Weak electricity of the nucleon in the chiral quark-soliton model

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The induced pseudotensor constant (weak electricity) of the nucleon is calculated in the framework of the chiral quark-soliton model. This quantity originates from G-parity violation and, hence, is proportional to $m_u - m_d$. We obtain for $m_u - m_d = -5$ MeV a value of $g_T/g_A = -0.0038$. [S0556-2821(97)00209-9]

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I. INTRODUCTION

Neutron β decay is a powerful tool to probe the structure of the nucleon. In particular, it provides a precise measurement of the triplet axial constant of the nucleon g_A, so that it is a touch stone for any model of nucleon structure. The underestimation of the nucleon axial charge in the solitonic picture of the nucleon was for a long main critical point of soliton models of the nucleon. Recently it was shown [1,2] that in the chiral quark-soliton model of the nucleon (χQSM) the rotational $1/N_c$ corrections to the g_A bring its value close to the experimental one. Also these corrections improve considerably the agreement of the electromagnetic characteristics of baryons [3–5] calculated in the χQSM with an experiment.

In the present paper we investigate the other than g_A axial characteristic of the nucleon-induced pseudotensor constant (weak electricity) of the nucleon g_T. The neutron-to-proton transition matrix element of the axial-vector current $J_\mu^5 = \overline{u}\gamma_\mu\gamma_5 d$ can be written in terms of three form factors:

$$\langle P(p')|J_\mu^5|N(p)\rangle = \overline{u_p}(p')\left\{ g_A\gamma_\mu\gamma_5 + \frac{g_T}{M_p+M_n} i\sigma_{\mu\nu}\gamma_5 q_\nu + g_P q_\mu \gamma_5 \right\} u_n(p), \quad q = p' - p, \tag{1}$$

where M_p (M_n) is the proton (neutron) mass and we use the convention of Bjorken and Drell for Dirac matrices and spinors. The axial-vector g_A and pseudoscalar g_P contants[1] were extensively analyzed theoretically and measured in experiments, while less is known about the pseudotensor constant g_T. The pseudotensor current has the opposite G parity to that of the axial vector current and hence is proportional to the parameter of isospin symmetry breaking. There are two different sources of isospin symmetry breaking: Electromagnetic interactions and u and d quark mass difference. In this work we calculate the hadronic part of the g_T proportional to $m_u - m_d$ in the limit of a large number of colors, $N_c \to \infty$.

Even though in reality $N_c = 3$, the limit of large N_c furnishes a useful guideline. At large N_c the nucleon is heavy and can be viewed as a classical soliton of the pion field [7]. An example of the dynamical realization of this idea is given by the Skyrme model [8]. A far more realistic effective chiral Lagrangian of the χQSM is based on the interaction of dynamically massive constituent quarks with pseudo Goldstone meson fields. It is given by the functional integral over the quark (ψ) in the background pion field [9–12]:

$$\exp(iS_{\rm eff}[\pi(x)]) = \int \mathcal{D}\psi\mathcal{D}\overline{\psi}\exp\left(i\int d^4x\, \overline{\psi} D\psi \right), \tag{2}$$

where D is the Dirac operator

$$D = i\partial\!\!\!/ - \hat{m} - MU^{\gamma_5}. \tag{3}$$

U^{γ_5} denotes the pseudoscalar chiral field

$$U^{\gamma_5} = \exp(i\pi^a\tau^a\gamma_5) = \frac{1+\gamma_5}{2}U + \frac{1-\gamma_5}{2}U^\dagger. \tag{4}$$

The $\hat{m}$ is the matrix of the current quark masses $\hat{m} = \mathrm{diag}(m_u, m_d)$. The M stands for the dynamical quark mass arising as a result of the spontaneous chiral symmetry breaking.

