Radiative corrections to the semileptonic Dalitz plot with angular correlation between polarized decaying and emitted hyperons

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We obtain an expression for the Dalitz plot of semileptonic decays of polarized hyperons including radiative corrections to order α and neglecting terms of the order $\alpha q/\pi M_1$, where q is the four-momentum transfer and M_1 is the mass of the decaying hyperon. Our results are specialized to exhibit the angular correlation between such polarization and the momentum of the emitted hyperon. The model dependence of radiative corrections is kept in a general form within this approximation which is suitable for model-independent experimental analysis. Our final result, valid for charged as well as for neutral hyperons, allows us to obtain the angular spin-asymmetry coefficient of the emitted hyperon. [S0556-2821(97)04709-7]

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I. INTRODUCTION

Measurements of form factors of hyperon semileptonic decays (HSD's) through the observation of the polarization of the decaying hyperon are more than just a complement of the measurements with unpolarized hyperons. Without the former not even parity violation could be established in such decays. Both types of measurements are necessary to achieve a determination of all the form factors that dress the weak interaction vertex in these decays. For this task it is required that, in addition to high statistics experiments, theoretical expressions as general and accurate as possible be available. In this latter respect radiative corrections must be taken into account. This has been done already for the Dalitz plot (DP) of unpolarized decaying hyperons [1]. However, radiative corrections depend on spins. It is the purpose of this paper to obtain the radiative corrections to the DP of polarized initial hyperons. In order to be able to integrate over the photon variables one is required to choose, in the center-of-mass frame of the decaying hyperon, the angular correlation between the polarization of the decaying hyperon and the direction of the emitted hyperon or charged lepton. We believe it is more useful to choose the former. So this is what we do here.

We shall follow the same approach of Ref. [1]. The virtual radiative corrections (Sec. II) will be separated into model-independent (MIP) and model-dependent (MDP) parts, according to the gauge-invariant analysis originally introduced by Sirlin [2] to study the radiative corrections to neutron β decay. The MIP is finite in the ultraviolet and fully contains [3] the infrared divergence. We shall not compromise our calculation to any one model. The MDP can be totally absorbed into the already existing strong-interaction form factors, without introducing new ones. Because of this, our results will be suitable for model-independent experimental analyses. The price for this is that the measured form factors will be not the pure strong-interaction form factors but new ones which are modified by the MDP of radiative corrections. There is no loss here, because strictly speaking it is only these modified form factors that can actually be experimentally determined. The resolution of which part of them is due to strong interactions only and which part belongs to radiative corrections is really a theoretical problem only. To deal with the bremsstrahlung contribution to radiative corrections (Sec. III) we shall use the Low theorem [4]. In our calculation we shall neglect terms of order $\alpha q/\pi M_1$, where q is the four-momentum transfer and M_1 is the mass of the decaying hyperons. No model dependence appears then in the bremsstrahlung correction.

Our results will be presented in two ways. We shall first obtain the radiative corrections in an integral form, i.e., leaving indicated the triple integration over the photon three-momentum. However, the infrared divergence of the bremsstrahlung part will be explicitly extracted (and canceled with its virtual counterpart) along with the finite contributions [5] that accompany it. This presentation will be ready for numerical integration and up to this point it will represent a first final result. Next (Sec. IV), we shall proceed to perform analytically the triple photon integrals. This will lead to our second final result (Sec. V): a completely analytical expression for the DP of initially polarized hyperons including order-α radiative corrections. Moreover, we also give the expression for the angular spin-asymmetry coefficient of the emitted hyperon with radiative corrections up to the order of approximation previously mentioned. In Appendix A we describe the manner in which the integrals were handled and reduced to forms already computed. For the sake of completeness, in Appendixes B and C we reproduce the main results of Refs. [1,6]. This makes this article self-contained. Appendix D contains an internal cross-check.

To our knowledge, there is one previous paper [7] addressing the radiative corrections to the DP of polarized decaying hyperons. In this paper the virtual corrections are handled also following Ref. [2]. However, the bremsstrahlung contributions calculated are very model dependent. First, the baryons are assumed to be pointlike. And second, the photon variables are integrated numerically in a, so to speak, global fashion by fixing the several form factors at prescribed values, given by the CVC (conserved vector current) and the PCAC (partially conserved axial-vector current) hypotheses. The final results of Ref. [7] are presented in numerical tables for fixed values of the energies of the lepton and emitted baryon, E and E_2. This method has shortcomings that limit its applicability considerably. The CVC and PCAC hypotheses are reliable only for $\Delta S=0$ decays. SU(3) breaking affects them appreciably in $\Delta S\neq 0$ decays and they are completely unreliable in charm decays. Baryons are not pointlike and extra contributions appear from their structure, introducing uncertainties of order $\alpha q/M_1$. Also, the fixed values of E and E_2 used in producing the numerical tables will not agree in general with the values used to define the bins of an experiment. Both methods, the one of Ref. [7] and ours, will lead to the same results for $\Delta S=0$ decays and still may give reasonable agreement with $\Delta S\neq 0$, but will differ considerably in $\Delta C\neq 0$ decays. A comparison for two $\Delta S\neq 0$ decays with Ref. [7], using exactly the same values of the corresponding form factors, is given in Appendix E. The agreement is quite reasonable.

The method we follow in this paper is not committed to any particular model. It exhibits the $\alpha q/M_1\approx 0$ approximation explicitly and does not require that the form factors be fixed at any particular values. Instead of producing numerical tables, we have produced detailed expressions that can be evaluated at any set of values of E and E_2 chosen by the user. Such an evaluation can be performed numerically or analytically within a Monte Carlo simulation. The advantage of the analytical expressions is to reduce enormously the computational effort involved in the Monte Carlo procedure.

Our long results have been organized to be used in a straightforward fashion. They will be reliable up to a precision around 0.5% over most of the DP. For high statistics experiments, with tens of thousands of events of HSD's not involving heavy quarks, this precision should be very satisfactory. If charm quarks are involved, our results should be acceptable in experiments with thousands of events [8]. If higher statistics experiments with these quarks are envisaged, our results represent a useful first approximation, but they should eventually be refined.

II. VIRTUAL RADIATIVE CORRECTIONS

We first introduce our notation and conventions and next we obtain the virtual radiative corrections. Let us consider the HSD

$$A\to B+l+\overline{\nu_l}, \tag{1}$$

and denote the four-momenta and masses of the decaying and emitted hyperons, the charged lepton, and the neutrino involved in the process by $p_1=(E_1,\mathbf{p}_1)$, $p_2=(E_2,\mathbf{p}_2)$, $l=(E,\mathbf{l})$, and $p_\nu=(E_\nu^0,\mathbf{p}_\nu)$ and by M_1, M_2, m, and m_ν, respectively. We will assume throughout this paper that $m_\nu=0$. $\hat{\mathbf{p}}_2$ will denote a unit vector along the direction of $\mathbf{p}_2$, etc. p_2, l, and p_ν will also denote the magnitudes of the corresponding three-momenta when specializing our calculations to the center-of-mass frame of A. No confusion is expected because in this case our expressions will not be manifestly covariant.

The uncorrected transition amplitude M_0 for process (1) is given by the product of matrix elements of the hadronic and leptonic currents,

$$\mathsf{M}_0=\frac{G_V}{\sqrt{2}}[\overline{u_B}(p_2)W_\mu(p_1,p_2)u_A(p_1)][\overline{u_l}(l)O_\mu v_\nu(p_\nu)], \tag{2}$$

where

$$W_\mu(p_1,p_2)=f_1(q^2)\gamma_\mu+\frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu+\frac{f_3(q^2)}{M_1}q_\mu + \left[g_1(q^2)\gamma_\mu+\frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q_\nu+\frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5. \tag{3}$$

Here $O_\mu=\gamma_\mu(1+\gamma_5)$ and $q=p_1-p_2$ is the four-momentum transfer. Our metric and γ-matrix convention are those of Ref. [1]. The leptonic current has the usual $(V-A)$ structure. For the baryon term the $(V-A)$ structure of the underlying quark transition is masked by the effects of the strong interactions. f_i and g_i, the conventional vector and axial-vector form factors, are functions of q^2 and, unless explicitly noted otherwise, we always deal with their values at $q^2\neq 0$.

The polarization of the initial hyperon can be taken into account by introducing the projection operator [9]

$$\Sigma(s_1)=\frac{1-\gamma_5\not{s}_1}{2}, \tag{4}$$

where the polarization four-vector s_1 obeys the conditions $s_1\cdot s_1=(s_1{}^0)^2-\mathbf{s}_1\cdot\mathbf{s}_1=-1$ and $s_1\cdot p_1=0$. In the center-of-mass frame of A, s_1 reduces to a purely spatial unit vector which gives the spin direction. The observable effects of spin polarization can be studied through the replacement

$$u_A(p_1)\to\Sigma(s_1)u_A(p_1) \tag{5}$$

in the corresponding spinor for the decaying hyperon.

The virtual radiative corrections to the DP of decay (1) can be obtained using a similar procedure to Ref. [1], which is an extension of the procedure introduced in Ref. [2]. Details are not needed to be repeated here. It is only important to point out that such corrections can be separated into a MIP M_v that is finite and calculable and into a MDP which can be absorbed into M_0 through the definition of effective form factors. Hereafter this fact will be denoted by putting a prime on M_0.

With the above considerations, the complete decay amplitude with virtual radiative corrections M_V is

$$\mathsf{M}_V=\mathsf{M}_0'+\mathsf{M}_v. \tag{6}$$

The MIP, given by Eq. (4) of Ref. [1], is

$$\mathsf{M}_v=\frac{\alpha}{2\pi}[\mathsf{M}_0\phi(E)+\mathsf{M}_{p_1}\phi'(E)], \tag{7}$$

where $\phi(E)$ and $\phi'(E)$, given, respectively, in Eqs. (5) and (6) of this reference, are

$$\phi(E)=2\left[\frac{1}{\beta}\mathrm{arctanh}\beta-1\right]\ln\left[\frac{\lambda}{m}\right]-\frac{1}{\beta}(\mathrm{arctanh}\beta)^2 + \frac{1}{\beta}L\left[\frac{2\beta}{1+\beta}\right]+\frac{1}{\beta}\mathrm{arctanh}\beta-\frac{11}{8} + \begin{cases} \pi^2/\beta+\frac{3}{2}\ln(M_2/m) & (\mathrm{NDH}), \\ \frac{3}{2}\ln(M_1/m) & (\mathrm{CDH}), \end{cases} \tag{8}$$

$$\phi'(E)=\left[\beta-\frac{1}{\beta}\right]\mathrm{arctanh}\beta. \tag{9}$$

According to our approximation, all the terms of the order $\alpha q/\pi M_1$ and $(\alpha q/\pi M_1)\ln(q/M_1)$ have been neglected in Eqs. (8) and (9). In these equations $\beta=l/E$ and L is the Spence function defined as

$$L(x)=\int_0^x dt\frac{1}{t}\ln(|1-t|). \tag{10}$$

λ is the infrared-divergence cutoff that will be canceled by its analogue in the bremsstrahlung contribution; NDH and CDH stand for neutral and charged decaying hyperons, respectively.

