\alpha}_{\alpha}+(D-2)L_m]$ is invariant.

It is very important to note that the terms depending on the matter fields in Eq. (19) as well as in Eq. (17) do not contain χ field, in contrast with the usual scalar-tensor theories, such as Brans-Dicke theory. As a result of this feature of the NGVE theory, the gravitational constant does not suffer space-time variations. However, the matter energy-momentum tensor $T_{\mu\nu}$ is not conserved. Actually, taking the covariant divergence of both sides of Eq. (19) and using the identity $\chi^{;\alpha}_{;\nu;\alpha}=(\Box\chi)_{,\nu}+\chi^{,\alpha}R_{\alpha\nu}$, Eqs. (19) and (18), we get the equation of matter nonconservation

$$T_{\mu\nu}{}^{;\mu}=-2\frac{\partial L_m}{\partial g^{\mu\nu}}g^{\mu\alpha}\partial_{\alpha}ln\chi. \tag{20}$$

We are interested now in studying the question whether there is an Einstein sector of solutions, that is are there solutions that satisfy Einstein's equations? First of all, we see that Eqs. (19) coincide with Einstein's equations only if the χ field is a constant. From Eq. (18) we conclude that this is possible only if an essential restriction on the matter model is imposed,

$$2M+T^{\alpha}_{\alpha}+(D-2)L_m\equiv 2\left[g^{\mu\nu}\frac{\partial(L_m-M)}{\partial g^{\mu\nu}}-(L_m-M)\right]=0, \tag{21}$$

which means that L_m-M is an homogeneous function of $g^{\mu\nu}$ of degree one, in any dimension. If condition (21) is satisfied then the equations of motion allow solutions of general relativity (GR) to be solutions of the model, that is $\chi=\text{const}$ and $G_{\mu\nu}=(\kappa/2)T_{\mu\nu}+Mg_{\mu\nu}$. It is interesting to observe that when condition (21) is satisfied, a new symmetry of the action (3) appears. We will call this symmetry ''Einstein symmetry'' [because Eq. (21) leads to the existence of an Einstein sector of solutions]. Such a symmetry consists of the scalings

$$g^{\mu\nu}\to\lambda g^{\mu\nu}, \tag{22}$$

$$\varphi_a\to\lambda^{-1/D}\varphi_a, \tag{23}$$

where $\lambda=\text{const}$. To see that this is indeed a symmetry, note that from definition of scalar curvature it follows that $R\to\lambda R$ when the transformations (22) and (23) are performed. Since condition (21) means that L_m is a homogeneous function of $g^{\mu\nu}$ of degree one, we see that under the transformations (22) and (23) the matter Lagrangian $L_m\to\lambda L_m$. From this we conclude that Eqs. (22) and (23) are indeed a symmetry of the action (3) when Eq. (21) is satisfied.

The situation described above can be realized for special kinds of bosonic matter models.

(1) Scalar fields without potentials, including fields subjected to nonlinear constraints, such as the σ model. The general coordinate-invariant action for these cases has the form $S_m = \int L_m \Phi d^D x$ where $L_m = \frac{1}{2}\sigma,_\mu \sigma,_\nu g^{\mu\nu}$.

(2) Matter consisting of fundamental bosonic strings. The condition (21) can be verified by representing the string action in the D-dimensional form where $g_{\mu\nu}$ plays the role of a background metric. For example, bosonic strings, according to our formulation, where the measure of integration in a D-dimensional space-time is chosen to be $\Phi d^D x$, will be governed by an action of the form

$$S_m = \int L_{\rm string} \Phi d^D x,$$

$$L_{\rm string} = -T \int d\sigma d\tau \frac{\delta^D(x - X(\sigma,\tau))}{\sqrt{-g}} \sqrt{\det(g_{\mu\nu} X^\mu_{,a} X^\nu_{,b})}, \tag{24}$$

where $\int L_{\rm string}\sqrt{-g} d^D x$ would be the action of a string embedded in a D-dimensional space-time in the standard theory; a,b label coordinates in the string world sheet and T is the string tension. Notice that under a scaling (22) (which means that $g_{\mu\nu} \to \lambda^{-1} g_{\mu\nu}$), $L_{\rm string} \to \lambda^{(D-2)/2} L_{\rm string}$; therefore, concluding that $L_{\rm string}$ is a homogeneous function of $g^{\mu\nu}$ of degree one, that is Eq. (21), is satisfied only if $D=4$.

(3) It is possible to formulate *the point particle model* of matter in a way such that Eq. (21) is satisfied. This is because for the free-falling point particle a variety of actions are possible (and are equivalent in the context of general relativity). The usual actions are taken to be $S = -m \int F(y) ds$, where $y = g_{\alpha\beta}(dX^\alpha/ds)(dX^\beta/ds)$ and s is determined to be an affine parameter except if $F = \sqrt{y}$, which is the case of reparametrization invariance. In our model we must take $S_m = -m \int L_{\rm part} \Phi d^4 x$ with $L_{\rm part} = -m \int ds\, \delta^4(x - X(s))/\sqrt{-g} F[y(X(s))]$ where $\int L_{\rm part}\sqrt{-g} d^4 x$ would be the action of a point particle in four dimensions in the usual theory. For the choice $F = y$, condition (21) is satisfied. Unlike the case of general relativity, different choices of F lead to inequivalent theories. Notice that in the case of point particles (taking $F = y$), a geodesic equation (and, therefore, the equivalence principle) is satisfied in terms of the metric $g^{\rm eff}_{\alpha\beta} \equiv \chi g_{\alpha\beta}$ even if χ is not constant. It is interesting also that in the four-dimensional case $g^{\rm eff}_{\alpha\beta}$ is invariant under the Einstein symmetry described by Eqs. (22) and (23).

