Effective Lagrangian of QED with a magnetic charge and dyon mass bounds

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The effective Lagrangian of QED coupled to dyons is calculated. The resulting generalization of the Euler-Heisenberg Lagrangian contains nonlinear P- and T-noninvariant terms corresponding to the virtual pair creation of dyons. The corresponding P- and T-violating part of the matrix element for light-by-light scattering is considered. This effect induces an electric dipole moment for the electron, of order M^{-2}, where M is the dyon mass. The current limit on the electric dipole moment of the electron yields the lower dyon mass bound $M > 1$ TeV. [S0556-2821(97)06109-2]

I. INTRODUCTION

Very precise measurements achieved during the last decade have opened up for a new approach in elementary particle physics. According to this, evidence of new particles can be extracted from indirect measurements of their virtual contribution to processes at energies which are too low for direct production. For example, the top quark mass as predicted from precision electroweak data [1] agrees to within 10% with direct experimental measurements [2].

This approach has recently been applied [3] for the estimation of possible virtual monopole contributions to observables at energies below the monopole mass. One-loop dyon-induced quantum corrections to the QED Lagrangian were discussed in [4]. Taking into account the violation of parity (and time-reversal symmetry) in a theory with monopoles [5], the emergence of an electric dipole moment was first pointed out by Purcell and Ramsey [6]. More recently, the effect due to monopole loop contributions has been discussed [7,8].

The calculation of quantum corrections due to the virtual pair creation of dyons is a very difficult problem because the standard diagram technique is not valid in this case. The difficulty is connected both to the large value of the magnetic charge of the dyon and to the lack of a consistent local Lagrangian formulation of electrodynamics with two types of charge (see, e.g., [9] and references therein). So, there is no possibility to use a perturbation expansion in a coupling constant. But one can apply the loop expansion which is just an expansion in powers of the Planck constant $\hbar$.

II. EFFECTIVE LAGRANGIAN

It is known (see, e.g., [10]) that the one-loop quantum correction to the QED Lagrangian can be calculated without the use of perturbation methods. The correction is just the change in the vacuum energy in an external field. Let us review the simple case of weak constant parallel electric and magnetic fields **E** and **H**. We impose the conditions

$e|\mathbf{E}|/m^2 \ll 1$ and $e|\mathbf{H}|/m^2 \ll 1$ such that the creation of particles is not possible. In this case the one-loop correction can be calculated by summing the one-particle modes, the solutions of the Dirac equation in the external electromagnetic field, over all quantum numbers [10,11]. For example, if there is just a magnetic field, $\mathbf{H} = (0,0,H)$, the corresponding equation is

$$[i\gamma^\mu(\partial_\mu + ieA_\mu) - m]\psi(x) = 0, \tag{1}$$

where the electromagnetic potential is $A^\mu = (0, -Hy, 0, 0)$. The solution to this equation gives the energy levels of an electron in a magnetic field [12,13]:

$$\varepsilon_n = \sqrt{m^2 + eH(2n - 1 + s) + k^2}, \tag{2}$$

where $n = 0, 1, 2, \ldots$, $s = \pm 1$, and k is the electron momentum along the field. In this case the correction to the Lagrangian is [10,12]

$$
\begin{aligned}
\Delta L_H &= \frac{eH}{2\pi^2} \int_0^\infty dk \left[(m^2 + k^2)^{1/2} + 2\sum_{n=1}^\infty (m^2 + 2eHn + k^2)^{1/2} \right] \\
&= -\frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-m^2 s} \left[(esH)\coth(esH) - 1 \right. \\
&\quad \left. - \frac{1}{3} e^2 s^2 H^2 \right],
\end{aligned}
\tag{3}
$$

where the terms independent of the external field **H** are dropped and a standard renormalization of the electron charge has been made [10].

If we consider simultaneously magnetic (**H**) and electric (**E**) homogeneous fields, then Eq. (1), as well as its classical analogue, can be separated into two uncoupled equations, each in two variables [13]. Indeed, in this case we can take $A^\mu = (Ez, -Hy, 0, 0)$ and the interactions of an electron with the fields **E** and **H** are determined independently. For such a configuration of electromagnetic fields the correction to the Lagrangian is (see [10], p. 787)

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$$\Delta L = \frac{eH}{2\pi^2} \sum_{n=1}^{\infty} \int_0^{\infty} dk \, \varepsilon_n^{(E)}(k). \tag{4}$$

Here $\varepsilon_n^{(E)}$ is the correction to the energy of an electron in the combined external magnetic and electric fields, which is in the first order proportional to $e^2 E^2$.