The second matrix element in Eq. (7) is

$$\mathsf{M}_{p_1}=\left(\frac{E}{mM_1}\right)\frac{G_V}{\sqrt{2}}[\overline{u_B}W_\lambda u_A][\overline{u_l}\not{p}_1 O_\lambda v_\nu]. \tag{11}$$

Within our order of approximation $W_\lambda(p_1,p_2)$ in Eqs. (7) and (11) is reduced to

$$W_\lambda(p_1,p_2)=f_1(0)\gamma_\lambda+g_1(0)\gamma_\lambda\gamma_5. \tag{12}$$

The DP with virtual radiative corrections is now obtained by leaving E_2 and E as the relevant variables in the differential decay rate for process (1). After making the replacement (5) in Eq. (6), squaring it, and rearranging terms, we can express

$$\sum_{\mathrm{spins}}|\mathsf{M}_V|^2=\frac{1}{2}\sum_{\mathrm{spins}}|\mathsf{M}'_V|^2-\frac{1}{2}\sum_{\mathrm{spins}}|\mathsf{M}_V^{(s)}|^2. \tag{13}$$

Here M'_V does not contain s_1 explicitly whereas $\mathsf{M}_V^{(s)}$ does. Notice that we keep in Eq. (13) the one-half factor and the minus sign arising from the definition of the projection operator, Eq. (4). This distinction is useful because it enables us to express the differential decay rate as

$$d\Gamma_V=\frac{dE_2dEd\Omega_2d\varphi_l}{(2\pi)^5}M_2mm_\nu\left[\frac{1}{2}\sum_{\mathrm{spins}}|\mathsf{M}'_V|^2-\frac{1}{2}\sum_{\mathrm{spins}}|\mathsf{M}_V^{(s)}|^2\right] \equiv d\Gamma'_V-d\Gamma_V^{(s)}, \tag{14}$$

where $d\Gamma'_V$ refers to the first term of the sum within the square brackets. It corresponds to the expression for the differential decay rate with virtual radiative corrections of unpolarized hyperons given by Eq. (10) of Ref. [1], with only a slight difference: We have chosen, without loss of generality, a coordinate frame in the center-of-mass system of A with the z axis along the emitted hyperon three-momentum.

After a long but otherwise standard calculation the decay rate is compactly given by

$$d\Gamma_V=\frac{G_V^2}{2}\frac{dE_2dEd\Omega_2d\varphi_l}{(2\pi)^5}2M_1\left\{A'_0+\frac{\alpha}{\pi}(A'_1\phi+A''_1\phi') -\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2\left[A''_0+\frac{\alpha}{\pi}(A'_2\phi+A''_2\phi')\right]\right\}. \tag{15}$$

A'_0, A'_1, A''_1, A''_0, A'_2, and A''_2 depend on the kinematical variables and are quadratic functions of the form factors. Their dependence on the kinematical variables is

$$A'_0=Q_1EE_\nu^0-Q_2Ep_2(p_2+ly_0)-Q_3l(p_2y_0+l) +Q_4E_\nu^0p_2ly_0-Q_5p_2^2ly_0(p_2+ly_0), \tag{16}$$

$$A'_1=D_1EE_\nu^0-D_2l(p_2y_0+l), \tag{17}$$

$$A''_1=D_1EE_\nu^0, \tag{18}$$

$$A''_0=Q_6Ep_2+Q_7Ely_0, \tag{19}$$

$$A'_2=-D_3E_\nu^0ly_0+D_4E(p_2+ly_0), \tag{20}$$

$$A''_2=D_4E(p_2+ly_0), \tag{21}$$

where y_0 is the cosine of the angle between the directions of emission of the charged lepton and B, and is given by

$$y_0=\frac{(E_\nu^0)^2-p_2^2-l^2}{2p_2l} \tag{22}$$

and by energy conservation

$$E_\nu^0=M_1-E_2-E. \tag{23}$$

The coefficients Q_i $(i=1,\ldots,7)$ are long quadratic functions of the form factors whereas the coefficients D_j $(j=1,\ldots,4)$ depend only on the leading form factors. Their explicit forms are given in Appendix B.

The primes in Eqs. (16)–(21) indicate that the form factors f'_1 and g'_1, containing the model dependence of the virtual radiative corrections, are the ones that appear in them. This involves a rearrangement of terms of second order in α in Eq. (15), but the remarkable fact is that to first order in α only f'_1 and g'_1 can be experimentally determined [1].

III. BREMSSTRAHLUNG AMPLITUDE AND INFRARED DIVERGENCE

In addition to the virtual radiative corrections, the bremsstrahlung counterpart must be included in order to obtain the complete radiative corrections to the DP of process (1) up to the order of approximation mentioned above. In this section we turn to the four-body decay

$$A \to B + l + \overline{\nu_l} + \gamma, \tag{24}$$

where γ represents a real photon with four-momentum $k=(w,\mathbf{k})$, and because of energy and momentum conservation, $E_1=E_2+E+E_\nu+w$ and $\mathbf{p}_1=\mathbf{p}_2+\mathbf{l}+\mathbf{p}_\nu+\mathbf{k}$.

First we will give the amplitude of process (24); next, we will extract the infrared divergence. Afterwards, we will give a complete expression for the differential bremsstrahlung decay rate which along with Eq. (15) gives the DP with radiative corrections of process (1).

Within our approximation and following the Low theorem [4] the transition amplitude M_B of process (24) is given by

$$\mathsf{M}_B = \frac{eG_V}{\sqrt{2}}[\overline{u_B}W_\lambda u_A][\overline{u_l}O_\lambda v_\nu]\left[\frac{2l\cdot\epsilon}{2l\cdot k+\lambda^2}+\frac{2p_1\cdot\epsilon}{\lambda^2-2p_1\cdot k}\right] + \frac{eG_V}{\sqrt{2}}[\overline{u_B}W_\lambda u_A][\overline{u_l}\not\epsilon \not k O_\lambda v_\nu]\left[\frac{1}{2l\cdot k+i\varepsilon}\right] \equiv \mathsf{M}_a+\mathsf{M}_b, \tag{25}$$

where M_a and M_b stand for the first and second summands, respectively. As in the virtual case, we have to make the substitution (5) in order to study spin effects. Observe that Eq. (25) is gauge invariant and model independent; the model dependence does not contribute to zeroth order in $\alpha q/\pi M_1$. W_λ should be the complete expression defined in Eq. (3); however, to our order of approximation we must use Eq. (12) instead. ϵ is the photon polarization four-vector and λ is a small mass given to the photon to regularize the infrared divergence.

Because we concern ourselves with the radiative corrections to process (1) and not with the process (24) itself, our analysis will be restricted to the three-body region of the DP of the latter defined by [1]

$$E_2^{\min} \leqslant E_2 \leqslant E_2^{\max} \tag{26}$$

and

$$m \leqslant E \leqslant E_m, \tag{27}$$

with

$$E_2^{\max,\,\min} = \frac{1}{2}(M_1-E\pm l)+\frac{M_2^2}{2(M_1-E\pm l)} \tag{28}$$

and

$$E_m = \frac{M_1^2-M_2^2+m^2}{2M_1}. \tag{29}$$

That is, although we assume that real photons are not detected, we also assume that events whose energies E and E_2 do not satisfy the three-body energy-momentum conservation restrictions of Eqs. (26)–(29) are rejected.

The bremsstrahlung contribution to the differential decay rate is given by [9]

$$d\Gamma_B = \frac{M_2 m m_\nu}{(2\pi)^8}\frac{d^3p_2}{E_2}\frac{d^3l}{E}\frac{d^3k}{2w}\frac{d^3p_\nu}{E_\nu} \times \sum_{\text{spins}} |\mathsf{M}_B|^2 \delta^4(p_1-p_2-l-p_\nu-k). \tag{30}$$

By analogy with the virtual case, the spin-independent amplitude M_B' can be separated from the spin-dependent one $\mathsf{M}_B^{(s)}$ so that

$$\sum_{\text{spins}} |\mathsf{M}_B|^2 = \frac{1}{2}\sum_{\text{spins}} |\mathsf{M}_B'|^2 - \frac{1}{2}\sum_{\text{spins}} |\mathsf{M}_B^{(s)}|^2, \tag{31}$$

where we also keep the one-half factor and the minus sign coming from Eq. (4). Thus

$$d\Gamma_B = d\Gamma_B' - d\Gamma_B^{(s)}. \tag{32}$$

With only minor changes, we can identify $d\Gamma_B'$ with the bremsstrahlung differential decay rate for unpolarized hyperons given by Eq. (55) of Ref. [1], which leaves the triple integration over the photon variables to be performed numerically. Such an expression can be reduced to the compact form (the upper indices a and b replace the indices I and II of this reference)

$$d\Gamma_B' = d\Gamma_B^{\text{ir}} + d\Gamma_B^a + d\Gamma_B^b. \tag{33}$$

The explicit form of each contribution is

$$d\Gamma_B^{\text{ir}} = \{A_1'[\hat{I}_0(k/\lambda)+C+C_1]+C_2\}d\Omega, \tag{34}$$

where

$$d\Omega = \frac{\alpha}{\pi}\frac{G_V^2}{2}\frac{dE_2\,dE\,d\Omega_2\,d\varphi_l}{(2\pi)^5}2M_1. \tag{35}$$

Here $\hat{I}_0(k/\lambda)$ fully contains the infrared divergence and C and C_1 are the finite terms that come along with it. We do not need their explicit forms here. Likewise, C_2 is given by

$$C_2 = D_2 p_2 l \frac{\beta^2}{2}(1+y_0)\int_{-1}^{1} dx \frac{1-x^2}{(1-\beta x)^2}, \tag{36}$$

where x is the cosine of the angle between $\mathbf{l}$ and $\mathbf{k}$.

On the other hand,

$$d\Gamma_B^a = -d\Omega \frac{p_2 l}{2\pi}\int_{-1}^{1} dx \frac{\beta^2}{2}\frac{1-x^2}{(1-\beta x)^2}(D_1E+D_2lx) \times \int_{-1}^{y_0} dy \int_0^{2\pi}\frac{d\varphi_k}{D} \tag{37}$$

and

$$d\Gamma_B^b = d\Omega \frac{p_2 l}{2\pi}\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\int_{-1}^{y_0}dy\frac{1}{2ED(1-\beta x)}$$
$$\times\left\{D_1\left[E_\nu w+(1+\beta x)EE_\nu-\frac{m^2E_\nu w}{l\cdot k}\right]\right.$$
$$\left.+D_2\mathbf{p}_\nu\cdot\left[\hat{\mathbf{k}}(E+w)+\mathbf{l}-\frac{m^2\mathbf{k}}{l\cdot k}\right]\right\}, \quad (38)$$

with

$$E_\nu = E_\nu^0 - w, \quad (39)$$

$$D = E_\nu^0 + (\mathbf{p}_2+\mathbf{l})\cdot\hat{\mathbf{k}}. \quad (40)$$

φ_k is the azimuthal angle of the photon and y is the cosine of the angle between $\mathbf{l}$ and $\mathbf{p}_2$. Furthermore,

$$w = \frac{F}{2D}, \quad (41)$$

where

$$F = 2p_2 l(y_0 - y). \quad (42)$$

Now we turn to study $d\Gamma_B^{(s)}$ and collect both results afterwards. For convenience, let us split $d\Gamma_B^{(s)}$ into two parts

$$d\Gamma_B^{(s)} = d\Gamma_B^{\mathrm{I}} + d\Gamma_B^{\mathrm{II}}, \quad (43)$$

where $d\Gamma_B^{\mathrm{I}}$ contains $\Sigma|\mathsf{M}_a^{(s)}|^2$ and $d\Gamma_B^{\mathrm{II}}$ contains $\Sigma(|\mathsf{M}_b^{(s)}|^2+2\,\mathrm{Re}[\mathsf{M}_a^{(s)}][\mathsf{M}_b^{(s)}]^\dagger)$. $d\Gamma_B^{\mathrm{I}}$ contains infrared-divergent terms as well as infrared-convergent ones and $d\Gamma_B^{\mathrm{II}}$ is infrared convergent.