Notice that the theory as formulated in this section makes sense even without condition (21) being satisfied. In contrast, we will see in the next section that when allowing torsion, the consistency of the equations of motion *dictates* a condition which generalizes the condition (21).

IV. THE NGVE THEORY—VIELBEIN-SPIN CONNECTION APPROACH

A. General consideration

We are now going to study the theory defined by the action (3) in the case that the scalar curvature is defined by Eqs. (4) and (5), which means that R may not coincide with the Riemannian scalar curvature and as a consequence we do not expect the NGVE-vielbein-spin connection (VSC) approach to coincide with the Riemannian approach of Sec. III.

As in Sec. II, variation with respect to the scalar fields φ_a leads to the equation

$$A^\mu_a \partial_\mu \left(-\frac{1}{\kappa} R(e,\omega) + L_m(e,\omega,\text{matter fields}) \right) = 0, \tag{25}$$

which implies, if $\Phi \neq 0$, that

$$-\frac{1}{\kappa} R(e,\omega) + L_m(e,\omega,\text{matter fields}) = M. \tag{26}$$

On the other hand, considering the equations obtained from the variation of the vielbeins, we get, if $\Phi \neq 0$,

$$-\frac{2}{\kappa} R_{\mu a}(e,\omega) + \frac{\partial L_m}{\partial e^{a\mu}} = 0, \tag{27}$$

where

$$R_{\mu a}(e,\omega) \equiv e^{b\nu} R_{\mu\nu ab}(\omega). \tag{28}$$

Notice that Eq. (27) is indeed invariant under the shift $L_m \to L_m + \text{const}$.

Since $R(e,\omega) \equiv e^{a\mu} R_{\mu a}(e,\omega)$, we can eliminate $R(\omega)$ from the Eqs. (26) and (27) after contracting the last one with $e_{a\mu}$. As a result we obtain *the nontrivial constraint*

$$e^{a\mu} \frac{\partial (L_m - M)}{\partial e^{a\mu}} - 2(L_m - M) = 0. \tag{29}$$

In the case $L_m = L_m(g_{\mu\nu}, \text{matter fields})$, we see that the form of the constraint (29) coincides with the condition (21) which provides the existence of the Einstein sector of solutions in the Riemannian approach. In contrast, *here it is not a choice but a consequence of the variational principle*.

The constraint (29) has to be satisfied for all components (in the functional space) of the function L_m. In particular, for the constant part denoted $\langle L_m \rangle$, we obtain:

$$\langle L_m \rangle - M = 0. \tag{30}$$

Therefore, one of the consequences of the constraint (29) is that it *dictates that the constant part of the matter Lagrangian* $\langle L_m \rangle$ *is compensated by the integration constant* M in the case of a maximally symmetric vacuum state.

We will see that constraint (29) can be satisfied in three possible ways: (1) Automatically, that is from the definition of L_m, without any dynamical consideration; (2) automatically after matter field equations *only* are used; (3) after all equations are used. All three matter model examples of the Sec. III belong to case (1).

As we saw in Sec. III, in the context of the Riemannian approach, the condition (21) is related to the Einstein symmetry, Eqs. (22) and (23). It is very interesting to see what kind of symmetry of the action (3) is associated with the constraint (29) in the context of the VSC approach. It turns out that when the constraint (29) is satisfied automatically (without using the equations of motion of matter), we obtain that a local version of Einstein symmetry holds. Furthermore, this *local Einstein symmetry* is nothing but diffeomorphism invariance in the space of the scalar fields φ_a, which has to be accompanied with a conformal transformation of the vielbeins:

$$\varphi_a \to \varphi_a' = \varphi_a'(\varphi_b), \tag{31}$$

$$e_{a\mu} \to e_{a\mu}' = J^{1/2} e_{a\mu}, \tag{32}$$

$$J \equiv \mathrm{Det}\left(\frac{\partial \varphi_a'}{\partial \varphi_b}\right). \tag{33}$$

In terms of $g^{\mu\nu}$ and Φ (and $\chi \equiv \Phi/\sqrt{-g}$), this symmetry has the form

$$g^{\mu\nu} \to g'^{\mu\nu} = J^{-1} g^{\mu\nu}, \tag{34}$$

$$\Phi \to \Phi' = J\Phi, \tag{35}$$

$$\chi \to \chi' = J^{1-D/2}\chi. \tag{36}$$

Since when $\Phi \neq 0$, we have that the transformation $\varphi_a = \varphi_a(x^\mu)$ is one to one, we obtain that by means of Eq. (35), Φ can be transformed to whatever we want, in particular $\Phi = \sqrt{-g}$ or, what is the same, $\chi = 1$ is a possible ''gauge'' if $\Phi \neq 0$.

B. Torsion in the absence of fermions

Let us now analyze what is the dependence of ω_μ^{ab} on $e_{a\mu}$ and χ. As a first step, let us consider the case when $L_m = L_m(g_{\mu\nu}$, matter fields) and the dimensionality of the space-time $D=4$. This of course excludes the possibility of fermions, but those can be incorporated without qualitative changes in the discussion.

Then, the variation of the action (3) with respect to ω_μ^{ab} gives

$$\varepsilon^{\mu\nu\lambda\rho}\varepsilon_{abcd}\left[\chi e_\lambda^c D_\nu e_\rho^d + \tfrac{1}{2} e_\lambda^c e_\rho^d \chi_{,\nu}\right] = 0, \tag{37}$$

where $D_\nu e_{a\rho} \equiv \partial_\nu e_{a\rho} + \omega_{\nu a}^d e_{d\rho}$.

The solution of Eq. (37) is

$$\omega_\mu^{ab} = \omega_\mu^{ab}(e) + K_\mu^{ab}, \tag{38}$$

where $\omega_\mu^{ab}(e)$ is the Riemannian spin connection [5,6] and K_μ^{ab} is the contorsion tensor [5,6] which in our case is given by

$$K_\mu^{ab} = \frac{1}{2}\sigma_{,\alpha}(e_\mu^a e^{b\alpha} - e_\mu^b e^{a\alpha}), \tag{39}$$

where $\sigma \equiv \ln\chi$. Notice that deviation of the new measure Φ from the GR measure $\sqrt{-g}$ (that is $\chi \neq \mathrm{const}$) is the origin of torsion.