So, the total Lagrangian is $L = L_0 + \Delta L$, where $L_0 = (\mathbf{E}^2 - \mathbf{H}^2)/2$ is just the Lagrangian of the free electromagnetic field in the tree approximation, and can be written as

$$L = \left(1 + \frac{\alpha}{3\pi} \int_0^{\infty} \frac{ds}{s} e^{-m^2 s}\right) \frac{\mathbf{E}^2 - \mathbf{H}^2}{2} + \Delta L'. \tag{5}$$

The logarithmic divergency can be removed by the standard renormalization of the external fields and the electron charge:

$$E_{\text{reg}} = Z_3^{-1/2} E, \quad H_{\text{reg}} = Z_3^{-1/2} H, \quad e_{\text{reg}} = Z_3^{1/2} e, \tag{6}$$

where $Z_3^{-1} = 1 + (\alpha/3\pi) \int_0^{\infty} (ds/s) e^{-m^2 s}$ is the usual QED renormalization factor. Thus the finite part of the correction to the Lagrangian $\Delta L'$ can be written in terms of physical quantities as (see [10], p. 790)

$$\Delta L' = -\frac{1}{8\pi^2} \int_0^{\infty} \frac{ds}{s^3} e^{-m^2 s} [(esE)(esH)\cot(esE) \\ \times \coth(esH) - 1], \tag{7}$$

which in the limit $E = 0$ reduces to the renormalized form of Eq. (3).

The series expansion of Eq. (7) in terms of the parameters $eE/m^2 \ll 1$, $eH/m^2 \ll 1$ yields the well-known Euler-Heisenberg correction [14]

$$\Delta L' \approx \frac{e^4}{360\pi^2 m^4} [(\mathbf{H}^2 - \mathbf{E}^2)^2 + 7(\mathbf{HE})^2], \tag{8}$$

where $e^2 = \alpha$.

Let us consider how the situation changes if we consider the virtual pair creation of dyons in the external electromagnetic field. Using an analogy with the classical Lorentz force on a dyon of velocity $\mathbf{v}$ with electric (Q) and magnetic (g) charges [9]

$$\mathbf{F} = Q\mathbf{E} + g\mathbf{H} + \mathbf{v} \times (Q\mathbf{H} - g\mathbf{E}), \tag{9}$$

we shall assume that the wave equation for this particle in an external electromagnetic field can be expressed as [15,4]

$$(i\gamma^\mu D_\mu - M)\psi(x) = 0, \tag{10}$$

where M is the dyon mass, and iD_μ a generalized momentum operator, with $D_\mu = \partial_\mu + iQA_\mu + igB_\mu$.

The potential A_μ and its dual B_μ are defined by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \varepsilon_{\mu\nu\rho\sigma} \partial^\rho B^\sigma$ where $\tilde{F}_{\mu\nu}$ is the electromagnetic field strength tensor[1] and $\varepsilon_{0123} = 1$. The potentials in the case of constant parallel electric and magnetic fields can be expressed as

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[1]This definition is consistent only if $\Box A_\mu = \Box B_\mu = 0$, i.e., for constant electromagnetic fields or for free electromagnetic waves.

$$A^\mu = (Ez, -Hy, 0, 0), \quad B^\mu = (Hz, Ey, 0, 0). \tag{11}$$

It is easily seen that the solution to the equation of motion for a dyon in an external electromagnetic field can be obtained from the solution to the equation for an electron, Eq. (1), by the substitution

$$eE \rightarrow QE + gH, \quad eH \rightarrow QH - gE. \tag{12}$$

Using the same substitution in Eqs. (5) and (7), we obtain the following expression for the quantum correction to the Lagrangian, due to the vacuum polarization caused by dyons:

$$L = \left(1 + \frac{Q^2}{12\pi^2} \int_0^{\infty} \frac{ds}{s} e^{-M^2 s} - \frac{g^2}{12\pi^2} \int_0^{\infty} \frac{ds}{s} e^{-M^2 s}\right) \\ \times \frac{\mathbf{E}^2 - \mathbf{H}^2}{2} + \Delta L', \tag{13}$$

where a total derivative has been dropped.