Explicitly we have

$$\sum_{\mathrm{spins}}|\mathsf{M}_a^{(s)}|^2 = \frac{e^2G_V^2}{2}\frac{4M_1}{M_2 m m_\nu}[-D_3\hat{\mathbf{s}}_1\cdot\mathbf{l}E_\nu^0 + D_4\hat{\mathbf{s}}_1\cdot\mathbf{p}_2 E$$
$$+D_4\hat{\mathbf{s}}_1\cdot\mathbf{l}E + D_3\hat{\mathbf{s}}_1\cdot\mathbf{l}w + D_4\hat{\mathbf{s}}_1\cdot\mathbf{k}E]$$
$$\times\sum_\epsilon\left(\frac{l\cdot\epsilon}{l\cdot k}-\frac{p_1\cdot\epsilon}{p_1\cdot k}\right)^2. \quad (44)$$

The infrared divergence is contained in the first three terms within the square brackets. The remaining two terms converge in the infrared. In order to deal with the infrared divergence properly, we will trace a close parallelism with the analysis of Ref. [10], which refers to the radiative corrections to the DP of $K_{e3}^{\pm}$ decays. This approach can be adapted to our case by using the invariant mass $\eta=(p_\nu+k)^2$, which in the center-of-mass frame of A is given by

$$\eta = M_1^2 + M_2^2 + m^2 - 2M_1E - 2M_1E_2 + 2EE_2 - 2p_2 l y. \quad (45)$$

y and η are related through

$$y = y_0 - \frac{\eta}{2p_2 l}. \quad (46)$$

On the other hand, the most general form of $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ depends on both θ_l and φ_l, the polar and azimuthal angles of $\mathbf{l}$, respectively. The terms directly proportional to $\cos\varphi_l$ drop off after integrating over φ_l from 0 to 2π. Hereafter those terms will not be considered. With the proper orientation of the coordinate axes ($\mathbf{p}_2$ along the z axis), the dependence on $\cos\theta_l$ is further reduced to y. Thus, $d\Gamma_B^{\mathrm{I}}$ will not show explicitly $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ any longer.

The sum over ϵ is still awaiting. In all infrared-convergent terms the ordinary covariant summation can be used. However, in the infrared-divergent terms the longitudinal degree of freedom must be included. We can accomplish this by using the Coester representation [11] in which

$$\sum_\epsilon (a\cdot\epsilon)(b\cdot\epsilon) = \mathbf{a}\cdot\mathbf{b} - \frac{(\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\cdot\mathbf{k})}{w^2}, \quad (47)$$

where a and b are arbitrary four-vectors and

$$w^2 = k^2 + \lambda^2. \quad (48)$$

The expression for $d\Gamma_B^{\mathrm{I}}$ once the explicit separation of the infrared-divergent terms from the infrared-convergent ones is performed becomes

$$d\Gamma_B^{\mathrm{I}} = d\Omega\,\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2 A_2'\frac{1}{8\pi}d\eta\frac{d^3k}{w}\frac{d^3p_\nu}{E_\nu}\delta^4(p_1-p_2-l-p_\nu-k)\left[\frac{2p_1\cdot l}{(p_1\cdot k)(l\cdot k)}-\frac{m^2}{(l\cdot k)^2}-\frac{M_1^2}{(p_1\cdot k)^2}\right]-d\Omega\frac{p_2 l}{4\pi}dy\frac{d^3k}{w}\frac{d^3p_\nu}{E_\nu}$$
$$\times\delta^4(p_1-p_2-l-p_\nu-k)\left\{(D_3E_\nu^0-D_4E)\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2\frac{\eta}{2p_2}+\left[D_3\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2 l\left(y_0-\frac{\eta}{2p_2 l}\right)+D_4\hat{\mathbf{s}}_1\cdot\hat{\mathbf{k}}E\right]w\right\}\frac{\beta^2}{w^2}\frac{1-(\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}. \quad (49)$$

The infrared-divergent integrand is easily identified as

$$I_0(E,E_2,\mathbf{k},\mathbf{p}_\nu,\eta) = \frac{1}{8\pi}d\eta\frac{d^3k}{w}\frac{d^3p_\nu}{E_\nu}\delta^4(p_1-p_2-l-p_\nu-k)\left[\frac{2p_1\cdot l}{(p_1\cdot k)(l\cdot k)}-\frac{m^2}{(l\cdot k)^2}-\frac{M_1^2}{(p_1\cdot k)^2}\right]. \quad (50)$$

The invariant mass is now integrated from a small photon mass λ^2 to its maximum η_m given by $\eta_m = 2p_2 l(1+y_0)$. The second part in Eq. (49) is infrared convergent. The use of y as an integration variable in such expressions will allow us to apply Eq. (38) of Ref. [1]. Therefore, after performing the integration over the δ function we have

$$d\Gamma_B^{\mathrm{I}} = d\Omega \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 A_2' I_0(E,E_2) + d\Omega \frac{1}{2\pi} \frac{\beta^2 l}{2} \int_{-1}^{y_0} dy \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \left[D_3 \left(E_\nu^0 + \frac{p_2 l y}{D} \right) \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 - D_4 E \left(\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 + \frac{p_2}{D} \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{k}} \right) \right] \frac{1-(\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}{(1-\beta \hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}. \tag{51}$$

The infrared-divergent integral is given explicitly in Eq. (27) of Ref. [10]. With only minor changes, it can be adapted to our notation as

$$I_0(E,E_2) = \frac{1}{\beta} \operatorname{arctanh}\beta \left[2\ln\left(\frac{2l}{\lambda}\right) + \ln\left(\frac{m\eta_m^2}{4(E+l)r_+}\right) \right] - \frac{1}{\beta} L\left(-\frac{a^2}{4r_+}\right) + \frac{1}{\beta} L\left(-\frac{4r_-}{a^2}\right) - 2\ln\left(\frac{m}{\lambda}\right) - \ln\left(\frac{\eta_m^2}{2mE_\nu^0(q^2-m^2)}\right), \tag{52}$$

where

$$(E+l)r_\pm = [E_\nu^0 l^2 (q^2-m^2) - a^2E/4] \pm \{[E_\nu^0 l^2(q^2-m^2) - a^2E/4]^2 - m^2a^4/16\}^{1/2}, \tag{53}$$

with

$$a^2 = \eta_m(4p_2 l - \eta_m), \tag{54}$$

$$q^2 = M_1^2 - 2M_1E_2 + M_2^2. \tag{55}$$

The alternative approach to extract the infrared divergence and the finite terms that come along with it of Ref. [1] allows us to verify that

$$I_0(E,E_2) = \hat{I}_0(k/\lambda) + C + C_1. \tag{56}$$

Thus, both approaches lead to the same results. In the above expressions, we have put a hat on $I_0(k/\lambda)$ to avoid confusion.

On the other hand, $d\Gamma_B^{\mathrm{II}}$ can be computed with no difficulty due to the fact that the trace calculation is carried out with standard techniques. Thus

$$d\Gamma_B^{\mathrm{II}} = d\Omega \frac{p_2\beta}{4\pi} \int_{-1}^{y_0} dy \int_{-1}^{1} dx \int_0^{2\pi} d\varphi_k \frac{1}{D} \frac{1}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})} \left\{ \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{l}} \left[\left(w - \frac{m^2}{E} \frac{1}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})} + (1+\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})E \right) l D_4 - E_\nu l D_3 \right] + \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 \left[w - \frac{m^2}{E} \frac{1}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})} + (1+\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})E \right] p_2 D_4 + \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{k}} \left[\left(w - \frac{m^2}{E} \frac{1}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})} + (1+\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})E \right) w D_4 - \left(w - \frac{m^2}{E} \frac{1}{(1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})} + E \right) E_\nu D_3 \right] \right\}. \tag{57}$$

The infrared-convergent parts of Eqs. (37), (38), (51), and (57) can be cast into a very compact form, namely,

$$d\Gamma_B^{\mathrm{ic}} = d\Omega [D_1 C_A + D_2 C_B - \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 (D_3 C_C + D_4 C_D)]. \tag{58}$$

The coefficients C_A, C_B, C_C, and C_D are collected in Appendix D.

At this point we have reached our first final result. The DP of polarized decaying hyperons with radiative corrections to order α and in the approximation of neglecting terms of order $\alpha q/\pi M_1$ is obtained by adding Eqs. (51) and (57) to get the result for Eq. (43) and then by using Eq. (33) along with this Eq. (43) to form Eq. (32). Finally, the sum of Eqs. (15) and (32) gives the desired formula. The integrations over the three-momentum of the real photon in Eqs. (37), (38), (51), and (57) can be performed numerically.

It turns out that the remaining photon integrals can be performed analytically. This we shall do in the next section. This way a completely analytical result will be obtained. This will be our second final result.

IV. ANALYTICAL INTEGRATIONS

The integrals over the real photon three-momentum indicated in Eqs. (51) and (57) can be performed analytically. To achieve this it is convenient to separate them in groups. Most of them can be identified with analytical integrals obtained in

Ref. [1]. Others are new, but they can be identified with integrals performed in Ref. [6] or reduced to combinations of such integrals. In Appendix A we give a complete list of the integrations to be performed and explain how they can be handled.

There are certain types of integrals arising from the terms depending on the spin of the decaying hyperon whose form is

$$\int_{-1}^{1} dx \int_{0}^{2\pi} d\varphi_k \frac{\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{r}}}{[E_\nu^0 + (\mathbf{p}_2 + \mathbf{l}) \cdot \hat{\mathbf{k}}]^m [1 - \beta \hat{\mathbf{l}} \cdot \hat{\mathbf{k}}]^n}, \tag{59}$$

where $\hat{\mathbf{r}}$ may denote $\hat{\mathbf{l}}$, $\hat{\mathbf{p}}_2$, or $\hat{\mathbf{k}}$, and $m=1,2,3$ and $n=0,1,2$. It is important to remark that not all the possible combinations of m and n occur. The above integrals can be performed analytically by exploiting the symmetries exhibited in the integrands, i.e., their transformation properties under rotations. The right orientation of the coordinate axes will simplify our task enormously because most integrals can be reduced to the forms computed in Refs. [1,6], namely, to the functions θ_i ($i=0,\ldots,16$) defined there and reproduced in Appendix C of the present paper for the sake of completeness. Other integrals are simplified by integrating first over the angular variables of the photon; the last integration step is over y. There are five different types of these integrals, viz.,

$$\eta_0 = \int_{-1}^{y_0} dy, \tag{60}$$

$$\eta_1 = \int_{-1}^{y_0} dy \frac{1}{G(y)}, \tag{61}$$

$$\eta_{2+j} = \int_{-1}^{y_0} dy [G(y)]^{1/2-j} \ln\left[\frac{E_\nu^0 + [G(y)]^{1/2}}{E_\nu^0 - [G(y)]^{1/2}}\right], \tag{62}$$

where $j=0,1,2$ and

$$G(y) = E_\nu^{0^2} + 2p_2 l(y - y_0). \tag{63}$$

In Eq. (62) a subtle point arises. The integrand has a vertical asymptote at $y=y_0$. We therefore integrate from -1 to $y_0-\varepsilon$, thus obtaining a function of ε, and then find the limit of this function when $\varepsilon\to 0$. We will give the explicit results at the end of this section.