If we insert this into the expression of $\Phi R(\omega,e)$, we obtain

$$\Phi R(\omega,e) \equiv \sqrt{-g}\chi R(\omega,e) = \sqrt{-g}[\chi R(g_{\mu\nu}) - 6\chi^{1/2}\Box\chi^{1/2}], \tag{40}$$

where $R(g_{\mu\nu})$ is the Riemannian scalar curvature. The conformal coupling form of the scalar field $\chi^{1/2}$ is apparent. This is not a surprise since the left-hand side is invariant under the local conformal rescalings (31)–(33) and the conformal coupling form in the right-hand side is the unique conformally invariant coupling between a scalar field and the Riemannian scalar curvature. The right-hand side represents the resulting second order formalism, that is, what is obtained after solving the spin connection in terms of the other fields and then replacing the result into the action. The appearance of the additional $-6\chi^{1/2}\Box\chi^{1/2}$ term in Eq. (40), which is absent in the approach developed in Sec. III, clearly shows the inequivalence of the two approaches in all cases, even when no assumptions are made concerning the validity of the local Einstein symmetry, Eqs. (31)–(33), or, what is the same, Eqs. (34)–(36).

This can be seen also by examining the shape of the equations of motion, even when the symmetry, Eqs. (34)–(36), is not assumed to hold. From Eqs. (27), (28), (4), (5), (38), and (39), we get

$$G_{\mu\nu}(g) + H_{\mu\nu} = \frac{\kappa}{2}(T_{\mu\nu} + Mg_{\mu\nu}), \tag{41}$$

where

$$H_{\mu\nu} \equiv 2\chi^{-1/2}[g_{\mu\nu}\Box\chi^{1/2} - (\chi^{1/2})_{,\mu;\nu}] + \chi^{-1}[4(\chi^{1/2})_{,\mu}(\chi^{1/2})_{,\nu} - g_{\mu\nu}(\chi^{1/2})_{,\alpha}(\chi^{1/2})^{,\alpha}], \tag{42}$$

and $G_{\mu\nu}(g) \equiv R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g)$ with $R_{\mu\nu}(g)$ and $R(g)$ being the Riemannian Ricci tensor and scalar. Taking the trace of Eq. (41), we get

$$\Box\chi^{1/2} - \frac{1}{6}\left[R(g) + \frac{\kappa}{2}(T_\alpha^\alpha + 4M)\right]\chi^{1/2} = 0. \tag{43}$$

Using that

$$T_\alpha^\alpha = e^{a\mu}\frac{\partial L_m}{\partial e^{a\mu}} - 4L_m, \tag{44}$$

and constraint (29), we get

$$\Box\chi^{1/2} - \frac{1}{6}[R(g) - \kappa(L_m - M)]\chi^{1/2} = 0. \tag{45}$$

As we expected, $\chi^{1/2}$ has an equation which is of the conformally coupled type.

In the vacuum (that is taking into account only constant part of L_m), because of the constraint (30), Eq. (45) takes the form

$$\Box\chi^{1/2} - \frac{1}{6}R(g)\chi^{1/2} = 0. \tag{46}$$

We can see then that Eqs. (46) and (41) are invariant under the transformations (34)–(36) (which in such a case play the role of conformal transformations). Therefore, χ field can be transformed into a constant and the resulting equation (41) becomes just vacuum Einstein's equations with zero cosmological constant. As an example how this is realized in a concrete model, see Sec. IV D below.

From Eqs. (41), (42), and (27), we get the equation of matter nonconservation

$$T_{\mu\nu}{}^{;\mu}=-\frac{\partial L_m}{\partial e_{a\mu}}g_{\mu\nu}e_a^{\alpha}\partial_\alpha ln\chi, \qquad (47)$$

where semicolon means covariant derivative in the Riemannian space-time with a metric $g_{\mu\nu}$. This equation coincides with Eq. (20) in the case where L_m depends on vielbeins only through $g_{\mu\nu}$. In cases where the local Einstein symmetry, Eqs. (31)–(33) [or, what is the same, Eqs. (34)–(36)] holds, the χ field can be transformed into a constant and then Eq. (47) becomes equation of covariant conservation of the energy-momentum tensor.

C. Study of constraint in fermionic models

As it is well known [5,6], one of the most attractive features of the vielbein formalism is its ability to incorporate fermions in the context of generally coordinate-invariant theories.

The simplest example of a fermion is that of spin-$\frac{1}{2}$ particles. In this case we regard the spinor field Ψ as a general coordinate scalar and transforming nontrivially with respect to local Lorentz transformation according to the spin-$\frac{1}{2}$ representation of the Lorentz group.

Considering the Hermitian action (which allows for the possibility of fermion self-interactions) of the form

$$S_f=\int L_f\Phi d^4x, \qquad (48)$$

where

$$L_f=\frac{i}{2}\overline{\Psi}[\gamma^a e_a^\mu(\vec{\partial}_\mu+\tfrac{1}{2}\omega_\mu^{cd}\sigma_{cd}) - (\overleftarrow{\partial}_\mu+\tfrac{1}{2}\omega_\mu^{cd}\sigma_{cd})\gamma^a e_a^\mu]\Psi+U(\overline{\Psi}\Psi). \qquad (49)$$

Here $\sigma_{cd}\equiv\frac{1}{4}[\gamma_c,\gamma_d]$.

Again, ω_μ^{cd} should be determined by the equation obtained from the variation of the full action with respect to ω_μ^{cd}. This in general will give rise to additional contribution to the torsion, as it is well known [5,6].