For the renormalization of this expression we can introduce the renormalization factors [16]

$$Z_e^{-1} = 1 + \frac{Q^2}{12\pi^2} \int_0^{\infty} \frac{ds}{s} e^{-M^2 s},$$

$$Z_g^{-1} = 1 - \frac{g^2}{12\pi^2} \int_0^{\infty} \frac{ds}{s} e^{-M^2 s}, \tag{14}$$

which are generalizations of the definition Z_3 of Eq. (6). In this case the fields and charges are renormalized as [16,17]

$$E_{\text{reg}}^2 = Z_e^{-1} Z_g^{-1} E^2, \quad H_{\text{reg}}^2 = Z_e^{-1} Z_g^{-1} H^2,$$

$$e_{\text{reg}}^2 = Z_e Z_g e^2, \quad g_{\text{reg}}^2 = Z_e^{-1} Z_g^{-1} g^2. \tag{15}$$

This relation (15) means that the vacuum of electrically charged particles shields the external electromagnetic field but the contribution from magnetically charged particles antishields it. This agrees with the results of [18,19].

Considering now the case of weak electromagnetic fields, the finite part of the Lagrangian $\Delta L'$ can, by analogy with Eq. (8), be written as

$$\Delta L' = \frac{1}{360\pi^2 M^4} \{[(Q^2 - g^2)^2 + 7Q^2 g^2](\mathbf{H}^2 - \mathbf{E}^2)^2 \\ + [16Q^2 g^2 + 7(Q^2 - g^2)^2](\mathbf{HE})^2 \\ + 6Qg(Q^2 - g^2)(\mathbf{HE})(\mathbf{H}^2 - \mathbf{E}^2)\}. \tag{16}$$

The expressions (8) and (16) describe nonlinear corrections to the Maxwell equations which correspond to photon-photon interactions. The principal difference between the formula (16) and the standard Euler-Heisenberg effective Lagrangian consists in the appearance of P- and T-noninvariant terms proportional to $(\mathbf{HE})(\mathbf{H}^2 - \mathbf{E}^2)$. It should, however, be noted that this term is invariant under charge conjugation C, since then *both* Q and g would change sign.

If we consider separately the virtual creation of dyon pairs, then because of invariance of the model under a dual transformation (see, e.g., [9]), the physics is determined not

by the values Q and g separately, but by the effective charge $\sqrt{Q^2+g^2}$. In the same way the operations of P and T inversions are modified. However, we will consider simultaneously the contributions from vacuum polarization by electron-positron and dyon pairs. In this case it is not possible to reformulate the theory in terms of just one effective charge by means of a dual transformation. Moreover, the Dirac charge quantization condition connects just the electric charge of the electron and the magnetic charge of a dyon: $eg=n/2$ whereas the electric charge Q is not quantized.

It is widely believed, based both on experimental bounds and theoretical predictions [20], that the dyon mass would be large, $M\gg m$, where m is the electron mass. Thus, in the one-loop approximation the first nonlinear correction to the QED Lagrangian from summing the contributions (8) and (16) can be written as

$$\Delta L' \approx \frac{e^4}{360\pi^2 m^4}[(\mathbf{H}^2-\mathbf{E}^2)^2+7(\mathbf{HE})^2]$$
$$+\frac{Qg(Q^2-g^2)}{60\pi^2 M^4}(\mathbf{HE})(\mathbf{H}^2-\mathbf{E}^2), \qquad (17)$$

where the P- and T-invariant terms corresponding to vacuum polarization by dyons have been dropped because they are suppressed by factors M^{-4}. Thus, their contribution to the effective Lagrangian will be of the same order as that of the ordinary QED multiloop amplitudes which we neglect.