Because of space limitations, it is impossible to detail the full steps of integration. We will follow a straightforward procedure and give only the relevant results.

We first obtain

$$d\Gamma_B^{\mathrm{ir}} + d\Gamma_B^{a} = d\Omega[(D_1+D_2)\theta' + D_2\theta'' + A_1'\theta_1], \tag{64}$$

where

$$\theta' = \frac{p_2 l E}{2}[(1-\beta^2)\theta_2 - 2\theta_3 + \theta_4], \tag{65}$$

$$\theta'' = \frac{p_2 l}{2}[2\theta_0 - E(1-\beta^2)\theta_3 + E\theta_4 + l\theta_5]. \tag{66}$$

In the above expressions we have used the definitions

$$\theta_0 = \frac{1}{2}\beta^2(1+y_0)\int_{-1}^{1} dx \frac{1-x^2}{(1-\beta x)^2}, \tag{67}$$

$$\theta_1 = I_0(E, E_2). \tag{68}$$

$\theta_2,\ldots,\theta_5$ are given in Appendix C. D_1 and D_2 can be found in Appendix B.

Next we obtain

$$d\Gamma_B^{b} = d\Omega[(D_1+D_2)\theta''' + D_2\theta^{\mathrm{IV}}], \tag{69}$$

where

$$\theta''' = \frac{p_2 l}{2}\left[-(E_\nu^0+E)(1-\beta^2)\theta_2 + \left[\frac{3-\beta^2}{2}E + E_\nu^0\right]\theta_3 - \frac{E}{2}\theta_4 - \frac{l}{2}\theta_5 + \frac{1-\beta^2}{2}\theta_6 - \frac{2E-E_\nu^0}{2E}\theta_7 + \frac{1}{2}\theta_8 - \frac{1}{4E}\theta_9\right], \tag{70}$$

$$\theta^{\mathrm{IV}} = \frac{p_2 l}{2}[-\theta_0 - E_\nu^0(1-\beta x_0)\theta_3 + E_\nu^0\theta_4], \tag{71}$$

with

$$x_0 = -\frac{p_2 y_0 + l}{E_\nu^0}. \tag{72}$$

$\theta_6,\ldots,\theta_9$ are given in Appendix C.

Similarly we get

$$d\Gamma_B^{\mathrm{I}} = d\Omega \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 [A_2'\theta_1 + D_3\rho_1 + D_4\rho_2] \tag{73}$$

and

$$d\Gamma_B^{\mathrm{II}} = d\Omega \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 [D_3\rho_3 + D_4\rho_4], \tag{74}$$

where

$$\rho_1 = \frac{l}{2}[2E_\nu^0\theta_0 - \zeta_{10} + 2\zeta_{11} + (\beta^2-1)\zeta_{12}], \tag{75}$$

$$\rho_2 = \frac{E}{2}[-2l\theta_0 - \chi_{10} + 2\chi_{11} + (\beta^2-1)\chi_{12}], \tag{76}$$

$$\rho_3 = \frac{\beta}{4}[-2E_\nu^0\zeta_{11} + \zeta_{21}] + \frac{1}{2}\left[-E_\nu^0\chi_{11} + \frac{E-E_\nu^0}{2E}\chi_{21} + \frac{\chi_{31}}{4E} + (1-\beta^2)E_\nu^0\chi_{12} - \frac{1}{2}(1-\beta^2)\chi_{22}\right], \tag{77}$$

$$\rho_4 = \frac{\beta}{2}\left[-E\zeta_{10} + 2E\zeta_{11} + \frac{\zeta_{21}}{2} - \frac{m^2}{E}\zeta_{12}\right] + \frac{p_2^2\beta}{2}\gamma_0 + \frac{1}{4}\left[-\chi_{20} + 2\chi_{21} + \frac{\chi_{31}}{2E} - \frac{m^2}{E^2}\chi_{22}\right]. \tag{78}$$

The ρ_i depend on other functions, viz., ζ_i, χ_i, and γ_0. The first two types of functions come from the integrands containing $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}$ and $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{k}}$, respectively, and γ_0 contains all the terms directly proportional to $\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2$. Their explicit forms are

$$\zeta_{10}=\tfrac{1}{2}[\eta_2-(p_2^2+l^2)\eta_3], \tag{79}$$

$$\frac{\zeta_{11}}{E}=-(E+E_\nu^0)(\theta_3-\theta_4)+l\theta_5+p_2\beta\theta_{12}, \tag{80}$$

$$\frac{\zeta_{21}}{2E^2}=-[\beta(l-p_2y_0)+(\beta^2-3)E_\nu^0-3E]\theta_3+[\beta(l-p_2y_0)-3(E_\nu^0+E)]\theta_4-3\beta(E+E_\nu^0)\theta_5+\frac{p_2\beta y_0}{2E}\theta_7-3E\beta^2\theta_{10}-\frac{1}{4E^2}\theta_{16}, \tag{81}$$

$$\frac{\zeta_{12}}{E}=-(E+E_\nu^0)(\theta_2-\theta_3)+E(\theta_3-\theta_4)+p_2\beta\theta_{11}, \tag{82}$$

$$\frac{\chi_{10}}{l}=\eta_0+(p_2^2-l^2)\eta_1-\frac{E_\nu^0}{2}[\eta_3+(p_2^2-l^2)\eta_4], \tag{83}$$

$$\chi_{20}=-E_\nu^0 l[\eta_0+(p_2^2-l^2)\eta_1]-\frac{l}{2}\eta_2+l^2(p_2y_0+l)\eta_3+\frac{l}{2}E_\nu^{02}(p_2^2-l^2)\eta_4, \tag{84}$$

$$\frac{\chi_{11}}{E}=(1+y_0)\ln\left[\frac{1+\beta}{1-\beta}\right]-\beta(E+E_\nu^0)\theta_3+l\theta_4, \tag{85}$$

$$\frac{\chi_{21}}{2El}=(p_2y_0\beta+2E_\nu^0+2E)\theta_3-2(E+E_\nu^0)\theta_4-2l\theta_5-\frac{E+E_\nu^0}{2E}\theta_7+\frac{1}{2}\theta_8-p_2\beta\theta_{12}, \tag{86}$$

$$\frac{\chi_{31}}{El}=[6E(\beta^2-3)(E+E_\nu^0)-8p_2ly_0]\theta_3+[18E(E+E_\nu^0)+2l(4p_2y_0-3l)]\theta_4+18l(E+E_\nu^0)\theta_5-\frac{E+E_\nu^0}{E}\theta_9+18l^2\theta_{10}+\theta_{15}+\frac{1}{E}\theta_{16}, \tag{87}$$

$$\chi_{12}=\frac{2l\eta_0}{1-\beta^2}-l(E+E_\nu^0)\theta_2+El\theta_3, \tag{88}$$

$$\frac{\chi_{22}}{l}=[2p_2ly_0+4E(E+E_\nu^0)]\theta_2-4E(2E+E_\nu^0)\theta_3+4E^2\theta_4-(E+E_\nu^0)\theta_6+E\theta_7-2p_2l\theta_{11}, \tag{89}$$

$$\gamma_0=-\frac{m^2}{E}\theta_2+E\theta_3+\frac{1}{2}\theta_7. \tag{90}$$

$\theta_{10},\ldots,\theta_{16}$ are given in Appendix C. Let us recall that all the functions θ_i are computed in Refs. [1,6]. On the other hand, the functions η_i defined by Eqs. (60)–(62) are explicitly given by

$$\eta_0=1+y_0, \tag{91}$$

$$\eta_1=\frac{1}{2p_2l}\ln\left[\frac{E_\nu^{0^2}}{(p_2-l)^2}\right], \tag{92}$$

$$\eta_2=\frac{E_\nu^{0^3}}{3p_2l}\left\{1-\left(\frac{p_2-l}{E_\nu^0}\right)^2+\ln\left[\frac{4E_\nu^{0^2}}{E_\nu^{0^2}-(p_2-l)^2}\right]-\left(\frac{p_2-l}{E_\nu^0}\right)^3\ln\left[\frac{E_\nu^0+p_2-l}{E_\nu^0-p_2+l}\right]\right\}, \tag{93}$$

$$\eta_3=\frac{E_\nu^0}{p_2l}\left\{\ln\left[\frac{4E_\nu^{0^2}}{E_\nu^{0^2}-(p_2-l)^2}\right]-\frac{p_2-l}{E_\nu^0}\ln\left[\frac{E_\nu^0+p_2-l}{E_\nu^0-p_2+l}\right]\right\}, \tag{94}$$

$$\eta_4=\frac{1}{p_2lE_\nu^0}\left\{\ln\left[\frac{E_\nu^{0^2}-(p_2-l)^2}{4(p_2-l)^2}\right]+\frac{E_\nu^0}{p_2-l}\ln\left[\frac{E_\nu^0+p_2-l}{E_\nu^0-p_2+l}\right]\right\}. \tag{95}$$

We can now collect our partial results and the bremsstrahlung decay rate finally becomes

$$\begin{aligned}d\Gamma_B&=d\Gamma_B'-d\Gamma_B^{(s)}\\&=d\Omega\{(D_1+D_2)(\theta'+\theta''')+D_2(\theta''+\theta^{\rm IV})+A_1'\theta_1\\&\quad-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2[A_2'\theta_1+D_3(\rho_1+\rho_3)+D_4(\rho_2+\rho_4)]\},\end{aligned} \tag{96}$$

where $d\Omega$ is given in Eq. (35),

$$\theta'+\theta'''=\frac{p_2l}{2}\left[-E_\nu^0(1-\beta^2)\theta_2+\left(E_\nu^0-\frac{1+\beta^2}{2}E\right)\theta_3+\frac{E}{2}\theta_4-\frac{l}{2}\theta_5+\frac{1-\beta^2}{2}\theta_6-\frac{2E-E_\nu^0}{2E}\theta_7+\frac{1}{2}\theta_8-\frac{1}{4E}\theta_9\right], \tag{97}$$

$$\theta''+\theta^{\rm IV}=\frac{p_2l}{2}[\theta_0-(E+E_\nu^0+\beta p_2y_0)\theta_3+(E_\nu^0+E)\theta_4+l\theta_5], \tag{98}$$

and

$$\theta_0=2(1+y_0)\left[\frac{\operatorname{arctanh}\beta}{\beta}-1\right]. \tag{99}$$

All the integrals that remained indicated in Eqs. (51) and (57) have now been analytically performed; i.e., these two equations have the analytical counterparts given in Eqs. (73) and (74), respectively. Thus, the complete bremsstrahlung differential decay rate is collected in Eq. (96).

As a cross-check, we also integrated numerically Eqs. (37), (38), (51), and (57) over the photon variables and compared with the corresponding analytical results in Eqs. (64), (69), (73), and (74). All this is included in Appendix D. One can observe that both results coincide remarkably well. Let us now turn to our closing section.