The effective chiral action given by Eq. (2) is known to contain automatically the Wess-Zumino term and the four-derivative Gasser-Leutwyler terms, with correct coefficients. Therefore, at least the first four terms of the gradient expansion of the effective chiral Lagrangian are correctly reproduced by Eq. (2), and chiral symmetry arguments do not leave much room for further modifications. Equation (2) has been derived from the instanton model of the QCD vacuum [12], which provides a natural mechanism of chiral symmetry breaking and enables one to express the dynamical mass M and the ultraviolet cutoff Λ intrinsic in Eq. (2) through the $\Lambda_{\rm QCD}$ parameter. It should be mentioned that Eq. (2) is of a general nature: one can use Eq. (2) without referring to the instantons.

An immediate implication of the effective chiral theory Eq. (2) is the quark-soliton model of baryons [13]. According to these ideas the nucleon can be viewed as a bound state of N_c (=3) *valence* quarks kept together by a hedgehoglike pion field whose energy coincides by definition with the ag-

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[1]The pseudoscalar axial constant g_P was calculated recently in Ref. [6] in the framework of the chiral quark soliton model.

TABLE I. Axial vector g_A and pseudotensor g_T constants of the nucleon as a function of the constituent quark mass M, $m_u - m_d = -5$ MeV.

M [MeV]	g_A [2]	g_T/g_A
370	1.26	−0.0029
400	1.24	−0.0035
420	1.21	−0.0038
450	1.16	−0.0040

gregate energy of quarks from the negative Dirac sea. Such a semiclassical picture of the nucleon is well justified in the limit $N_c \to \infty$—in line with more general arguments by Witten [7]. Further studies showed that the χQSM is successful in reproducing the static properties and form factors of the baryons using just one parameter set (see the recent review [14]). The powerful numerical method to carry out the calculation of the $N-\Delta$ splitting and nucleon matrix elements of arbitrary quark bilinear operators has been developed in Refs. [15,16]. This method is also used in the present paper.

II. COMPUTING WEAK ELECTRICITY

The transition matrix element Eq. (1) can be computed as the Euclidean functional integral in the χQSM:

$$\langle P|\overline{u}\gamma_\mu\gamma_5 d|N\rangle = \frac{1}{\mathcal{Z}}\lim_{T\to\infty}\exp\left(ip_0\frac{T}{2} - ip_0'\frac{T}{2}\right) \times \int d^3x d^3y \exp(-i\vec{p}\,'\cdot\vec{y} + i\vec{p}\cdot\vec{x}) \times \int \mathcal{D}U \int \mathcal{D}\psi \int \mathcal{D}\psi^\dagger \times J_p(\vec{y},T/2)\overline{u}\gamma_\mu\gamma_5 d J_n^\dagger(\vec{x},-T/2) \times \exp\left[\int d^4z\, \psi^\dagger D \psi\right]. \quad (5)$$

The nucleon current J_N ($N=p,n$) is built of N_c quark fields:

$$J_N(x) = \frac{1}{N_c!}\epsilon_{i_1\cdots i_{N_c}}\Gamma^{\alpha_1\cdots\alpha_{N_c}}_{JJ_3TT_3}\psi_{\alpha_1 i_1}(x)\cdots\psi_{\alpha_{N_c} i_{N_c}}(x). \quad (6)$$

$\alpha_1\cdots\alpha_{N_c}$ denote spin-flavor indices, while $i_1\cdots i_{N_c}$ designate color indices. The matrices $\Gamma^{\alpha_1\cdots\alpha_{N_c}}_{JJ_3TT_3}$ are taken to endow the corresponding current with the quantum numbers JJ_3TT_3.

In the large N_c limit the integral over Goldstone fields U in Eq. (5) can be calculated by the steepest descent method (semiclassical approximation). The corresponding saddle point equation admits a static soliton solution, an example of which is the hedgehog field configuration

$$U_s(\vec{x}) = \exp[i\vec{n}\cdot\vec{\tau}P(r)]. \quad (7)$$

$P(r)$ denotes the profile function satisfying the boundary condition $P(0)=\pi$ and $P(\infty)=0$, which is determined by solving the saddle point equations (for details see Ref. [14]). The soliton is quantized by introducing collective coordinates corresponding to $SU(2)_I$ isospin rotations of the soliton [and simultaneously $SU(2)_{\rm spin}$ in spin space]:

$$U(t,\vec{x}) = R(t)U_s(\vec{x})R^\dagger(t), \quad (8)$$

where $R(t)$ is a time-dependent SU(2) matrix. The quantum states arising from this quantization have the quantum numbers corresponding to the nucleon and Δ.