III. PHOTON-PHOTON SCATTERING

Expression (17) yields the matrix element for low-energy photon-photon scattering. In order to determine it, we substi-

tute into Eq. (17) the expansion

$$F_{\mu\nu}(x)=\frac{i}{(2\pi)^4}\int d^4q(q_\mu A_\nu-q_\nu A_\mu)e^{iqx}. \qquad (18)$$

Corresponding to the second term of Eq. (17), we find

$$\frac{Qg(Q^2-g^2)}{480\pi^2 M^4}\int d^4x\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}F_{\alpha\beta}F^{\alpha\beta}$$
$$=\frac{1}{(2\pi)^{12}}\int d^4q_1 d^4q_2 d^4q_3 d^4q_4\delta(q_1+q_2+q_3+q_4)$$
$$\times A_\mu(q_1)A_\nu(q_2)A_\rho(q_3)A_\sigma(q_4)\widetilde{M}^{\mu\nu\rho\sigma}, \qquad (19)$$

where

$$\widetilde{M}^{\mu\nu\rho\sigma}=\widetilde{M}^{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4)$$
$$=\frac{Qg(Q^2-g^2)}{60\pi^2 M^4}\varepsilon_{\alpha\beta}{}^{\mu\nu}q_1^\alpha q_2^\beta[q_4^\rho q_3^\sigma-g^{\rho\sigma}(q_3 q_4)]. \qquad (20)$$

Symmetrizing this pseudotensor one obtains the P- and T-violating part of the matrix element for light-by-light scattering. With all momenta flowing inwards, $k_1+k_2+k_3+k_4=0$, the matrix element takes the form

$$M'_{\mu\nu\rho\sigma}=\tfrac{1}{6}[\widetilde{M}_{\mu\nu\rho\sigma}(k_1,k_2,k_3,k_4)+\widetilde{M}_{\mu\rho\nu\sigma}(k_1,k_3,k_2,k_4)+\widetilde{M}_{\mu\sigma\nu\rho}(k_1,k_4,k_2,k_3)+\widetilde{M}_{\nu\rho\mu\sigma}(k_2,k_3,k_1,k_4)$$
$$+\widetilde{M}_{\nu\sigma\mu\rho}(k_2,k_4,k_1,k_3)+\widetilde{M}_{\rho\sigma\mu\nu}(k_3,k_4,k_1,k_2)]$$
$$=\frac{Qg(Q^2-g^2)}{60\pi^2 M^4}[\varepsilon_{\alpha\beta}{}^{\mu\nu}k_1^\alpha k_2^\beta k_3^\sigma k_4^\rho+\varepsilon_{\alpha\beta}{}^{\mu\rho}k_1^\alpha k_2^\sigma k_3^\beta k_4^\nu+\varepsilon_{\alpha\beta}{}^{\mu\sigma}k_1^\alpha k_2^\rho k_3^\nu k_4^\beta+\varepsilon_{\alpha\beta}{}^{\nu\rho}k_1^\sigma k_2^\alpha k_3^\beta k_4^\mu+\varepsilon_{\alpha\beta}{}^{\nu\sigma}k_1^\rho k_2^\alpha k_3^\mu k_4^\beta$$
$$+\varepsilon_{\alpha\beta}{}^{\rho\sigma}k_1^\nu k_2^\mu k_3^\alpha k_4^\beta-\varepsilon_{\alpha\beta}{}^{\mu\nu}g^{\rho\sigma}(k_3 k_4)k_1^\alpha k_2^\beta-\varepsilon_{\alpha\beta}{}^{\mu\rho}g^{\nu\sigma}(k_2 k_4)k_1^\alpha k_3^\beta-\varepsilon_{\alpha\beta}{}^{\mu\sigma}g^{\rho\nu}(k_2 k_3)k_1^\alpha k_4^\beta-\varepsilon_{\alpha\beta}{}^{\nu\rho}g^{\mu\sigma}(k_1 k_4)k_2^\alpha k_3^\beta$$
$$-\varepsilon_{\alpha\beta}{}^{\nu\sigma}g^{\mu\rho}(k_1 k_3)k_2^\alpha k_4^\beta-\varepsilon_{\alpha\beta}{}^{\rho\sigma}g^{\mu\nu}(k_1 k_2)k_3^\alpha k_4^\beta]. \qquad (21)$$