V. FINAL RESULTS AND DISCUSSIONS

The differential decay rate of HSD's in the variables E and E_2, that is, the DP, with nonzero polarization of the initial hyperon including radiative corrections to order α, is given by

$$d\Gamma(A\to Bl\overline{\nu_l})=d\Gamma_V+d\Gamma_B. \qquad (100)$$

$d\Gamma_V$ is given by Eq. (15). For $d\Gamma_B$ we have two forms. In the first one the triple integration over the real photon variables remains to be performed numerically. It is given by the sum of Eqs. (37), (38), (51), and (57). The infrared divergence and the finite terms that accompany it have been explicitly and analytically extracted, however. The second form of $d\Gamma_B$ is completely analytical; all integrations over the photon variables have been explicitly performed. It is given by the sum of Eqs. (64), (69), (73), and (74).

Our analytical result can be cast into the compact form

$$d\Gamma(A\to Bl\overline{\nu_l})=\frac{G_V^2}{2}\frac{dE_2dEd\Omega_2}{(2\pi)^4}2M_1\left\{A_0'+\frac{\alpha}{\pi}\Phi_1-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2\times\left[A_0''+\frac{\alpha}{\pi}\Phi_2\right]\right\}, \qquad (101)$$

where

$$\Phi_1=A_1'(\phi+\theta_1)+A_1''\phi'+(D_1+D_2)(\theta'+\theta''')+D_2(\theta''+\theta^{\mathrm{IV}}), \qquad (102)$$

$$\Phi_2=A_2'(\phi+\theta_1)+A_2''\phi'+D_3(\rho_1+\rho_3)+D_4(\rho_2+\rho_4). \qquad (103)$$

A_0', A_1', ϕ, θ_0, A_1'', ϕ', D_1, D_2, $\theta'+\theta'''$, and $\theta''+\theta^{\mathrm{IV}}$ have been previously computed [1] and are given in Eqs. (16), (17), (8), (99), (18), (9), (B8), (B9), (97), and (98) in the present paper. A_0'', A_2', θ_1, A_2'', D_3, D_4, ρ_1, ρ_2, ρ_3, and ρ_4 are new expressions and are, respectively, given in Eqs. (19), (20), (68), (21), (B10), (B11), (75), (76), (77), and (78).

Despite its length, the analytical form of Eq. (101) is basically simple and organized in a way that is easy to handle. Its main usefulness lies in that it can be incorporated into a Monte Carlo simulation of an experimental analysis, reducing considerably the computational effort required by the triple integration pending in the first form of our result.

From the DP equation (101) we can proceed to obtain the total transition rate R and the spin-asymmetry coefficient of the emitted hyperon α_B. This last quantity is defined as

$$\alpha_B=2\frac{N(\theta_B<\pi/2)-N(\theta_B>\pi/2)}{N(\theta_B<\pi/2)+N(\theta_B>\pi/2)}, \qquad (104)$$

where $N(\theta_B<\pi/2)$ denotes the number of emitted hyperons with momenta in the forward hemisphere with respect to the polarization of the decaying hyperon, etc.

The uncorrected differential decay rate can be obtained immediately if the terms proportional to α/π are dropped from Eq. (101), i.e.,

$$d\Gamma^0=\frac{G_V^2}{2}\frac{dE_2dEd\Omega_2}{(2\pi)^4}2M_1[A_0'-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2A_0''], \qquad (105)$$

with A_0' and A_0'' given by Eqs. (16) and (19), respectively. When we integrate this expression over the kinematical variables restricted to the three-body region of the DP we obtain, for the uncorrected decay rate R^0,

$$R^0=\frac{G_V^2}{4\pi^3}M_1B_1 \qquad (106)$$

and, for the uncorrected angular spin-asymmetry coefficient of the emitted hyperon,

$$\alpha_B^0=-\frac{B_2}{B_1}, \qquad (107)$$

where

$$B_1=\int_m^{E_m}\int_{E_2^{\mathrm{min}}}^{E_2^{\mathrm{max}}}A_0'dE_2dE, \qquad (108)$$

$$B_2=\int_m^{E_m}\int_{E_2^{\mathrm{min}}}^{E_2^{\mathrm{max}}}A_0''dE_2dE. \qquad (109)$$

Within our approximations, the radiatively corrected integrated observables starting from Eq. (101), turn out to be

$$R=\frac{G_V^2}{4\pi^3}M_1\left[B_1+\frac{\alpha}{\pi}a_1\right] \qquad (110)$$

and

$$\alpha_B=-\frac{B_2+(\alpha/\pi)a_2}{B_1+(\alpha/\pi)\,a_1}, \qquad (111)$$

with

$$a_1=\int_m^{E_m}\int_{E_2^{\mathrm{min}}}^{E_2^{\mathrm{max}}}\Phi_1dE_2dE, \qquad (112)$$

$$a_2=\int_m^{E_m}\int_{E_2^{\mathrm{min}}}^{E_2^{\mathrm{max}}}\Phi_2dE_2dE. \qquad (113)$$

To our knowledge Eq. (101) is the only analytical expression available in the literature. There is, however, one previous paper [7] addressing the radiative corrections to the DP of polarized decaying hyperons. In order to make a detailed comparison we also need to produce numerical results. This is done in Appendix E. The main advantage of the closed analytical expression of the one of Sec. III, which is to be integrated numerically, is that in a Monte Carlo simulation it provides an enormous reduction of computer effort. The feeding of such analytical results into a computer is not worse than the feeding of the numerical results of Ref. [7]. Thus in practice, the use of the analytical results is of great advantage over the use of the one to be integrated numerically or the use of numerical tables.

A generalized practice in experimental setups is the application of kinematical cuts to the observed electron an emitted hyperon kinematical variables. They result in that only a region of points and not the full DP is accessible in an experiment. However, on each one point of the DP the photon momentum integration limits do depend on the values of $(E,\mathbf{l})$ and $(E_2,\mathbf{p}_2)$ of that point, i.e., $k_{\rm min}(E,\mathbf{l},E_2,\mathbf{p}_2)$ and $k_{\rm max}(E,\mathbf{l},E_2,\mathbf{p}_2)$. Thus, the common kinematical cuts are automatically taken into account in the integration limits of the emitted photons at each point. Therefore, our complete expression Eq. (101) is appropriate for Monte Carlo simulation.

Concerning the integrated observables the situation is different. In as much as experiments quote measurements of the decay rate and asymmetry coefficients, the effects of the particular cuts and biases of an experiment are already taken into account in the error bars of that experiment. However, the definition of these observables is theoretical and independent of the peculiarities and limitations of a specific experimental setup. Accordingly, the theoretical expression of those observables must be fully integrated, regardless of particular cuts and biases. This must be done so whether radiative corrections are included or not.

Let us stress that our results are model independent and are not compromised to any particular values of the different form factors. All of the model dependence of radiative corrections has been incorporated into f_1 and g_1 form factors, in our approximation of neglecting contributions of order $\alpha q/\pi M_1$. This is indicated by putting a prime on them. For nonheavy hyperons our results are reliable up to a precision of around 0.5%. This precision is useful for experiments involving several thousands of events. For experiments involving several hundreds of thousands of events or in decays involving charm or heavier quarks Eq. (101) gives a good first approximation. In these latter cases our results can be improved following the approach of this paper. This, however, involves a non-negligible extra effort. We shall attempt this elsewhere.

To conclude, let us remark that our results are valid both for neutral or charged decaying hyperons and whether the emitted charged lepton is an electron or a muon.

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APPENDIX A

Because of the inclusion of the polarization of the decaying hyperon, some integrals that have not been computed yet [1,6] appear. They are mainly of two kinds:

$$\int_{-1}^{y_0} dy[F(y)]^{m-1}\int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1\cdot\hat{\mathbf{k}}}{[E_\nu^0+(\mathbf{p}_2+\mathbf{l})\cdot\hat{\mathbf{k}}]^m[1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}]^n} \equiv 2\pi\frac{(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)}{p_2 l}\chi_{mn}, \tag{A1}$$

where $y=\hat{\mathbf{p}}_2\cdot\hat{\mathbf{l}}$ and $F(y)$ was defined in Eq. (42). S^2 stands for the surface of a unit sphere in three dimensions and $d\Omega_k$ is the element of solid angle of the photon. $m=1,2,3$ and $n=0,1,2$, discarding the values χ_{30} and χ_{32}.

The second kind is

$$\int_{-1}^{y_0} dy[F(y)]^{p-1}\int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1\cdot\hat{\mathbf{l}}}{[E_\nu^0+(\mathbf{p}_2+\mathbf{l})\cdot\hat{\mathbf{k}}]^p[1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}]^q} \equiv 2\pi\frac{(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)}{p_2 l}\zeta_{pq}, \tag{A2}$$

with $p=1,2$ and $q=0,1,2$, without considering the values ζ_{20} and ζ_{22}.

Let us notice that the integrand in all these integrals remains a scalar under rotations of the coordinates. Therefore, we may choose the coordinate systems to compute the integrations in which the integrand acquires the simplest form.

First, let us consider the particular case

$$I_1=\int_{-1}^{y_0} dy\int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1\cdot\hat{\mathbf{k}}}{E_\nu^0+(\mathbf{p}_2+\mathbf{l})\cdot\hat{\mathbf{k}}}. \tag{A3}$$

After integrating over $d\Omega_k$ we get

$$I_1=2\pi\int_{-1}^{y_0} dy\frac{\hat{\mathbf{s}}_1\cdot(\mathbf{p}_2+\mathbf{l})}{G'}\left\{2-\frac{E_\nu^0}{\sqrt{G'}}\ln\left[\frac{E_\nu^0+\sqrt{G'}}{E_\nu^0-\sqrt{G'}}\right]\right\}, \tag{A4}$$

where

$$G'=p_2^2+l^2+2\mathbf{p}_2\cdot\mathbf{l}. \tag{A5}$$

If we return to our initial system, G' reduces to $G(y)$ defined by Eq. (63). The integral over $d\varphi_l$ is trivial; it amounts to an overall factor of 2π. Thus,

$$I_1=2\pi(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)\int_{-1}^{y_0} dy\frac{p_2+ly}{G(y)}\left\{2-\frac{E_\nu^0}{\sqrt{G(y)}}\times\ln\left[\frac{E_\nu^0+\sqrt{G(y)}}{E_\nu^0-\sqrt{G(y)}}\right]\right\} \equiv 2\pi\frac{(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)}{p_2 l}\chi_{10}, \tag{A6}$$

TABLE I. Values of C_A in $\Sigma^- \rightarrow n e \nu$ decay by (a) integrating numerically and (b) using the corresponding analytical expressions. The dimensions of C_A are GeV^2.

σ					(a)					
0.8078	0.0545	0.0593	0.0429	0.0186	-0.0072	-0.0303	-0.0469	-0.0537	-0.0472	-0.0227
0.8036		0.1053	0.0739	0.0340	-0.0056	-0.0383	-0.0589	-0.0630	-0.0464	-0.0064
0.7994		0.1162	0.0833	0.0420	0.0020	-0.0297	-0.0478	-0.0482	-0.0276	
0.7952			0.0899	0.0486	0.0096	-0.0200	-0.0351	-0.0317	-0.0069	
0.7909				0.0531	0.0159	-0.0106	-0.0216	-0.0133		
0.7867					0.0191	-0.0022	-0.0063			
					(b)					
0.8078	0.0544	0.0592	0.0428	0.0185	-0.0074	-0.0304	-0.0470	-0.0538	-0.0474	-0.0228
0.8036		0.1049	0.0729	0.0327	-0.0068	-0.0393	-0.0600	-0.0644	-0.0482	-0.0064
0.7994		0.1163	0.0828	0.0414	0.0015	-0.0303	-0.0489	-0.0496	-0.0283	
0.7952			0.0906	0.0492	0.0102	-0.0196	-0.0351	-0.0317	-0.0062	
0.7909				0.0554	0.0182	-0.0088	-0.0201	-0.0119		
0.7867					0.0229	0.0010	-0.0041			
δ	0.0500	0.1500	0.2500	0.3500	0.4500	0.5500	0.6500	0.7500	0.8500	0.9500
$\sigma^{\max}$	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078
$\sigma^{\min}$	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

with χ_{10} defined by Eq. (83).