Here, in the context of the matter models (48) and (49), we focus on the conditions where the constraint (29) is satisfied, while χ remains unspecified (i.e., remains unphysical).

From Eq. (49) and using the equations of motion derived from the actions (48) and (49), we get

$$e_a^\mu\frac{\partial L_f}{\partial e_a^\mu}-2L_f=\overline{\Psi}\Psi U'-2U, \qquad (50)$$

where U' is the derivative of U with respect to its argument. We see that the constraint (29) is satisfied on the mass shell (since the fermion equations of motion are used) with $M=0$ for L_f defined by Eq. (49) if, for example, $U=c(\overline{\Psi}\Psi)^2$. Any other quartic interaction, such as $\overline{\Psi}\gamma_a\Psi\overline{\Psi}\gamma^a\Psi$, $\overline{\Psi}\sigma_{ab}\Psi\overline{\Psi}\sigma^{ab}\Psi$, $(\overline{\Psi}\gamma_5\Psi)^2$, etc., would also satisfy the constraint (29) on the mass shell with $M=0$. In particular, the Nambu–Jona-Lasinio model [7] would also satisfy the constraint (29) on the mass shell with $M=0$.

It is interesting to compare this kind of fermionic models where the constraint (29) is satisfied with $M=0$, with the models discussed already at the end of the Sec. III. Here the constraint is satisfied only on the mass shell while in those previous examples the constraint (29) is satisfied automatically, using only the definition of the Lagrangian.

D. Example with scalar field

Now let us consider cases when the constraint (29) is not satisfied without restrictions on the dynamics of the matter fields. Nevertheless, the constraint (29) holds as a consequence of the variational principle in any situation.

A simple case where the constraint (29) is not automatic is the case of a scalar field with a nontrivial potential $V(\phi)$. In this case the constraint (29) implies

$$V(\phi)+M=0. \qquad (51)$$

Therefore, we conclude that, provided $\Phi\neq 0$, there is no dynamics for the theory of a single scalar field, since constraint (51) forces this scalar field to be a constant. This means that the effective cosmological constant $V(\phi)+M$ in the Eq. (41) vanishes identically provided $\Phi\neq 0$.

The constraint (51) has to be solved, together with the equation of motion,

$$\Box\phi+\sigma,_\mu\phi^{,\mu}+\frac{\partial V}{\partial\phi}=0, \qquad (52)$$

where $\sigma=\ln\chi$. From Eqs. (51) and (52) we conclude that the ϕ field has to be located at an extremum of the potential $V(\phi)$. Since the constraint (51) eliminates the dynamics of the scalar field ϕ, we cannot really say that we have a situation where the symmetry, Eqs. (31)–(33) [or, what is the same, in the forms (34)–(36)] is actually broken, since after solving the constraint, together with the equation of motion (i.e., on the mass shell), the symmetry remains true.

Then using the constraint (51) in the equation of motion for $\chi^{1/2}$ [Eq. (45)], we get

$$\Box\chi^{1/2}-\frac{1}{6}R(g)\chi^{1/2}=0. \qquad (53)$$

By using the obvious conformal invariance of Eq. (53) and of all other equations, the χ field can be transformed into a constant, for example one [the correspondent conformal transformation is in fact the particular case of the local Einstein symmetry, Eqs. (34)–(36) with $J(\varphi_a(x))=\chi(x)$]. Notice that in this simple matter model, Eq. (47) takes the trivial form 0=0.

V. THE INCORPORATION OF VECTOR BOSONS INTO THE NGVE THEORY IN THE VSC APPROACH

A. General notions

As it is well known, interactions between elementary particles appear to be well described by the exchange of vector bosons. The incorporation of vector bosons is, therefore, an important subject which has to be dealt with in the context of the new gravitational theory developed in this paper.

As we have seen in the case of the point particle models, different formulations of a matter model, which in the case of GR are physically equivalent, can in fact be the origin of inequivalent theories when formulated in the framework of the NGVE theory. As we will see in this chapter, a similar situation arises in the case of vector bosons. We will discuss here (and in the next section) several options, some consistent with local Einstein symmetry and others which are not. As it is well known, the vielbein formalism allows us to regard a vector in different ways: (i) GVLS: a vector under general coordinate transformations, while being a scalar under local Lorentz transformations. (ii) GSLV: a scalar under general coordinate transformations, while being a vector under local Lorentz transformations. Let A_μ be a GVLS. We can then always define a GSLV as $A_a = e_a^\mu A_\mu$.

B. Model of vector boson with the local Einstein symmetry

Here we will choose the GSLV variables as the fundamental Lagrangian variables. Defining the Lorentz tensor and generally coordinate-invariant scalar field strength

$$F_{ab} = e_a^\mu D_\mu A_b - e_b^\mu D_\mu A_a, \tag{54}$$

where $D_\mu A_a = \partial_\mu A_a + \omega_{\mu ab} A^b$, we choose the following matter Lagrangian for massless vector bosons:

$$L_{v.b.} = -\tfrac{1}{4}\,\eta^{ac}\eta^{bd} F_{ab} F_{cd}. \tag{55}$$

In the first order formalism it is understood that $\omega_{\mu bc}$ is regarded as an independent variable, to be determined from the equations of motion obtained by variating $\omega_{\mu bc}$. Notice that the matter Lagrangian (55) is in fact homogeneous of degree two in the vielbeins. Therefore, a theory incorporating only $L_{v.b.}$ as a matter model, is consistent with the local Einstein symmetry, Eqs. (31)–(33), and satisfies in an automatic form the constraint (29) with $M=0$. As a consequence of the symmetry, Eqs. (31)–(33), we obtain of course that in this model χ can be taken to be one if $\Phi \neq 0$ everywhere. If we do this, then we can see immediately that in the approximation where $\omega_{\mu ab} = \omega_{\mu ab}(e)$ $\{\omega_{\mu ab}(e)$ is the Riemannian-spin connection [see also Eq. (38)]$\}$, the Lagrangian density (55) with F_{ab} defined by Eq. (54) is invariant under the gauge transformations

$$A_a \rightarrow A_a + e_a^\mu \frac{\partial \Lambda}{\partial x^\mu}. \tag{56}$$

If we do not fix χ field, then the form of the gauge transformations is modified, but the model is still gauge invariant in the same approximation.