Since the interaction contains an ε tensor, the coupling between two of the photons is different from that involving the other two, and the familiar pairwise equivalence of the six terms does not hold. The matrix element satisfies gauge invariance (with respect to any of the four photons),

$$k_1^\mu M'_{\mu\nu\rho\sigma}(k_1,k_2,k_3,k_4)=0, \quad \text{etc.} \qquad (22)$$

We note that the above contribution to the matrix element is proportional to the fourth power of the inverse dyon mass, $M'_{\mu\nu\rho\sigma}\propto M^{-4}$. However, this result is only valid at low energies, where the photon momenta are small compared to

M, being obtained from an effective, nonrenormalizable theory.

Thus, as a result of interference between two one-loop diagrams corresponding to loops with dyons and those with simply electrically charged particles there is an asymmetry between the processes of photon splitting and photon coalescence [4]. The physical effect of this asymmetry will depend on the photon spectrum and the directions of the photon momenta with respect to the magnetic field. In particular, the asymmetry vanishes when these are perpendicular, i.e., for $\cos\theta=0$. Furthermore, the asymmetry is linear in the product of the dyon charges, and proportional to the fourth power of the electron to dyon mass ratio.
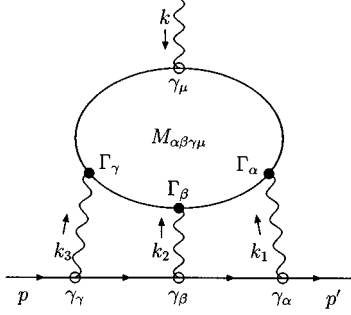
FIG. 1. Typical three-loop vertex diagram. The closed line represents a dyon loop.

IV. ELECTRIC DIPOLE MOMENT

The contribution of this matrix element (21) breaks the P and T invariance of ordinary electrodynamics. Thus, among the sixth-order radiative corrections to the electron-photon vertex there are terms containing this photon-photon scattering subdiagram with a dyon-loop contribution (see Fig. 1), that induce an electric dipole moment of the electron [8].[2]

Indeed, one can write the contribution of this diagram to the electron-photon vertex as[3]

$$\Lambda_\mu(p',p) = \frac{e^2}{(2\pi)^8} \int d^4k_1 d^4k_3 \frac{1}{k_1^2+i\epsilon} \frac{1}{k_2^2+i\epsilon} \frac{1}{k_3^2+i\epsilon}$$

$$\times M_{\alpha\beta\gamma\mu}(k_1,k_2,k_3,k)$$

$$\times \gamma^\alpha \frac{p\!\!\!/'+k\!\!\!/_1+m}{(p'+k_1)^2-m^2+i\epsilon}$$

$$\times \gamma^\beta \frac{p\!\!\!/-k\!\!\!/_3+m}{(p-k_3)^2-m^2+i\epsilon} \gamma^\gamma, \quad (23)$$

where $M_{\alpha\beta\gamma\mu}(k_1,k_2,k_3,k)$ is the polarization pseudotensor representing the dyon box diagram contribution to the photon-photon scattering amplitude, the low-energy limit of which is given by the pseudotensor $M'_{\alpha\beta\gamma\mu}$ of Eq. (21).

In order to extract the electric dipole moment from the general expression (23), it is convenient, according to the approach by [22], to exploit the identity

$$M'_{\alpha\beta\gamma\mu}(k_1,k_2,k_3,k) = -k^\nu \frac{\partial}{\partial k^\mu} M_{\alpha\beta\gamma\nu}(k_1,k_2,k_3,k), \quad (24)$$

which can be obtained upon differentiating the gauge invariance condition of the polarization tensor [cf. Eq. (22)] with respect to k^μ.

Substituting Eq. (24) into Eq. (23), we can write the $ee\gamma$ matrix element as

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[2]This has been noted by Khriplovich [21], see also a recent paper by Flambaum and Murray [7].