A similar procedure can be followed to compute

$$I_2 = \int_{-1}^{y_0} dy F(y) \int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{k}}}{[E_\nu^0 + (\mathbf{p}_2 + \mathbf{l}) \cdot \hat{\mathbf{k}}]^2} \equiv 2\pi \frac{(\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2)}{p_2 l} \chi_{20} \tag{A7}$$

and so on.

It is important to remark that with this approach we are able to improve two previous results. First, the value of η_3, given by Eq. (94), corresponds to θ_4 given by Eq. (102) of Ref. [1]. Thus, the former value supersedes the latter. A further simplification is

$$\theta_8 - 2l\theta_5 = 2(1 + y_0). \tag{A8}$$

The above results may help reduce the numerical computational effort involved.

On the other hand, the other integrals can be computed with no difficulty if we consider the most general case

$$J_1(\alpha_1, \alpha_2) = \int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{k}}}{[E_\nu^0 \alpha_1 + (\mathbf{p}_2 + \mathbf{l}) \cdot \hat{\mathbf{k}}][\alpha_2 - \beta \hat{\mathbf{l}} \cdot \hat{\mathbf{k}}]}, \tag{A9}$$

where α_1 and α_2 are real parameters ranging from 0 to 1. Observe that

$$\frac{\partial}{\partial \alpha_1} J_1(\alpha_1, \alpha_2) = -E_\nu^0 \int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{k}}}{[E_\nu^0 \alpha_1 + (\mathbf{p}_2 + \mathbf{l}) \cdot \hat{\mathbf{k}}]^2 [\alpha_2 - \beta \hat{\mathbf{l}} \cdot \hat{\mathbf{k}}]} \tag{A10}$$

and

$$\frac{\partial}{\partial \alpha_2} J_1(\alpha_1, \alpha_2) = -\int_{S^2} d\Omega_k \frac{\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{k}}}{[E_\nu^0 \alpha_1 + (\mathbf{p}_2 + \mathbf{l}) \cdot \hat{\mathbf{k}}][\alpha_2 - \beta \hat{\mathbf{l}} \cdot \hat{\mathbf{k}}]^2} \tag{A11}$$

and so forth. Therefore, the remaining integrals can be obtained by differentiating with respect either to α_1 or to α_2, and setting both of them equal 1. At this point we can save a considerable amount of work if we use the previous results presented in Refs. [1,6].

APPENDIX B

The coefficients Q_i and D_j involved in Eqs. (16)–(21) are quadratic functions of the form factors. We shall repeat the explicit forms of Ref. [1] for completeness,

TABLE II. Values of C_B in $\Sigma^- \to n e \nu$ decay by (a) integrating numerically and (b) using the corresponding analytical expressions. The dimensions of C_B are GeV^2.

σ					(a)					
0.8078	0.0545	0.0595	0.0432	0.0191	-0.0067	-0.0297	-0.0463	-0.0531	-0.0466	-0.0223
0.8036		0.0825	0.0630	0.0305	-0.0042	-0.0338	-0.0527	-0.0562	-0.0408	-0.0053
0.7994		0.0604	0.0529	0.0273	-0.0022	-0.0272	-0.0416	-0.0410	-0.0228	
0.7952			0.0392	0.0224	-0.0006	-0.0201	-0.0299	-0.0257	-0.0054	
0.7909				0.0177	0.0017	-0.0124	-0.0177	-0.0101		
0.7867					0.0052	-0.0035	-0.0044			
					(b)					
0.8078	0.0545	0.0595	0.0432	0.0190	-0.0068	-0.0299	-0.0465	-0.0533	-0.0468	-0.0224
0.8036		0.0821	0.0622	0.0294	-0.0052	-0.0347	-0.0537	-0.0578	-0.0428	-0.0054
0.7994		0.0607	0.0529	0.0271	-0.0026	-0.0281	-0.0433	-0.0433	-0.0242	
0.7952			0.0405	0.0232	-0.0004	-0.0209	-0.0315	-0.0272	-0.0050	
0.7909				0.0195	0.0023	-0.0127	-0.0184	-0.0098		
0.7867					0.0067	-0.0027	-0.0034			
δ	0.0500	0.1500	0.2500	0.3500	0.4500	0.5500	0.6500	0.7500	0.8500	0.9500
$\sigma^{\max}$	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078
$\sigma^{\min}$	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

$$Q_1 = F_1^2\left[\frac{2E_2 - M_2}{M_1}\right] + \frac{1}{2}F_2^2\left[\frac{M_2+E_2}{M_1}\right] + F_1F_2\left[\frac{M_2+E_2}{M_1}\right] + F_1F_3\left[1+\frac{M_2}{M_1}\right]\left[1-\frac{E_2}{M_1}\right] + F_2F_3\left[\frac{M_2+E_2}{M_1}\right]\left[1-\frac{E_2}{M_1}\right]$$
$$+ G_1^2\left[\frac{2E_2+M_2}{M_1}\right] - \frac{1}{2}G_2^2\left[\frac{M_2-E_2}{M_1}\right] + G_1G_2\left[\frac{M_2-E_2}{M_1}\right] + G_1G_3\left[\frac{M_2}{M_1}-1\right]\left[1-\frac{E_2}{M_1}\right] - G_2G_3\left[\frac{M_2-E_2}{M_1}\right]\left[1-\frac{E_2}{M_1}\right]$$
$$+ M_1^2 Q_5\left\{\left[\frac{M_1-E_2}{M_1}\right]^2 - \frac{1}{2}\frac{q^2}{M_1{}^2}\right\}, \tag{B1}$$

$$Q_2 = -\frac{F_1^2}{M_1} - \frac{G_1^2}{M_1} - \frac{F_1F_2}{M_1} + \frac{G_1G_2}{M_1} + \frac{F_1F_3}{M_1}\left[1+\frac{M_2}{M_1}\right] + \frac{F_2F_3}{M_1}\left[\frac{M_2+E_2}{M_1}\right] + \frac{G_1G_3}{M_1}\left[\frac{M_2}{M_1}-1\right] - \frac{G_2G_3}{M_1}\left[\frac{M_2-E_2}{M_1}\right] + 2\frac{F_1G_1}{M_1}$$
$$+ M_1 Q_5\left[\frac{M_1-E_2}{M_1}\right], \tag{B2}$$

$$Q_3 = Q_1 - 2F_1^2\left[\frac{E_2-M_2}{M_1}\right] - 2G_1^2\left[\frac{E_2+M_2}{M_1}\right] - M_1^2 Q_5\left\{\left[1-\frac{E_2}{M_1}\right]^2 - \frac{q^2}{M_1^2}\right\}, \tag{B3}$$

$$Q_4 = Q_2 - 4\frac{F_1G_1}{M_1}, \tag{B4}$$

$$Q_5 = \frac{F_3^2}{M_1^2}\left[\frac{M_2+E_2}{M_1}\right] - \frac{G_3^2}{M_1^2}\left[\frac{M_2-E_2}{M_1}\right] - 2\frac{F_1F_3}{M_1^2} + 2\frac{G_1G_3}{M_1^2}. \tag{B5}$$

The new coefficients are

TABLE III. Values of C_C in $\Sigma^- \to n e \nu$ decay by (a) integrating numerically and (b) using the corresponding analytical expressions. The dimensions of C_C are GeV^2.

σ										
					(a)					
0.8078	0.0547	0.0597	0.0434	0.0191	-0.0068	-0.0300	-0.0467	-0.0535	-0.0471	-0.0227
0.8036		0.1197	0.0972	0.0582	0.0156	-0.0221	-0.0483	-0.0573	-0.0445	-0.0063
0.7994		0.1436	0.1336	0.0951	0.0480	0.0047	-0.0261	-0.0377	-0.0249	
0.7952			0.1770	0.1399	0.0869	0.0356	-0.0022	-0.0180	-0.0056	
0.7909				0.2039	0.1388	0.0721	0.0212	-0.0020		
0.7867					0.2426	0.1236	0.0309			
					(b)					
0.8078	0.0547	0.0597	0.0434	0.0191	-0.0069	-0.0300	-0.0467	-0.0536	-0.0471	-0.0226
0.8036		0.1196	0.0971	0.0580	0.0157	-0.0217	-0.0478	-0.0575	-0.0457	-0.0069
0.7994		0.1432	0.1329	0.0946	0.0480	0.0050	-0.0262	-0.0387	-0.0263	
0.7952			0.1756	0.1391	0.0872	0.0362	-0.0021	-0.0188	-0.0061	
0.7909				0.2032	0.1398	0.0738	0.0222	-0.0021		
0.7867					0.2456	0.1273	0.0325			
δ	0.0500	0.1500	0.2500	0.3500	0.4500	0.5500	0.6500	0.7500	0.8500	0.9500
$\sigma^{\rm max}$	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078
$\sigma^{\rm min}$	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

$$Q_6=F_1^2\left[\frac{E_2-M_2}{M_1}-\frac{p_2\beta y_0}{M_1}\right]+G_1^2\left[\frac{E_2+M_2}{M_1}-\frac{p_2\beta y_0}{M_1}\right]+2F_1G_1\left[\frac{E_2-p_2\beta y_0}{M_1}\right]+(G_1G_2-F_1F_2)\left[\frac{p_2\beta y_0}{M_1}\right]+F_2G_2\left[-1+(1+\beta^2)\frac{E}{M_1}+\frac{E_2}{M_1}+\frac{p_2\beta y_0}{M_1}\right]+F_1G_2\left[-1+\frac{M_2}{M_1}+(1+\beta^2)\frac{E}{M_1}+\frac{p_2\beta y_0}{M_1}\right]-G_1F_2\left[-1-\frac{M_2}{M_1}+(1+\beta^2)\frac{E}{M_1}+\frac{p_2\beta y_0}{M_1}\right]-F_3G_3\left[\frac{m^2}{M_1^2}\left(1-\frac{E_2}{M_1}-(1-\beta^2)\frac{E}{M_1}+\frac{p_2\beta y_0}{M_1}\right)\right]+F_1G_3\left[\frac{m^2}{M_1E}\left(-1+\frac{M_2}{M_1}+\frac{E}{M_1}\right)\right]-F_3G_1\left[\frac{m^2}{M_1E}\left(-1-\frac{M_2}{M_1}+\frac{E}{M_1}\right)\right]-(F_2G_3+F_3G_2)\left[\frac{m^2}{M_1E}\left(\frac{M_1-E_2-E}{M_1}\right)\right], \tag{B6}$$

$$Q_7=F_1^2\left[\frac{(M_1+M_2)(E_2-M_2)}{M_1E}\right]+G_1^2\left[\frac{(M_1-M_2)(E_2+M_2)}{M_1E}\right]+2F_1G_1\left[\frac{M_1(-M_1+E_2+2E)-m^2}{M_1E}\right]+F_1G_2\left(\frac{E_2-M_2}{M_1}\right)\times\left(\frac{M_1-2E-E_2}{E}\right)-G_1F_2\left(\frac{E_2+M_2}{M_1}\right)\left(\frac{M_1-2E-E_2}{E}\right)+F_3G_1\left(\frac{E_2+M_2}{M_1}\right)\left(\frac{m^2}{M_1E}\right)-G_3F_1\left(\frac{E_2-M_2}{M_1}\right)\left(\frac{m^2}{M_1E}\right)+(F_1F_2-G_1G_2)\left(\frac{E_2^2-M_2^2}{M_1E}\right). \tag{B7}$$