We should point out that together with the obvious advantages which this formulation of the theory of massless vector bosons has, this approach leads to weak violations (of gravitational strength) of the gauge invariance principle. This is a consequence of the first order formalism, where the spin connection is determined from its equation of motion and we obtain in fact that there will be a contribution to the torsion from the vector boson itself. For example, it turns out that this gravitational back reaction of the vector bosons on gravity (i.e., on $\omega_{\mu ab}$) is proportional to the gravitational constant κ and violates the gauge invariance. Contribution to $\omega_{\mu ab}$ from fermions would also produce violations of gauge invariance in Eq. (55).

C. Gauge fields from extra dimensions in the VSC approach

Here we will see that in the framework of higher-dimensional unification, the VSC approach can incorporate gauge fields. It is important to notice that in the context of the NGVE theories only the VSC alternative can successfully implement the idea of higher-dimensional unification. The Riemannian approach, developed in [4] and reviewed in Sec. III, is not suitable for this task.

Let us see first of all that the purely Riemannian approach to the NGVE theories does not provide a successful formulation of the higher-dimensional unification. To see this, we start from the five-dimensional NGVE-Riemannian action

$$S_5 = -\frac{1}{\kappa_5}\int \Phi R_5(\gamma_{ab}) d^5x, \tag{57}$$

where

$$\Phi \equiv \varepsilon_{abcde}\varepsilon^{ABCDE}(\partial_A \varphi_a)(\partial_B \varphi_b)\cdots(\partial_E \varphi_e), \tag{58}$$

γ_{ab} is the five-dimensional metric, and R_5 is the Riemannian scalar curvature in the five-dimensional space-time.

Our choice of parametrizing the five-dimensional metric γ_{AB} is [8]

$$\gamma_{AB} = \begin{pmatrix} g_{\mu\nu} + e^2\kappa^2 v A_\mu A_\nu & e\kappa v A_\mu \\ e\kappa v A_\nu & v \end{pmatrix},$$

where $g_{\mu\nu}$, v, and A_μ do not depend on the fifth dimension x^5, which is taken to be compactified. Doing the integration over x^5, we get

$$S_5 = \frac{1}{\kappa}\int \Phi[-R_4 + e^2 v g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}] d^4x, \tag{59}$$

where R_4 is the scalar curvature of a four-dimensional space-time with the metric $g_{\mu\nu}$, $\kappa = \kappa_5/2\pi\rho$, ρ being the size of the extra dimension, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

We see now that in contrast with the usual Kaluza-Klein theories, variation with respect to v leads to the nontrivial constraint for the gauge field $g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} = 0$. Such a constraint is of course inconsistent with a phenomenologically successful theory of gauge fields showing, therefore, the failure of the Riemannian approach to the Kaluza-Klein unification in the context of the nongravitating vacuum energy theories. We now turn our attention to higher-dimensional unification in the context of the VSC approach.

Let us consider then the action (3) in the five-dimensional case ($D=5$), but where the scalar curvature is defined by Eq. (4), which means that R may not coincide with the Riemannian definition, as we have verified it is the case in the four-dimensional theory.

Let us now consider the dependence of the spin connection ω_{Aab} on the vielbein e_a^A and on $\chi \equiv \Phi/\sqrt{\gamma}$ (we follow the same steps we went through in the four-dimensional case). Variation of the action (3) with R defined as in Eq. (4) in five dimensions gives

$$\varepsilon_{abcdf}\varepsilon^{ABCDF}(\chi e^c_C e^d_D D_B e^f_F + \tfrac{1}{3}\, e^c_C e^d_D e^f_F \chi,_B)=0. \quad (60)$$

The solution of Eq. (60) is now

$$\omega^{ab}_A = \omega^{ab}_A(e) + K^{ab}_A, \quad (61)$$

where $\omega^{ab}_A(e)$ is the Riemannian-spin connection of the five-dimensional space-time and K^{ab}_A is the contorsion tensor which in our case is given by

$$K^{ab}_A = \frac{1}{3}\sigma,_B(e^a_A e^{bB} + e^b_A e^{aB}). \quad (62)$$

If we insert this into the expression for $\Phi R_5(e,\omega)$, we obtain

$$\Phi R_5(e,\omega) \equiv \sqrt{\gamma}\chi R_5(e,\omega) = \sqrt{\gamma}[\chi R_5(\gamma_{AB}) - \tfrac{16}{3}\chi^{1/2}\Box\chi^{1/2}] = 0. \quad (63)$$

Here $R_5(\gamma_{AB})$ is the ordinary scalar curvature in the five-dimensional Riemannian space-time with the metric γ_{AB}. Again, we find a conformal coupling appropriate to $D=5$, for the field $\chi^{1/2}$.