[3]Of course, there are more diagrams.

$$M_{ee\gamma}(p',p,k) = e^\mu(k)\bar{u}(p')\Lambda_\mu(p',p)u(p)$$

$$= e^\mu(k)k^\nu\bar{u}(p')\Lambda_{\mu\nu}(p',p)u(p), \quad (25)$$

where $e^\mu(k)$ is the photon polarization vector and

$$\Lambda_{\mu\nu}(p',p) = -\frac{e^2}{(2\pi)^8}\int d^4k_1 d^4k_3 \frac{1}{k_1^2+i\epsilon} \frac{1}{k_2^2+i\epsilon} \frac{1}{k_3^2+i\epsilon}$$

$$\times \frac{\partial}{\partial k^\mu} M_{\alpha\beta\gamma\nu}(k_1,k_2,k_3,k)\gamma^\alpha$$

$$\times \frac{p\!\!\!/'+k\!\!\!/_1+m}{(p'+k_1)^2-m^2+i\epsilon}$$

$$\times \gamma^\beta \frac{p\!\!\!/-k\!\!\!/_3+m}{(p-k_3)^2-m^2+i\epsilon} \gamma^\gamma. \quad (26)$$

Since the matrix element (25) is already proportional to the external photon momentum k, one can put $k=0$ in $\Lambda_{\mu\nu}$ after differentiation to obtain the static electric dipole moment.

Then, following [22], we note that due to Lorentz covariance of $\Lambda_{\mu\nu}$, it can be written in the form

$$\Lambda_{\mu\nu}(p',p) = (\widetilde{A}g_{\mu\nu} + \widetilde{B}\sigma_{\mu\nu} + \widetilde{C}P_\mu\gamma_\nu + \widetilde{D}P_\nu\gamma_\mu$$

$$+ \widetilde{E}P_\mu P_\nu)\gamma_5 + \cdots, \quad (27)$$

where we have omitted terms that do not violate parity, as well as those proportional to k_μ, and where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$, and $P_\mu = p_\mu + p'_\mu$.

Substituting this expression into the matrix element $M_{ee\gamma}(p',p,k)$ of Eq. (25), one can see that there are two contributions to the P-violating part, arising from the $\widetilde{B}$ and $\widetilde{C}$ terms. In order to project out the dipole moment from Eq. (25), one has to compare Eq. (27) with the phenomenological expression for the electric dipole moment d_e [23]:

$$M_{ee\gamma}(p',p,k) = e^\mu(k)k^\nu\bar{u}(p')\frac{d_e}{2m}\gamma_5\sigma_{\mu\nu}u(p). \quad (28)$$

In the nonrelativistic limit it corresponds to the interaction Hamiltonian $-(d_e/2m)\boldsymbol{\sigma}\mathbf{E}$. Thus, multiplying Eq. (27) by $\sigma_{\mu\nu}\gamma_5$ and taking the trace we have

$$d_e = -\frac{m}{24}\text{Tr}[\sigma_{\mu\nu}\gamma_5\Lambda^{\mu\nu}]. \quad (29)$$

In order to provide an estimate of the induced electric dipole moment we need to estimate $\Lambda^{\mu\nu}$. The first task is to evaluate the polarization pseudotensor $M_{\alpha\beta\gamma\mu}$ corresponding to the virtual dyon one-loop subdiagram. If we were to substitute for $M_{\alpha\beta\gamma\mu}$ the low-energy form $M'_{\alpha\beta\gamma\mu}$ of Eq. (21) into Eq. (23), we would obtain a quadratically divergent integral.

On the other hand, straightforward application of the Feynman rules in QED with magnetic charge (see, e.g., [15]) would give for the photon-by-photon scattering subdiagram[4] in Fig. 1:

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[4]It should be noted that the expression (21) contains contributions from such loop diagrams with all possible combinations of either three or one magnetic-coupling vertex Γ_ρ.

$$M_{\alpha\beta\gamma\mu}(k_1,k_2,k_3,k)=\frac{Qg^3}{2\pi^4}\int d^4q\,\mathrm{Tr}\left(\Gamma_\alpha\frac{1}{\slashed{q}+\slashed{k}_1-M}\Gamma_\beta\frac{1}{\slashed{q}-\slashed{k}_3-\slashed{k}-M}\Gamma_\gamma\frac{1}{\slashed{q}-\slashed{k}-M}\gamma_\mu\frac{1}{\slashed{q}-M}\right). \qquad (30)$$

Here Γ_α represents the magnetic coupling of the photon to the dyon, which we take according to Ref. [16] to be

$$\Gamma_\mu=-i\varepsilon_{\mu\nu\rho\sigma}\frac{\gamma^\nu k^\rho n^\sigma}{(n\cdot k)}. \qquad (31)$$

The vertex function depends on k^ρ, the photon momentum entering the vertex, and on n^σ, a unit constant spacelike vector corresponding to the Dirac singularity line. It was shown by Zwanziger [24] that although the matrix element depends on n, the cross section as well as other physical quantities are n independent.