The D_1 and D_2 coefficients of Eqs. (17) and (18) are [1]

$$D_1=f_1'^2+3g_1'^2, \tag{B8}$$

$$D_2=f_1'^2-g_1'^2. \tag{B9}$$

and the new ones, D_3 and D_4, introduced in Eqs. (20) and (21), are

$$D_3=2(-g_1'^2+f_1'g_1'), \tag{B10}$$

$$D_4=2(g_1'^2+f_1'g_1'). \tag{B11}$$

In the above equations we have used the definitions

$$F_1=f_1'+[1+M_2/M_1]f_2,\quad G_1=g_1'-[1-M_2/M_1]g_2,$$

$$F_2=-2f_2,\quad G_2=-2g_2,$$

$$F_3=f_2+f_3,\quad G_3=g_2+g_3,$$

and

$$\beta = \frac{l}{E}.$$

APPENDIX C

In order to make this paper self-contained, we shall give in this appendix the different functions θ_i that were computed in Refs. [1,6]. They are very long, but they have been organized as

$$\theta_i = \frac{1}{p_2}(T_i^+ + T_i^-), \tag{C1}$$

where $i=2, \ldots, 16$, and

$$T_2^\pm = \pm \frac{1 \mp a^\pm}{(1\pm\beta)(1+\beta a^\pm)} \ln\left[\frac{1\mp\beta}{1-\beta x_0}\right] \pm \frac{(1\pm x_0)\ln(1\pm x_0)}{(1\pm\beta)(1-\beta x_0)} \pm \frac{1\pm a^\pm}{(1\mp\beta)(1+\beta a^\pm)}\ln(1\pm a^\pm) - \frac{(x_0+a^\pm)\ln(\pm x_0 \pm a^\pm)}{(1+\beta a^\pm)(1-\beta x_0)}, \tag{C2}$$

$$T_3^+ = T_3^- = \frac{1}{2\beta}\left\{ L\left[\frac{1-\beta}{1-\beta x_0}\right] - L\left[\frac{1-\beta x_0}{1+\beta}\right] - L\left[\frac{1+\beta a^-}{1-\beta x_0}\right] + L\left[\frac{1+\beta a^-}{1+\beta}\right] + L\left[\frac{1-\beta x_0}{1+\beta a^+}\right] - L\left[\frac{1-\beta}{1+\beta a^+}\right] + \ln\left[\frac{1-\beta x_0}{1-\beta}\right]\ln\left[\frac{1+\beta a^+}{1+\beta}\right]\right\}, \tag{C3}$$

$$T_4^\pm = (x_0\pm1)\ln(1\pm x_0) \pm (1\pm a^\pm)\ln(1\pm a^\pm) - (x_0+a^\pm)\ln(\pm x_0 \pm a^\pm), \tag{C4}$$

$$T_5^\pm = -\frac{1}{2}\{(1-x_0^2)\ln(1\pm x_0) + (x_0\mp1)a^\pm + 1 - (1-a^{\pm 2})\ln(1\pm a^\pm) + (x_0^2 - a^{\pm 2})\ln[\pm(x_0+a^\pm)]\}, \tag{C5}$$

$$T_6^\mp = \left[-l + p_2 \pm \frac{\beta E_\nu^0(x_0+a^\mp)}{1+\beta a^\mp}\right] I_4 \pm \frac{\beta E_\nu^0 (x_0+a^\mp)}{(1+\beta a^\mp)^2} I_1 + \left[E_\nu^0 - \frac{\beta E_\nu^0(x_0+a^\mp)}{1+\beta a^\mp}\right] J_4 - \frac{\beta E_\nu^0(x_0+a^\mp)}{(1+\beta a^\mp)^2} J_1 \pm \frac{E_\nu^0 (x_0+a^\mp)}{(1+\beta a^\mp)^2} I_2^\mp - \frac{E_\nu^0(x_0+a^\mp)}{(1+\beta a^\mp)^2} J_2^\mp, \tag{C6}$$

$$T_7^\pm = \left[p_2 - l \mp \frac{\beta E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm}\right] I_1 \mp \frac{E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm} I_2^\pm + \left[E_\nu^0 - \frac{\beta E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm}\right] J_1 - \frac{E_\nu^0(x_0+a^\pm)}{1+\beta a^\pm} J_2^\pm, \tag{C7}$$

$$T_8^\pm = -2(l - p_2 + E_\nu^0 x_0) \mp E_\nu^0 (x_0+a^\pm) I_2^\pm - E_\nu^0(x_0+a^\pm) J_2^\pm, \tag{C8}$$

$$\frac{T_9^\pm}{4l^2} = -\frac{3E}{2l^2}(l-p_2+E_\nu^0 x_0) + \left[\frac{3(l-p_2)}{4\beta l} + \frac{3E_\nu^0 p_2}{4l^2} + \beta G^\pm\right] I_1 \mp \frac{E_\nu^{0^2}(x_0+a^\pm)^2}{4l^2(1+\beta a^\pm)} I_3^\pm - \frac{E_\nu^{0^2}(x_0+a^\pm)^2}{4l^2(1+\beta a^\pm)} J_3{}^\pm + G^\pm I_2^\pm + \left[-\frac{3E_\nu^0}{4\beta l} + \frac{3E_\nu^0(E_\nu^0 + l x_0)}{4l^2} \pm \beta G^\pm\right] J_1 \pm G^\pm J_2^\pm, \tag{C9}$$

$$T_{10}^\mp = \tfrac{1}{3}(x_0^3 \mp 1)\ln(1\mp x_0) + \tfrac{1}{3}[(a^\mp)^3 \mp 1]\ln(1\mp a^\mp) - \tfrac{1}{3}[x_0^3 + (a^\mp)^3]\ln[\mp(x_0+a^\mp)] + \tfrac{1}{6}(1-x_0^2)(a^\mp \pm 1) - \tfrac{1}{3}(x_0\pm1)[1-(a^\mp)^2], \tag{C10}$$

$$T_{11}^+ = T_{11}^- = \frac{1}{2p_2\beta}\{E_\nu^0[(1-\beta x_0)J_4 - J_1] - (\beta E_\nu^0 + l - p_2) I_4 + (l-p_2) I_1\}, \tag{C11}$$

$$T_{12}^+ = T_{12}^- = \frac{1}{2p_2\beta}[E_\nu^0(1-\beta x_0)J_1 + 2E_\nu^0 x_0 + 2(l-p_2) - (\beta E_\nu^0 + l - p_2) I_1], \tag{C12}$$

$$T_{13}^+ = T_{13}^- = -\frac{1}{2p_2}E_\nu^0(1-x_0^2), \tag{C13}$$

$$T_{14}^\pm = E_\nu^0[1 + x_0^2 + 2a^\pm(x_0\mp1) \pm a^\pm(x_0+a^\pm)(I_2^\pm \pm J_2^\pm)], \tag{C14}$$

$$T_{15}^\pm = 3E_\nu^0[2p_2(1+y_0) + l(1-x_0^2)] - (E_\nu^0)^2(x_0+a^\pm)^2 \times (J_3^\pm \pm I_3^\pm) - 2lE_\nu^0(x_0+a^\pm)a^\pm(J_2^\pm \pm I_2^\pm), \tag{C15}$$

$$T_{16}^\pm = 4l^2\left[\frac{3}{2\beta^2}[2(l-p_2+E_\nu^0 x_0) + \beta E_\nu^0(1-x_0^2)] + \left(-\frac{3(l-p_2+\beta E_\nu^0)}{2\beta^2} - p_2(1+y_0) + \frac{p_2(E_\nu^0)^2}{2l^2}\right) I_1 - \frac{(E_\nu{}^0)^2(x_0+a^\pm)^2}{2l(1+\beta a^\pm)}(\beta J_1 + J_2^\pm \pm \beta I_1 \pm I_2^\pm) + \left(\frac{3E_\nu^0(1-\beta x_0)}{2\beta^2} + \frac{(E_\nu^0)^2(E_\nu^0 + l x_0)}{2l^2}\right) J_1\right]. \tag{C16}$$

The following definitions are used in these expressions:

$$x_0 = -\frac{p_2 y_0 + l}{E_\nu^0}, \quad a^\pm = \frac{E_\nu^0 \pm p_2}{l},$$

and

$$G^\pm = \mp \frac{\beta E_\nu^{0^2}(x_0+a^\pm)^2}{4l^2(1+\beta a^\pm)^2} \mp \frac{a^\pm(a^{\pm 2}-1)}{4(1+\beta a^\pm)},$$

TABLE IV. Values of C_D in $\Sigma^- \to n e \nu$ decay by (a) integrating numerically and (b) using the corresponding analytical expressions. The dimensions of C_D are GeV^2.

σ					(a)					
0.8078	0.0544	0.0591	0.0427	0.0184	-0.0074	-0.0304	-0.0469	-0.0536	-0.0470	-0.0224
0.8036		0.0894	0.0535	0.0101	-0.0312	-0.0636	-0.0814	-0.0799	-0.0554	-0.0070
0.7994		0.0834	0.0398	-0.0097	-0.0539	-0.0846	-0.0956	-0.0825	-0.0427	
0.7952			0.0230	-0.0330	-0.0791	-0.1058	-0.1065	-0.0767	-0.0146	
0.7909				-0.0619	-0.1098	-0.1285	-0.1101	-0.0498		
0.7867					-0.1646	-0.1566	-0.0765			
					(b)					
0.8078	0.0544	0.0591	0.0427	0.0184	-0.0074	-0.0304	-0.0469	-0.0536	-0.0470	-0.0223
0.8036		0.0894	0.0534	0.0103	-0.0306	-0.0627	-0.0808	-0.0802	-0.0570	-0.0077
0.7994		0.0833	0.0399	-0.0092	-0.0531	-0.0843	-0.0966	-0.0851	-0.0455	
0.7952			0.0233	-0.0324	-0.0787	-0.1067	-0.1094	-0.0807	-0.0159	
0.7909				-0.0613	-0.1103	-0.1312	-0.1146	-0.0531		
0.7867					-0.1666	-0.1612	-0.0804			
δ	0.0500	0.1500	0.2500	0.3500	0.4500	0.5500	0.6500	0.7500	0.8500	0.9500
$\sigma^{\rm max}$	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078	0.8078
$\sigma^{\rm min}$	0.8043	0.7978	0.7925	0.7884	0.7857	0.7847	0.7854	0.7884	0.7939	0.8023

$$I_1 = \frac{2}{\beta} \text{arctanh}\beta,$$

$$I_2^{\pm} = \ln\left| \frac{a^{\pm}+1}{a^{\pm}-1} \right|,$$

$$I_3^{\pm} = \frac{2}{a^{\pm 2}-1},$$

$$I_4 = \frac{2}{1-\beta^2},$$

$$J_1 = -\frac{1}{\beta}\left\{ \ln\left[\frac{1+\beta}{1-\beta x_0}\right] + \ln\left[\frac{1-\beta}{1-\beta x_0}\right]\right\},$$

$$J_2^{\pm} = \ln\left|\frac{a^{\pm}-1}{a^{\pm}+x_0}\right| + \ln\left|\frac{a^{\pm}+1}{a^{\pm}+x_0}\right|,$$

$$J_3^{\pm} = -2\left[\frac{a^{\pm}}{a^{\pm 2}-1} - \frac{1}{a^{\pm}+x_0}\right],$$

$$J_4 = \frac{2}{\beta}\left[\frac{1}{1-\beta^2} - \frac{1}{1-\beta x_0}\right].$$

APPENDIX D

In order to be sure that our calculations are correct, we have cross-checked our results by performing numerically the triple integrals involved in Eqs. (37), (38), (51), and (57) and then comparing these results with their analytical counterparts in Eqs. (64), (69), (73), and (74).