The other equations of motion, obtained from the variation with respect to e^A_a, after some algebraic manipulations, are

$$G_{(5)AB}(e,\omega) = G_{(5)AB}(\gamma_{CD}) + H_{(5)AB}(\chi^{1/2}) = 0, \quad (64)$$

where $G_{(5)AB} \equiv R_{(5)AB} - \frac{1}{2}\gamma_{AB}R_5$ and

$$H_{(5)AB}(\chi^{1/2}) = \frac{2}{\chi^{1/2}}\left[\gamma_{AB}\Box\chi^{1/2} - \tfrac{2}{3}(\chi^{1/2}),_{A;B}\right] - \frac{2}{3\chi}[\gamma_{AB}(\chi^{1/2}),_C(\chi^{1/2})^{,C} - 2(\chi^{1/2}),_A(\chi^{1/2}),_B]. \quad (65)$$

As in the four-dimensional case, if $\Phi \neq 0$, using the symmetry Eqs. (31)–(33) [which in terms of χ and γ_{AB} appears as a conformal transformation, see Eqs. (34)–(36), where $g_{\mu\nu}$ should be replaced by γ_{AB}], we can set the gauge $\chi=1$, obtaining then equations identical to those of ordinary general relativity, in this case for $D=5$, however. The Kaluza-Klein mechanism for gauge field generation works then as usual.

When considering non-Abelian compactifications, things work most straightforward when the matter Lagrangian that produces the compactification satisfies the constraint (29). This is the case if the compactification is achieved, for example, through some hedgehog configuration [9] which corresponds to the identity mapping from the extra-dimensional sphere into a space of scalar fields satisfying nonlinear σ model-type equations. In addition, instead of sphere some other finite area noncompact manifolds can be considered [10]. The problem in this case is associated with the large mass generation which the Kaluza-Klein gauge fields get in this kind of approaches.

VI. BREAKING LOCAL EINSTEIN SYMMETRY

A. A gauge+matter fields system

In some cases, the constraint (29) is not automatically satisfied, in fact. In those cases, in order for the constraint to be satisfied, the χ field becomes determined, therefore, breaking the symmetry, Eqs. (31)–(33).

To see how this works, we study a model of a gauge field (now formulated in the GVLS way, in contrast with what we did in Sec. V B) and a neutral scalar field. Now gauge invariance is evident, however, local Einstein symmetry is broken. Although the model is not very realistic we study it only to get an insight how the theory works rather than to get a correct description of nature.

Therefore, we study the model (3) with the particular choice of the matter Lagrangian density L_m given by

$$L_m = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - \frac{1}{4}F_{\mu\nu}F_{\alpha\beta}g^{\mu\alpha}g^{\nu\beta}, \quad (66)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$.

Notice that the action (3) with the matter Lagrangian (66) is *not* invariant under the local Einstein symmetry Eqs. (31)–(33). However, the nontrivial constraint

$$-\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + 2[U(\phi) + M] = 0 \quad (67)$$

is still satisfied as a result of the equations of motion.

We can study now how the theory works in several types of solutions. First of all, if we are interested in radiation-type solutions, where $F_{\mu\nu}F^{\mu\nu}=0$, the situation becomes identical (from the point of view of symmetries) to that when no gauge field was considered (see Sec. IV D).

If we look, for example, for static purely electric spherically symmetric solutions of the equations of motion, Eq. (67) tells us that ϕ is a function of the electric field, not just a constant as in Sec. IV D. After this, the equation of motion for the ϕ field (52) allows us to solve $d\sigma/dr$ as a function of $\phi=\phi(F_{0r})$ and its first and second derivatives (as well as function of the metric). Finally, this solution for $d\sigma/dr$ has to be inserted in the equation for $E \equiv F_{0r}$, which involves $d\sigma/dr$. The resulting problem is a highly nonlinear one but a well-defined one which shows the role of the $\sigma \equiv \ln\chi$ field in the enforcement of the constraint. Notice that the χ field is now not arbitrary. However, away from the sources, that is in a vacuum state that satisfies constraint (51), Eq. (53) holds, which is an equation with conformal invariance. Therefore, χ is there totally arbitrary and, therefore, unphysical.

B. Breaking the local Einstein symmetry by the gravitational sector of the theory

Breaking of the local Einstein symmetry is possible also in the gravitational sector of the theory in a case of an appearance of higher order terms in the curvature (for example, as could be the case for quantum corrections). These terms usually give rise to noncausal propagation and ghosts. In our case, however, the fact that the measure of integration is Φ instead of $\sqrt{-g}$ allows us to consider contributions which are meaningless in the usual theory. This is the case when in the Lagrangian density we consider possible Euler (ρ) and Hirzebruch-Pontryagin (ξ) contributions

$$\rho \equiv \frac{1}{\sqrt{-g}} \varepsilon^{\alpha\beta\mu\nu} \varepsilon^{abcd} R_{\alpha\beta ab} R_{\mu\nu cd}, \tag{68}$$

$$\xi \equiv \frac{1}{\sqrt{-g}} \varepsilon^{\alpha\beta\mu\nu} R_{\alpha\beta ab} R_{\mu\nu cd} \eta^{ac} \eta^{bd}. \tag{69}$$

$\rho\sqrt{-g}$ and $\xi\sqrt{-g}$ are total divergences even in the presence of torsion [11] (our conventions for $\varepsilon^{\alpha\beta\mu\nu}$ are different from those of Ref. [11]) and, therefore, irrelevant in the standard approaches. However, since the measure of integration is Φ, $\rho\Phi$ and $\xi\Phi$ are contributions to the action that can be considered in our case. These give nontrivial contributions to the equations of motion. Since the contribution of ξ into the total action violates the parity symmetry, we do not consider it in this first analysis. Furthermore, we present here only a sketch about the main features of the theory in the presence of the Euler contribution into the Lagrangian density.

It is known [12] that when studying space-time of dimensionalities bigger than four, the corresponding generalization of the four-dimensional Euler density gives a nontrivial contribution to the equations of motion; however, it does not give rise to ghosts. In our case, this is still true and the proof follows the same lines of what is done in the higher-dimensional case [12].