Calculations using this technique are very complicated and can only be done in a few simple situations [15], for example, in the case of the charge-monopole scattering problem [25]. We will here avoid this approach.

While the integration over q in Eq. (30) is logarithmically divergent [the magnetic couplings in Eq. (30] are dimensionless), after renormalization the sum of such contributions must, in the low-energy limit, reduce to the form given in Eq. (21). We also note that the substitution of Eq. (30) into Eq. (26) yields a convergent integral. Thus, the following method for evaluating $\Lambda_{\mu\nu}$ suggests itself. We divide the region of integration into two domains: (i) the momenta k_1 and k_3 are small compared to M, and (ii) the momenta are of order M (or larger).

In the first region, the form (21) can be used, but since the integral is quadratically divergent, the integral will be proportional to M^2. Together with the overall factor M^{-4}, this will give a contribution $\propto M^{-2}$. For large values of the photon momenta, the other form, Eq. (30), can be used. This gives a convergent integral, and dimensional arguments determine the scale to be M^{-2}. It means that

$$|\Lambda_{\mu\nu}|\sim\frac{e^2Qg(Q^2-g^2)}{(4\pi^2)^3M^2}. \qquad (32)$$

The numerical coefficient has been estimated as $1/(4\pi^2)^3$, one factor $1/4\pi^2$ from each loop, and the $1/24$ of Eq. (29) is assumed canceled by a combinatorial factor from the number of diagrams involved. This is of course a very rough assessment.

Now we can estimate the order of magnitude of the electron dipole moment generated by virtual dyons. It is obvious from Eqs. (29) and (32) that in order of magnitude one can write

$$d_e\sim\frac{e^2Qg(Q^2-g^2)}{(4\pi^2)^3}\frac{m}{M^2}. \qquad (33)$$

This estimate can be used to obtain a new bound on the dyon mass. Indeed, recent experimental progress in the search for an electron electric dipole moment [26] gives a rather strict upper limit: $d_e<9\times10^{-28}e$ cm. If we suppose that $Q\sim e$, then from Eq. (33) one can obtain $M\geqslant2\times10^6m\approx10^3$ GeV. This estimate shows that the dyon mass belongs at least to the electroweak scale.

The above estimate coincides with the bound obtained by De Rújula [3] for monopoles, from an analysis of data from the CERN e^+e^- collider LEP, but it is weaker than the result given in [7], where the limit $M\geqslant10^5$ GeV was obtained. The authors of Ref. [7] used the hypothesis that a radial magnetic field could be induced due to virtual dyon pairs. In order to estimate the effect, they used the well-known formula for the Ueling correction to the electrostatic potential, simply replacing the electron charge and mass with those of the monopole. But the Ueling term is just a correction to the scalar Coulomb potential due to vacuum polarization and cannot itself be considered as a source of a radial magnetic field. Indeed, there is only one second-order term in the effective Lagrangian that can violate the P and T invariance of the theory, namely, $\Delta L'\propto\mathbf{EH}$. But in the framework of QED there is no reason to consider such a correction because it is just a total derivative. The reference to the θ term, used in [7] to estimate the electric charge of the dyon, is only relevant in the context of a nontrivial topology (e.g., in the 't Hooft–Polyakov monopole model) where their limit applies. In this case there are arguments in favor of stronger limits on the monopole (dyon) mass (see, e.g., [20]).

One should note that the dyon-loop diagram considered above can also contribute to the neutron electric dipole moment. The experimental value $d_n<1.1\times10^{-25}e$ cm [27] will, in the naive quark model with $m\approx10$ MeV, allow us to obtain an estimate of the dyon lower mass bound which is similar to the one obtained for the electron.

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