From the former set of equations, the infrared-convergent part of the bremsstrahlung decay rate can be rewritten in a more convenient way as

$$d\Gamma_B^{\rm ic} = d\Omega[D_1 C_A + D_2 C_B - \hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 (D_3 C_C + D_4 C_D)], \tag{D1}$$

where $d\Omega$ is given by Eq. (35); $D_1, \ldots, D_4$ depend on $f_1'(0)$ and $g_1'(0)$ and are given by Eqs. (B8), (B9), (B10), and (B11), respectively. C_A, C_B, C_C, and C_D are easily extracted from that set of equations: namely,

$$C_A = \frac{p_2 l}{2\pi}\int_{-1}^{1} dx \int_{-1}^{y_0} dy \int_0^{2\pi} d\varphi_k \frac{1}{2ED(1-\beta x)}\left\{ -l^2 \frac{1-x^2}{1-\beta x} + E_\nu\left[w + (1+\beta x)E - \frac{m^2}{E}\,\frac{1}{1-\beta x}\right]\right\}, \tag{D2}$$

$$C_B = \frac{p_2 l}{2\pi}\int_{-1}^{1} dx \int_{-1}^{y_0} dy \int_0^{2\pi} d\varphi_k \frac{1}{2ED(1-\beta x)} \times\left\{ -\beta l^2 \frac{x(1-x^2)}{1-\beta x} + \mathbf{p}_\nu \cdot \left[\hat{\mathbf{k}}\left(E + w - \frac{m^2}{E}\,\frac{1}{1-\beta x}\right) + \mathbf{l}\right]\right\}, \tag{D3}$$

TABLE V. Numerical values of the integrated model-independent radiative corrections to two hyperon semileptonic decays. The dimensions of Φ_{Di} are GeV^4.

Decay	Φ_{D1}	Φ_{D2}	Φ_{D3}	Φ_{D4}
$\Sigma^-\to ne\nu$	-2.04 $\times 10^{-4}$	1.08 $\times 10^{-4}$	-4.51 $\times 10^{-5}$	-1.08 $\times 10^{-4}$
$\Lambda\to pe\nu$	5.68 $\times 10^{-5}$	8.44 $\times 10^{-6}$	4.42 $\times 10^{-5}$	3.54 $\times 10^{-5}$

$$\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2 C_C=\frac{\beta p_2}{4\pi}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{1}{1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}}\times\left\{\beta l\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2\left[\frac{E_\nu^0}{p_2}+\frac{ly}{D}\right]\frac{1-(\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}{1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}}-\frac{E_\nu}{D}\left[\hat{\mathbf{s}}_1\cdot\mathbf{l}+\hat{\mathbf{s}}_1\cdot\mathbf{k}\left(1-\frac{m^2}{Ew}\frac{1}{1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}}+\frac{E}{w}\right)\right]\right\}, \tag{D4}$$

$$\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2 C_D=\frac{\beta}{4\pi}\int_{-1}^{y_0}dy\int_{-1}^{1}dx\int_0^{2\pi}d\varphi_k\frac{1}{1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}}\left\{l^2\left[-\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2+\hat{\mathbf{s}}_1\cdot\hat{\mathbf{k}}\frac{p_2}{D}\right]\frac{1-(\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})^2}{1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}}-\frac{p_2}{D}\hat{\mathbf{s}}_1\cdot\mathbf{p}_\nu\times\left[w-\frac{m^2}{E}\frac{1}{1-\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}}}+(1+\beta\hat{\mathbf{l}}\cdot\hat{\mathbf{k}})E\right]\right\}, \tag{D5}$$

where E_ν, E_ν^0, w, F, D, and y_0 are given by Eqs. (39), (23), (41), (42), (40), and (22), respectively.

In order to handle the scalar products containing $\hat{\mathbf{s}}_1$ on the right-hand side of Eqs. (D4) and (D5), one can follow the procedure of Ref. [7] and make the replacements

$$\hat{\mathbf{s}}_1\cdot\mathbf{p}\to(\hat{\mathbf{s}}_1\cdot\hat{\mathbf{p}}_2)(\mathbf{p}\cdot\hat{\mathbf{p}}_2)\quad(\mathbf{p}=\mathbf{l},\mathbf{p}_2,\mathbf{k},\mathbf{p}_\nu) \tag{D6}$$

or one can follow the approach discussed in the present paper. Both procedures lead to the same results.

From Eq. (96) the analytical counterparts of C_A, C_B, C_C, and C_D are

$$C_A=\theta'+\theta''', \tag{D7}$$

$$C_B=\theta'+\theta'''+\theta''+\theta^{\rm IV}, \tag{D8}$$

$$C_C=\rho_1+\rho_3, \tag{D9}$$

$$C_D=\rho_2+\rho_4, \tag{D10}$$

where $\theta'+\theta'''$, $\theta''+\theta^{\rm IV}$, ρ_1, ρ_2, ρ_3, and ρ_4 are defined in Eqs. (97), (98), (75), (76), (77), and (78).

The evaluation of such coefficients is presented in Tables I, II, III, and IV respectively. In those tables, the dimensionless quantities δ and σ are defined as

$$\delta=\frac{E}{E_m},\quad \sigma=\frac{E_2}{M_1},$$

where E_m is given by Eq. (29); $\sigma^{\rm max}$ and $\sigma^{\rm min}$ denote the maximum and minimum values of σ within the three-body region of the DP. For this comparison we shall work with the $\Sigma^-\to ne\nu$ decay for definiteness.

APPENDIX E

The approach implemented in Ref. [7] to compute the radiative corrections to the DP of polarized decaying hyperons differs from ours mainly in two aspects.

First, in that reference it was assumed that real photons are emitted by pointlike hadrons. This introduces model-dependent uncertainties of the order $\alpha q/\pi M_1$. This fact makes those results unreliable beyond $\alpha q/\pi M_1$, just as ours.

Second, the dependence on form factors of the radiative corrections is handled in that reference by fixing the form factors. Unfortunately, if the CVC and PCAC hypotheses are used to fix them, these assumptions are reliable in $\Delta S=0$ decays; they are affected by SU(3) breaking in $\Delta S\neq 0$, and are completely unreliable in charm decays. Contrarily, our approach is not compromised to particular values of the several form factors.

Now, to make a comparison with Ref. [7] we need to produce some numerical results.

From Eqs. (112) and (113), let

$$a_1=D_1\Phi_{D1}+D_2\Phi_{D2}, \tag{E1}$$

$$a_2=D_3\Phi_{D3}+D_4\Phi_{D4}, \tag{E2}$$

where

$$\Phi_{D1}=\int_m^{E_m}\int_{E_2^{\rm min}}^{E_2^{\rm max}}[EE_\nu^0(\phi+\theta_1+\phi')+\theta'+\theta''']dE_2dE, \tag{E3}$$

$$\Phi_{D2}=\int_m^{E_m}\int_{E_2^{\rm min}}^{E_2^{\rm max}}[-l(p_2y_0+l)(\phi+\theta_1)+\theta'+\theta'''+\theta''+\theta^{\rm IV}]dE_2dE, \tag{E4}$$

$$\Phi_{D3}=\int_m^{E_m}\int_{E_2^{\rm min}}^{E_2^{\rm max}}[-E_\nu^0 ly_0(\phi+\theta_1)+\rho_1+\rho_3]dE_2dE, \tag{E5}$$

TABLE VI. Numerical values of the relevant factors involved in our calculation. The several form factors are fixed as mentioned in the text. V_{us} is taken from Ref. [8]. R is in units of 10^6 s^{-1}.

Decay	$(\alpha/\pi)a_1$ (GeV^4)	$(\alpha/\pi)(a_1/B_1)$	$(\alpha/\pi)a_2$ (GeV^4)	$(\alpha/\pi)(a_2/B_2)$	R	$100\delta R$
$\Sigma^-\to ne\nu$	-4.175 $\times 10^{-7}$	-6.953 $\times 10^{-3}$	2.086 $\times 10^{-7}$	-5.205 $\times 10^{-3}$	5.972	-0.7
$\Lambda\to pe\nu$	3.464 $\times 10^{-7}$	1.635 $\times 10^{-2}$	2.454 $\times 10^{-7}$	1.976 $\times 10^{-2}$	2.944	1.6

TABLE VII. Comparison with Ref. [7].

Decay	$100\alpha_B^0$		$100\delta\alpha$	
	This paper	Ref. [7]	This paper	Ref. [7]
$\Sigma^- \to ne\nu$	66.7	66.7	0.1	0.0
$\Lambda \to pe\nu$	-58.6	-58.6	-0.2	-0.1

$$\Phi_{D4}=\int_m^{E_m}\int_{E_2^{\min}}^{E_2^{\max}}[E(p_2+ly_0)(\phi+\theta_1+\phi')+\rho_2 + \rho_4]dE_2dE. \tag{E6}$$

The numerical computation of the quantities Φ_{Di}, being independent of the different form factors, can be made once and for all. In Table V we have tabulated the numerical values of Φ_{Di} for some decays as an intermediate step in our calculation.

In Table VI we display the numerical values of some relevant coefficients as well as the relative radiative corrections to the decay rate. Notice that the corrections $(\alpha/\pi)(a_1/B_1)$ and $(\alpha/\pi)(a_2/B_2)$ are both small and of the same order of magnitude. Table VII contains radiative corrections to the angular spin-asymmetry coefficient of the emitted hyperon. The masses of the particles involved are those given in Ref. [8]. The values of the form factors were fixed as [7]

$$\Sigma^-\to ne\nu,\quad g_1/f_1=-0.34,\quad f_2/f_1=-0.97,$$

$$\Lambda\to pe\nu,\quad g_1/f_1=0.72,\quad f_2/f_1=0.97.$$

We have also neglected the q^2 dependence of the form factors and the contributions from f_3, g_2, and g_3. In those tables, we define

$$\delta R=(R-R^0)/R^0,$$

$$\delta\alpha=\alpha_B-\alpha_B^0.$$

The $\delta\alpha$ and δR corrections to $\Sigma^-\to ne\nu$ and $\Lambda\to pe\nu$ decays are in acceptable agreement, within our approximations, with those computed in Refs. [7] and [12], respectively.

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