Considering small perturbations of e_a^μ and χ around $e_a^\mu = \delta_a^\mu$ and of $\chi = 1$, we obtain from the Euler contribution into the Lagrangian density:

$$S_E = \int \Phi d^4 x \partial_\alpha j^\alpha, \tag{70}$$

where

$$j^\alpha = 4[2\sigma_{;\beta}^{,\alpha} \sigma^{,\beta} - 2\sigma^{,\alpha} \Box \sigma - \sigma^{,\beta} \sigma_{,\beta} \sigma^{,\alpha}] \tag{71}$$

and $\sigma = \ln\chi$. As in Ref. [12], the purely gravitational effects vanish in the quadratic approximation. We see, for example, that $(\partial^2\sigma/\partial t^2)^2$ is absent in the integrand of Eq. (70). Second derivatives with respect to time appear in the integrand of Eq. (70) only linearly.

Notice that when the Euler density is present, the constraint (29) becomes now a dynamical equation for σ:

$$2\rho + e^{a\mu} \frac{\partial L_m}{\partial e^{a\mu}} - 2L_m + 2M = 0, \tag{72}$$

where ρ is given by Eq. (68). Equation (72) is a dynamical equation for σ rather than a constraint because second order time derivatives of σ appear in it.

Finally, notice that in the context of the modifications in the gravitational sector, described in this subsection, flat space-time is still always a solution.

C. General relativity limit as freezing the χ degree of freedom

As we have seen, the vanishing of the cosmological constant relies on the existence of the χ field and the nontrivial constraint (29) associated with the new measure of integration and with the scalar fields φ_a from which this measure (and the χ field) is built.

Now, we want to see in what limit the model studied here becomes undistinguishable from GR. This is important from the point of view of the correspondence principle. This question has obviously to do with what is assumed for the dynamics of the χ field.

When the local symmetry, Eqs. (31)–(33), exists, the χ field does not represent a physical degree of freedom and it is in fact arbitrary and not determined by the equations of motion. When the constraint (29) is nontrivially satisfied, the χ field has to be determined so as to make things work. Finally, as we explained in Sec. VI B, we found a way to turn the χ field into a dynamical field by introducing the Euler term.

Obviously, we should be able to obtain GR if the dynamics of the χ field is turned into a trivial one, that is, if only $\chi = \text{const}$ is allowed. This is of course the exact opposite to the case of unbroken symmetry. This is possible within the general form (3) of the theory, provided we add to L_m a Lagrange multiplier term that enforces $\chi = \text{const}$. The form of this contribution to the Lagrangian density is

$$L_{\text{freezing}} = \frac{1}{\sqrt{-g}} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu E_{\nu\alpha\beta}, \tag{73}$$

where we assume that all components of $E_{\nu\alpha\beta}$ are to be varied without restriction, i.e., $E_{\nu\alpha\beta}$ is a new fundamental field. The variation of the action with respect to $E_{\nu\alpha\beta}$ gives in fact

$$\partial_\mu \chi = \text{const}, \tag{74}$$

that is the only possible configuration for the χ field is $\chi = \text{const}$.

The variation of the action with respect to $e^{a\mu}$ gives

$$-\frac{2}{\kappa} R_{a\mu}(\omega) + \frac{\partial L_m}{\partial e^{a\mu}} + e_{a\mu} L_{\text{freezing}} = 0. \tag{75}$$

Contracting this with $e^{a\mu}$, we obtain

$$-\frac{2}{\kappa} R(\omega) + e^{a\mu} \frac{\partial L_m}{\partial e^{a\mu}} + 4L_{\text{freezing}} = 0. \tag{76}$$

Also, the variation with respect to the fields φ_a leads to (if $\Phi \neq 0$)

$$-\frac{1}{\kappa} R(e,\omega) + L_m(e,\omega,\text{matter fields}) + L_{\text{freezing}} = M. \tag{77}$$

From Eqs. (76) and (77), we get

$$e^{a\mu} \frac{\partial L_m}{\partial e^{a\mu}} - 2(L_m - L_{\text{freezing}} - M) = 0, \tag{78}$$

which now is not a constraint but rather determines the Lagrange multiplier term L_{freezing}.

Contracting Eq. (75) with e^a_ν, we obtain

$$-\frac{2}{\kappa}R_{\mu\nu}(\omega)+e^a_\nu\frac{\partial L_m}{\partial e^{a\mu}}+g_{\mu\nu}L_{\text{freezing}}=0. \tag{79}$$

From Eqs. (79) and (76), we get

$$-\frac{2}{\kappa}\left[R_{\mu\nu}(\omega)-\frac{1}{2}g_{\mu\nu}R(\omega)\right]+e^a_\nu\frac{\partial L_m}{\partial e^{a\mu}}-g_{\mu\nu}L_m+g_{\mu\nu}M=0, \tag{80}$$

or, what is the same,

$$G_{\mu\nu}=\frac{\kappa}{2}\left(e^a_\nu\frac{\partial L_m}{\partial e^{a\mu}}-g_{\mu\nu}L_m+g_{\mu\nu}M\right)=\frac{\kappa}{2}(T_{\mu\nu}+Mg_{\mu\nu}), \tag{81}$$

where $G_{\mu\nu}\equiv R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R$.

Taking into account that the χ degree of freedom is frozen now [see Eq. (74)], we conclude that Eqs. (81) are the Einstein equations of GR (or Einstein-Cartan equations if fermions are included in the model), where an arbitrary integration constant M playing the role of a cosmological constant appears.

No information concerning the vanishing of the cosmological constant is obtained now, because the constraint that used to do this job contains now the Lagrange multiplier field $E_{\nu\alpha\beta}$ which is not determined by any other equation, therefore, Eq. (78) is now not a constraint at all.

VII. DISCUSSION AND CONCLUSIONS

Here we have developed the consequences of changing the measure of integration from $\sqrt{-g}d^Dx$ to $d\varphi_1\wedge d\varphi_2\wedge\cdots\wedge d\varphi_D$, when the mapping from the scalars φ_a ($a=1,2,\ldots,D$) to the coordinates is not known *a priori*. This means that the measure of integration is determined dynamically and not assumed to have a particular form as it is done in GR. Such model [4] has been called ‘‘nongravitating vacuum energy (NGVE) theory’’ because if we change the integrand (i.e., the Lagrange density) by a constant, which in GR is associated with a vacuum energy, no change in the equations of motion is obtained (ignoring possible boundary effects).