Calculating the functional integral Eq. (5) we obtain the following expression for the neutron to proton transition element of the axial-vector current:

$$\langle P|\overline{u}\gamma_\mu\gamma_5 d|N\rangle = N_c(M_p+M_n)\int d^3x e^{i\mathbf{q}\cdot\mathbf{x}}\int dR\,\phi_p^*(R) \times \int\frac{d\omega}{2\pi}\mathrm{tr}\left(\left\langle \mathbf{x}\left|\frac{1}{\omega+iH+i(m_u-m_d)R^\dagger\tau^3 R}\times\gamma_0\gamma_\mu\gamma_5 R^\dagger\tau^{1+i2}R\right|\mathbf{x}\right\rangle\right)\phi_n(R), \quad (9)$$

where $\phi^{S=T}_{S_3T_3}(R)$ is the rotational wave function of the nucleon ($\phi_p = \phi^{(1/2)}_{(1/2)(1/2)}$, $\phi_n = \phi^{(1/2)}_{(1/2)-(1/2)}$) given by the Wigner finite-rotation matrix [8,13]

$$\phi^{S=T}_{S_3T_3}(R) = \sqrt{2S+1}(-1)^{T+T_3}\mathcal{D}^{S=T}_{-T_3,S_3}(R), \quad (10)$$

and the integral over the SU(2) group is normalized by $\int dR = 1$. The one-particle Dirac Hamiltonian H in a background of the static pion field Eq. (7) has a form

$$H = \gamma^0\gamma^k\partial_k + iM\gamma^0 U_s^{\gamma_5} + \frac{1}{2}(m_u+m_d). \quad (11)$$

Projecting the general expression Eq. (9) onto the pseudotensor structure one obtains

$$\frac{g_T(q^2)}{M_p+M_n} = N_c\int d^3x e^{i\mathbf{qx}}\frac{q^3}{|\mathbf{q}|^2}\int dR\,\phi_p^*(R) \times \int\frac{d\omega}{2\pi}\mathrm{tr}\left(\left\langle \mathbf{x}\left|\frac{1}{\omega+iH+i(m_u-m_d)R^\dagger\tau^3 R}\times\gamma_5 R^\dagger\tau^{1+i2}R\right|\mathbf{x}\right\rangle\right)\phi_n(R). \quad (12)$$

Let us now show that the above expression is zero in the isospin symmetry limit ($m_u=m_d$). To prove this we introduce the following unitary transformation of the Dirac and Pauli matrices connecting them to the transposed ones:

$$W\gamma_\mu W^{-1} = \gamma_\mu^T, \quad W\tau^a W^{-1} = -(\tau^a)^T. \quad (13)$$

Evidently then $WHW^{-1} = H^T$. Using properties of the trace $\mathrm{tr}(M^T) = \mathrm{tr}(M)$ and $\mathrm{tr}(WMW^{-1}) = \mathrm{tr}(M)$ one can write

$$\mathrm{tr}\left(\left\langle \mathbf{x}\left|\frac{1}{\omega+iH+i(m_u-m_d)R^\dagger\tau^3R}\,\gamma_5R^\dagger\tau^{1+i2}R\right|\mathbf{x}\right\rangle\right)=\mathrm{tr}\left(W\left\langle \mathbf{x}\left|\frac{1}{\omega+iH+i(m_u-m_d)R^\dagger\tau^3R}\,\gamma_5R^\dagger\tau^{1+i2}R\right|\mathbf{x}\right\rangle\right)W^{-1T}$$
$$=-\mathrm{tr}\left(\left\langle \mathbf{x}\left|\frac{1}{\omega+iH-i(m_u-m_d)R^\dagger\tau^3R}\,\gamma_5R^\dagger\tau^{1+i2}R\right|\mathbf{x}\right\rangle\right).$$