Moreover, we have discovered in this paper, that when using the vielbein-spin connection first order formalism in the context of the models based on the NGVE principle, a nontrivial constraint (29) appears as a result of equations of motion. The first order formalism based on $g_{\mu\nu}$ (or e^a_μ) and the connection $\Gamma^\lambda_{\mu\nu}$ as independent variables is physically equivalent and will be studied in a subsequent publication.

In our previous paper on NGVE theory [4] we have seen that the constant part of the vacuum energy does not affect gravitational properties, but an integration constant appears and it plays a role similar to that of an effective cosmological term (although a maximally symmetric de Sitter space did not exist there). Now, allowing for the possibility of torsion, such integration constant appears too, but it is determined *dynamically* so as to cancel any possible constant part of the vacuum energy, which is present in the starting formulation.

The existence of a nontrivial constraint does, however, modify the dynamics of the matter fields in a nontrivial way. This can be avoided, as we have seen in Sec. VI D, by making the dynamics of the χ field trivial if we introduce a Lagrange multiplier. In this case the constraint becomes only a definition of the Lagrange multiplier and, therefore, fails to give additional information on the effective cosmological constant. This version of the theory coincides physically with the models discussed by the authors of Refs. [13] where the cosmological constant is an integration constant. It is here obtained from the general formalism by enforcing from outside the triviality of the χ field, which is of course not a natural thing to do. This version of the theory does not answer the question of the vanishing of the cosmological constant, which is again an arbitrary integration constant, with no particular reason to pick the vanishing value as has been pointed out by Weinberg [3].

We have seen in Sec. VI A that terms that violate the local Einstein symmetry can be incorporated and they do not alter the basic conclusions of the model, that is, the vanishing of the cosmological constant term. They do give rise, however, to a nontrivial dynamics for the field χ which acquires a physical meaning because of these breaking terms. Furthermore, the constraint (29) is satisfied anyway by dynamically adjusting the field χ, as we saw in the particular example of Sec. VI A.

Incorporating masses, for example, will modify this constraint so that the masses will enter in the constraint. In the case of fermions, we have seen in Sec. IV C that if we start from Nambu–Jona-Lasinio– (NJL-) type models [7], the constraint (29) is satisfied on the mass shell without restriction on the matter field dynamics. However, a spontaneous symmetry-breaking mechanism originates from quantum corrections and as a result masses of fermions appear. So if our classical arguments concerning the satisfaction of the constraint (29) in the NJL model survive the quantum corrections, we would then expect that the fermion masses may not enter in the constraint at all. If they do, they contribute to a nontrivial χ dynamics. Which alternative is the right one requires a nontrivial analysis.

In addition, in the context of some model resembling the standard model, the constraint (29) seems to give a basic condition which tells us that the Higgs boson field is a composite of the other fields appearing in the theory in a way that resembles what we have studied in Sec. VI A.

A way to avoid the constraint from having a big effect on the dynamics of any single matter field is to introduce a large number of fields, most of them interacting with one another only gravitationally and of course through the constraint. Since the whole L_m enters in the constraint, enlarging the number of fields diminishes the ‘‘job’’ each individual field has to do. In such a way we expect to recover the local symmetry, Eqs. (31)–(33), at long distances, i.e., the triviality of the constraint at long distances. This would be a way to realize the infrared dynamical symmetry restoration of gauge symmetries as it has been discussed in the literature [14].

Keeping a nontrivial constraint (that is, avoiding the introduction of the Lagrange multiplier that trivializes the χ dynamics), in Secs. III–V, we have formulated several models (including fermions, scalar field, and vector bosons) where the constraint (29) is satisfied at least on the mass shell. However, it follows from the equations of motion that the only possible configuration of a single scalar field with a potential is a constant scalar field located in the extremum of the potential. In this case *the constraint dictates that this extremal value of the potential is compensated by the integration constant, thus providing the mechanism for the nonexistence of the cosmological constant on the mass shell*. Those models respect the local Einstein symmetry. Therefore, we can set the gauge $\chi=1$ and in this case Eq. (47) becomes the equation of the covariant conservation of the energy-momentum tensor.

The infinite-dimensional symmetries (7) and (12) impose strict restrictions on the possible induced terms in the quantum effective action, if no anomalies appear in this effective action. In particular, symmetry under the transformation (12) seems to prevent the appearance of terms of the form $f(\chi)\Phi$ [except for $f(\chi)\propto 1/\chi$] in the effective action which although is invariant under volume-preserving transformations (7), breaks symmetry (12). The case $f(\chi)\propto 1/\chi$ is not forbidden by symmetry (12) and appearance of such a term would mean inducing a ''real'' cosmological term, i.e., a term of the form $\sqrt{-g}\Lambda$ in the effective action. However, appearance of such a term seems to be ruled out because of having opposite parity properties to those of the action given in Eq. (3). Furthermore, in the absence of Euler-like terms (of Sec. VI B), the variational principle gives now $\Lambda=0$ in the vacuum if such term is ''forced'' into the theory. Of course, in the absence of a consistently quantized theory, such arguments are only preliminary. Nevertheless, it is interesting to note that if all these symmetry arguments are indeed applicable, this would imply that the scalar fields φ_a can appear in the effective action only in the integration measure, that is, they preserve their geometrical role.

Finally, it is very interesting that in attempts to build a model which respects both the local Einstein symmetry and the gauge invariance, we have succeeded in finding it only in the framework of the Kaluza-Klein unification. It is a clue that the resolution of the cosmological constant problem and the problem of unifying the fundamental forces of nature are intrinsically intertwined.

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