This immediately implies that the pseudotensor constant given by Eq. (12) is zero in the isosymmetric limit and the first nonzero result appears expanding Eq. (12) in m_u-m_d to linear order. The result for the pseudotensor constant g_T in the leading order of $1/N_c$ expansion ($g_T\sim N_c$) and the linear order in m_u-m_d has a form [17]

$$\frac{g_T}{M_p+M_n}=\frac{iN_c(m_u-m_d)}{24}\int\frac{d\omega}{2\pi}\mathrm{Sp}\left(\frac{1}{\omega+iH}\,\gamma_0\tau^i\,\frac{1}{\omega+iH}\,\varepsilon_{ijk}\tau^jx^k\gamma_5\right)\varepsilon_{ab3}\int dR\,\phi_p^*(R)\mathcal{D}^{(1)}_{1+i2,a}(R)\mathcal{D}^{(1)}_{3,b}(R)\,\phi_n(R). \tag{14}$$

The integral over soliton orientations in the second line of Eq. (14) can be easily calculated by using the relations

$$\varepsilon_{ab3}\mathcal{D}^{(1)}_{1\pm i2,a}(R)\mathcal{D}^{(1)}_{3,b}(R)=\pm i\mathcal{D}^{(1)}_{1\pm i2,3}(R) \tag{15}$$

and

$$\int dR\,\phi_p^*(R)\mathcal{D}^{(1)}_{1+i2,3}(R)\,\phi_n(R)=-2/3. \tag{16}$$

The functional trace in the first line of Eq. (14) was estimated in Ref. [17] by means of the gradient expansion

$$\begin{aligned}\frac{g_T}{M_p+M_n}\approx&\frac{N_c(m_u-m_d)}{9\times96\pi^2M}\mathrm{Im}\int d^3x\Big\{\varepsilon_{klm}\mathrm{tr}\,(U\partial_kU^\dagger\partial_lU\tau^m)\\&-\frac{i}{4}\varepsilon_{klm}\varepsilon^{abm}\mathrm{tr}\,[\tau^b\partial_kU(\tau^aU-U^\dagger\tau^a)\partial_lU^\dagger]\\&-\frac{i}{2}x^i\varepsilon_{klm}\varepsilon^{abi}\mathrm{tr}\,[\tau^bU^\dagger\partial_kU(\tau^a\partial_lU\\&-\partial_lU^\dagger\tau^a)U^\dagger\partial_mU]-\frac{i}{2}x^i\varepsilon_{klm}\varepsilon^{abi}\mathrm{tr}\,[(\tau^bU^\dagger\tau^a\\&-\tau^aU^\dagger\tau^b)\partial_kU^\dagger\partial_lU\partial_mU^\dagger U]\Big\}.\end{aligned} \tag{17}$$

This approximation is justified only for a soliton of large size $RM\gg1$. The real nucleon has a radius of order $1/M$ and hence the Eq. (17) can be used only as an order of magnitude estimate.

III. NUMERICAL RESULTS AND CONCLUSION

In order to evaluate exactly the functional trace in Eq. (14), we diagonalize the Hamiltonian H, Eq. (11), numerically in the Kahana-Ripka discretized basis [18]. The constituent quark mass M is fixed to 420 MeV in our model by reproducing best many static baryon observables and form factors in the model (in particular, the isospin mass splittings for octet and decuplet baryons [19,14]). To make sure of the numerical calculation, we compare our results for g_T with the analytical ones of the gradient expansion Eq. (17) justified in the limit of large soliton size. Our numerical procedure is in good agreement within a few percent with the analytical results of the gradient expansion in the large soliton size limit.

The results of our calculation are summarized in Table I. For completeness we give in Table I also results for g_A obtained in [2]. Let us note that the present result is comparable to a recent calculation of the nucleon pseudotensor constant with the QCD sum rule technique [20] which gives $g_T/g_A=-0.0151\pm0.0053$. Both the QCD sum rule result and ours are in agreement with the bag model calculation ($g_T/g_A=-0.00455$) [20,21], whereas they seem to be smaller than preliminary experimental data [22] ranging from -0.21 ± 0.14 to 0.14 ± 0.10. However, the accuracy of the experiment is not enough to be compared in a reasonable way with the results of theoretical calculations.

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